K200477
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(1)

Date

EXERCISE # 3.2.

Q#1

$y'' - 4y' + 4y = 0$ $\qquad y_1 = e^{2x}$

$P(x) = -4.$

$y_2(x) = y_1(x) \int \frac{e^{-\int P(x)dx}}{y_1^2(x)}$

$= e^{2x} \int \frac{e^{\int 4 dx}}{e^{4x}} dx$

$= e^{2x} \int \frac{e^{4x}}{e^{4x}} dx$

$= e^{2x} \int dx$

$y_2(x) = xe^{2x}$

Q#2

$y'' + 2y' + y = 0$ $\qquad y_1(x) = e^{-x}x$

$P(x) = 2$

$y_2(x) = xe^{-x} \int \frac{e^{-\int 2dx}}{(xe^{-x})^2} dx.$

$= xe^{-x} \int \frac{e^{-2x}}{x^2 e^{-2x}} dx$

$= xe^{-x} \int x^{-2} dx$

$= xe^{-x}\left(-\frac{1}{x}\right)$

$y_2(x) = -e^{-x}.$

$y_2(x) = e^{-x}$

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Q#3

$y'' + 16y = 0$ $y_1 = \cos 4x$.

$P(x) = 0$.

$$y_2(x) = \cos 4x \int \frac{e^{-\int 0 dx}}{(\cos 4x)^2} dx$$

$$= \cos 4x \int \frac{c}{\cos^2 4x} dx$$

$$= \cos 4x \int c \sec^2 4x \, dx.$$

$$= \cos 4x \quad c \frac{\tan 4x}{4}$$

$$= \frac{c}{4} \cancel{\cos 4x} \frac{\sin 4x}{\cancel{\cos 4x}}$$

$$= \frac{c \sin 4x}{4}$$

$y_2(x) = \sin 4x$.

Q#4

$y'' + 9y = 0$ $y_1 = \sin 3x$

$P(x) = 0$

$$y_2(x) = \sin 3x \int \frac{e^{-\int 0 dx}}{(\sin 3x)^2} dx.$$

$$= \sin 3x \int \frac{c}{\sin^2 3x} dx$$

$$= \sin 3x \int c \operatorname{cosec}^2 3x \, dx$$

$$= \sin 3x \; c\left(\frac{-\cot 3x}{3}\right)$$

$$= -\frac{c}{3} \cdot \cancel{\sin 3x} \cdot \frac{\cos 3x}{\cancel{\sin 3x}}$$

$$= -\frac{c}{3} \cos 3x$$

$y_2(x) = \cos 3x$.

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Q#5

$y'' - y = 0$ $\qquad y_1 = \cosh x$.

$P(x) = 0$

$$y_2(x) = \cosh x \int \frac{e^{-\int 0 dx}}{(\cosh x)^2} dx$$

$$= \cosh x \int \frac{c}{\cosh^2 x} dx$$

$$= \cosh x \int c \operatorname{sech}^2 x \, dx$$

$$= \cosh x \; c \tanh x$$

$$= c \cdot \cancel{\cosh x} \cdot \frac{\sinh x}{\cancel{\cosh x}}$$

$$= c \cdot \sinh x$$

$$y_2(x) = \sinh x.$$

Q#6

$y'' - 25y = 0$ $\qquad y_1(x) = e^{5x}$

$p(x) = 0$

$$y_2(x) = e^{5x} \int \frac{e^{-\int 0 dx}}{(e^{5x})^2} dx$$

$$= e^{5x} \int \frac{c}{e^{10x}} dx$$

$$= e^{5x} - \left(\frac{1}{10 e^{10x}} \right) c$$

$$= -\frac{c}{10} \cdot e^{-5x}$$

$$y_2(x) = e^{-5x}.$$

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Q#7

$9y'' - 12y' + 4y = 0 \qquad y_1 = e^{2x/3}$

$y'' - \frac{12}{9}y' + \frac{4}{9}y = 0$

$P(x) = -\frac{12}{9} = -\frac{4}{3}$

$y_2(x) = e^{2x/3} \int \frac{e^{-\int -4/3\,dx}}{(e^{2x/3})^2}\,dx$

$= e^{2x/3} \int \frac{e^{4x/3}}{e^{4x/3}}\,dx$

$= e^{2x/3} \int dx$

$y_2(x) = xe^{2x/3}$

Q#8

$6y'' + y' - y = 0 \qquad y_1 = e^{x/3}$

$y'' + \frac{1}{6}y' - \frac{y}{6} = 0$

$P(x) = \frac{1}{6}$

$y_2(x) = e^{x/3} \int \frac{e^{-\int 1/6\,dx}}{e^{2x/3}}\,dx$

$= e^{x/3} \int \frac{e^{-x/6}}{e^{2x/3}}\,dx$

$= e^{x/3} \int e^{-x/6 - 2x/3}\,dx$

$= e^{x/3} \int e^{-5x/6}\,dx$

$= e^{x/3} \left(\frac{-6}{5}\right) e^{-5x/6}$

$= -\frac{6}{5} e^{x/3 - 5x/6}$

$y_2(x) = -\frac{6}{5} e^{-x/2}$

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Q#9

$$x^2y'' - 7xy' + 16y = 0 \qquad y_1 = x^4$$

$$y'' - \frac{7y'}{x} + \frac{16}{x^2}y = 0$$

$$P(x) = -7/x$$

$$y_2(x) = x^4 \int \frac{e^{\int 7/x\,dx}}{(x^4)^2} dx$$

$$= x^4 \int \frac{e^{7\ln x\,dx}}{x^8} dx$$

$$= x^4 \int \frac{x^7}{x^8} dx$$

$$= x^4 \int \frac{dx}{x}$$

$$y_2(x) = x^4 \ln x$$

Q#10

$$x^2y'' + 2xy' - 6y = 0 \qquad y_1 = x^2$$

$$y'' + \frac{2y'}{x} - \frac{6y}{x^2} = 0$$

$$P(x) = 2/x$$

$$y_2(x) = x^2 \int \frac{e^{-\int 2/x\,dx}}{(x^2)^2} dx$$

$$= x^2 \int \frac{e^{-2\ln x}}{x^4} dx$$

$$= x^2 \int \frac{x^{-2}}{x^4} dx$$

$$= x^2 \int \frac{dx}{x^6}$$

$$= x^2 \left(- \frac{1}{5x^5} \right)$$

$$y_2(x) = -\frac{1}{5} x^{-3} = x^{-3}.$$

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Q#11

$xy'' + y' = 0$ $\qquad y_1 = \ln x$

$y'' + \frac{1}{x} y' = 0$

$P(x) = 1/x$.

$y_2(x) = \ln x \int \frac{e^{-\int 1/x\,dx}}{(\ln x)^2} dx$.

$= \ln x \int \frac{e^{-\ln x} \cdot dx}{(\ln x)^2}$

$= \ln x \int \frac{x^{-1}}{(\ln x)^2} dx$

$= \ln x \int \frac{1/x}{(\ln x)^2} dx$,

$u = \ln x$

$\frac{du}{dx} = 1/x$.

$du = \frac{1}{x} dx$

$= \ln x \int \frac{du}{u^2}$

$= \ln x \left(-\frac{1}{u} \right)$

$= \ln x \left(-\frac{1}{\ln x} \right)$

$= -1$

$y_2(x) = 1$.

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Q#12.

$4x^2 y'' + y = 0$ $\qquad y_1 = x^{1/2}\ln x$

$y'' + \frac{1}{4x^2} y = 0$

$P(x) = 0$

$y_2(x) = x^{1/2}\ln x \int \frac{e^{-\int 0\, dx}}{(x^{1/2}\ln x)^2} dx$

$= x^{1/2}\ln x \int \frac{e}{x(\ln x)^2} dx$

$u = \ln x$

$\frac{du}{dx} = 1/x$

$du = 1/x\, dx$.

$= x^{1/2}\ln x\, c \int \frac{du}{u}$

$= x^{1/2}\ln x\, c \left(-1/u\right)$

$= x^{1/2}\ln x\, c\left(-1/\ln x\right)$

$= -c x^{1/x}$

$y_2(x) = x^{1/2}$.

Q#13

$x^2 y'' - 3xy' + 5y = 0$ $\qquad y_1 = x\sin(\ln x)$

$y'' - \frac{3}{x} y' + 5/x^2\, y = 0$

$P(x) = -3/x$

$y_2(x) = x\sin(\ln x) \int \frac{e^{\int 3/x\, dx}}{(x\sin(\ln x))^2} dx$.

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$$y_2(x) = x\sin(\ln x)\int \frac{e^{\int 1/x\,dx}}{x^2\sin^2(\ln x)}\,dx$$

$$= x\sin(\ln x)\int \frac{e^{\ln x}}{x^2\sin^2(\ln x)}\,dx$$

$$= x\sin(\ln x)\int \frac{x}{x^2\sin^2(\ln x)}\,dx$$

$$= x\sin(\ln x)\int \frac{dx}{x\sin^2(\ln x)}$$

$$= x\sin(\ln x)\int \frac{1}{x}\operatorname{cosec}^2(\ln x)\,dx$$

$u = \ln x$

$\frac{du}{dx} = 1/x$

$du = 1/x\,dx.$

$$= x\sin(\ln x)\int \operatorname{cosec}^2 u\,du$$

$$= x\sin(\ln x)(-\cot u)$$

$$= x\sin(\ln x)\left(-\frac{\cos(\ln x)}{\sin(\ln x)}\right)$$

$$= -x\cos(\ln x)$$

$$y_2(x) = x\cos(\ln x)$$

Q#14

$x^2y'' - 3xy' + 5y = 0.$ $\qquad y_1 = x^2\cos(\ln x)$

$y'' - \frac{3}{x}y' + \frac{5}{x^2}y = 0.$

$P(x) = -3/x.$

$$y_2(x) = x^2\cos(\ln x)\int \frac{e^{\int 3/x\,dx}}{x^4\cos^2(\ln x)}\,dx$$

$$= x^2\cos(\ln x)\int \frac{e^{3\ln x}}{x^4\cos^2(\ln x)}\,dx$$

~~$= x^2\cos(\ln x)$~~ ~~x^3~~

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$$= x^2 \cos(\ln x) \int \frac{x^3}{x^4 \cos^2(\ln x)} \, dx.$$

$$= x^2 \cos(\ln x) \int \frac{dx}{x \cos^2(\ln x)}$$

$$= x^2 \cos(\ln x) \int \frac{1}{x} \sec^2(\ln x) \, dx$$

$$u = \ln x$$

$$\frac{du}{dx} = \frac{1}{x}$$

$$du = \frac{dx}{x}.$$

$$= x^2 \cos(\ln x) \int \sec^2(u) \, du.$$

$$= x^2 \cos(\ln x) (\tan u)$$

$$= x^2 \cos(\ln x) \left(\frac{\sin(\ln x)}{\cos(\ln x)} \right)$$

$$y_2(x) = x^2 \sin(\ln x)$$

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