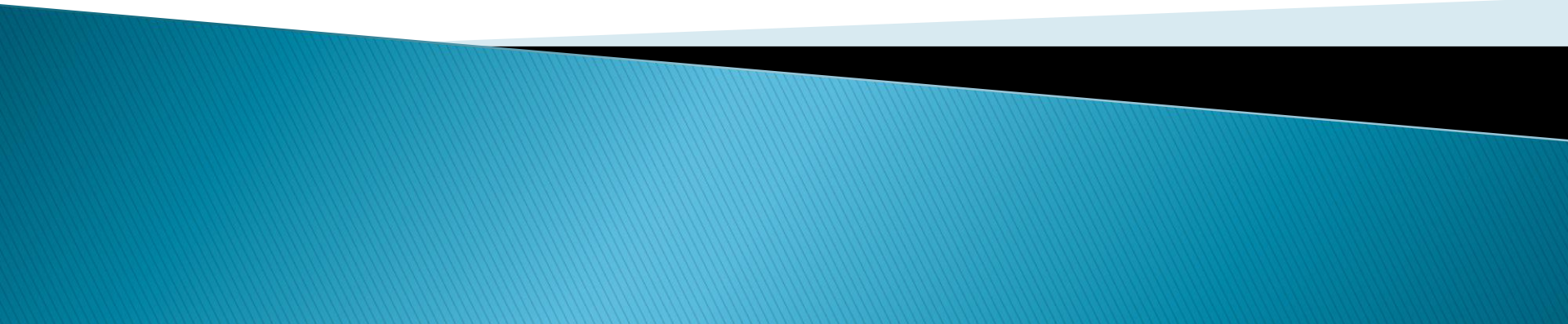


Chapter-5

Some Discrete Probability Distribution



Discrete probability distributions.

[Binomial probability distribution](#)

[Hypergeometric probability distribution](#)

[Multinomial probability distribution](#)

[Poisson probability distribution](#)

Continuous probability distributions.

[Normal probability distribution](#)

[Student's t distribution](#)

[Chi-square distribution](#)

[F distribution](#)

The Bernoulli Process

Bernoulli process must possess the following properties:

1. The experiment consists of repeated trials.
2. Each trial results in an outcome that may be classified as a success or a failure.
3. The probability of success, denoted by p , remains constant from trial to trial.
4. The repeated trials are independent.

Binomial Distribution A Bernoulli trial can result in a success with probability p and a failure with probability $q = 1 - p$. Then the probability distribution of the binomial random variable X , the number of successes in n independent trials, is

$$b(x; n, p) = \binom{n}{x} p^x q^{n-x}, \quad x = 0, 1, 2, \dots, n.$$

Theorem 5.1: The mean and variance of the binomial distribution $b(x; n, p)$ are $\mu = np$ and $\sigma^2 = npq$.

PROCEDURE 5.1 To Find a Binomial Probability Formula

Assumptions

1. n trials are to be performed.
2. Two outcomes, success or failure, are possible for each trial.
3. The trials are independent.
4. The success probability, p , remains the same from trial to trial.

Step 1 Identify a success.

Step 2 Determine p , the success probability.

Step 3 Determine n , the number of trials.

Step 4 The binomial probability formula for the number of successes, X , is

$$P(X = x) = \binom{n}{x} p^x (1 - p)^{n-x}.$$

The probability that a certain kind of component will survive a shock test is $3/4$. Find the probability that exactly 2 of the next 4 components tested survive.

Solution:

$$b\left(2; 4, \frac{3}{4}\right) = \binom{4}{2} \left(\frac{3}{4}\right)^2 \left(\frac{1}{4}\right)^2 = \left(\frac{4!}{2! 2!}\right) \left(\frac{3^2}{4^4}\right) = \frac{27}{128}.$$

Example 5.2: The probability that a patient recovers from a rare blood disease is 0.4. If 15 people are known to have contracted this disease, what is the probability that (a) at least 10 survive, (b) from 3 to 8 survive, and (c) exactly 5 survive?

Find the mean and variance of the binomial random variable

Solution: Let X be the number of people who survive.

$$\begin{aligned} \text{(a)} \quad P(X \geq 10) &= 1 - P(X < 10) = 1 - \sum_{x=0}^9 b(x; 15, 0.4) = 1 - 0.9662 \\ &= 0.0338 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad P(3 \leq X \leq 8) &= \sum_{x=3}^8 b(x; 15, 0.4) = \sum_{x=0}^8 b(x; 15, 0.4) - \sum_{x=0}^2 b(x; 15, 0.4) \\ &= 0.9050 - 0.0271 = 0.8779 \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad P(X = 5) &= b(5; 15, 0.4) = \sum_{x=0}^5 b(x; 15, 0.4) - \sum_{x=0}^4 b(x; 15, 0.4) \\ &= 0.4032 - 0.2173 = 0.1859 \end{aligned}$$

Table A.1 (continued) Binomial Probability Sums $\sum_{x=0} b(x; n, p)$

<i>n</i>	<i>r</i>	<i>p</i>									
		0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.80	0.90
15	0	0.2059	0.0352	0.0134	0.0047	0.0005	0.0000				
	1	0.5490	0.1671	0.0802	0.0353	0.0052	0.0005	0.0000			
	2	0.8159	0.3980	0.2361	0.1268	0.0271	0.0037	0.0003	0.0000		
	3	0.9444	0.6482	0.4613	0.2969	0.0905	0.0176	0.0019	0.0001		
	4	0.9873	0.8358	0.6865	0.5155	0.2173	0.0592	0.0093	0.0007	0.0000	
	5	0.9978	0.9389	0.8516	0.7216	0.4032	0.1509	0.0338	0.0037	0.0001	
	6	0.9997	0.9819	0.9434	0.8689	0.6098	0.3036	0.0950	0.0152	0.0008	
	7	1.0000	0.9958	0.9827	0.9500	0.7869	0.5000	0.2131	0.0500	0.0042	0.0000
	8		0.9992	0.9958	0.9848	0.9050	0.6964	0.3902	0.1311	0.0181	0.0003
	9		0.9999	0.9992	0.9963	0.9662	0.8491	0.5968	0.2784	0.0611	0.0022
	10		1.0000	0.9999	0.9993	0.9907	0.9408	0.7827	0.4845	0.1642	0.0127
	11			1.0000	0.9999	0.9981	0.9824	0.9095	0.7031	0.3518	0.0556
	12				1.0000	0.9997	0.9963	0.9729	0.8732	0.6020	0.1841
	13					1.0000	0.9995	0.9948	0.9647	0.8329	0.4510
	14						1.0000	0.9995	0.9953	0.9648	0.7941
	15							1.0000	1.0000	1.0000	1.0000

Experiments yielding numerical values of a random variable X , the number of outcomes occurring during a given time interval or in a specified region, are called **Poisson experiments**. The given time interval may be of any length, such as a minute, a day, a week, a month, or even a year. For example, a Poisson experiment

Poisson Probability Formula

Probabilities for a random variable X that has a Poisson distribution are given by the formula

$$P(X = x) = e^{-\lambda} \frac{\lambda^x}{x!}, \quad x = 0, 1, 2, \dots,$$

where λ is a positive real number and $e \approx 2.718$. (Most calculators have an e key.) The random variable X is called a **Poisson random variable** and is said to have the **Poisson distribution** with parameter λ .

Mean and Standard Deviation of a Poisson Random Variable

The mean and standard deviation of a Poisson random variable with parameter λ are

$$\mu = \lambda \quad \text{and} \quad \sigma = \sqrt{\lambda},$$

respectively.

Example:

During a laboratory experiment, the average number of radioactive particles passing through a counter in 1 millisecond is 4. What is the probability that 6 particles enter the counter in a given millisecond?

$$\begin{aligned} p(6; 4) &= \frac{e^{-4} 4^6}{6!} \\ &= \sum_{x=0}^6 p(x; 4) - \sum_{x=0}^5 p(x; 4) \\ &= 0.8893 - 0.7851 = 0.1042. \end{aligned}$$

Example:

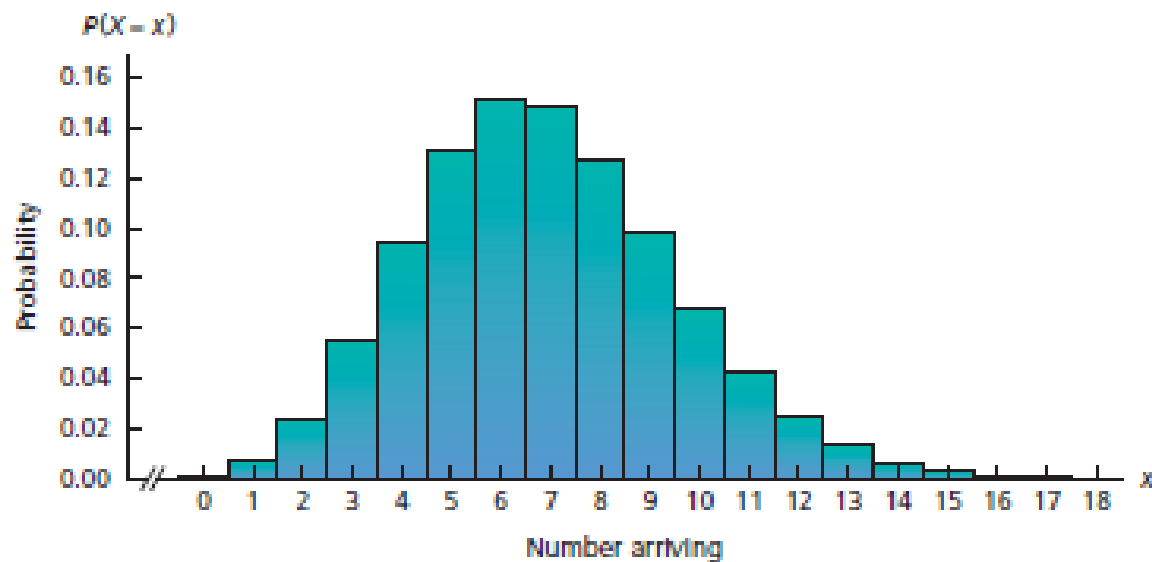
Emergency Room Traffic Desert Samaritan Hospital keeps records of emergency room (ER) traffic. Those records indicate that the number of patients arriving between 6:00 P.M. and 7:00 P.M. has a Poisson distribution with parameter $\lambda = 6.9$. Determine the probability that, on a given day, the number of patients who arrive at the emergency room between 6:00 P.M. and 7:00 P.M. will be

- exactly 4.
- at most 2.
- between 4 and 10, inclusive.
- Obtain a table of probabilities for the random variable X , the number of patients arriving between 6:00 P.M. and 7:00 P.M. Stop when the probabilities become zero to three decimal places.
- Use part (d) to construct a (partial) probability histogram for X .
- Identify the shape of the probability distribution of X .

Answers:

- 0.095
- 0.032
- 0.821

Number arriving x	Probability $P(X = x)$	Number arriving x	Probability $P(X = x)$
0	0.001	10	0.068
1	0.007	11	0.043
2	0.024	12	0.025
3	0.055	13	0.013
4	0.095	14	0.006
5	0.131	15	0.003
6	0.151	16	0.001
7	0.149	17	0.001
8	0.128	18	0.000
9	0.098		



Poisson Approximation to the Binomial Distribution

Recall that the binomial probability formula is

$$P(X = x) = \binom{n}{x} p^x (1 - p)^{n-x}.$$

To Approximate Binomial Probabilities
by Using a Poisson Probability Formula

Step 1 Find n , the number of trials, and p , the success probability.

Step 2 Continue only if $n \geq 100$ and $np \leq 10$.

Step 3 Approximate the binomial probabilities by using the Poisson probability formula

$$P(X = x) = e^{-np} \frac{(np)^x}{x!}.$$

Example:

IMR in Finland The infant mortality rate (IMR) is the number of deaths of children under 1 year old per 1000 live births during a calendar year. From the *World Factbook*, the Central Intelligence Agency's most popular publication, we found that the IMR in Finland is 3.5. Use the Poisson approximation to determine the probability that, of 500 randomly selected live births in Finland, there are

- a. no infant deaths.
- b. at most three infant deaths.

5.17: During a laboratory experiment, the average number of radioactive particles passing through a counter in 1 millisecond is 4. What is the probability that 6 particles enter the counter in a given millisecond?

Solution:

$$p(6; 4) = \frac{e^{-4}4^6}{6!} = \sum_{x=0}^6 p(x; 4) - \sum_{x=0}^5 p(x; 4) = 0.8893 - 0.7851 = 0.1042.$$

Class Practice:

5.20: In a manufacturing process where glass products are made, defects or bubbles occur, occasionally rendering the piece undesirable for marketing. It is known that, on average, 1 in every 1000 of these items produced has one or more bubbles. What is the probability that a random sample of 8000 will yield fewer than 7 items possessing bubbles?

5.11 The probability that a patient recovers from a delicate heart operation is 0.9. What is the probability that exactly 5 of the next 7 patients having this operation survive?

5.12 A traffic control engineer reports that 75% of the vehicles passing through a checkpoint are from within the state. What is the probability that fewer than 4 of the next 9 vehicles are from out of state?

Multinomial Distribution

If a given trial can result in the k outcomes E_1, E_2, \dots, E_k with probabilities p_1, p_2, \dots, p_k , then the probability distribution of the random variables X_1, X_2, \dots, X_k , representing the number of occurrences for E_1, E_2, \dots, E_k in n independent trials, is

$$f(x_1, x_2, \dots, x_k; p_1, p_2, \dots, p_k, n) = \binom{n}{x_1, x_2, \dots, x_k} p_1^{x_1} p_2^{x_2} \cdots p_k^{x_k},$$

with

$$\sum_{i=1}^k x_i = n \text{ and } \sum_{i=1}^k p_i = 1.$$

5.7: The complexity of arrivals and departures of planes at an airport is such that computer simulation is often used to model the “ideal” conditions. For a certain airport with three runways, it is known that in the ideal setting the following are the probabilities that the individual runways are accessed by a randomly arriving commercial jet:

Runway 1: $p_1 = 2/9$,

Runway 2: $p_2 = 1/6$,

Runway 3: $p_3 = 11/18$.

What is the probability that 6 randomly arriving airplanes are distributed in the following fashion?

Runway 1: 2 airplanes,

Runway 2: 1 airplane,

Runway 3: 3 airplanes

Hypergeometric Distribution

as a **hypergeometric experiment**, that is, one that possesses the following two properties:

1. A random sample of size n is selected without replacement from N items.
2. Of the N items, k may be classified as successes and $N - k$ are classified as failures.

Hypergeometric Distribution The probability distribution of the hypergeometric random variable X , the number of successes in a random sample of size n selected from N items of which k are labeled **success** and $N - k$ labeled **failure**, is

$$h(x; N, n, k) = \frac{\binom{k}{x} \binom{N-k}{n-x}}{\binom{N}{n}},$$

5.9: Lots of 40 components each are deemed unacceptable if they contain 3 or more defectives. The procedure for sampling a lot is to select 5 components at random and to reject the lot if a defective is found. What is the probability that exactly 1 defective is found in the sample if there are 3 defectives in the entire lot?

Solution:

Using the hypergeometric distribution with

$$h(x; N, n, k) = \frac{\binom{k}{x} \binom{N-k}{n-x}}{\binom{N}{n}},$$

$n = 5$, $N = 40$, $k = 3$, and $x = 1$

$$h(1; 40, 5, 3) = \frac{\binom{3}{1} \binom{37}{4}}{\binom{40}{5}} = 0.3011.$$

The mean and variance of the hypergeometric distribution $h(x; N, n, k)$ are

$$\mu = \frac{nk}{N} \text{ and } \sigma^2 = \frac{N-n}{N-1} \cdot n \cdot \frac{k}{N} \left(1 - \frac{k}{N}\right).$$

Geometric Distribution

If repeated independent trials can result in a success with probability p and a failure with probability $q = 1 - p$, then the probability distribution of the random variable X , the number of the trial on which the first success occurs, is

$$g(x; p) = pq^{x-1}, \quad x = 1, 2, 3, \dots$$

Example:

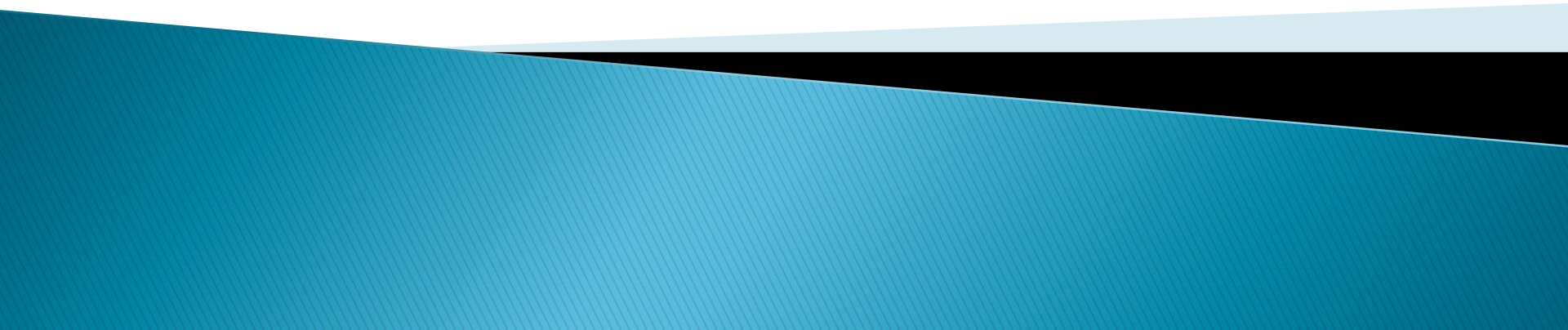
For a certain manufacturing process, it is known that, on the average, 1 in every 100 items is defective. What is the probability that the fifth item inspected is the first defective item found?

with $x = 5$ and $p = 0.01$,

$$g(5; 0.01) = (0.01)(0.99)^4 = 0.0096.$$

Chapter-6

Some Continuous Probability Distribution



1-Discrete probability distributions.

[Binomial probability distribution](#)

[Hyper geometric probability distribution](#)

[Multinomial probability distribution](#)

[Poisson probability distribution](#)

2-Continuous probability distributions.

[Normal probability distribution](#)

[Student's t distribution](#)

[Chi-square distribution](#)

[F distribution](#)

Continuous Prob.Distribution

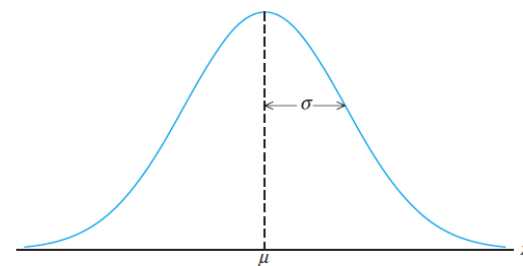


Figure 6.2: The normal curve.

The most important continuous probability distribution in the entire field of statistics is the **normal distribution**. Its graph, called the **normal curve**, is the

Normal Distribution The density of the normal random variable X , with mean μ and variance σ^2 , is

$$n(x; \mu, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}, \quad -\infty < x < \infty,$$

where $\pi = 3.14159\dots$ and $e = 2.71828\dots$

Theorem 6.2: The mean and variance of $n(x; \mu, \sigma)$ are μ and σ^2 , respectively. Hence, the standard deviation is σ .

Area under Normal Curve

$$P(x_1 < X < x_2) = \int_{x_1}^{x_2} n(x; \mu, \sigma) dx = \frac{1}{\sqrt{2\pi}\sigma} \int_{x_1}^{x_2} e^{-\frac{1}{2\sigma^2}(x-\mu)^2} dx$$

is represented by the area of the shaded region.

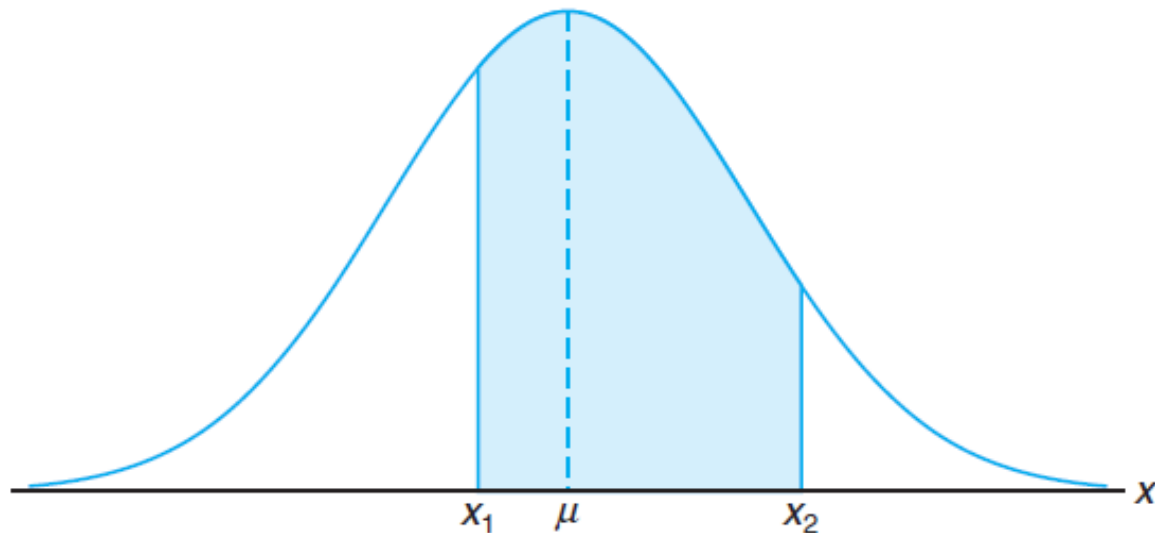


Figure 6.6: $P(x_1 < X < x_2) = \text{area of the shaded region.}$

Z-curves

$$Z = \frac{X - \mu}{\sigma}.$$

Definition 6.1: The distribution of a normal random variable with mean 0 and variance 1 is called a standard normal distribution.

6.2: Given a standard normal distribution, find the area under the curve that lies

- (a) to the right of $z = 1.84$ and
- (b) between $z = -1.97$ and $z = 0.86$.

Use table

$$1 - 0.9671 = 0.0329.$$

$$0.8051 - 0.0244 = 0.7807.$$

6.4: Given a random variable X having a normal distribution with $\mu = 50$ and $\sigma = 10$, find the probability that X assumes a value between 45 and 62.

$$P(45 < X < 62) = P(-0.5 < Z < 1.2)$$

Use table

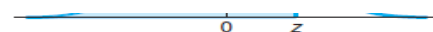
$$\begin{aligned} &= P(Z < 1.2) - P(Z < -0.5) \\ &= 0.8849 - 0.3085 = 0.5764. \end{aligned}$$

6.5: Given that X has a normal distribution with $\mu = 300$ and $\sigma = 50$, find the probability that X assumes a value greater than 362.

$$P(X > 362) = P(Z > 1.24)$$

$$\begin{aligned} &= 1 - P(Z < 1.24) = 1 - 0.8925 \\ &= 0.1075. \end{aligned}$$

Table A.3 Areas under the Normal Curve



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Summary

Binomial distribution

n = number of trials

p = probability of success

$$P(x) = \binom{n}{x} p^x q^{n-x}$$

$$\mathbf{E}(X) = np$$

$$\text{Var}(X) = npq$$

Geometric distribution

p = probability of success

$$P(x) = (1 - p)^{x-1} p, \quad x = 1, 2, \dots$$

$$\mathbf{E}(X) = \frac{1}{p}$$

$$\text{Var}(X) = \frac{1 - p}{p^2}$$

Summary

Poisson distribution

λ = frequency, average number of events

$$P(x) = e^{-\lambda} \frac{\lambda^x}{x!}, \quad x = 0, 1, 2, \dots$$

$$\mathbf{E}(X) = \lambda$$

$$\text{Var}(X) = \lambda$$

Normal distribution

μ = expectation, location parameter

σ = standard deviation, scale parameter

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp \left\{ \frac{-(x - \mu)^2}{2\sigma^2} \right\}, \quad -\infty < x < \infty$$

$$\mathbf{E}(X) = \mu$$

$$\text{Var}(X) = \sigma^2$$

1- TRY TO READ AND UNDERSTAND BINOMIAL ,
POISSON AND NORMAL DISTRIBUTION TABLE

2- DO ATLEAST 5-10 QUESTIONS FROM WALPOLE
OF EACH DISTRIBUTION

ANY
Questions?