Date:

$x \equiv 4 \pmod 5$ $\quad m_1 = 5$ $\quad M_1 y_1 \equiv 1 \pmod{m_1}$

$x \equiv 6 \pmod 8$ $\quad m_2 = 8$ $\quad M_2 y_2 \equiv 1 \pmod{m_2}$

$x \equiv 8 \pmod 9$ $\quad m_3 = 9$ $\quad M_3 y_3 \equiv 1 \pmod{m_3}$

product $= 5 \times 8 \times 9 = 360$

$M_1 = \frac{360}{m_1} = \frac{360}{5} = 72$

$M_2 = \frac{360}{m_2} = \frac{360}{8} = 45$

$M_3 = \frac{360}{m_3} = \frac{360}{9} = 40$

$M_1 y_1 \equiv 1 \pmod{m_1}$ $\quad M_2 y_2 \equiv 1 \pmod{m_2}$ $\quad M_3 y_3 \equiv 1 \pmod{m_3}$

① $72 y_1 \equiv 1 \bmod 5$ $\quad$ ② $45 y_2 \equiv 1 \pmod 8$ $\quad$ ③ $40 y_3 \equiv 1 \pmod 9$

$y_1 = 3$ $\quad y_2 = 5$ $\quad y_3 = 7$.

① $72 y_1 \equiv 1 \pmod 5$

$72 > 5$ → greater number on left side

$72 \equiv 5 \cdot 14 + 2$ — $2 \equiv 72 - 5(14)$ — (i)

$5 \equiv 2 \cdot 2 + 1$ $\quad 1 \equiv 5 - 2(2)$ — (ii)

$2 \equiv 1 \cdot 2 + 0$

(ii) $1 \equiv 5 - 2(2)$ • substitute value of 2 from eq (i)

$1 \equiv 5 - 2(72 - 5(14))$

$1 \equiv 1(5) - 2(72) + 28(5)$

$1 \equiv -2(72) + 29(5)$

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Now -2 is the inverse so multiply -2 on both sides of (1)

$72(-2) \equiv 1(-2) \pmod 5$

$y_1 \equiv -2 \bmod 5$ $\quad -2 \bmod 5 = 3$

$y_1 \equiv 3 \pmod 5$

$\boxed{y_1 = 3}$

(2) $45y_2 = 1 \pmod 8$

$45 > 8$

$45 = 8.5 + 5 \longrightarrow 5 = 45 - 5(8)$ (i)

$8 = 5.1 + 3 \longrightarrow 3 = 8 - 1(5)$ (ii)

$5 = 3.1 + 2 \longrightarrow 2 = 5 - 1(3)$ (iii)

$3 = 2.1 + 1 \longrightarrow 1 = 3 - 1(2)$ (iv)

$2 = 1.2 + 0$

(iv) $1 = 3 - 1(2)$ — substitute eq (iii), val of 2

$1 = 3 - 1(5 - 1(3))$

$1 = 3 - 1(5) + 1(3)$

$1 = 1(3) - 1(5) + 1(3)$

$1 = -5 + 2(3)$ — substitute eq (ii), val of 3

$1 = -5 + 2(8 - 1(5))$

$1 = -5 + 2(8) - 2(5)$

$1 = -3(5) + 2(8)$ ⟶ substitute eq (i), val of 5

$1 = 2(8) - 3(45 - 5(8))$

$1 = 2(8) - 3(45) + 15(8)$

$1 = 17(8) - 3(45)$

-3 is the inverse multiply -3 on both sides

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② $45 y_2 \equiv 1 \pmod 8$

$45(-3) \equiv -3 \pmod 8$

$y_2 \equiv -3 \pmod 8$

$\boxed{y_2 \equiv 5}$ $\quad y_2 \equiv 5 \pmod 8$

③ $40 y_3 \equiv 1 \pmod 9$

$40 > 9$

$40 = 9.4 + 4 \longrightarrow 4 = 40 - 9(4) - (i)$

$9 = 4.2 + 1 \longrightarrow 1 = 9 - 2(4) - (ii)$

$4 = 1.4 + 0$

(ii) $1 = 9 - 2(4)$ — substitute eq i, val of 4

$1 = 9 - 2(40 - 9(4))$

$1 = 1(9) - 2(40) + 8(9)$

~~$1 = -3(9) - 2$~~

$1 = 9(9) - 2(40)$

-2 is inverse multiply it on both sides

$40 y_3 \equiv 1 \pmod 9$

$40(-2) y_3 \equiv -2 \pmod 9$

$y_3 \equiv -2 \bmod 9$

$y_3 = 7 \bmod 9$

$-2 \bmod 9 = 7$

$\boxed{y_3 = 7}$

a_1 a_2 a_3 Date:

$x \equiv 4 \bmod 5$ $x \equiv 6 \bmod 8$ $x \equiv 8 \bmod 9$

$y_1 = 3, y_2 = 5, y_3 = 7$ $M_1 = 72, M_2 = 45, M_3 = 40$

$x = a_1 M_1 y_1 + a_2 M_2 y_2 + a_3 M_3 y_3$

$x = (4)(72)(3) + 6(45)(5) + 8(40)(7)$

$x = 864 + 1350 + 2240$

$\boxed{x = 4454}$

$x = 4454 \pmod{360}$

$x \equiv 134 \pmod{360}$

$4454 \bmod 360 = 134$

Hence x is 134.