# EXERCISE SET 5.4

1. Mid Point formula
2. Modify Euler method
3. Heun’s method
4. 4-RK method

$$w_{i+1} = y_i + hf\left(t_i + \frac{h}{2}, y_i + \frac{h}{2} f(t_i, y_i)\right)$$

## Midpoint Method

$$w_0 = \alpha,$$

$$w_{i+1} = w_i + hf\left(t_i + \frac{h}{2}, w_i + \frac{h}{2} f(t_i, w_i)\right), \quad \text{for } i = 0, 1, \ldots, N-1.$$

OR

$$\left.\begin{aligned} &\text{Consider } k_1 = f(t_i, w_i) \\ &k_2 = f\left(t_i + \frac{h}{2}, w_i + \frac{h}{2} k_1\right) \\ &w_{i+1} = wi + h\, k_2 \end{aligned}\right\} \text{Alternate form of MidPoint method}$$

$$\boxed{w_{i+1} = y_i + \frac{h}{2}\left(f(t_i, y_i) + f(t_i + h, y_i + hf(t_i, y_i))\right)}$$

## Modified Euler Method

$$w_0 = \alpha,$$

$$w_{i+1} = w_i + \frac{h}{2}[f(t_i, w_i) + f(t_{i+1}, w_i + hf(t_i, w_i))], \quad \text{for} \quad i = 0, 1, \ldots, N-1.$$

OR

$$\left.\begin{aligned} &\text{Consider } k_1 = hf(t_i, w_i) \\ &k_2 = hf(t_{i+1}, w_i + k_1) \\ &w_{i+1} = w_i + \frac{1}{2}(k_1 + k_2) \end{aligned}\right\} \text{Alternate form of Improved Euler method}$$

**Example 2** Use the Midpoint method and the Modified Euler method with $N = 10$, $h = 0.2$, $t_i = 0.2i$, and $w_0 = 0.5$ to approximate the solution to our usual example,

$$y' = y - t^2 + 1, \quad 0 \le t \le 2, \quad y(0) = 0.5.$$

for each $i = 0, 1, \ldots, 9$. The first two steps of these methods give

$$\text{Midpoint method:} \quad w_1 = 1.22(0.5) - 0.0088(0)^2 - 0.008(0) + 0.218 = 0.828;$$

$$\text{Modified Euler method:} \quad w_1 = 1.22(0.5) - 0.0088(0)^2 - 0.008(0) + 0.216 = 0.826,$$

MP $$\boxed{w_{i+1} = w_i + hf\left(t_i + \frac{h}{2}, w_i + \frac{h}{2} f(t_i, w_i)\right)}$$

$$\boxed{w_{i+1} = w_i + \frac{h}{2}[f(t_i, w_i) + f(t_{i+1}, w_i + hf(t_i, w_i))],}$$ ME

$$\begin{aligned} \text{Midpoint method:} \quad w_2 &= 1.22(0.828) - 0.0088(0.2)^2 - 0.008(0.2) + 0.218 \\ &= 1.21136; \end{aligned}$$

$$\begin{aligned} \text{Modified Euler method:} \quad w_2 &= 1.22(0.826) - 0.0088(0.2)^2 - 0.008(0.2) + 0.216 \\ &= 1.20692, \end{aligned}$$

exact values given by $y(t) = (t+1)^2 - 0.5e^t$.

| $t_i$ | $y(t_i)$ | Midpoint Method | Error | Modified Euler Method | Error |
|---|---|---|---|---|---|
| 0.0 | 0.5000000 | 0.5000000 | 0 | 0.5000000 | 0 |
| 0.2 | 0.8292986 | 0.8280000 | 0.0012986 | 0.8260000 | 0.0032986 |
| 0.4 | 1.2140877 | 1.2113600 | 0.0027277 | 1.2069200 | 0.0071677 |
| 0.6 | 1.6489406 | 1.6446592 | 0.0042814 | 1.6372424 | 0.0116982 |
| 0.8 | 2.1272295 | 2.1212842 | 0.0059453 | 2.1102357 | 0.0169938 |
| 1.0 | 2.6408591 | 2.6331668 | 0.0076923 | 2.6176876 | 0.0231715 |
| 1.2 | 3.1799415 | 3.1704634 | 0.0094781 | 3.1495789 | 0.0303627 |
| 1.4 | 3.7324000 | 3.7211654 | 0.0112346 | 3.6936862 | 0.0387138 |
| 1.6 | 4.2834838 | 4.2706218 | 0.0128620 | 4.2350972 | 0.0483866 |
| 1.8 | 4.8151763 | 4.8009586 | 0.0142177 | 4.7556185 | 0.0595577 |
| 2.0 | 5.3054720 | 5.2903695 | 0.0151025 | 5.2330546 | 0.0724173 |

**Example:**

## Modified Euler Method

$$y' = ty + t^3 \quad \text{initial condition } y(0) = 1.$$

The exact solution $y(t) = 3e^{t^2/2} - t^2 - 2$

**Solution:**

$$f(t, y) = ty + t^3$$

$$w_0 = y_0 = 1$$

$$\boxed{w_{i+1} = w_i + \frac{h}{2}(f(t_i, w_i) + f(t_i + h, w_i + hf(t_i, w_i)))}$$

$$= w_i + \frac{h}{2}(t_i y_i + t_i^3 + (t_i + h)(w_i + h(t_i y_i + t_i^3)) + (t_i + h)^3).$$

Using step size $h = 0.1$,

| step | $t_i$ | $w_i$ | $y_i$ | $e_i$ |
|---|---|---|---|---|
| 0 | 0.0 | 1.0000 | 1.0000 | 0.0000 |
| 1 | 0.1 | 1.0051 | 1.0050 | 0.0001 |
| 2 | 0.2 | 1.0207 | 1.0206 | 0.0001 |
| 3 | 0.3 | 1.0483 | 1.0481 | 0.0002 |
| 4 | 0.4 | 1.0902 | 1.0899 | 0.0003 |
| 5 | 0.5 | 1.1499 | 1.1494 | 0.0005 |
| 6 | 0.6 | 1.2323 | 1.2317 | 0.0006 |
| 7 | 0.7 | 1.3437 | 1.3429 | 0.0008 |
| 8 | 0.8 | 1.4924 | 1.4914 | 0.0010 |
| 9 | 0.9 | 1.6890 | 1.6879 | 0.0011 |
| 10 | 1.0 | 1.9471 | 1.9462 | 0.0010 |

## Activity:

Use the modified Euler's method to obtain an approximate solution of $\frac{dy}{dt} = -2ty^2$, $y(0) = 1$, in the interval $0 \le t \le 0.5$ using $h = 0.1$. Compute the error and the percentage error. Given the exact solution is given by $y = \frac{1}{(1+t^2)}$.

Verify ?

| n | $t_n$ | | Modified Euler $y_n$ | Exact value | Error | Percentage Error |
|---|---|---|---|---|---|---|
| 0 | 0 | | 1 | 1 | 0 | 0 |
| 1 | 0.1 | | 0.9900 | 0.9901 | 0.0001 | 0.0101 |
| 2 | 0.2 | | 0.9614 | 0.9615 | 0.0001 | 0.0104 |
| 3 | 0.3 | | 0.9173 | 0.9174 | 0.0001 | 0.0109 |
| 4 | 0.4 | | 0.8620 | 0.8621 | 0.0001 | 0.0116 |
| 5 | 0.5 | | 0.8001 | 0.8000 | 0.0001 | 0.0125 |

Use the improved Euler's method to obtain the approximate value of $y(1.5)$ for the solution of the initial-value problem $y' = 2xy$, $y(1) = 1$. Compare the results for $h = 0.1$ and $h = 0.05$.

the solution $y = e^{x^2-1}$

Verify ?

Improved Euler's Method with $h = 0.1$

| $x_n$ | $y_n$ | Actual value | Abs. error | % Rel. error |
|---|---|---|---|---|
| 1.00 | 1.0000 | 1.0000 | 0.0000 | 0.00 |
| 1.10 | 1.2320 | 1.2337 | 0.0017 | 0.14 |
| 1.20 | 1.5479 | 1.5527 | 0.0048 | 0.31 |
| 1.30 | 1.9832 | 1.9937 | 0.0106 | 0.53 |
| 1.40 | 2.5908 | 2.6117 | 0.0209 | 0.80 |
| 1.50 | 3.4509 | 3.4904 | 0.0394 | 1.13 |

Improved Euler's Method with $h = 0.05$

| $x_n$ | $y_n$ | Actual value | Abs. error | % Rel. error |
|---|---|---|---|---|
| 1.00 | 1.0000 | 1.0000 | 0.0000 | 0.00 |
| 1.05 | 1.1077 | 1.1079 | 0.0002 | 0.02 |
| 1.10 | 1.2332 | 1.2337 | 0.0004 | 0.04 |
| 1.15 | 1.3798 | 1.3806 | 0.0008 | 0.06 |
| 1.20 | 1.5514 | 1.5527 | 0.0013 | 0.08 |
| 1.25 | 1.7531 | 1.7551 | 0.0020 | 0.11 |
| 1.30 | 1.9909 | 1.9937 | 0.0029 | 0.14 |
| 1.35 | 2.2721 | 2.2762 | 0.0041 | 0.18 |
| 1.40 | 2.6060 | 2.6117 | 0.0057 | 0.22 |
| 1.45 | 3.0038 | 3.0117 | 0.0079 | 0.26 |
| 1.50 | 3.4795 | 3.4904 | 0.0108 | 0.31 |

## Heun's method.

(DIY: Home Activity)

$$w_0 = \alpha$$

$$w_{i+1} = w_i + \tfrac{h}{4}\left(f(t_i, w_i) + 3f\left(t_i + \tfrac{2h}{3}, w_i + \tfrac{2h}{3} f\left(t_i + \tfrac{h}{3}, w_i + \tfrac{h}{3} f(t_i, w_i)\right)\right)\right),$$

$$\text{for} \quad i = 0, 1, \ldots, N-1.$$

### 3rd-Order Heun Method

$$\begin{cases} k_1 = f(t_i, y_i) \\ k_2 = f(t_i + \frac{1}{3}h, y_i + \frac{1}{3}k_1 h) \\ k_3 = f(t_i + \frac{2}{3}h, y_i + \frac{2}{3}k_2 h) \end{cases}$$

$$y_{i+1} = y_i + \frac{1}{4}(k_1 + 3k_3)h$$

Alternate form of Heun's

Applying Heun's method with $N = 10$, $h = 0.2$, $t_i = 0.2i$, and $w_0 = 0.5$ to approximate the solution to our usual example,

$$y' = y - t^2 + 1, \quad 0 \le t \le 2, \quad y(0) = 0.5.$$

(verify :Home Activity)

exact values given by $y(t) = (t+1)^2 - 0.5e^t$.

**Table 5.7**

| $t_i$ | $y(t_i)$ | Heun's Method | Error |
|---|---|---|---|
| 0.0 | 0.5000000 | 0.5000000 | 0 |
| 0.2 | 0.8292986 | 0.8292444 | 0.0000542 |
| 0.4 | 1.2140877 | 1.2139750 | 0.0001127 |
| 0.6 | 1.6489406 | 1.6487659 | 0.0001747 |
| 0.8 | 2.1272295 | 2.1269905 | 0.0002390 |
| 1.0 | 2.6408591 | 2.6405555 | 0.0003035 |
| 1.2 | 3.1799415 | 3.1795763 | 0.0003653 |
| 1.4 | 3.7324000 | 3.7319803 | 0.0004197 |
| 1.6 | 4.2834838 | 4.2830230 | 0.0004608 |
| 1.8 | 4.8151763 | 4.8146966 | 0.0004797 |
| 2.0 | 5.3054720 | 5.3050072 | 0.0004648 |

## Runge-Kutta Order Four

$$w_0 = \alpha,$$

$$k_1 = hf(t_i, w_i),$$

$$k_2 = hf\left(t_i + \frac{h}{2}, w_i + \frac{1}{2}k_1\right),$$

$$k_3 = hf\left(t_i + \frac{h}{2}, w_i + \frac{1}{2}k_2\right),$$

$$k_4 = hf(t_{i+1}, w_i + k_3),$$

$$w_{i+1} = w_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4),$$

**Example 3** Use the Runge-Kutta method of order four with $h = 0.2$, $N = 10$, and $t_i = 0.2i$ to obtain approximations to the solution of the initial-value problem

$$y' = y - t^2 + 1, \quad 0 \le t \le 2, \quad y(0) = 0.5.$$

***Solution*** The approximation to $y(0.2)$ is obtained by

$$w_0 = 0.5$$

$$k_1 = 0.2f(0, 0.5) = 0.2(1.5) = 0.3$$

$$k_2 = 0.2f(0.1, 0.65) = 0.328$$

$$k_3 = 0.2f(0.1, 0.664) = 0.3308$$

$$k_4 = 0.2f(0.2, 0.8308) = 0.35816$$

$$\boxed{w_{i+1} = w_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4),}$$

$$w_1 = 0.5 + \frac{1}{6}(0.3 + 2(0.328) + 2(0.3308) + 0.35816) = 0.8292933.$$

the exact solution is $y(t) = (t+1)^2 - 0.5e^t$.

(Home Activity)

**Table 5.8**

| $t_i$ | Exact $y_i = y(t_i)$ | Runge-Kutta Order Four $w_i$ | Error $\lvert y_i - w_i \rvert$ |
|---|---|---|---|
| 0.0 | 0.5000000 | 0.5000000 | 0 |
| 0.2 | 0.8292986 | 0.8292933 | 0.0000053 |
| 0.4 | 1.2140877 | 1.2140762 | 0.0000114 |
| 0.6 | 1.6489406 | 1.6489220 | 0.0000186 |
| 0.8 | 2.1272295 | 2.1272027 | 0.0000269 |
| 1.0 | 2.6408591 | 2.6408227 | 0.0000364 |
| 1.2 | 3.1799415 | 3.1798942 | 0.0000474 |
| 1.4 | 3.7324000 | 3.7323401 | 0.0000599 |
| 1.6 | 4.2834838 | 4.2834095 | 0.0000743 |
| 1.8 | 4.8151763 | 4.8150857 | 0.0000906 |
| 2.0 | 5.3054720 | 5.3053630 | 0.0001089 |

Verify ?

Write a code for solution of

**Differential Equation**

1- Midpoint method

2-Modified Euler method

3-Heun’s method

4-RK-4 method

## EXERCISE SET 5.4

1. Use the Modified Euler method to approximate the solutions to each of the following initial-value problems, and compare the results to the actual values.

   a. $y' = te^{3t} - 2y, \quad 0 \le t \le 1, \quad y(0) = 0,$ with $h = 0.5$; actual solution $y(t) = \frac{1}{5}te^{3t} - \frac{1}{25}e^{3t} + \frac{1}{25}e^{-2t}$.

   b. $y' = 1 + (t - y)^2, \quad 2 \le t \le 3, \quad y(2) = 1,$ with $h = 0.5$; actual solution $y(t) = t + \frac{1}{1-t}$.

   c. $y' = 1 + y/t, \quad 1 \le t \le 2, \quad y(1) = 2,$ with $h = 0.25$; actual solution $y(t) = t \ln t + 2t$.

5. Repeat Exercise 1 using the Midpoint method.

9. Repeat Exercise 1 using Heun's method.

13. Repeat Exercise 1 using the Runge-Kutta method of order four.

# Solution: (Home Activity)

Q1 (c)

| $t$ | Modified Euler | $y(t)$ |
|---|---|---|
| 1.25 | 2.7750000 | 2.7789294 |
| 1.50 | 3.6008333 | 3.6081977 |
| 1.75 | 4.4688294 | 4.4793276 |
| 2.00 | 5.3728586 | 5.3862944 |

Q5 (c)

| $t$ | Midpoint | $y(t)$ |
|---|---|---|
| 1.25 | 2.7777778 | 2.7789294 |
| 1.50 | 3.6060606 | 3.6081977 |
| 1.75 | 4.4763015 | 4.4793276 |
| 2.00 | 5.3824398 | 5.3862944 |

Q9 (c)

| | Heun | |
|---|---|---|
| $t_i$ | $w_i$ | $y(t_i)$ |
| 1.25 | 2.7767857 | 2.7789294 |
| 1.50 | 3.6042017 | 3.6081977 |
| 1.75 | 4.4736520 | 4.4793276 |
| 2.00 | 5.3790494 | 5.3862944 |

Q13 (c)

| | Runge-Kutta | |
|---|---|---|
| $t_i$ | $w_i$ | $y(t_i)$ |
| 1.25 | 2.7789095 | 2.7789294 |
| 1.50 | 3.6081647 | 3.6081977 |
| 1.75 | 4.4792846 | 4.4793276 |
| 2.00 | 5.3862426 | 5.3862944 |

# RUNGE – KUTTA METHOD

These are computationally, most efficient methods in terms of accuracy. They were developed by two German mathematicians, Runge and Kutta.
They are distinguished by their orders in the sense that they agree with Taylor's series

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

where

$$k_1 = hf(t_n, y_n)$$

$$k_2 = hf\left(t_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$$

$$k_3 = hf\left(t_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right)$$

$$k_4 = hf(t_n + h, y_n + k_3)$$

## More Practice with 4R-K

**Example**

Solve the following differential equation

$\frac{dy}{dt} = t + y$ with the initial condition y(0) = 1, using fourth- order Runge-Kutta method from t = 0 to t = 0.4 taking h = 0.1

In this problem,

$$f(t,y) = t + y,\ \ h = 0.1,\ \ t_0 = 0,\ \ y_0 = 1.$$

$$k_1 = hf(t_0, y_0) = 0.1(1) = 0.1$$

$$k_2 = hf(t_0 + 0.05, y_0 + 0.05)$$

$$= hf(0.05, 1.05) = 0.1[0.05 + 1.05] = 0.11$$

$$k_3 = hf(t_0 + 0.05, y_0 + 0.055)$$

$$= 0.1(0.05 + 1.055) = 0.1105$$

$$k_4 = 0.1(0.1 + 1.1105) = 0.12105$$

$$y_1 = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$= 1 + \frac{1}{6}(0.1 + 0.22 + 0.2210 + 0.12105)$$

$$= 1.11034$$

Therefore y(0.1) = y1=1.1103

In the second step, we have to find y2 = y(0.2)
We compute

$$k_1 = hf(t_1, y_1) = 0.1(0.1 + 1.11034) = 0.121034$$

$$k_2 = hf\left(t_1 + \frac{h}{2}, y_1 + \frac{k_1}{2}\right)$$

$$= 0.1[0.15 + (1.11034 + 0.060517)] = 0.13208$$

$$k_3 = hf\left(t_1 + \frac{h}{2}, y_1 + \frac{k_2}{2}\right)$$

$$= 0.1[0.15 + (1.11034 + 0.06604)] = 0.132638$$

$$k_4 = hf(t_1 + h, y_1 + k_3)$$

$$= 0.1[0.2 + (1.11034 + 0.132638)] = 0.1442978$$

$$y_2 = 1.11034 + \frac{1}{6}[0.121034 + 2(0.13208)$$

$$+2(0.132638) + 0.1442978] = 1.2428$$

Similarly we calculate,

$$k_1 = hf(t_2, y_2) = 0.1[0.2 + 1.2428] = 0.14428$$

$$k_2 = hf\left(t_2 + \frac{h}{2}, y_2 + \frac{k_1}{2}\right) = 0.1[0.25 + (1.2428 + 0.07214)] = 0.156494$$

$$k_3 = hf\left(t_1 + \frac{h}{2}, y_1 + \frac{k_2}{2}\right) = 0.1[0.3 + (1.2428 + 0.078247)] = 0.1571047$$

$$k_4 = hf(t_2 + h, y_2 + k_3) = 0.1[0.3 + (1.2428 + 0.1571047)] = 0.16999047$$

$$y(0.3) = y_3 = y_2 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) = 1.399711$$

Finally, we calculate

$$k_1 = hf(t_3, y_3) = 0.1[0.3 + 1.3997] = 0.16997$$

$$k_2 = hf\left(t_3 + \frac{h}{2}, y_3 + \frac{k_1}{2}\right) = 0.1[0.35 + (1.3997 + 0.084985)] = 0.1834685$$

$$k_3 = hf\left(t_3 + \frac{h}{2}, y_3 + \frac{k_2}{2}\right) = 0.1[0.35 + (1.3997 + 0.091734)] = 0.1841434$$

$$k_4 = hf(t_3 + h, y_3 + k_3) = 0.1[0.4 + (1.3997 + 0.1841434)] = 0.19838434$$

$$\begin{aligned} y(0.4) &= y_4 \\ &= y_3 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \\ &= 1.58363 \end{aligned}$$

Find an approximate solution to the initial value problem $\frac{dy}{dt} = 1 - t + 4y,\ y(0) = 1$, in the initial $0 \le t \le 1$ using Runge-Kutta method of order four with $h = 0.1$. Compute the exact value given by $y = \frac{-9}{16} + \frac{1}{4}t + \frac{19}{16}e^{4t}$.

Compute the absolute error and the percentage relative error.

For $n = 0$,

$$K_1 = f(x_0, y_0) = 5$$
$$K_2 = f(0 + 0.05,\ 1 + 0.25) = 5.95$$
$$K_3 = f(0 + 0.05,\ 1 + 0.2975) = 6.14$$
$$K_4 = f(0.1,\ 1 + 0.614) = 7.356$$

| n | $t_n$ | Runge-Kutta $y_n$ | Exact value | Absolute error | Percentage relative error |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | | |
| 1 | 0.1 | 1.6089 | 1.6090 | 0.0001 | 0.0062 |
| 2 | 0.2 | 2.5050 | 2.5053 | 0.0002 | 0.0119 |
| 3 | 0.3 | 3.8294 | 3.8301 | 0.0007 | 0.07 |
| 4 | 0.4 | 5.7928 | 5.7942 | 0.0014 | 0.14 |
| 5 | 0.5 | 8.7093 | 8.7120 | 0.0027 | 0.27 |

$$y_{n+1} = y_n + \frac{1}{6}[K_1 + 2K_2 + 2K_3 + K_4]$$

Verify ?

$$y_1 = 1 + \frac{0.1}{6}[5 + 2(5.95) + 2(6.14) + 7.356] = 1.6089$$

Use the RK4 method with $h = 0.1$ to obtain an approximation to $y(1.5)$ for the solution of $y' = 2xy$, $y(1) = 1$.

$$k_1 = f(x_0, y_0) = 2x_0y_0 = 2$$

$$k_2 = f\left(x_0 + \tfrac{1}{2}(0.1), y_0 + \tfrac{1}{2}(0.1)2\right)$$

$$= 2\left(x_0 + \tfrac{1}{2}(0.1)\right)\left(y_0 + \tfrac{1}{2}(0.2)\right) = 2.31$$

$$k_3 = f\left(x_0 + \tfrac{1}{2}(0.1), y_0 + \tfrac{1}{2}(0.1)2.31\right)$$

$$= 2\left(x_0 + \tfrac{1}{2}(0.1)\right)\left(y_0 + \tfrac{1}{2}(0.231)\right) = 2.34255$$

$$k_4 = f(x_0 + (0.1), y_0 + (0.1)2.34255)$$

$$= 2(x_0 + 0.1)(y_0 + 0.234255) = 2.715361$$

and therefore

$$y_1 = y_0 + \frac{0.1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$= 1 + \frac{0.1}{6}(2 + 2(2.31) + 2(2.34255) + 2.715361) = 1.23367435.$$

known solution $y(x) = e^{x^2-1}$,

| $x_n$ | $y_n$ | Actual value | Abs. error | % Rel. error |
|---|---|---|---|---|
| 1.00 | 1.0000 | 1.0000 | 0.0000 | 0.00 |
| 1.10 | 1.2337 | 1.2337 | 0.0000 | 0.00 |
| 1.20 | 1.5527 | 1.5527 | 0.0000 | 0.00 |
| 1.30 | 1.9937 | 1.9937 | 0.0000 | 0.00 |
| 1.40 | 2.6116 | 2.6117 | 0.0001 | 0.00 |
| 1.50 | 3.4902 | 3.4904 | 0.0001 | 0.00 |

Can you Verify ?


ANY
Questions?