MOHAMMAD BASIL ALI KHAN (1)
20K-0477

Date

EXERCISE # 1.1

Q#1

Second order Differential Equation is linear.

Q#2

Third order Differential Equation is Non-Linear $\Rightarrow \left(\frac{dy}{dx}\right)^4$.

Q#3

Forth order Differential Equation is Linear.

Q#4

Second order Differential Equation is Non-Linear $\Rightarrow \cos(r+u)$

Q#5

Second order Differential Equation is Non-Linear $\Rightarrow \sqrt{1+\left(\frac{dy}{dx}\right)^2}$

Q#6

Second order Differential Equation is Non-Linear $\Rightarrow -\frac{K}{R^2}$

Q#7

Third order Differential Equation is Linear.

Q#8

Second order Differential Equation is Non-Linear $\Rightarrow \frac{1}{3}\dot{x}^2\dot{x}$

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Q#11

$$2y' + y = 0 \text{ -- (1)}; \quad y = e^{-x/2}$$

$$y = e^{-x/2}$$

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{-x/2}\right)$$

$$y' = -1/2\left(e^{-x/2}\right)$$

(1) $\Rightarrow$ $2y' + y = 0$

$$2\left(-\tfrac{1}{2}e^{-x/2}\right) + e^{-x/2} = 0$$

$$-e^{-x/2} + e^{-x/2} = 0$$

$$0 = 0$$

Proved

Q#12

$$\frac{dy}{dt} + 20y = 24 \text{ -- (1)}; \quad y = 6/5 - 6/5\, e^{-20t}$$

$$y = 6/5 - 6/5\, e^{-20t}$$

$$y' = 0 - 6/5\,(-20t)\,e^{-20t}$$

$$y' = +\,120/5\, t\,(e^{-20t})$$

$$y' = 24e^{-20t}$$

(1) $\Rightarrow$ $\frac{dy}{dt} + 20y = 24$

$$24e^{-20t} + 20\left(\frac{6}{5} - \frac{6}{5}e^{-20t}\right) = 24$$

$$\cancel{24e^{-20t}} + 24 - \cancel{24e^{-20t}} = 24$$

$$24 = 24$$

Proved

$\frac{d}{dx}\cos x \Rightarrow -\sin x\, x'$ (3)

$\frac{d}{dx}\sin x \Rightarrow \cos x\, x'$

Date

Q#13

$$y'' - 6y' + 13y = 0 \quad -① ; \quad y = e^{3x}\cos 2x$$

$$y = e^{3x}\cos 2x$$

$$y' = \cos 2x \frac{d}{dx} e^{3x} + e^{3x}\frac{d}{dx}(\cos 2x)$$

$$y' = 3e^{3x}\cos 2x - 2e^{3x}\sin 2x$$

$$y'' = \cos 2x \frac{d}{dx}(3e^{3x}) + 3e^{3x}\frac{d}{dx}(\cos 2x) - \sin 2x \frac{d(2e^{3x})}{dx} + 2e^{3x}\frac{d}{dx}(\sin 2x)$$

$$y'' = 9e^{3x}\cos 2x - 6e^{3x}\sin 2x - 6e^{3x}\sin 2x - 4e^{3x}\cos 2x$$

$$y'' = 5e^{3x}\cos 2x - 12e^{3x}\sin 2x$$

$$① \Rightarrow y'' - 6y' + 13y = 0$$

$$5e^{3x}\cos 2x - 12e^{3x}\sin 2x - 6(3e^{3x}\cos 2x - 2e^{3x}\sin 2x) + 13(e^{3x}\cos 2x) = 0$$

$$5e^{3x}\cos 2x - 12e^{3x}\sin 2x - 18e^{3x}\cos 2x + 12e^{3x}\sin 2x + 13e^{3x}\cos 2x = 0$$

$$5e^{3x}\cos 2x + 13e^{3x}\cos 2x - 18e^{3x}\cos 2x + 12e^{3x}\sin 2x - 12e^{3x}\sin 2x = 0$$

$$0 = 0$$

Proved.

Q#14

$$y'' + y = \tan x \quad -① ; \quad y = -(\cos x)\ln(\sec x + \tan x)$$

$$y = -(\cos x)\ln(\sec x + \tan x)$$

$$y' = \ln(\sec x + \tan x)\frac{d}{dx}(-\cos x) - \cos x \frac{d}{dx}[\ln(\sec x + \tan x)]$$

$$y' = \sin x \ln(\sec x + \tan x) - \cos x \left(\frac{1}{\sec x + \tan x}\right)\left[\frac{d}{dx}(\sec x + \tan x)\right]$$

$\frac{d}{dx}\sec x \Rightarrow \sec x \tan x$

$\frac{d}{dx}\tan x \Rightarrow \sec^2 x$.

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$$y' = \sin x \ln(\sec x + \tan x) - \cos x \left(\frac{\sec x \tan x + \sec^2 x}{\sec x + \tan x}\right)$$

$$y' = \sin x \ln(\sec x + \tan x) - \cos x \sec x \left(\frac{\tan x + \sec x}{\sec x + \tan x}\right)$$

$$y' = \sin x \ln(\sec x \tan x) - \cos x \left(\frac{1}{\cos x}\right)$$

$$y' = \sin x \ln(\sec x \tan x) - 1$$

$$y'' = \ln(\sec x + \tan x)\frac{d}{dx}\sin x + \sin x \frac{d}{dx}\ln(\sec x + \tan x) - 0$$

$$y'' = \cos x \ln(\sec x + \tan x) + \sin x \frac{1}{\sec x + \tan x}\frac{d}{dx}(\sec x + \tan x)$$

$$y'' = \cos x \ln(\sec x + \tan x) + \sin x \left(\frac{\sec x \tan x + \sec^2 x}{\sec x + \tan x}\right)$$

$$y'' = \cos x \ln(\sec x + \tan x) + \sin x \sec x \left(\frac{\tan x + \sec x}{\sec x + \tan x}\right)$$

$$y'' = \cos x \ln(\sec x + \tan x) + \tan x.$$

① $\Rightarrow$ $y'' + y = \tan x$;

$$\cos x \ln(\sec x + \tan x) + \tan x - \cos x \ln(\sec x + \tan x) = \tan x$$

$$\cos x \ln(\sec x + \tan x) + \tan x - \cos x \ln(\sec x + \tan x) = \tan x$$

$$\tan x = \tan x$$

Proved.

Date

Q#15

$(y-x)y' = y-x+8 \quad -① \; ; \quad y = x+4\sqrt{x+2}$

$x+2 \geq 0$
$x \geq -2$
$x \in [-2, \infty)$

$y = x+4\sqrt{x+2}$
$y' = 1 + 4\left(\frac{1}{2}\right)(x+2)^{1/2-1}$

$y' = 1 - \frac{2}{\sqrt{x+2}}$ it is not defined on -2.

$① \Rightarrow (y-x)y' = y-x+8$
$(x+4\sqrt{x+2}-x)\left(1-\frac{2}{\sqrt{x+2}}\right) = x+4\sqrt{x+2}-x+8$
$4\sqrt{x+2} - \frac{8\sqrt{x+2}}{\sqrt{x+2}} = 4\sqrt{x+2}+8$
$4\sqrt{x+2} - 8 = 4\sqrt{x+2}+8$
Proved

largest interval $\Rightarrow x \in (-2, \infty)$.

Q#16

$y' = 25+y^2 \quad -① \; ; \quad y = 5\tan 5x$

$5x$ should not be equal to $90°$ because function of $\tan x$ is infinity (udefined).

$5x \neq \frac{\pi}{2} + n(2\pi)$

$x \neq \pi/10 + \frac{n2\pi}{5}$

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$y' = 5(5\sec^2 5x)$
$y' = 25\sec^2 5x$

$① \Rightarrow 25\sec^2 5x = 25 + (5\tan 5x)^2$
$25\sec^2 5x = 25 + 25\tan^2 5x$
$25(1+\tan^2 5x) = 25 + 25\tan^2 5x$
$25 + 25\tan^2 5x = 25 + 25\tan^2 5x$.
Proved.

Q#17

$y' = 2xy^2 \quad —① ; \quad y = 1/(4-x^2)$
$4 - x^2 \neq 0$
$x^2 \neq 4$
$x \neq \pm 2$
$(-\infty,-2) \cup (-2,2) \cup (2,\infty)$

$y = \dfrac{1}{4-x^2}$
$y' = (4-x^2)^{-1}$
$y' = -1(4-x^2)^{-1-1}(-2x)$
$y' = \dfrac{2x}{(4-x^2)^2}$
$① \Rightarrow \dfrac{2x}{(4-x^2)^2} = 2x\left(\dfrac{1}{4-x^2}\right)^2$
$\dfrac{2x}{(4-x^2)^2} = \dfrac{2x}{(4-x^2)^2}$.
Proved.
largest interval $\Rightarrow (-\infty,-2)\cup(-2,2)\cup(2,\infty)$

Q#18

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$2y' = y^3 \cos x$ — ① ; $y = (1-\sin x)^{-1/2}$

$1-\sin x > 0$

$\sin x < 1$ (can never be greater than 1)

$\sin x \neq 1$.

$x \neq \pi/2 + n\,2\pi$

$y' = -1/2\,(1-\sin x)^{-1/2-1}\,(1-\sin x)'$

$y' = \dfrac{\cos x}{2(1-\sin x)^{3/2}}$

$\cancel{2}\left(\dfrac{\cos x}{\cancel{2}(1-\sin x)^{3/2}}\right) = \left[(1-\sin x)^{-1/2}\right]^3 \cos x$

$\dfrac{\cos x}{(1-\sin x)^{3/2}} = (1-\sin x)^{-3/2} \cos x$

$\dfrac{\cos x}{(1-\sin x)^{3/2}} = \dfrac{\cos x}{(1-\sin x)^{3/2}}$

Proved

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Q#21.

$$\frac{dP}{dt} = P(1-P) \quad — \; ① ; \qquad P = \frac{c_1 e^t}{1+c_1 e^t}$$

$$P = \frac{c_1 e^t}{1+c_1 e^t}$$

$$\frac{dP}{dt} = \frac{(1+c_1 e^t)\frac{d}{dt} c_1 e^t - c_1 e^t \frac{d}{dt}(1+c_1 e^t)}{(1+c_1 e^t)^2}$$

$$\frac{dP}{dt} = \frac{(1+c_1 e^t)\, c_1 e^t - c_1 e^t (c_1 e^t)}{(1+c_1 e^t)^2}$$

$$\frac{dP}{dt} = \frac{c_1 e^t (1+c e_1^t) - c_1^2 e^{2t}}{(1+c_1 e^t)^2}$$

$$\frac{dP}{dt} = \frac{c_1 e^t}{(1+c_1 e^t)}\left(1 - \frac{c_1 e^t}{1+c_1 e^t}\right)$$

$$\frac{dP}{dt} = P(1-P)$$

Q#22

$$\frac{dy}{dx} + 2xy = 1 \quad — \; ① ; \qquad y = e^{-x^2}\int_0^x e^{t^2} dt + c_1 e^{-x^2}$$

$$y = e^{-x^2}\int_0^x e^{t^2} dt + c_1 e^{-x^2}$$

$$y' = e^{-x^2}\left[\frac{d}{dx}\left(\int_0^x e^{t^2} dt\right)\right] + \int_0^x e^{t^2} dt \frac{d}{dx}(e^{-x^2}) + c_1 \frac{d}{dx}(e^{-x^2})$$

$$y' = e^{-x^2}(e^{x^2}) + \int_0^x e^{t^2} dt\,(-2xe^{-x^2}) + c_1(-2xe^{-x^2})$$

$$y' = e^0 - 2xe^{-x^2}\int_0^x e^{t^2} dt - 2x c_1 e^{-x^2}$$

$$y' = 1 - 2xe^{-x^2}\int_0^x e^{t^2} dt - 2xe^{-x^2} c_1$$

$$① \Rightarrow \frac{dy}{dx} + 2xy = 1$$

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$$1 - 2xe^{-x^2}\int_0^x e^{t^2}dt - 2xe^{-x^2}c_1 + 2x\left(e^{-x^2}\int_0^x e^{t^2}dt + c_1e^{-x^2}\right) = 1$$

$$1 - 2xe^{-x^2}\int_0^x e^{t^2}dt - 2xe^{-x^2}c_1 + 2xe^{-x^2}\int_0^x e^{t^2}dt + 2c_1e^{-x^2} = 1$$

$$1 = 1$$
(Proved)

Q #23

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0 \quad (1); \quad y = c_1e^{2x} + c_2xe^{2x}$$

$$y = c_1e^{2x} + c_2xe^{2x}$$

$$y' = c_1\frac{d}{dx}e^{2x} + c_2\left(x\frac{d}{dx}e^{2x} + e^{2x}\frac{d}{dx}x\right)$$

$$y' = 2c_1e^{2x} + c_2e^{2x} + 2c_2xe^{2x}$$

$$y'' = 4c_1e^{2x} + 2c_2e^{2x} + 2c_2\left(x\frac{d}{dx}e^{2x} + e^{2x}\frac{d}{dx}x\right)$$

$$y' = 4c_1e^{2x} + 2c_2e^{2x} + 2c_2(2e^{2x}x + e^{2x})$$

$$y' = 4c_1e^{2x} + 2c_2e^{2x} + 4c_2e^{2x} + 2c_2e^{2x}$$

$$y' = 4c_1e^{2x} + 4c_2e^{2x} + 4c_2e^{2x}$$

$$(1) \Rightarrow \frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$$

$$4c_1e^{2x} + 4c_2e^{2x} + 4c_2xe^{2x} - 4(2c_1e^{2x} + c_2e^{2x} + 2c_2e^{2x}x) + 4(c_1e^{2x} + c_2xe^{2x}) = 0$$

$$4c_1e^{2x} + 4c_2e^{2x} + 4c_2xe^{2x} - 8c_1e^{2x} - 4c_2e^{2x} - 8c_2e^{2x}x + 4c_1e^{2x} + 4c_2xe^{2x} = 0$$

$$0 = 0$$
(Proved)

Date

Q#24

$$x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 12x^2 \quad (1);$$

$$y = c_1 x^{-1} + c_2 x + c_3 x \ln x + 4x^2.$$

$$y = c_1 x^{-1} + c_2 x + c_3 x \ln x + 4x^2.$$

$$y' = -c_1 x^{-2} + c_2 + c_3\left(x \frac{d}{dx}\ln x + \ln x \frac{d}{dx} x\right) + 8x$$

$$y' = -c_1 x^{-2} + c_2 + c_3 + c_3 \ln x + 8x.$$

$$y'' = 2c_1 x^{-3} + c_3 x^{-1} + 8$$

$$y''' = -6c_1 x^{-4} - c_3 x^{-2}$$

$$(1) \Rightarrow x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 12x^2.$$

$$x^3(-6c_1x^{-4} - c_3x^{-2}) + 2x^2(2c_1x^{-3} + c_3x^{-1} + 8) - x(-c_1x^{-2} + c_2 + c_3 + c_3\ln x + 8x) + (c_1x^{-1} + c_2x + c_3x\ln x + 4x^2) = 12x^2$$

$$-6c_1x^{-1} - c_3x + 4c_1x^{-1} + 2c_3x + 16x^2 + c_1x^{-1} - c_2x - c_3x - c_3x\ln x - 8x^2 + c_1x^{-1} + c_2x + c_3x\ln x + 4x^2 = 12x^2$$

$$\cancel{-6c_1x^{-1} + 4c_1x^{-1} + c_1x^{-1} + c_1x^{-1}}$$

$$12x^2 = 12x^2$$

(Proved)

Date

Q#27

$y' + 2y = 0$ — ①
$y = e^{mx}$ — ②
$y' = me^{mx}$

① => $me^{mx} + 2(e^{mx}) = 0$
$me^{mx} + 2e^{mx} = 0$
$e^{mx}(m+2) = 0$
$m + 2 = 0$
$m = -2$

② => $y = e^{2x}$.

Q#28

$3y' = 4y$ — ①
$y = e^{mx}$ — ②
$y' = me^{mx}$

① => $3(me^{mx}) = 4(e^{mx})$
$3me^{mx} = 4e^{mx}$
$3me^{mx} - 4e^{mx} = 0$
$e^{mx}(3m - 4) = 0$
$3m - 4 = 0$
$3m = 4$
$m = 4/3$

② => $y = e^{4/3 x}$

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Q#29

$$y'' - 5y' + 6y = 0 \quad -① $$

$$y = e^{mx} \quad -②$$

$$y' = me^{mx}$$

$$y'' = m^2 e^{mx}$$

① ⇒ $y'' - 5y' + 6y = 0$

$$m^2 e^{mx} - 5me^{mx} + 6e^{mx} = 0$$

$$e^{mx}(m^2 - 5m + 6) = 0$$

$$m^2 - 5m + 6 = 0$$

$$m^2 - 3m - 2m + 6 = 0$$

$$m(m-3) - 2(m-3) = 0$$

$$(m-3)(m-2) = 0$$

$$m = 3 \quad m = 2$$

② ⇒ $y_1 = e^{3x}$

$$y_2 = e^{2x}$$

Q#30

$$2y'' + 9y' - 5y = 0 \quad -①$$

$$y = e^{mx} \quad -②$$

$$y' = me^{mx}$$

$$y'' = m^2 e^{mx}$$

① ⇒ $2(m^2 e^{mx}) + 9(me^{mx}) - 5(e^{mx}) = 0$

$$2m^2 e^{mx} + 9me^{mx} - 5e^{mx} = 0$$

$$e^{mx}(2m^2 + 9m - 5) = 0$$

$$2m^2 + 9m - 5 = 0$$

$$2m^2 + 10m - m - 5 = 0$$

$$2m(m+5) - 1(m+5) = 0$$

$$(m+5)(2m-1) = 0$$

$$m+5 = 0 \quad 2m - 1 = 0$$

$$m = -5 \quad m = 1/2$$

② ⇒ $y_1 = e^{-5x}$

$$y_2 = e^{1/2 x}$$

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Q#31

$$xy'' + 2y' = 0 \quad — ①$$

$$y = x^m \quad — ②$$

$$y' = mx^{m-1}$$

$$y'' = m(m-1)x^{m-1-1}$$

$$y'' = (m^2 - m)x^{m-2}$$

$① \Rightarrow$ $x\left[(m^2-m)x^{m-2}\right] + 2(mx^{m-1}) = 0$

$$(m^2-m)x^{m-1} + 2(mx^{m-1}) = 0$$

$$x^{m-1}(m^2 - m + 2m) = 0$$

$$m^2 + m = 0$$

$$m(m+1) = 0$$

$$m = 0 \qquad m+1 = 0$$

$$m = -1$$

$② \Rightarrow$ $y_1 = x^0 \qquad y_2 = x^{-1}$

$$y_1 = 1$$

Q#32

$$x^2y'' - 7xy' + 15y = 0 \quad — ①$$

$$y = x^m \quad — ②$$

$$y' = mx^{m-1}$$

$$y'' = (m^2-m)x^{m-2}$$

$① \Rightarrow$ $x^2\left((m^2-m)x^{m-2}\right) - 7x(mx^{m-1}) + 15(x^m) = 0$

$$(m^2-m)x^m - 7mx^m + 15x^m = 0$$

$$x^m(m^2 - m - 7m + 15) = 0$$

$$m^2 - 8m + 15 = 0$$

$$m^2 - 5m - 3m + 15 = 0$$

$$m(m-5) - 3(m-5) = 0$$

$$(m-5)(m-3) = 0$$

$$m = 5 \qquad m = 3$$

$② \Rightarrow$ $y_1 = x^5 \qquad y_2 = x^3$

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Q#37

$$\frac{dx}{dt} = x + 3y \quad \text{— (1)} \qquad x = e^{-2t} + 3e^{6t}$$

$$\frac{dy}{dt} = 5x + 3y \quad \text{— (2)} \qquad y = -e^{-2t} + 5e^{6t}$$

$x = e^{-2t} + 3e^{6t}$

$x' = -2e^{-2t} + 18e^{6t}$

$y = -e^{-2t} + 5e^{6t}$

$y' = 2e^{-2t} + 30e^{6t}$

(1) $\Rightarrow \frac{dx}{dt} = x + 3y$.

$-2e^{-2t} + 18e^{6t} = e^{-2t} + 3e^{6t} + 3(-e^{-2t} + 5e^{6t})$

$-2e^{-2t} + 18e^{6t} = e^{-2t} + 3e^{6t} - 3e^{-2t} + 15e^{6t}$.

$-2e^{-2t} + 18e^{6t} = -2e^{-2t} + 18e^{6t}$.

Proved.

(2) $\Rightarrow \frac{dy}{dt} = 5x + 3y$

$2e^{-2t} + 30e^{6t} = 5(e^{-2t} + 3e^{6t}) + 3(-e^{-2t} + 5e^{6t})$

$2e^{-2t} + 30e^{6t} = 5e^{-2t} + 15e^{6t} - 3e^{-2t} + 15e^{6t}$

$2e^{-2t} + 30e^{6t} = 2e^{-2t} + 30e^{6t}$

Proved

Date

Q#38

$$\frac{d^2x}{dt^2} = 4y + e^t \quad —(1)$$

$$\frac{d^2y}{dt^2} = 4x - e^t \quad —(2)$$

$x = \cos 2t + \sin 2t + \frac{1}{5}e^t$

$y = -\cos 2t - \sin 2t - \frac{1}{5}e^t$

$x = \cos 2t + \sin 2t + \frac{1}{5}e^t$

$x' = -2\sin 2t + 2\cos 2t + \frac{1}{5}e^t$

$x'' = -4\cos 2t - 4\sin 2t + \frac{1}{5}e^t$

$y = -\cos 2t - \sin 2t - \frac{1}{5}e^t$

$y' = 2\sin 2t - 2\cos 2t - \frac{1}{5}e^t$

$y'' = 4\cos 2t + 4\sin 2t - \frac{1}{5}e^t$

$(1) \Rightarrow \frac{d^2x}{dt^2} = 4y + e^t$

$-4\cos 2t - 4\sin 2t + \frac{1}{5}e^t = 4\left(-\cos 2t - \sin 2t - \frac{1}{5}e^t\right) + e^t$

$-4\cos 2t - 4\sin 2t + \frac{1}{5}e^t = -4\cos 2t - 4\sin 2t - \frac{4}{5}e^t + e^t$

$-4\cos 2t - 4\sin 2t + \frac{1}{5}e^t = -4\cos 2t - 4\sin 2t + \frac{1}{5}e^t$

Proved

$(2) \Rightarrow \frac{d^2y}{dt^2} = 4x - e^t$

$4\cos 2t + 4\sin 2t - \frac{1}{5}e^t = 4\left(\cos 2t + \sin 2t + \frac{1}{5}e^t\right) - e^t$

$4\cos 2t + 4\sin 2t - \frac{1}{5}e^t = 4\cos 2t + 4\sin 2t + \frac{4}{5}e^t - e^t$

$4\cos 2t + 4\sin 2t - \frac{1}{5}e^t = 4\cos 2t + 4\sin 2t - \frac{1}{5}e^t$

Proved.

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Q#44

$$y'' + 2y' + 4y = 5\sin t \quad \text{--- (1)} \quad ; \quad y = A\sin t + B\cos t$$

$y = A\sin t + B\cos t$ — (A)

$y' = A\cos t - B\sin t$

$y'' = -A\sin t - B\cos t$

(1) => $y'' + 2y' + 4y = 5\sin t$

$-A\sin t - B\cos t + 2(A\cos t - B\sin t) + 4(A\sin t + B\cos t) = 5\sin t$

$-A\sin t - B\cos t + 2A\cos t - 2B\sin t + 4A\sin t + 4B\cos t = 5\sin t$

$-A\sin t - 2B\sin t + 4A\sin t - B\cos t + 2A\cos t + 4B\cos t = 5\sin t$

$3A\sin t - 2B\sin t + 3B\cos t + 2A\cos t = 5\sin t$

$\sin t(3A - 2B) + \cos t(2A + 3B) = 5\sin t$

$3A - 2B = 5$ — (2) $\quad 2A + 3B = 0$ — (3)

Multiply eq (2) by 3

Multiply eq (3) by 2.

(2) => $9A - 6B = 15$

(3) => $6A + 6B = 0$

$13A = 15$

$A = 15/13$

Put A in eq (3)

$2(15/13) + 3B = 0$

$30/13 + 3B = 0$

$-3B = -30/13$

$B = 10/13$

substitute value of A & B in eq (A)

(A) => $y = \frac{15}{13}\sin t - \frac{10}{13}\cos t$

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Q#47

$$x^3 + y^3 = 3cxy \qquad \frac{dy}{dx} = \frac{y(y^3 - 2x^3)}{x(2y^3 - x^3)}$$

$$x^3 + y^3 = 3cxy$$

$$\frac{x^3 + y^3}{xy} = 3c$$

$$\frac{d}{dx}\left(\frac{x^3 + y^3}{xy}\right) = \frac{d}{dx} 3c$$

$$\frac{xy\left(3x^2 + 3y^2 \frac{dy}{dx}\right) - \left(y + x\frac{dy}{dx}\right)(x^3 + y^3)}{(xy)^2} = 0$$

$$\left(3x^3y + 3xy^3 \frac{dy}{dx}\right) - \left(yx^3 + y^4 + x^4\frac{dy}{dx} + y^3 x\frac{dy}{dx}\right) = 0$$

$$3x^3y + 3xy^3\frac{dy}{dx} - yx^3 - y^4 - x^4\frac{dy}{dx} - y^3x\frac{dy}{dx} = 0$$

$$3xy^3\frac{dy}{dx} - x^4\frac{dy}{dx} - y^3x\frac{dy}{dx} = -3x^3y + x^3y + y^4$$

$$\frac{dy}{dx}\left(3y^3x - x^4 - y^3x\right) = -3x^3y + x^3y + y^4$$

$$\frac{dy}{dx}\left(2y^3x - x^4\right) = -2x^3y + y^4$$

$$\frac{dy}{dx} = \frac{-2x^3y + y^4}{2y^3x - x^4}$$

$$\frac{dy}{dx} = \frac{y(y^3 - 2x^3)}{x(2y^3 - x^3)}$$

Proved