Normative theory

The theory of how we should choose
among possible actions under ideal conditions

What is the best decision?
the decision that allows us to achieve our goals

→ BUT Every decision implies a **conflict**

Normative theory

Conflicts

Between desirability and probability
Between goals
Between your own and other's goals

Decision process has to resolve such conflicts

Normative theory

How?

→ By trying to "maximize total utility"

Choose the option that will yield the greatest total utility

UTILITY

A measure of the desirability of the consequences of each option

- *how much the option allows to achieve goal* -

Normative theory

UTILITY

! Variable concept !

Whatever people want to achieve

$\neq$ pleasure
Utility $\neq$ money
$\neq$ satisfaction

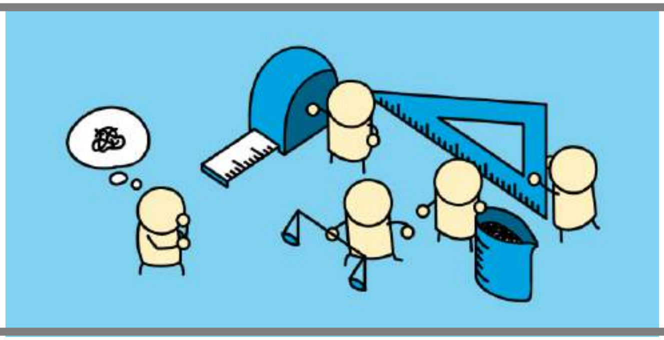
=

how consequences realize
ultimate values or goals

Normative theory

from the17th century with the development of economic theories
(to analyze the behavior of buyers and sellers)

Psychologists became interested around the 1950



Utility theory
became the normative benchmark
to evaluate decisions

Normative theory

Tradeoffs between utility and **X**

?

Normative theory

Choose the option that will yield the greatest total utility

Important: Uncertainty has to be taken into account

… we do not know the future and
must base our decisions on probabilities

Normative theory

Tradeoffs between utility and **probability**

?

Normative theory

Tradeoffs between utility and probability

?

… we frequently have to base our decisions on probabilities

UNCERTAINTY

known probabilities → decision under **risk**

unknown probabilities → decision under **ambiguity**

Normative theory

Choose the option that will yield the greatest total utility

Important: Uncertainty has to be taken into account

… we do not know the future and
must base our decisions on probabilities

→ **Expected Utility Theory**

Expected Utility Theory

For simple gambles that involve money

Expected value

$EV = p$ of outcome x v of the payoff

I pay you 4 €, if you get a 6

Expected Utility Theory

For simple gambles that involve money

Expected value

$EV = p$ of outcome x v of the payoff

I pay you 4 €, if you get a 6

p of getting a 6 = 1/6

v = 4 €

.17 * 4 € = $\boxed{.68\ €}$

Average winning per play

Expected Utility Theory

For simple gambles that involve money

Expected value

$$EV = \sum_i p_i * vi$$

I pay you 4 € if you get a 6, and 2€ if you get a 1

$$(.17*4) + (.17*2) = \boxed{1.02}$$

Expected Utility Theory

For simple gambles that involve money

Expected value

If you have to choose between two gambles,
choose the one with the highest expected value

A

4 € if you get 6
4 € if you get a 1

?

B

3 € if you get 1
3 € if you get 2
2 € if you get 3

Expected Utility Theory

How to determine the expected utility of an option?

$$UA_h = \underbrace{\sum_{i=1}^{m} p_i}_{\text{probability}} \cdot \underbrace{u_i}_{\text{utility}} = p_1 \cdot u_1 + p_2 \cdot u_2 + \ldots + p_m \cdot u_m$$

highly subjective

since it depends on individual desires and preferences

= different individuals, different expected utilities

Expected Utility Theory

STATES OF THE WORLD
(only one can be true)

Option	Traffic jam (30%)	No traffic jam (70%)
Train		
Drive		

Expected Utility Theory

STATES OF THE WORLD
(only one can be true)

Option	Traffic jam (30%)	No traffic jam (70%)
Train	On time, annoyed	On time, annoyed
Drive	Not on time, very annoyed	On time, not annoyed

Possible outcomes

Expected Utility Theory

Option	Traffic jam (30%)	No traffic jam (70%)
Train	On time, annoyed	On time, annoyed
Drive	Not on time, very annoyed	On time, not annoyed

→ Assign numerical values using an appropriate scale

[utilities]

Expected Utility Theory

Option	Traffic jam (30%)	No traffic jam (70%)
Train	On time, annoyed 60	On time, annoyed 60
Drive	Not on time, very annoyed [0]	On time, not annoyed 100

Reference point

Expected Utility Theory

Option	Traffic jam (30%)	No traffic jam (70%)
Train	On time, annoyed 60	On time, annoyed 60
Drive	Not on time, very annoyed 0	On time, not annoyed 100

$$EU_{\text{train}} = p(\text{traffic jam})*u_{\text{train}}(\text{traffic jam}) + p(\text{no traffic jam})*u_{\text{train}}(\text{no traffic jam})$$

$$EU_{\text{drive}} = p(\text{traffic jam})*u_{drive}(\text{traffic jam}) + p(\text{no traffic jam})*u_{drive}(\text{no traffic jam})$$

Expected Utility Theory

	Option	Traffic jam (30%)	No traffic jam (70%)
60	Train	On time, annoyed 60	On time, annoyed 60
70	Drive	Not on time, very annoyed 0	On time, not annoyed 100

$$EU_{\text{train}} = p(\text{traffic jam})*u_{\text{train}}(\text{traffic jam}) + p(\text{no traffic jam})*u_{\text{train}}(\text{no traffic jam})$$

$$EU_{\text{drive}} = p(\text{traffic jam})*u_{drive}(\text{traffic jam}) + p(\text{no traffic jam})*u_{drive}(\text{no traffic jam})$$

Expected Utility Theory

Exercise

Expected Utility Theory

"What test is better?"

Test 1	Healthy	Sick
Positive	.10	.40
Negative	.40	.10

Test 2	Healthy	Sick
Positive	.25	.50
Negative	.25	0

Suppose that what matters are the "costs":

treating the healthy or failing to treat the sick

- 10 - 100

Expected Utility Theory

"What test is better?"

Test 1	Healthy	Sick
Positive	.10	.40
Negative	.40	.10

Test 2	Healthy	Sick
Positive	.25	.50
Negative	.25	0

$$EU_{\mathrm{test1}} = (.10)(-10) + (.10)(-100)$$

$$EU_{\mathrm{test2}} = (.25)(-10) + (0)(-100)$$

Expected Utility Theory

"What test is better?"

Test 1	Healthy	Sick
Positive	.10	.40
Negative	.40	.10

Test 2	Healthy	Sick
Positive	.25	.50
Negative	.25	0

$$EU_{\text{test1}} = (.10)(-10) + (.10)(-100) = -11$$

$$EU_{\text{test2}} = (.25)(-10) + (0)(-100) = -2.5$$

Expected Utility Theory

The Utility of Money

Is the expected utility of a gamble simply its expected value?

You win 4 € if a coin comes up heads
on two out of two tosses

EV? 1 €

Would you pay any amount less than 1€ to play this gamble? Probably NO

If you have a ticket for this gamble, would you sell for any amount over 1€? Probably YES

Expected Utility Theory

The Utility of Money

Is the expected utility of a gamble simply its expected value?

St. Petersburg paradox
(Bernoulli, 1738)

Repeated throws. You pay an entry fee to play the game. If it comes head at the first throw, you win 1 €, 2€ if it comes on the second throw, 4€ if is comes on the third throw, etc.

How much would you pay to play this game?

No more than 3-4 €

Expected Utility Theory

The Utility of Money

Is the expected utility of a gamble simply its expected value?

St. Petersburg paradox
(Bernoulli, 1738)

Repeated throws. If it comes head at the first throw, you win 1 €, 2€ if it comes on the second throw, 4€ if is comes on the third throw, etc.

… but the expected value of this gamble is infinite

EV = ½*1 + ¼*2 + 1/8*4 + 1/16*8 + ..

Expected Utility Theory

The Utility of Money

Is the expected utility of a gamble simply its expected value?

St. Petersburg paradox
(Bernoulli, 1738)

Repeated throws. If it comes head at the first throw, you win 1 €, 2€ if it comes on the second throw, 4€ if is comes on the third throw, etc.

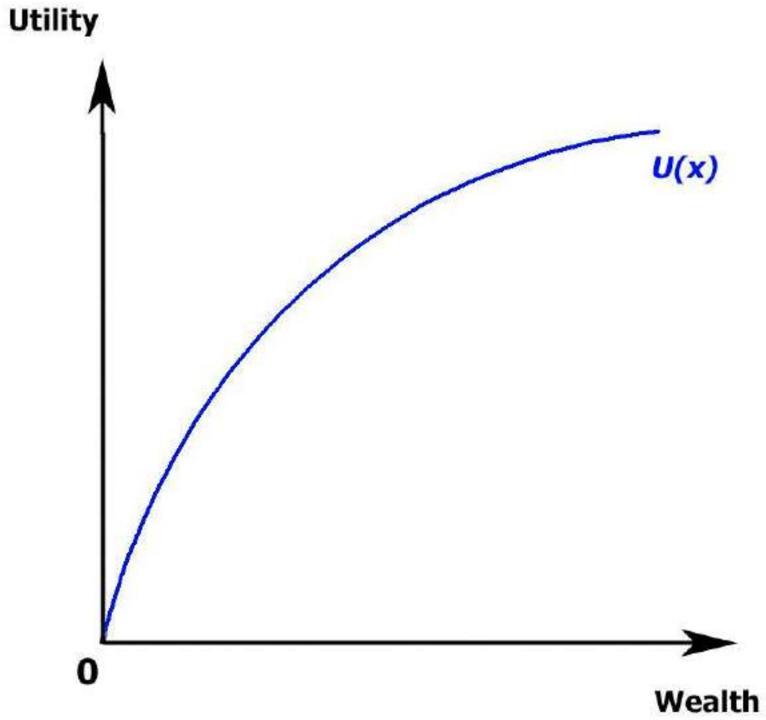
We do not evaluate gambles on their expected value

Expected Utility Theory

The Utility

is not simply its monetary value

The function is marginally declining: Its slope decreases

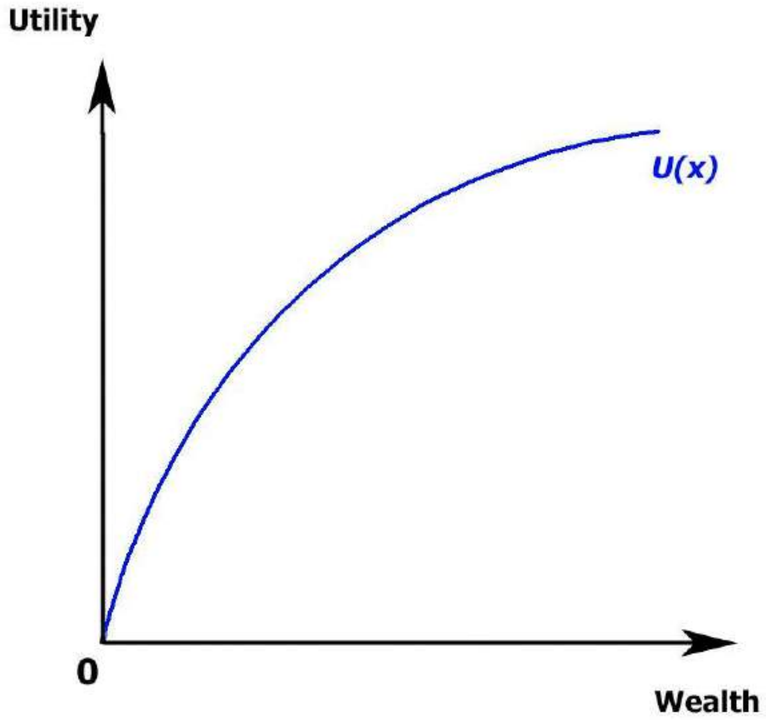


Expected Utility Theory

The Utility

is not simply its monetary value

The expected utility of a gamble is about the logarithm of its utility



$$EU = \sqrt{V}$$

Expected Utility Theory

The Utility

10 € if it comes up head

$$EV = \sum_i p_i * vi$$

$$EV = .5 * 10 = 5$$

$$EU = \sum_i p_i * \sqrt{v_i}$$

$$EU = .5 * \sqrt{10} = .5 * 3.16 = 1.58$$

Is Utility Theory normative?

- Attackers: people are generally rational, better descriptive model will lead to better normative models

- Defenders: irrationality exists, better descriptive models can only tell us where people fail

Is Utility Theory normative?

yes.

FIRST ARGUMENT

In the long run,
this decision rule is the one that helps us achieve our goals to a greater extent

Is Utility Theory normative?

If a heart is drawn
You win 4 €
$EV = (.25)(4) = 1$

?

Sufficiently
=
INFINITE

If a red card is drawn
You win 1 €
$EV = (.50)(1) = .5$

In a suffic?ently large number of plays, I will do best to choose the first gamble

Is Utility Theory normative?

yes.

SECOND ARGUMENT

Expected utility is implied by certain principles (“axioms”) that are closely related to the idea of rational decision making

Whatever helps us achieve our goals

Is Utility Theory normative?

WEAK ORDERING
Two outcomes can always be compared ($X > Y$; $Y > X$; $X = Y$)

TRANSITIVITY
Mathematical property concerning relations among elements ($X > Y$; $Y > Z$; $X > Z$)

SURE THING PRINCIPLE
If there is some state of the world that leads to the same outcome no matter what choice you make, then your choice should not depend on that outcome

Option	win	lose
Lottery1	W Europe	WE London
Lottery2	W Caribbean	WE London

Is Utility Theory normative?

WEAK ORDERING

Two outcomes can always be compared (X > Y; Y < X; X = Y)

TRANSITIVITY

Mathematical property concerning relations among elements (X > Y; Y > Z; X > Z)

SURE THING PRINCIPLE

If there is some state of the world that leads to the same outcome no matter what choice you make, then your choice should not depend on that outcome

Option	win	lose
Lottery1	W Europe	V
Lottery2	W Caribbean	V