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20K-0477

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ASSIGNMENT # 03.

Q#1

1) $f(t) = \begin{cases} e^t & t \leq 2 \\ 3 & t > 2 \end{cases}$

$$\mathcal{L}(f(t)) = \int_0^2 e^{-st} \cdot e^t \, dt + \int_2^\infty 3e^{-st} \, dt$$

$$= \int_0^2 e^{-t(s-1)} \, dt + \int_2^\infty 3e^{-st} \, dt$$

$$= -\frac{e^{-t(s-1)}}{(s-1)}\Big|_0^2 + \frac{3e^{-st}}{-s}\Big|_2^\infty$$

$$= \left[-\frac{e^{-2(s-1)}}{s-1} + \frac{1}{s-1}\right] + \left[\frac{3e^{-\infty}}{-s} - \frac{3e^{-2s}}{-s}\right]$$

$$= \left[\frac{1 - e^{-2(s-1)}}{s-1}\right] + \left[\frac{3e^{-2s}}{s}\right]$$

2) $f(t) = 3 + 2t^2$

$$\mathcal{L}(f(t)) = \int_0^\infty e^{-st} \cdot (3 + 2t^2) \, dt$$

$uv - \int v \, du$

$u = 3 + 2t^2$ $\quad dv = e^{-st}$

$du = 4t \, dt$ $\quad v = \frac{e^{-st}}{-s}$

$$\left[(3+2t^2)\left(\frac{e^{-st}}{-s}\right)\right]_0^\infty - \int_0^\infty \left(\frac{e^{-st}}{-s}\right) 4t \, dt$$

$$\underbrace{(3+2t^2)\left(\frac{e^{-st}}{-s}\right)\Big|_0^\infty} + \frac{4}{s}\int_0^\infty te^{-st} \, dt$$

$\hookrightarrow \mathcal{L}[t] = 1/s^2$

$e^{-\infty} = 0$

$e^0 = 1$

$$\left[\frac{3+2(\infty)^2 e^{-s(\infty)}}{-s} + \frac{3+2(0)^2 (e^{-s(0)})}{s}\right] + \frac{4}{s}\left[\frac{1}{s^2}\right]$$

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$$= \left[\frac{3}{s}\right] + \left[\frac{4}{s^3}\right]$$

$$= \frac{3}{s} + \frac{4}{s^2}.$$

3) $f(t) = 5\sin 3t - 17e^{-2t}$

$$\mathcal{L}(f(t)) = \int_0^\infty e^{-st}\,(5\sin 3t - 17e^{-2t})\,dt$$

$$= \int_0^\infty 5e^{-st}\sin 3t\,dt - \int_0^\infty 17e^{-st}e^{-2t}\,dt$$

$$= 5\int_0^\infty e^{-st}\sin 3t\,dt - 17\int_0^\infty e^{-2t}\cdot e^{-st}\,dt$$

$$\sin kt = \frac{k}{s^2+k^2} \qquad e^{\alpha t} = \frac{1}{s-a}$$

$$k = 3 \qquad \alpha = 2$$

$$= 5\left[\frac{3}{s^2+3^2}\right] - 17\left[\frac{1}{s-(-2)}\right]$$

$$= \frac{15}{s^2+9} - \frac{17}{s+2}.$$

4) $f(t) = te^{4t}$

$\alpha = 4$

$f(t) = t$

$f(s) = 1/s^2$

$$= \frac{1}{(s-4)^2}$$

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Q#2.

1) $\mathcal{L}^{-1}\left\{\dfrac{1}{s(s^2+2s+5)}\right\}$

$\dfrac{1}{s(s^2+2s+5)} = \dfrac{A}{s} + \dfrac{Bs+C}{s^2+2s+5}$.

$1 = A(s^2+2s+5) + sBs + sC$.

$1 = As^2 + 2As + 5A + sBs + sC$.

$s = 0$

$5A = 1$ $\qquad 0 = 2As + + sC$

$A = 1/5$ $\qquad 0 = 2A + C$

$\qquad 0 = 2(1/5) + C$

$As^2 + Bs^2 = 0.$ $\qquad C = -2/5$

$A + B = 0$

$1/5 + B = 0$

$B = -1/5$.

$= \dfrac{1}{5s} - \left[\dfrac{s+2}{5(s^2+2s+5)}\right]$

$= \dfrac{1}{5s} - \left[\dfrac{s+2}{5(s^2+1+2s+4)}\right]$

$= \dfrac{1}{5s} - \left[\dfrac{s+1+1}{5((s+1)^2+4)}\right]$.

$= \mathcal{L}^{-1}\left[\frac{1}{5}\left(\frac{1}{s}\right)\right] - \frac{1}{5}\mathcal{L}^{-1}\left[\dfrac{s+1}{(s+1)^2+4}\right] + \mathcal{L}^{-1}\left[\dfrac{1}{(s+2)^2+4}\right]$

$a = 1$

$= \frac{1}{5} - \frac{1}{5} e^{-t}\cos 2t + \frac{1}{5 \cdot 2}\mathcal{L}^{-1}\left[\dfrac{2}{(s+2)^2+2^2}\right]$

$= \frac{1}{5} - \frac{1}{5} e^{-t}\cos 2t + \frac{1}{10} e^{-t}\cos 2t$.

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2) $\mathcal{L}^{-1}\left\{\frac{7s-1}{(s+1)(s+2)(s-3)}\right\}$

$$\frac{7s-1}{(s+1)(s+2)(s+3)} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s-3}.$$

$$7s-1 = A(s+2)(s-3) + B(s+1)(s-3) + C(s+1)(s+2)$$

Put $s = 3$

$$7s-1 = 0 + 0 + C(s^2+2s+s+2)$$

$$7s-1 = C(s^2+3s+2).$$

$$7s-1 = Cs^2+3Cs+2C.$$

$$21-1 = 9C+9C+2C$$

$$21-1 = 20C$$

$$C = 1$$

Put $s=-1$

$$7(-1)-1 = A(-1+2)(-1-3).$$

$$-7-1 = A(1)(-4).$$

$$-8 = -4A$$

$$A = 2$$

$$As^2+Bs^2+Cs^2 = 0.$$

$$A+B+C=0.$$

$$2+1+c=0$$

$$B = -3.$$

$$= \mathcal{L}^{-1}\left\{\frac{2}{s+1} + \frac{-3}{s+2} + \frac{1}{s-3}\right\}.$$

$$= 2e^{-t} - 3e^{-2t} + e^{3t}$$

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3) $\mathcal{L}^{-1}\left\{\dfrac{s^2+9s+2}{(s-1)^2(s+3)}\right\}$

$\dfrac{s^2+9s+2}{(s-1)^2(s+3)} = \dfrac{A}{s-1} + \dfrac{B}{(s-1)^2} + \dfrac{C}{s+3}.$

$s^2+9s+2 = A(s-1)(s+3) + B(s+3) + C(s-1)^2$

Put $s = -3$

$(-3)^2+9(-3)+2 = 0+0+(-3-1)^2$

$9 - 27 + 2 = 16C$

$-16 = 16C$

$C = -1$

Put $s = 1$

$(1)^2+9(1)+2 = B(1+3)$

$1+9+2 = 4B.$

$12 = 4B$

$B = 3.$

$1 = A + C.$

$1 = A - 1$

$A = 2.$

$= \mathcal{L}^{-1}\left\{\dfrac{2}{s-1} + \dfrac{3}{(s-1)^2} - \dfrac{1}{s+3}.\right\}.$

$= 2e^t + 3e^t t - e^{-3t}$

4) $\mathcal{L}^{-1}\left\{\dfrac{2s^2+10s}{(s^2-2s+5)(s+1)}\right\}.$

$\dfrac{2s^2+10s}{s^2-2s+5} = \dfrac{A}{(s+1)} + \dfrac{Bs+C}{s^2-2s+5}$

$2s^2+10s = A(s^2-2s+5)+(s+1)(Bs+C)$

Put $s = -1$

$2(-1)^2+10(-1) = A[(-1)^2-2(-1)+5]$

$2 - 10 = A(1+2+5)$

$-8 = 8A$

$A = -1$

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$2 = A\cancel{0}^2 + B\cancel{0}$. $\quad 0 = 5A + C$

$2 = -1 + B \quad 0 = 5(-1) + C$

$B = 3 \quad C = 5$.

$= \mathcal{L}^{-1}\left\{ \frac{-1}{s+1} + \frac{3s+5}{s^2-2s+5} \right\}$.

$= -e^{-t} + 3\mathcal{L}^{-1}\left[\frac{(s-1)}{(s-1)^2+4}\right] + 4\mathcal{L}^{-1}\left[\frac{2}{(s-1)^2+4}\right]$.

$= -e^{-t} + 3e^{t}\cos 2t + 4e^{t}\sin 2t$

Q#3

1) $y' - 5y = e^{5x}$. $\quad y(0) = 0$

$s\,Y(s) - y(0) - 5\,Y(s) = \frac{1}{s-5}$

$Y(s)(s-5) = \frac{1}{s-5}$

$Y(s) = \frac{1}{(s-5)^2}$

$y(x) = \mathcal{L}^{-1}\left(\frac{1}{(s-5)^2}\right)$.

$\frac{n!}{s+1}$

$n = 1$

$f(s) \Rightarrow (s-5)$

$a = 5$

$= x\,e^{5x}$.

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2) $y' + y = \sin x \qquad y(0) = 1$

$$sY(s) - y(0) + Y(s) = \frac{1}{s+1}$$

$$sY(s) - 1 + Y(s) = \frac{1}{s^2+1}$$

$$sY(s) + Y(s) = \frac{1}{s^2+1} + 1$$

$$Y(s)(s+1) = \frac{1}{s^2+1} + 1$$

$$Y(s) = \frac{1}{(s^2+1)(s+1)} + \frac{1}{s+1}$$

$$y(x) = \mathcal{L}^{-1}\left[\frac{1}{(s^2+1)(s+1)} + \frac{1}{s+1}\right].$$

$$\frac{1}{(s^2+1)(s+1)} = \frac{(As+B)}{(s^2+1)} + \frac{C}{s+1}$$

$$1 = (As+B)(s+1) + C(s^2+1)$$

Put $s = -1$

$$1 = 0 + C\{(-1)^2+1\}$$

$$1 = 2C$$

$C = 1/2 \qquad\qquad 1 = B + C$

$1 = 1/2 + B$

$B = 1/2$

$$0 = As^2 + Bs^2$$

$$0 = A + B$$

$$A = -1/2$$

$$\frac{1-s}{2(s^2+1)} + \frac{1}{2(s+1)} + \frac{1}{s+1}$$

$$\mathcal{L}^{-1}\left[\frac{1-s}{2(s^2+1)} + \frac{3}{2(s+1)}\right].$$

$$\frac{1}{2}\left\{\mathcal{L}^{-1}\left[\frac{1}{s^2+1}\right] - \mathcal{L}^{-1}\left[\frac{s}{s^2+1}\right]\right\} + \frac{3}{2}e^{-x}$$

$$\frac{1}{2}\sin x - \frac{1}{2}\cos x + \frac{3}{2}e^{-x}$$

3) $y'' - y' = 2x$

$$s^2 F(s) - s f(0) - f'(0) - \left[sF(s) - f(0)\right] = \frac{2}{s^2}$$

$$s^2 F(s) - s + 2 - sF(s) + 1 = \frac{2}{s^2}$$

$$F(s)(s^2 - s) - s + 3 = \frac{2}{s^2}$$

$$F(s)(s^2 - s) = \frac{2}{s^2} - 3 + s$$

$$F(s) = \frac{2}{s^2(s^2 - s)} - \frac{3}{(s^2 - s)} + \frac{s}{s^2 - s}$$

$$F(s) = \frac{2}{s^3(s-1)} + \frac{s - 3}{s(s-1)}$$

$$y(x) = \mathcal{L}^{-1}\left[\frac{2}{s^3(s-1)}\right] + \mathcal{L}^{-1}\left[\frac{s}{s(s-1)}\right] - \mathcal{L}^{-1}\left[\frac{3}{s(s-1)}\right]$$

$$\frac{3}{s(s-1)} = \frac{A}{s} + \frac{B}{s-1}$$

$$3 = A(s-1) + Bs$$

Put $s = 1$

$3 = B$

$-A = 3$

$A = -3$

$$= \mathcal{L}^{-1}\left[-\frac{3}{s} + \frac{3}{s-1}\right]$$

$$= -3 + 3e^{x}$$

$$\frac{2}{s^3(s-1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{D}{s-1}$$

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$2 = A(s^2)(s-1) + B(s)(s-1) + C(s-1) + D(s^3)$

Put $s = 0$

$2 = C(0-1)$

$C = -2$

$0 = C - B$

$0 = -2 - B$

$B = -2$

Put $s = 1$

$2 = D(1)^3$

$D = 2$

$0 = A + D$

$0 = A + 2$

$A = -2$

$= \mathcal{L}^{-1}\left[\frac{-2}{s} - \frac{2}{s^2} - \frac{2}{s^3} + \frac{2}{s-1} \right]$

$= -2 - 2x - x^2 + 2e^x$

$y(x) = -2 - 2x - x^2 + 2e^x + 3 - 2e^x$

$y(x) = 1 - x^2 - 2x$

4) $y'' - 2y' + 5y = 8e^{7-x}$

$t = x + 7$

$x = t + 7$

$y'' - 2y' + 5y = -8e^{-(t+7)+7}$

$y'' - 2y' + 5y = -8e^{-t}$

$s^2 Y(s) - sY(s) - y'(7) - 2sY(s) + 2y(7) + 5Y(s) = \frac{-8}{s+1}$

$s^2 Y(s) - 2s + 5Y(s) - 12 - 2sY(s) + (4) = \frac{-8}{s+1}$

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$$Y(s)\,(s^2-2s+5) = \frac{-8}{s+1} + 8 + 2s$$

$$Y(s) = \frac{-8}{[s^2-2s+5](s+1)} + \frac{8}{s^2-2s+5} + \frac{2s}{s^2-2s+5}$$

$$= \frac{-8+8s+8+2s^2+2s}{(s+1)(s^2-2s+5)}$$

$$= \frac{2s^2+10s}{(s+1)(s^2-2s+5)}$$

$$\frac{2s^2+10s}{(s+1)(s^2-2s+5)} = \frac{A}{s+1} + \frac{Bs+C}{s^2-2s+5}$$

$$2s^2+10s = A(s^2-2s+5) + (s+1)(Bs+C)$$

Put $s=-1$

$$2(-1)^2+10(-1) = A[(-1)^2-(2)(-1)+5]$$

$$2-10 = A(1+2+5)$$

$$-8 = 8A$$

$$A = -1.$$

$$2s^2 = As^2 + Bs^2$$

$$2 = A+B$$

$$2 = -1+B$$

$$B = 3.$$

$$0 = 5A + C$$

$$0 = C+5(-1)$$

$$C = 5.$$

$$= L^{-1}\left[\frac{-1}{s+1}\right] + L^{-1}\left[\frac{3s+5}{s^2-2s+5}\right]$$

$$= e^{-t} + 3L^{-1}\left[\frac{(s-1)}{(s-1)^2+4}\right] + 4L^{-1}\left[\frac{2}{(s-1)^2+4}\right]$$

$$= e^{-t} = 3e^{t}\cos 2t + 4e^{t}\sin 2t$$

$$= e^{-(x-\pi)} + 3e^{x-\pi}\cos 2(x-\pi) + 4e^{x-\pi}\sin 2(x-\pi)$$

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ANAYTICAL METHOD.

Q #3

1) $y' - y = e^{5x}$

$p(x) = 5$

$= \int e^{-5x}$

$= e^{-5x}$

$e^{-5x} y' - e^{5x} y = 1$

$\frac{d}{dx}(y \cdot e^{-5x}) = 1$

$\int d\, y \cdot e^{-5x} = \int 1\, dx$

$y \cdot e^{-5x} = x$

$y = x e^{5x}$

2) $y' + y = \sin x$

$p(x) = 1$

$= e^{\int p(x)}$

$= e^{\int 1}$

$= e^{x}$

$e^{x} y' + e^{x} y = e^{x} \sin x$

$\int \frac{d}{dx}(ye^{x}) = \int e^{x} \sin x\, dx$

$ye^{x} = \int e^{x} \sin x\, dx$

$uv - \int v\, du$

$u = e^{x}$ $\quad dv = \sin x$

$du = e^{x} dx$ $\quad v = -\cos x$

$-e^{x} \cos x + \int e^{x} \cos x\, dx$

$u = e^{x}$ $\quad v = \cos x$

$u = e^{x}$ $\quad v = \sin x$

$\cancel{-e^{x} \cos x + e^{x} \sin x - \int e^{x} \sin x}$

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$\int e^x \sin x\,dx = -e^x \cos x + e^x \sin x - \int e^x \sin x\,dx$

$2\int e^x \sin x\,dx = -e^x \cos x + e^x \sin x$

$e^x \sin y = \frac{1}{2}(e^x \sin x - e^x \cos x) + c.$

$y = \frac{\sin x}{2} - \frac{\cos x}{2} + c \cdot e^{-x}$

$y(0) = 1$

$1 = \frac{-1}{2} + c$

$c = 3/2$

$y = \frac{\sin x}{2} - \frac{\cos x}{2} + \frac{3e^{-x}}{2}.$

3) $y'' - y' = 2x.$

$m^2 - m = 0$

$m_1 = 0, \quad m_2 = 1$

$y = c_1 + c_2 e^x.$

$y' = 2ax + b$

$y'' = 2a$

$2a - (2ax + b) = 2x$

$-2ax - b + 2a = 2x$

$a = -1$

$-b + 2a = 0$

$-b = 2a$

$-b = 2(-1)$

$b = 2.$

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$y = c_1 + c_2 e^x - x^2 - 2x.$

$1 = c_1 + c_2$ $\quad y' = c_2 e^x - 2x - 2.$

$1 = c_1$ $\quad -2 = c_2 - 2.$

$c_2 = 0$

$y = -x^2 - 2x + 1$

4) $y'' - 2y' + 5y = -8e^{7x}.$

$m^2 - 2m + 5 = 0$

$m = 1 \pm 2i$

$y_c = e^t(c_1\cos 2t + c_2\sin 2t).$

$y_p = -8e^{-t}$

$t = x - 7$

$y_p = Ae^{-t}$

$y'_p = -Ae^{-t}$

$y''_p = -1\cdot(-Ae^{-t})$

$Ae^{-t} - 2(-Ae^{-t}) + 5(Ae^{-t}) = -8e^{-t}$

$8Ae^{-t} = -8e^{-t}$

$8A = -8$

$A = -1$

$y_p = -e^{-t}$

$y = e^t(c_1\cos 2t + c_2\sin 2t) - e^{-t}$

$y(7) = 2 \Rightarrow y(0) = 7$

$y = e^0(c_1\cos 2(0) + c_2\sin(0)) - e^{-0}$

$2 = c_1 - 1$

$c_1 = 3.$

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$y'(7) = 12 \Rightarrow y'(0) = 112$

$y' = -6 \sin 2t + 3e^{t} \cos 2t + e^{t}$
$C_2 \sin 2t + 2C_2 \cos 2t$
$+e^{t}$

$y'(0)$ $12 = 3 + 2 \cdot C_2 + 1$

$8/2 = C_2$

$C_2 = 4$

$y = e^{t}(3\cos 2t + 4\sin 2t) - e^{-t}$

$y = e^{x-7}(3\cos 2(x-7) + 4\sin 2(x-7)) - e^{-(x-7)}$