Exercise # 3.3

Q#1

$4y'' + y' = 0$

$a = 4, \quad b = 1$

$4m^2 + m = 0$

$m(4m+1) = 0$

$m = 0 \qquad 4m+1 = 0$

$m = -1/4.$

$y = c_1 e^{m_1 x} + c_2 e^{m_2 x}$

$y = c_1 e^{0x} + c_2 e^{-1/4 x}$

$y = c_1 + c_2 e^{-x/4}$

Q#2

$y'' - 36y = 0$

$m^2 - 36 = 0$

$(m-6)(m+6) = 0$

$m = 6 \qquad m = -6.$

$y = c_1 e^{6x} + c_2 e^{-6x}$

Q#3

$y'' - y' - 6y = 0$

$m^2 - m - 6m = 0$

$m^2 - 3m + 2m - 6m = 0$

$m(m-3) + 2(m-3) = 0$

$(m-3)(m+2) = 0.$

$m = 3 \qquad m = -2.$

$y = c_1 e^{3x} + c_2 e^{-2x}$

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Q#4

$y'' - 3y' + 2y = 0$

$m^2 - 3m + 2 = 0$

$m^2 - 2m - m + 2 = 0$

$m(m-2) - 1(m-2) = 0$

$(m-2)(m-1) = 0$

$m = 2 \quad m = 1$

$y = c_1 e^{2x} + c_2 e^{1x}$

Q#5

$y'' + 8y' + 16y = 0$

$m^2 + 8m + 16 = 0$

$m^2 + 4m + 4m + 16 = 0$

$m(m+4) + 4(m+4) = 0$

$(m+4)(m+4) = 0$

$(m+4)^2 = 0$

$m + 4 = 0$

$m = -4$

$y = c_1 e^{-4x} + c_2 e^{-4x}$

Q#6

$y'' + 4y' - y = 0$

$m^2 + 4m - 1 = 0$

$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$m = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(-1)}}{2(1)}$

$m = \frac{-4 \pm \sqrt{16 + 4}}{2}$

$m = \frac{-4 + \sqrt{20}}{2}$

$m = \frac{2(-2 \pm \sqrt{5})}{2}$

$m = -2 \pm \sqrt{5}$

$y = c_1 e^{(-2-\sqrt{5})x} + c_2 e^{(-2+\sqrt{5})x}$

Q#7

$12y'' - 5y' - 2y = 0$

$12m^2 - 5m - 2 = 0$

$12m^2 - 8m + 3m - 2 = 0$

$4m(3m-2) + 1(3m-2) = 0$

$(3m-2)(4m+1) = 0$

$3m - 2 = 0 \quad 4m + 1 = 0$

$m = 2/3 \quad m = -1/4$

$y = c_1 e^{2/3x} + c_2 e^{-x/4}$

Q#16

$y'' - 10y' + 25y = 0$

$m^2 - 10m + 25 = 0$

$m^2 - 5m - 5m + 25 = 0$

$m(m-5) - 5(m-5) = 0$

$(m-5)(m-5) = 0$

$(m-5)^2 = 0$

$m - 5 = 0$

$m = 5$

$y = c_1 e^{5x} + c_2 e^{5x}$

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Q#9

$y'' + 9y = 0$

$m^2 + 9 = 0$

$m^2 = -9$

$m = \pm 3i$

$\alpha = 0, \beta = 3$

$y = e^{\alpha x}(c_1 \cos \beta x + c_1 \sin \beta x)$

$y = e^{0}(c_1 \cos 3x + c_2 \sin 3x)$

$y = c_1 \cos 3x + c_2 \sin 3x$

Q#10

$3y'' + y = 0$

$3m^2 + 1 = 0$

$3m^2 = -1$

$m^2 = -1/3$

$m = \pm \sqrt{1/3}\, i$

$m = \pm 1/\sqrt{3}\, i$

$y = c_1 \cos 1/\sqrt{3}\, x + c_2 \sin 1/\sqrt{3}\, x$

Q#11

$y'' - 4y' + 5y = 0$

$m^2 - 4m + 5 = 0$

$m = \dfrac{4 \pm \sqrt{(-4)^2 - 4(1)(5)}}{2(1)}$

$m = \dfrac{4 \pm \sqrt{16 - 20}}{2}$

$m = \dfrac{4 \pm \sqrt{-4}}{2}$

$m = \dfrac{2(2 \pm i)}{2}$

$m = 2 \pm i$

$y = e^{2x}(c_1 \cos x + c_2 \sin x)$

Teacher's Signature.

$2y'' + 2y' + y = 0$

$2m^2 + 2m + 1 = 0$

$m = \frac{-2 \pm \sqrt{(2)^2 - 4(2)(1)}}{2(2)}$

$m = \frac{-2 \pm \sqrt{4 - 8}}{4}$

$m = \frac{-2 \pm 2i}{4}$

$m = -\frac{1}{2} \pm \frac{i}{2}$

$y = e^{x/2}\left(C_1 \cos x/2 + C_2 \sin x/2\right)$

Q#13

$3y'' + 2y' + y = 0$

$3m^2 + 2m + 1 = 0$

$m = \frac{-2 \pm \sqrt{(2)^2 - 4(3)(1)}}{2(3)}$

$m = \frac{-2 \pm \sqrt{-8}}{8}$

$m = \frac{-2 \pm 2\sqrt{2}i}{8}$

$m = -\frac{1}{4} \pm \frac{\sqrt{2}\,i}{4}$

$y = e^{x/4}\left(C_1 \cos \sqrt{2}/4\, x + C_2 \sin \sqrt{2}/4\, x\right)$

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Q#14

$$2y'' - 3y' + 4y = 0$$

$$2m^2 - 3m + 4 = 0$$

$$m = \frac{3 \pm \sqrt{(-3)^2 - 4(2)(4)}}{2(2)}$$

$$m = \frac{3 \pm \sqrt{9 - 32}}{4}$$

$$m = \frac{3 \pm \sqrt{23}\, i}{4}$$

$$m_1 = \frac{3}{4} + \frac{\sqrt{23}}{4} i \qquad m_2 = \frac{3}{4} + \frac{\sqrt{23}}{4} i$$

$$\alpha = 3/4 \quad ; \quad \beta = \sqrt{23}/4$$

$$y = e^{3x/4}\left(c_1 \cos \frac{\sqrt{23}}{4} x + c_2 \sin \frac{\sqrt{23}}{4} x\right)$$

Q#15

$$y''' - 4y'' - 5y' = 0$$

$$m^3 - 4m^2 - 5m = 0$$

$$m(m^2 - 4m - 5) = 0$$

$$m(m^2 - 5m + m - 5) = 0$$

$$m(m(m-5) + 1(m-5)) = 0$$

$$m(m-5)(m+1) = 0$$

$$m_1 = 0 \;, \; m_2 = 5 \;, \; m_3 = -1$$

$$y = c_1 e^{m_1 x} + c_2 e^{m_2 x} + c_3 e^{m_3 x}$$

$$y = c_1 + c_2 e^{5x} + c_3 e^{-x}$$

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Q#16

$$y''' - y = 0$$

$$m^3 - 1 = 0$$

$$(m-1)(m^2+m+1) = 0$$

$m_1 = 1$; $m = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(1)}}{2(1)}$

$$m = \frac{-1 \pm \sqrt{3}\, i}{2}$$

$$m_2 = -\frac{1}{2} + \frac{\sqrt{3}}{2} i \,, \quad m_3 = -\frac{1}{2} - \frac{\sqrt{3}}{2} i$$

$$\alpha = -\frac{1}{2} \,, \quad \beta = \frac{\sqrt{3}}{2}$$

$$y = c_1 e^{mx} + e^{\alpha x}\left(c_1 \cos \beta x + c_2 \sin \beta x\right).$$

$$y = c_1 e^{x} + e^{-x/2}\left(c_1 \cos \frac{\sqrt{3}}{2} x + c_2 \sin \frac{\sqrt{3}}{2} x\right).$$

Q#17

$$y''' - 5y'' + 3y' + 9y = 0$$

$$m^3 - 5m^2 + 3m + 9 = 0$$

$$(m+1)(m^2 - 6m + 9) = 0$$

$m_1 = -1$ $\quad m - 3 = 0$

$m = 3.$

$m_1 = -1 \quad m_2 = m_3 = 3.$

$$y = c_1 e^{x} + c_2 e^{3x} + c_3 e^{3x}.$$

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Q#18

$y''' - 5y'' + 3y' + 9y = 0$

$m^3 - 5m^2 + 3m + 9 = 0$

$y''' + 3y'' - 4y' - 12y = 0$

$m^3 + 3m^2 - 4m - 12 = 0$

$m^3 - 4m + 3m^2 - 12 = 0$

$m(m^2 - 4) + 3(m^2 - 4) = 0$

$(m^2 - 4)(m + 3) = 0$

$(m - 2)(m + 2)(m + 3) = 0$

$m_1 = 2, \quad m_2 = -2, \quad m_3 = -3.$

$y = c_1 e^{2x} + c_2 e^{-2x} + c_3 e^{-3x}.$

Q#19

$\frac{d^3y}{dt^3} + \frac{d^2u}{dt^2} - 2u = 0$

$m^3 + m^2 - 2 = 0$

$m^2 + 2m + 2$

$m - 1 \;\big]\; m^3 + m^2 - 2$

$\pm m^3 \mp m^2$

$2m^2 - 2$

$\pm 2m^2 \mp 2m$

$2m - 2$

$\pm 2m \mp 2$

0

$(m - 1)(m^2 + 2m + 2) = 0$

$m_1 = 1 \qquad m = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(2)}}{2(1)}$

$m = \frac{-2 \pm \sqrt{-4}}{2}$

$m_2 = -1 + i \qquad m_3 = -1 - i$

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$y = c_1 e^{t} + e^{-t}(c_2 \cos t + c_3 \sin t)$

Q#20

$$\frac{d^3x}{dt^3} + \frac{d^2x}{dt^2} - 4x = 0$$

$$m^3 + m^2 - 4 = 0$$

$$m-2 \;\big)\; m^3 - m^2 - 4 \;\big(\; m^2 + m + 2$$

$$\pm m^3 \mp 2m^2$$

$$m^2 - 4$$

$$\pm m^2 \pm 2m$$

$$2m - 4$$

$$\mp 2m \pm 4$$

$$0$$

$$(m-2)(m^2+m+2) = 0$$

$m_1 = 2$

$$m = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(2)}}{2(1)}$$

$$m = \frac{-1 \pm \sqrt{7}\, i}{2}$$

$$m_2 = -\frac{1}{2} + \frac{\sqrt{7}}{2} i \;, \quad m_3 = -\frac{1}{2} - \frac{\sqrt{7}}{2} i$$

$$y = c_1 e^{2t} + e^{-t/2}\left(c_2 \cos \sqrt{7}/2\, t + c_3 \sin \sqrt{7}/2\, t\right).$$

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Q#21

$$y''' + 3y'' + 3y' + y = 0$$

$$m^3 + 3m^2 + 3m + 1 = 0$$

$$m+1 \;\big|\; m^3 + 3m^2 + 3m + 1$$

$$m^2 + 2m + 1$$

$$\pm m^3 \pm m^2$$

$$2m^2 + 3m$$

$$\pm 2m^2 \pm 2m$$

$$m + 1$$

$$\pm m \pm 1$$

$$0$$

$$(m+1)(m^2 + 2m + 1) = 0$$

$$(m+1)(m^2 + m + m + 1) = 0$$

$$(m+1)(m+1)(m+1) = 0$$

$$(m+1)^3 = 0$$

$$m + 1 = 0$$

$$m = -1$$

$$m_1 = m_2 = m_3 = -1$$

$$y = c_1 e^{-x} + c_2 x e^{-x} + c_3 x^2 e^{-x}$$

Q#22

$$y''' - 6y'' + 12y' - 8y = 0$$

$$m^3 - 6m^2 + 12m - 8 = 0$$

$$m-2 \;\big|\; m^3 - 6m^2 + 12m - 8$$

$$m^2 - 4m + 4$$

$$\pm m^3 \mp 2m^2$$

$$-4m^2 + 12m$$

$$\mp 4m^2 \pm 8m$$

$$4m - 8$$

$$\pm 4m \mp 8$$

$$0$$

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$(m-2)(m^2-4m+4)$

$(m-2)(m-2)(m-2)=0$

$(m-2)^3=0$

$m=2.$

$m_1=m_2=m_3=2.$

$y=c_1e^{2x}+c_2xe^{2x}+c_3x^2e^{2x}$

Q#23

$y^{(4)}+y'''+y''=0$

$m^4+m^3+m^2=0$

$m^2(m^2+m+1)=0$

$m^2=0$ $\quad$ $m^2+m+1=0$

$m=0$ $\quad$ $m=\frac{-1\pm\sqrt{(1)^2-4(1)(1)}}{2(1)}$

$m_1=m_2=0.$ $\quad$ $m=\frac{-1\pm\sqrt{3}i}{2}$

$m_3=\frac{-1}{2}+\frac{\sqrt{3}}{2}i$, $m_4=\frac{-1}{2}-\frac{\sqrt{3}}{2}i$

$y=c_1+c_2x+e^{-x/2}\left(c_3\cos\frac{\sqrt{3}}{2}x+c_4\sin\frac{\sqrt{3}}{2}x\right)$

Q#24

$y^{(4)}-2y''+1y=0$

$m^4-2m^2+1=0$

$m^4-m^2-m^2+1=0$

$m^2(m^2-1)-1(m^2-1)=0$

$(m^2-1)(m^2-1)=0$

$(m-1)(m+1)(m-1)(m+1)=0$

$(m-1)^2(m+1)^2=0$

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$(m-1)^2=0$ $\quad$ $(m+1)^2=0$

$m-1=0$ $\quad$ $m+1=0$

$m=1$ $\quad$ $m=-1$

$m_1=m_2=1$ $\quad$ $m_3=m_4=-1$

$$y=c_1e^x+c_2xe^x+c_3e^{-x}+c_4xe^{-x}$$

Q#25

$$16\frac{d^4y}{dx^4}+24\frac{d^2y}{dx^2}+9y=0$$

$$16m^4+24m^2+9=0$$

$$16m^4+12m^2+12m^2+9=0$$

$$4m^2(4m^2+3)+3(4m^2+3)=0$$

$$(4m^2+3)(4m^2+3)=0$$

$$(4m^2+3)=0$$

$$4m^2=-3$$

$$m^2=-3/4$$

$$m=\sqrt{3/4}\,i$$

$$m_1=m_2=m_3=m_4=\sqrt{3}/2\,i$$

$$y=c_1\cos\frac{\sqrt{3}}{2}x+c_2\sin\frac{\sqrt{3}}{2}x+c_3x\cos\frac{\sqrt{3}}{2}x+c_4x\sin\frac{\sqrt{3}}{2}x$$

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Q#29

$y'' + 16y = 0$, $y(0) = 2$, $y'(0) = -2$

$m^2 + 16 = 0$

$m^2 = -16$

$m = \pm 4i$

$m_1 = 4i$ $m_2 = -4i$

$\alpha = 0$, $\beta = 4$

$y = e^{0x}(c_1 \cos 4x + c_2 \sin 4x)$

$y = c_1 \cos 4x + c_2 \sin 4x$ — (1)

$y' = -4c_1 \sin 4x + 4c_2 \cos 4x$ — (2)

(1) $\Rightarrow$ $2 = c_1 \cos 4(0) + c_2 \sin 4(0)$

$2 = c_1(1) + c_2(0)$

$c_1 = 2$

(2) $\Rightarrow$ $-2 = -4c_1 \sin 4(0) + 4c_2 \cos 4(0)$

$-2 = (0)c_1 + 4c_2(1)$

$c_2 = -2/4 = -1/2$

$y = 2\cos 4x - \frac{1}{2}\sin 4x$.

Q#30

$\frac{d^2y}{dx^2} + y = 0$, $y(\pi/3) = 0$, $y'(\pi/3) = 2$

$m^2 + 1 = 0$

$m^2 = -1$

$m = \pm i$

$m_1 = i$, $m_2 = -i$

$\alpha = 0$, $\beta = 1$

$y = c_1 \cos x + c_2 \sin x$ — (1)

$y' = -c_1 \sin x + c_2 \cos x$ — (2)

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(1) $\Rightarrow$ ~~$x = c_1 \sin(\pi/3) + c_2 \cos(\pi/3)$~~

$0 = c_1 \cos(\pi/3) + c_2 \sin(\pi/3)$

$0 = c_1 (1/2) + c_2 (\sqrt{3}/2)$ — (3)

(2) $\Rightarrow$ $2 = -c_1 (\sqrt{3}/2) + c_2 (1/2)$

$4 = -c_1 \sqrt{3} + c_2$

$c_2 = 4 + c_1 \sqrt{3}$ — (4)

(3) $\Rightarrow$ $0 = c_1 + c_2 \sqrt{3}$.

$0 = c_1 + (4 + c_1 \sqrt{3})(\sqrt{3})$

$0 = c_1 + 4\sqrt{3} + 3c_1$

$4c_1 = -4\sqrt{3}$

$c_1 = -\sqrt{3}$

(4) $\Rightarrow$ $c_2 = 4 + (-\sqrt{3})(\sqrt{3})$

$c_2 = 4 - 3$

$c_2 = 1$

$y = -\sqrt{3} \cos\theta + \sin\theta$.

Q#31

$\frac{d^2y}{dt^2} - 4\frac{dy}{dt} - 5y = 0$, $y(1) = 0$, $y'(1) = 2$

$m^2 - 4m - 5 = 0$

$m^2 - 5m + m - 5 = 0$

$m(m-5) + 1(m-5) = 0$

$(m-5)(m+1) = 0$

$m_1 = +5$, $m_2 = -1$

$y = c_1 e^{5t} + c_2 e^{-t}$ — (1)

$y' = 5c_1 e^{5t} - c_2 e^{-t}$ — (2).

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$y(1) = 0$

(1) $\Rightarrow$ $0 = c_1 e^5 + c_2 e^{-1}$ — (3)

$y'(1) = 2$

(2) $\Rightarrow$ $2 = 5c_1 e^5 - c_2 e^{-1}$ — (4)

Add eq (3) & eq (4)

$c_1 e^5 + c_2 e^{-1} = 0$

$5c_1 e^5 - c_2 e^{-1} = 2$

$6c_1 e^5 = 2$

$c_1 e^5 = 2/6$

$c_1 e^5 = 1/3$

$c_1 = 1/3 e^{-5}$

Substitute in (1)

(1) $\Rightarrow$ $0 = 1/3 e^{-5} e^5 + c_2 e^{-1}$

$0 = c_2 e^{-1} + 1/3$

$c_2 e^{-1} = -1/3$

$c_2 = -1/3 e$

$y = 1/3 e^{5t-5} - 1/3 e^{1-t}$

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#32

$4y''' - 4y'' + -3y = 0 \;,\; y(0) = 1, \; y'(0) = 5$

$4m^2 - 4m - 3 = 0$

$4m^2 - 6m + 2m - 3 = 0$

$2m(2m-3) + 1(2m-3) = 0$

$(2m-3)(2m+1) = 0$

$m = 3/2 \;,\; m = -1/2$

$y = c_1 e^{3x/2} + c_2 e^{-x/2} \quad - (1)$

$y' = 3/2\, c_1 e^{3x/2} - 1/2\, c_2 e^{-x/2} \quad - (2)$

$y(0) = 1$

(1) $\Rightarrow$ $1 = c_1 e^{0} + c_2 e^{0}$

$1 = c_1 + c_2 \quad - (3)$

$y'(0) = 5$

(2) $\Rightarrow$ $5 = 3/2\, c_1 - 1/2\, c_2$

$10 = 3c_1 - c_2 \quad - (4)$

Add (3) & (4)

$c_1 + c_2 = 1$

$3c_1 - c_2 = 10$

$4c_1 = 11$

$c_1 = 11/4$

$11/4 + c_2 = 1$

$c_2 = -7/4$

$y = 11/4\, e^{3x/2} - 7/4\, e^{-x/2}$

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Q#33

$y'' + y' + 2y = 0 \quad , y(0) = 0 \; , y'(0) = 0$

$m^2 + m + 2 = 0$

$m = \dfrac{-1 \pm \sqrt{(1)^2 - 4(1)(2)}}{2(1)}$

$m = \dfrac{-1 \pm \sqrt{7}\, i}{2}$

$m_1 = -\frac{1}{2} + \frac{\sqrt{7}}{2} i \qquad m_2 = -\frac{1}{2} - \frac{\sqrt{7}}{2} i$

$y = e^{-x/2}\left(c_1 \cos \frac{\sqrt{7}}{2} x + c_2 \sin \frac{\sqrt{7}}{2} x\right) \quad - (1)$

$y' = -\frac{1}{2} e^{-x/2}\left(c_1 \cos \frac{\sqrt{7}}{2} x + c_2 \sin \frac{\sqrt{7}}{2} x\right) +$

$e^{-x/2}\left(\frac{\sqrt{7}}{2} c_1 \sin \frac{\sqrt{7}}{2} x + c_2 \cos \frac{\sqrt{7}}{2} x\right) \quad - (2)$

$y(0) = 0$

$(1) \Rightarrow 0 = e^0\left(c_1 \cos(0) + c_2 \sin(0)\right)$

$0 = c_1 + 0$

$c_1 = 0$

$y'(0) = 0$

$(2) \Rightarrow 0 = -\frac{1}{2} e^0\left(c_1 \cos(0) + c_2 \sin(0)\right) + e^0\left(\frac{-\sqrt{7}}{2} \sin(0) + \frac{\sqrt{2}}{2} c_2 \cos(0)\right)$

$0 = -\frac{1}{2}(0 + 0) + \left(0 + \frac{\sqrt{7}}{2} c_2\right)$

$\frac{\sqrt{7}}{2} c_2 = 0$

$c_2 = 0$

$y = 0$

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Q#34

$y'' - 2y' + y = 0$, $y(0) = 5$, $y'(0) = 10$.

$m^2 - 2m + 1 = 0$

$m^2 - m - m + 1 = 0$

$m(m-1) - 1(m-1) = 0$

$(m-1)(m-1) = 0$

$(m-1)^2 = 0$

$m - 1 = 0$

$m = 1$

$m_1 = m_2 = 1$

$y = c_1 e^x + c_2 x e^x$ — (1).

$y = c_1 e^x + c_2 (e^x + x e^x)$ — (2)

$y(0) = 5$

(1) $\Rightarrow$ $5 = c_1 e^0 + 0$

$c_1 = 5$

(2) $\Rightarrow$ $y'(0) = 10$

(2) $\Rightarrow$ $10 = c_1 e^0 + c_2 (e^0 + 0)$

$10 = 5 + c_2$

$c_2 = 10 - 5$

$c_2 = 5$

$y = 5e^x + 5xe^x$

Q#35

$y''' + 12y'' + 36y' = 0$, $y(0) = 0$, $y'(0) = 1$, $y''(0) = -7$

$m^3 + 12m^2 + 36m = 0$.

$m(m^2 + 12m + 36) = 0$.

$m_1 = 0$ $\quad m^2 + 12m + 36 = 0$

$(m+6)^2 = 0$

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$m+6=0$

$m=-6.$

$m_2=m_3=-6.$

$y=c_1e^{-6x}+c_2e^{-6x}+c_3xe^{-6x}$

$y'=-6c_1e^{-6x}+c_3(e^{-6x}-6xe^{-6x})$

$y''=36c_1e^{-6x}+c_3(-6e^{-6x}-6e^{-6x}+36xe^{-6x})$

$y(0)=0$

$0=c_1+c_2e^0+0$

$0=c_1+c_2 \quad \text{---} (1)$

$y'(0)=1$

$1=-6c_1e^0+(c_3e^0-0)$

$1=-6c_2+c_3. \Rightarrow (2)$

$y''(0)=-7$

$-7=36c_2+c_3(-12e^0+0)$

$-7=36c_2-12c_3 \quad \text{---} (3)$

$(2)\Rightarrow 12=-12c_2+12c_3$

$-7=36c_2-12c_3$

$-36c_2=5$

$c_2=-5/36$

$(1)\Rightarrow c_1-5/36=0$

$c_1=5/36$

$(2)\Rightarrow -6(-5/36)+c_3=1$

$c_3=1-5/6$

$c_3=1/6.$

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$$y = \frac{5}{36} - \frac{5}{6} e^{-6x} + \frac{1}{6} x e^{-6x}$$

Q#2

$$y''' + 2y'' - 5y' - 6y = 0, \quad y(0) = 0, \quad y'(0) = 0, \quad y''(0) = 1$$

$$m^3 + 2m^2 - 5m - 6 = 0$$

$$m+1 \;\big)\; m^3 + 2m^2 - 5m - 6 \;\big(\; m^2 + m - 6$$

$$\pm m^3 \pm m^2$$

$$m^2 - 5m$$

$$\pm m^2 \pm m$$

$$-6m + 6$$

$$\mp 6m \mp 6$$

$$0$$

$$(m+1)(m^2 + m - 6) = 0$$

$$(m+1)(m-2)(m+3) = 0$$

$$m_1 = -1, \quad m_2 = 2, \quad m_3 = -3.$$

$$y = c_1 e^{-x} + c_2 e^{2x} + c_3 e^{-3x}$$

$$y' = -c_1 e^{-x} + 2c_2 e^{2x} - 3c_3 e^{-3x}$$

$$y'' = c_1 e^{-x} + 4c_2 e^{2x} + 9c_3 e^{-3x}$$

$$y(0) = 0.$$

$$0 = c_1 e^0 + c_2 e^0 + c_3 e^0$$

$$c_1 + c_2 + c_3 = 0 \quad - (1)$$

$$y'(0) = 0.$$

$$0 = -c_1 e^0 + 2c_2 e^0 - 3c_3 e^0$$

$$-c_1 + 2c_2 - 3c_3 = 0 \quad - (2)$$

$$y''(0) = 1$$

$$1 = c_1 e^0 + 4c_2 e^0 + 9c_3 e^0$$

$$1\, c_1 + 4c_2 + 9c_3 = 1 \quad - (3)$$

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Adding (2) & (3).

$$-c_1 + 2c_2 - 3c_3 = 0$$
$$+c_1 + 4c_2 + 9c_3 = 0$$
$$6c_2 + 6c_3 = 0 \quad \text{— (4)}$$

Adding (1) & (2).

$$c_1 + c_2 + c_3 = 0$$
$$-c_1 + 2c_2 - 3c_3 = 0$$
$$3c_2 - 2c_3 = 0 \quad \text{— (5)}$$

(4) → $6c_2 + 6c_3 = 0$

(5) → $9c_2 - 6c_3 = 0$

$$15c_2 = 0$$
$$c_2 = 0$$

(5) → $3(0) - 2c_3 = 0$

$$-2c_3 = 0$$
$$c_3 = 0$$

(1) → $c_1 + c_2 + c_3 = 0$

$$c_1 = 0$$

$$y = 0 e^{x} + 0 e^{2x} + 0 e^{3x}$$

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Q#37

$y'' - 10y' + 25y = 0 \quad , y(0) = 1 \quad , y(1) = 0$

$m^2 - 10m + 25 = 0$

$m^2 - 5m - 5m + 25 = 0$

$m(m-5) - 5(m-5) = 0$

$(m-5)(m-5) = 0$

$(m-5)^2 = 0$

$m = 5$

$y = c_1 e^{5x} + c_2 e^{5x}$

$y(0) = 1$

$1 = c_1 e^0 + 0$

$c_1 = 1$

$y(1) = 0$

$0 = (1)e^5 + c_2(1)e^5$

$0 = e^5 + e^5 c_2$

$c_2 e^5 = -e^5$

$c_2 = -1$

$y = e^{5x} - x e^{5x}$

Q#38

$y'' + 4y = 0 \quad , y(0) = 0 \quad , y(\pi) = 0$

$m^2 + 4 = 0$

$m^2 = -4$

$m = \pm 2i$

$m_1 = 2i \quad m_2 = -2i$

$y = c_1 \cos 2x + c_2 \sin 2x$

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$y(0) = 0$

$0 = c_1 \cos(0) + c_2 \sin(0)$

$0 = c_1 + 0$

$c_1 = 0$

$y(\pi) = 0$

$0 = c_1 \cos\pi + c_2 \sin\pi$

$0 = c_2 0 + 0$

$c_2 = 0$

$y = 0$

Q#39

$y'' + y = 0 \quad , y'(0) = 0 \quad , y'(\pi/2) = 0$

$m^2 + 1 = 0$

$m^2 = 1$

$m = \pm i$

$m_1 = i \quad m_2 = -i$

$y = c_1 \cos x + c_2 \sin x$

$y' = -c_1 \sin x + c_2 \cos x$

$y'(0) = 0$

$0 = -c_1 \sin 0 + c_2 \cos 0$

$0 = c_2 + 0$

$c_2 = 0$

$y'(\pi/2) = 0$

$0 = -c_1 \sin \pi/2 + c_2 \cos \pi/2$

$0 = -c_1 + 0$

$c_1 = 0$

$y = 0$

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Q#40

$y'' - 2y' + 2y = 0$, $y(0) = 1$, $y(\pi) = 1$

m^2

$m^2 - 2m + 2 = 0$.

$m = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)}$

$m = \frac{2 \pm \sqrt{4 - 8}}{2}$

$m = \frac{2 \pm 2i}{2}$

$m = 1 \pm i$

$m_1 = 1 + i \quad m_2 = 1 - i$

$\alpha = 1 \quad \beta = 1$

$y = e^x (C_1 \cos x + C_2 \sin x)$

$y(0) = 1$

$1 = e^0 (C_1 \cos 0 + C_2 \sin 0)$

$1 = C_1(1) + C_2(0)$

$C_1 = 0$

$y(\pi) = 1$

$1 = e^{\pi} (C_1 \cos \pi + C_2 \sin \pi)$

$1 = e^{\pi}(-1 + 0)$

$e^{\pi} = -1$

Not possible

BVP has no solution

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