(1)

# ASSIGNMENT # 04.

Date

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## EXERCISE # 5.2

**Q#8**

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$\lambda I - A$

$$= \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}.$$

$$= \begin{bmatrix} \lambda-1 & 0 & 0 \\ 0 & \lambda-1 & -1 \\ 0 & -1 & \lambda-1 \end{bmatrix}.$$

$\det(\lambda I - A)$

$$= \begin{vmatrix} \lambda-1 & 0 & 0 \\ 0 & \lambda-1 & -1 \\ 0 & -1 & \lambda-1 \end{vmatrix}$$

$$\lambda-1 \begin{vmatrix} \lambda-1 & -1 \\ -1 & \lambda-1 \end{vmatrix} - 0 + 0 = 0$$

$$\lambda-1 \left[ (\lambda-1)^2 - (-1)^2 \right] = 0$$

$$\lambda-1 \left[ \lambda^2 - 2\lambda + 1 - 1 \right] = 0.$$

$$(\lambda-1)(\lambda-2)\lambda = 0.$$

$$\lambda = 0, \quad \lambda = 1, \quad \lambda = 2$$

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For $\lambda = 0$:

$$\begin{bmatrix} 0-1 & 0 & 0 \\ 0 & 0-1 & -1 \\ 0 & -1 & 0-1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & -1 \\ 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

$\Rightarrow -R_1$ & $-R_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

$\Rightarrow R_3 + R_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

$x_1 = 0$

$x_3 = t$

$x_2 = -t$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = t \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

For $\lambda = 1$

$$\begin{bmatrix} 1-1 & 0 & 0 \\ 0 & 1-1 & -1 \\ 0 & -1 & 1-1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow$ Interchang $R_1$ and $R_3$ and the $-R_1$ & $-R_2$.

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_1 = t$ $\quad x_3 = 0$

$x_2 = 0$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = t \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

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FOR $\lambda = 2$.

$$\begin{bmatrix} 2-1 & 0 & 0 \\ 0 & 2-1 & -1 \\ 0 & -1 & 2-1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

$\Rightarrow R_3 + R_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

$x_1 = 0$

$x_3 = t$

$x_2 = t$.

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = t \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$$P = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

using calculator

$$P^{-1} = \begin{bmatrix} 0 & -1/2 & 1/2 \\ 1 & 0 & 0 \\ 0 & 1/2 & 1/2 \end{bmatrix}.$$

$$P^{-1}AP = \begin{bmatrix} 0 & -1/2 & 1/2 \\ 1 & 0 & 0 \\ 0 & 1/2 & 1/2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}.$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \Rightarrow \text{Using calculator}$$

$\hookrightarrow$ Eigen values.

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Q#10

$$\begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 1 & 2 \end{bmatrix}$$

a) Find the eigen values.

$(\lambda I - A)$

$$= \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} \lambda-3 & 0 & 0 \\ 0 & \lambda-2 & 0 \\ 0 & -1 & \lambda-2 \end{bmatrix}$$

Lower triangle matrix

$\det(\lambda I - A) = 0.$

$$\begin{vmatrix} \lambda-3 & 0 & 0 \\ 0 & \lambda-2 & 0 \\ 0 & -1 & \lambda-2 \end{vmatrix} = 0$$

$(\lambda-3)(\lambda-2)^2 = 0$

$\lambda = 3 \qquad \lambda = 2$

b) Rank of matrix $\lambda I - A$

FOR $\lambda = 2$ : (Algebraic Multiciply $\Rightarrow$ 2)

$$\begin{bmatrix} 2-3 & 0 & 0 \\ 0 & 2-2 & 0 \\ 0 & -1 & 2-2 \end{bmatrix}$$

$\Rightarrow -R_1$ & $-R_3$ and interchange $R_2$ & $R_3$.

$$\begin{vmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Rank = 2

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For $\lambda = 3$. : Algebraic multiplicity $\Rightarrow 1$

$$\begin{bmatrix} 3-3 & 0 & 0 \\ 0 & 3-2 & 0 \\ 0 & -1 & 3-2 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}.$$

$\Rightarrow$ interchange $R_1$ & $R_2$

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$\Rightarrow$ Interchange $R_2$ & $R_3$

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$\Rightarrow$ $R_2 + R_1$

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Rank = 2.

c) A is diagonalizable or not

Geometric Multiplicity of $\lambda = 3$.

$\lambda - \lambda_0 = 3 - 2 = 1$

$1 == 1$.

Geometric Multiplicity of $\lambda = 2$.

$\lambda - \lambda_0 = 3 - 2 = 1$

$1 < 2$.

Therefore A is not diagonalizable

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Q#15

a) $(\lambda-1)(\lambda+3)(\lambda-5)=0$

Roots $\Rightarrow 3$

The degree of characteristics polynomial $\Rightarrow 3$.

Size of matrix $\Rightarrow 3\times3$.

All three eigen spaces have dimension $\Rightarrow 1$

b) $\lambda^2(\lambda-1)(\lambda-2)^3=0$

Roots $\Rightarrow 6$

The degree of characteristics polynomial $\Rightarrow 6$

Size of matrix $\Rightarrow 6\times6$.

Possible dimension of eigenspaces of $\lambda=0$ are 1 or 2

The dimensions of eigenspaces of $\lambda=1$ must be 1.

Possible dimension of eigenspaces of $\lambda=2$ are 1, 2 or 3.

Q#20

$$A=\begin{bmatrix}1&-2&8\\0&-1&0\\0&0&-1\end{bmatrix}\quad P=\begin{bmatrix}1&-4&1\\1&0&0\\0&1&0\end{bmatrix}$$

$P^{-1}AP =$

Calculating $P^{-1}$ using calculator

$$P^{-1}=\begin{bmatrix}0&1&0\\0&0&1\\1&-1&4\end{bmatrix}$$

$$P^{-1}AP=\begin{bmatrix}0&1&0\\0&0&1\\1&-1&4\end{bmatrix}\begin{bmatrix}1&-2&8\\0&-1&0\\0&0&-1\end{bmatrix}\begin{bmatrix}1&-4&1\\1&0&0\\0&1&0\end{bmatrix}$$

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$$P^{-1}AP = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

A is a diagnolizable
P diagnolizes A.

a) $A^{1000}$

$$PD^{1000}P^{-1} = \begin{bmatrix} 1 & -4 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} (-1)^{1000} & 0 & 0 \\ 0 & (-1)^{1000} & 0 \\ 0 & 0 & (1)^{1000} \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

b) $A^{-1000}$

$$PD^{-1000}P^{-1} = \begin{bmatrix} 1 & -4 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} (-1)^{-1000} & 0 & 0 \\ 0 & (-1)^{-1000} & 0 \\ 0 & 0 & (1)^{-1000} \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -4 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

c) $A^{2301}$

$$PD^{2301}P^{-1} = \begin{bmatrix} 1 & -4 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} (-1)^{2301} & 0 & 0 \\ 0 & (-1)^{2301} & 0 \\ 0 & 0 & (1)^{2301} \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 4 \end{bmatrix}.$$

$$= \begin{bmatrix} 1 & -2 & 8 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

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d) $A^{-2301}$

$$P D^{-2301} P^{-1} = \begin{bmatrix} 1 & -4 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} (-1)^{-2301} & 0 & 0 \\ 0 & (-1)^{-2301} & 0 \\ 0 & 0 & (1)^{-2301} \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 4 \end{bmatrix}.$$

$$= \begin{bmatrix} 1 & -2 & 8 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

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EXERCISE # 6.1

Q#2

$$\langle u, v \rangle = \tfrac{1}{2} u_1 v_1 + 5 u_2 v_2$$
$u = (1,1)$, $v = (3,2)$, $w(0,-1)$, $k = 3$.

a) $\langle u, v \rangle$
$= \frac{1}{2}(1)(3) + 5(1)(2)$
$= 3/2 + 10$
$= 23/2$

b) $\langle kv, w \rangle$
$= \frac{1}{2}((3)(3)(0)) + 5((3)(2)(-1))$
$= 0 + 5(-6)$
$= -30$

c) $\langle u+v, w \rangle$
$= \frac{1}{2}(1+3)(0) + 5(1+2)(-1)$
$= 0 + 5(-3)$
$= -15$

d) $\|v\|$
$\langle v, v \rangle^{1/2} = [\frac{1}{2}(3)(3) + 5(2)(2)]^{1/2}$
$= (9/2 + 20)^{1/2}$
$= \sqrt{49/2}$
$= 7/\sqrt{2}$

e) $d(u,v)$
$\|u - v\| = \langle (1-3), (1-2) \rangle^{1/2}$
$= (\frac{1}{2}(-2)(-2) + 5(-1)(-1))^{1/2}$
$= \sqrt{2 + 5}$
$= \sqrt{7}$

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f) $||u - kv||$

$= \langle (1,1) - 3(3,2) \rangle^{1/2}$.

$= \langle (1,1) - (9,6) \rangle^{1/2}$

$= \langle -8, -5 \rangle^{1/2}$

$= \left(\frac{1}{2}(-8)(-8) + 5(-5)(-5)\right)^{1/2}$.

$= \sqrt{32 + 125}$

$= \sqrt{157}$

Q#13

$u = (u_1, u_2)$ and $v = (v_1, v_2)$.

$\langle u, v \rangle = 3u_1v_1 + 5u_2v_2$

$w_1 = 3$

$w_2 = 5$

$$\begin{bmatrix} \sqrt{3} & 0 \\ 0 & \sqrt{5} \end{bmatrix}$$

Q#15

$p = x + x^3 \quad q = 1 + x^2$

$x_0 = -2,\ x_1 = -1,\ x_2 = 0,\ x_3 = 1$

$\langle p, q \rangle = p(-2)q(-2) + p(-1)q(-1) + p(0)q(0) + p(1)q(1)$

$= (-10)(5) + (-2)(2) + (0)(1) + (2)(2)$.

$= -50 + (-4) + 0 + 4$.

$= -50$.

Q#22

$U = \begin{bmatrix} 1 & 2 \\ -3 & 5 \end{bmatrix}, \quad V = \begin{bmatrix} 4 & 6 \\ 0 & 8 \end{bmatrix}$

$||U|| = \langle u, u \rangle^{1/2} = (tr(u^T u))^{1/2}$

$= \left(tr\left(\begin{bmatrix} 1 & -3 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -3 & 5 \end{bmatrix}\right)\right)^{1/2}$.

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$$= \sqrt{tr\begin{bmatrix} 10 & -13 \\ -13 & 29 \end{bmatrix}}.$$

$$= \sqrt{10+29}$$

$$= \sqrt{39}$$

$d(u,v) = ||u-v||$

$$u-v = \begin{bmatrix} 1 & 2 \\ -3 & 5 \end{bmatrix} - \begin{bmatrix} 4 & 6 \\ 0 & 8 \end{bmatrix}.$$

$$= \begin{bmatrix} -3 & -4 \\ -3 & -3 \end{bmatrix}$$

$$(u-v)^T = \begin{bmatrix} -3 & -3 \\ -4 & -3 \end{bmatrix}.$$

$$||u-v|| = \sqrt{tr\left((u-v)^T(u-v)\right)}$$

$$= \sqrt{tr\left(\begin{bmatrix} -3 & -3 \\ -4 & -3 \end{bmatrix}\begin{bmatrix} -3 & -4 \\ -3 & -3 \end{bmatrix}\right)}$$

$$= \sqrt{tr\begin{bmatrix} 18 & 21 \\ 21 & 25 \end{bmatrix}}$$

$$= \sqrt{18+25}$$

$$= \sqrt{43}$$

Q#26

$u = (-1, 2)$ , $v = (2, 5)$.

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$$

$$||u|| = \langle u, u \rangle^{1/2}$$

$$= \sqrt{\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} -1 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} -1 \\ 2 \end{bmatrix}}.$$

$$= \sqrt{\begin{bmatrix} 3 \\ 7 \end{bmatrix}\begin{bmatrix} 3 \\ 7 \end{bmatrix}}.$$

$$= \sqrt{9+49} = \sqrt{58}.$$

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$d(u,v) = \|u-v\|$

$u-v = \begin{bmatrix} -1 \\ 2 \end{bmatrix} - \begin{bmatrix} 2 \\ 5 \end{bmatrix}$

$= \begin{bmatrix} -3 \\ -3 \end{bmatrix}$.

$d(u,v) = \sqrt{\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} -3 \\ -3 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} -3 \\ -3 \end{bmatrix}}$

$= \sqrt{\begin{bmatrix} -9 \\ -6 \end{bmatrix} \cdot \begin{bmatrix} -9 \\ -6 \end{bmatrix}}$

$= \sqrt{81+36}$     117

$= \sqrt{117}$

$= 3\sqrt{13}$

Q#28

$\langle u,v\rangle = 2$, $\langle v,w\rangle = -6$ $\langle u,w\rangle = -3$.

$\|u\| = 1$, $\|v\| = 2$, $\|w\| = 7$

a) $\langle u-v-2w, 4u+v\rangle$

$= \langle u, 4u+v\rangle - \langle v, 4u+v\rangle - \langle 2w, 4u+v\rangle$

$= \langle u, 4u\rangle + \langle u,v\rangle - \langle v,4u\rangle - \langle v,v\rangle - \langle 2w, 4u\rangle - \langle 2w + v\rangle$

$= 4\langle u,u\rangle + \langle u,v\rangle - 4\langle v,u\rangle - \langle v,v\rangle - (2)(4)\langle w,u\rangle - 2\langle w,v\rangle$

$\langle u,u\rangle$

$= 4\langle u,u\rangle - 3\langle u,v\rangle - \langle v,v\rangle - 8\langle w,u\rangle - 2\langle w,v\rangle$

$= 4(1) - 3(2) - 4 - 8(-3) - 2(-6)$

$= 4 - 6 - 4 + 24 + 12$

$= 30$

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b) $\|2w - v\|$

$= \sqrt{\langle 2w-v, 2w-v \rangle}$

$= \sqrt{\langle 2w, 2w-v \rangle - \langle v, 2w-v \rangle}$

$= \sqrt{\langle 2w, 2w \rangle - \langle 2w, v \rangle - \langle v, 2w \rangle + \langle v, v \rangle}$

$= \sqrt{(2)(2)\langle w, w \rangle - 2\langle w, v \rangle - 2\langle v, w \rangle + \langle v, v \rangle}$

$= \sqrt{4.\langle w, w \rangle - 2\langle w, v \rangle - 2\langle v, w \rangle + \langle v, v \rangle}$

$= \sqrt{4(49) - 4(-6) + 4}$

$= \sqrt{196 + 24 + 4}$

$= \sqrt{224}$

EXERCISE # 6.2.

Q#2

a) $u = (1, -3)$, $v = (2, 4)$ (Mistakenly done part a of Q#1)

$\cos\theta = \dfrac{\langle u, v \rangle}{\|u\| \|v\|}$ — ①

$\langle u, v \rangle = (1)(2) + (-3)(4)$

$= 2 + (-12)$

$= -10.$

$\|u\| = \sqrt{(1)^2 + (-3)^2}$

$= \sqrt{1 + 9} = \sqrt{10}.$

$\|v\| = \sqrt{(2)^2 + (4)^2}$

$= \sqrt{4 + 16} = \sqrt{20}$

$\cos\theta = \dfrac{-10}{\sqrt{10}\sqrt{20}} = \dfrac{-10}{\sqrt{200}} = -\dfrac{\sqrt{2}}{2} \times \dfrac{\sqrt{2}}{\sqrt{2}}$

$= -\dfrac{2}{2\sqrt{2}} = -\dfrac{1}{\sqrt{2}}$

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b) $u = (-1, 0)$, $v = (3, 8)$

$$\cos\theta = \frac{(-1)(3) + (0)(8)}{\sqrt{(-1)^2 + (0)^2} \sqrt{(3)^2 + (8)^2}}$$

$$= \frac{-3}{\sqrt{1}\ \sqrt{73}}$$

$$= \frac{-3}{\sqrt{73}}.$$

c) $u = (2, 1, 7, -1)$, $v = (4, 0, 0, 0)$

$$\cos\theta = \frac{(2)(4) + (1)(0) + 7(0) + (-1)(0)}{\sqrt{(2)^2 + (1)^2 + (7)^2 + (-1)^2} \sqrt{(4)^2 + (0)^2 + (0)^2 + (0)^2}}$$

$$= \frac{8}{\sqrt{55}\ \sqrt{16}}$$

$$= \frac{8}{4\sqrt{55}} = \frac{2}{\sqrt{55}}$$

Q#4

$p = x - x^2$, $q = 7 + 3x + 3x^2$

$$\cos\theta = \frac{(0)(7) + (1)(3) + (-1)(3)}{\sqrt{(1)^2 + (-1)^2} \sqrt{(7)^2 + (3)^2 + (3)^2}}$$

$$= \frac{3 - 3}{\sqrt{2}\ \sqrt{67}}$$

$$= 0$$

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## Q#6

$$A = \begin{bmatrix} 2 & 4 \\ -1 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} -3 & 1 \\ 4 & 2 \end{bmatrix}$$

$$tr(A^T B) = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} -3 & 1 \\ 4 & 2 \end{bmatrix}$$

$$= tr\begin{bmatrix} -10 & 0 \\ 0 & 10 \end{bmatrix}$$

$$= -10 + 10 = 0$$

$$\cos\theta = \frac{0}{\|A\| \, \|B\|} = 0$$

## Q#8

a) $u = (u_1, u_2, u_3)$, $v = (0, 0, 0)$

$\langle u, v \rangle = (u_1)(0) + (u_2)(0) + (u_3)(0)$

$= 0$

Orthogonal.

b) $u = (-4, 6, -10, 1)$, $v = (2, 1, -2, 9)$

$\langle u, v \rangle = (-4)(2) + (6)(1) + (-10)(-2) + (1)(9)$

$= -8 + 6 + 20 + 9$

$= 27 \neq 0$

Not orthogonal

c) $u = (a, b, c)$, $v = (-c, 0, a)$

$\langle u, v \rangle = (a)(-c) + (b)(0) + (c)(a)$

$= -ac + 0 + ac$

$= 0$

Orthogonal.

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Q#12

$$u = \begin{bmatrix} 5 & -1 \\ 2 & -2 \end{bmatrix}, \quad v = \begin{bmatrix} 1 & 3 \\ -1 & 0 \end{bmatrix}$$

$$\langle u, v \rangle = (5)(1) + (-1)(3) + (2)(-1) + (-2)(0).$$
$$= 5 - 3 - 2 + 0.$$
$$= 0$$

Orthogonal

Q#18

$$u = \begin{bmatrix} 3 \\ 3 \end{bmatrix} \text{ and } v = \begin{bmatrix} 5 \\ -8 \end{bmatrix}.$$

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

$$\langle u, v \rangle = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 5 \\ -8 \end{bmatrix}.$$
$$= \begin{bmatrix} 9 \\ 6 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$
$$= 18 - 18$$
$$= 0.$$