DATE T W T F M YAO ASSIGNMENT # 02. DAY M T W T F S S

BASIL ALI KHAN
20K-0477

Question # 01.

$n = 13$.
$P = 0.7$
$q = 1 - P = 1 - 0.7 = 0.3$.

a) $P(X = 5)$
$= {}^{13}C_5 (0.7)^5 (0.3)^8$.
$= 0.01419$

b) $P(X \leq 7)$
$= \sum_{x=0}^{7} \left({}^{13}C_x (0.7)^x (0.3)^{13-x} \right)$
$= 0.1654$.

c) Mean and Standard deviation.
mean $= np = (13)(0.7) = 9.1$

S.D $= \sqrt{npq} = \sqrt{(13)(0.7)(0.3)} = 1.652$.

Question # 02

$\lambda = 6$

a) $P(X = 6)$

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$$= \frac{e^{-\lambda} \lambda^x}{x!}$$

$$= \frac{e^{-6}\, 6^6}{6!} = 0.1606$$

b) $P(X < 5)$

$$= \sum_{x=0}^{4} \left(\frac{e^{-6} \times 6^x}{x!} \right)$$

$$= 0.2850$$

c) $P(X \geq 4)$

$$= 1 - P(X < 4)$$

$$= 1 - \sum_{x=0}^{3} \left(\frac{e^{-6} \times 6^x}{x!} \right)$$

$$= 1 - 0.1512$$

$$= 0.8488$$

Question # 03

$X \Rightarrow$ No of defective MC

(a)

$X = 0, 1, 2$ (Possibilities of defective MC).

(b)

x	0	1	2
$P(X=x)$	0.4167	0.5	0.0833

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$$P(X=0) = \frac{{}^{3}C_0 * {}^{6}C_2}{{}^{9}C_2}$$

$$= 0.4167$$

$$P(X=1) = \frac{{}^{3}C_1 \times {}^{6}C_1}{{}^{9}C_2}$$

$$= 0.5$$

$$P(X=2) = \frac{{}^{3}C_2 \times {}^{6}C_0}{{}^{9}C_2}$$

$$= 0.0833$$

c) $P(1 \leq x \leq 2)$

$= 0.5 + 0.0833$

$= 0.5833$

$P(x \leq 1)$

$= 0.4167 + 0.5$

$= 0.9167$

$P(0 < x \leq 2)$

$= 0.5 + 0.0833$

$= 0.5833$

$f(2)$

$= 0.0833$

$f(5)$

$= 0$

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Question # 04.

x	0	1	2
$f(x)$	$10/28$	$15/28$	$3/28$
$xf(x)$	0	$15/28$	$6/28$
$x^2 f(x)$	0	$15/28$	$12/28$

$$E(X^2) = \Sigma x^2 f(x)$$
$$= 15/28 + 12/28$$
$$= 27/28.$$

$$E(X) = \Sigma x f(x)$$
$$= 15/28 + 6/28$$
$$= 21/28.$$

a) $Var(x) = E(X^2) - [E(X)]^2$
$= 27/28 - (21/28)^2$
$= 0.4017$

b) $F(1)$
$= 10/28 + 15/28$
$= 25/28$

c) $P(1 \leq x \leq 2)$
$= 15/28 + 3/28$
$= 18/28$

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Question # 05

(a)

$P = 0.16$

$q = 1 - p = 1 - 0.16 = 0.84$

$$P(X \leq 1) = \sum_{x=0}^{1} \left(\binom{15}{x} (0.16)^x (0.84)^{15-x} \right)$$

$$= 0.2821$$

(b)

x	0	1	2	3
$P(X=x)$	$1/35$	$12/35$	$18/35$	$4/35$

$X \Rightarrow$ number of girls in team

$$P(X=0) = \frac{{}^3C_3 \, {}^4C_0}{{}^7C_3} = \boxed{1/35}$$

$$P(X=1) = \frac{{}^3C_2 \, {}^4C_1}{{}^7C_3} = \boxed{12/35}$$

$$P(X=2) = \frac{{}^3C_1 \, {}^4C_2}{{}^7C_3} = \boxed{18/35}$$

$$P(X=3) = \frac{{}^3C_0 \, {}^4C_3}{{}^7C_3} = \boxed{4/35}$$

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Mean $\Rightarrow$ $E(X) = \sum_{x=0}^{3} x f(x)$

$= (0)\left(\frac{1}{35}\right) + (1)\left(\frac{12}{35}\right) + (2)\left(\frac{18}{35}\right) + (3)\left(\frac{4}{35}\right)$

$= 12/7$

$\boxed{= 1.7143}$

$E(X^2) = \sum_{x=0}^{3} x^2 f(x)$

$= (0)^2\left(\frac{1}{35}\right) + (1)^2\left(\frac{12}{35}\right) + (2)^2\left(\frac{18}{35}\right) + (3)^2\left(\frac{4}{35}\right)$

$= 120/35 = 24/7$

$\boxed{= 3.428}$

Variance $= E(X^2) - [E(X)]^2$

$= 24/7 - \left(12/7\right)^2$

$= 24/49$

$\boxed{= 0.4898}$

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Question #06

(Discarded)

Question #07.

$$Cov(x,y) = E(XY) - E(X)E(Y) \quad \text{— (A)}$$

$$g(x) = \frac{2}{7}\int_0^2 x + 2y \, dy$$

$$= \frac{2}{7}\left(xy + y^2 \Big|_0^2 \right)$$

$$= \frac{2}{7}\left[2x + 4 \right]$$

$$h(y) = \frac{2}{7}\int_0^1 x + 2y \, dx$$

$$= \frac{2}{7}\left(\frac{x^2}{2} + 2xy \Big|_0^1 \right)$$

$$= \frac{2}{7}\left(\frac{1}{2} + 2y \right)$$

$$E(X) = \int_0^1 x\, g(x)\, dx$$

$$= \frac{2}{7}\int_0^1 x(2x+4)\, dx$$

$$= \frac{16}{21}$$

$$E(Y) = \int_0^2 y\, h(y)\, dy$$

$$= \frac{2}{7}\int_0^2 y\left(\frac{1}{2} + 2y\right) dy$$

$$= \frac{38}{21}$$

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$$E(XY) = \frac{2}{7}\int_0^2\int_0^1 x^2y + 2xy^2\,dx\,dy.$$

$$= \frac{2}{7}\int_0^2 \frac{x^3}{3}y + x^2y^2\Big|_0^1 dy.$$

$$= \frac{2}{7}\int_0^2 \frac{8}{3}y + 4y^2\,dy$$

$$= \frac{32}{7}$$

Put in eq (1)

$$= \frac{32}{7} - \left(\frac{16}{21}\right)\left(\frac{38}{21}\right)$$

$$= \frac{1408}{441} = 3.1927$$

$$\sigma_{xy} = \sqrt{E(X^2) - [E(X)]^2}$$

$$E(X^2) = \frac{2}{7}\int_0^1 x^2(x+2y)\,dx$$

$$= \frac{2}{7}\int_0^1 x^3 + 2yx^2\,dx$$

$$= \frac{11}{21}.$$

$$\sigma_x = \sqrt{\frac{11}{21} - \left(\frac{16}{21}\right)^2}$$

$$= \text{root}(-).$$

S.D of x not possible.

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(b)

$$\text{Correlation} = \frac{\sigma_{xy}}{\sigma_x \sigma_y}$$

~~$E(XY) = \sum x y f(x)$~~

$$= \text{[illegible]}$$

x \ y	0	1	2	
0	0.12	0.04	0.04	1/5
1	0.08	0.19	0.05	8/25
2	0.06	0.12	0.30	12/25
	13/50	7/20	39/100	

$$E(X) = \sum x f(x)$$
$$= 0\left(\tfrac{1}{5}\right) + 1\left(\tfrac{8}{25}\right) + 2\left(\tfrac{12}{25}\right)$$
$$= \tfrac{32}{25}$$

$$E(Y) = \sum y f(y)$$
$$= 0\left(\tfrac{13}{50}\right) + 1\left(\tfrac{7}{20}\right) + 2\left(\tfrac{39}{100}\right)$$
$$= \frac{113}{100}$$

$$E(XY) = \sum xy f(x_i y_i)$$
$$= (0)(0)(0.12) + (0)(1)(0.04) + (0)(2)(0.04)$$
$$(1)(0)(0.08) + (1)(1)(0.19) + (1)(2)(0.05)$$
$$(2)(0)(0.06) + (2)(1)(0.12) + (2)(2)(0.30)$$
$$= \tfrac{173}{100}$$

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$$Cov(x,y) = \frac{173}{100} - \left(\frac{32}{25}\right)\left(\frac{113}{100}\right)$$

$$= \frac{709}{2500}$$

$$\sigma_x = \sqrt{E(x^2) - [E(x)]^2} \quad - (A)$$

$$E(x^2) = \sum x^2 f(x)$$

$$= (0)^2\left(\frac{1}{5}\right) + (1)^2\left(\frac{8}{25}\right) + (2)^2\left(\frac{12}{25}\right)$$

$$= 56/25$$

Put in (A)

$$\sigma_x = \sqrt{\frac{56}{25} - \left(\frac{32}{25}\right)^2}$$

$$= 0.7756$$

$$\sigma_y = \sqrt{E(y^2) - [E(y)]^2} \quad - (B)$$

$$E(y^2) = \sum y^2 f(y)$$

$$= (0)^2\left(\frac{13}{50}\right) + (1)^2\left(\frac{7}{20}\right) + (2)^2\left(\frac{39}{100}\right)$$

$$= 1.91$$

Put in (B)

$$\sigma_y = \sqrt{1.91 - 1.132}$$

$$= 0.7957$$

$$\sigma_{xy} = \frac{0.2536}{0.7957 \times 0.7756} \quad = 0.4952$$

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Question #08

$P = 8/200 = 0.04$

$q = 1 - P = 0.96$

$n = 8$

a) $P(X=0)$

$= \binom{8}{0} (0.04)^0 \times (0.96^8)$

$= 0.7214$

b) $P(X \geq 03)$

$= 1 - P(X \leq 2)$

$= 1 - \sum_{x=0}^{2} \binom{8}{x} (0.04)^x (0.96)^{8-x}$

$= 1 - 0.9969$

$= 0.00307$

c) Mean

$= np = 100 \times 0.04$

$= 4$ people

Question #09.

$\lambda = 5$

$= \frac{e^{-\lambda} \times \lambda^x}{x!}$

a) $P(X=0)$

$= \frac{e^{-5} \times 5^0}{0!}$

$= 0.00673$

b) $P(X=4)$

$= \frac{e^{-5} \times (5)^4}{4!}$

$= 0.17547$

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c) $P(X \leq 6) = \sum_{x=0}^{6} \frac{e^{-5} \times 5^x}{x!}$

$= 0.76218$

(b)

$\lambda = 5$

$t = 12.$

$P(X \leq 3) = \sum_{x=0}^{2} \frac{e^{-5(12)} \times 5(12)^x}{x!}$

$= 1.6296 \times 10^{-23}$

Question # 10.

(a)

$n = 100$

$\mu = 174.5$

$\sigma = 6.9$

a) $P(X < 159.75) = P\left(Z < \frac{159.75 - 174.5}{6.9}\right)$

$= P(Z < -2.1377).$

$= 0.0163.$

$= 16$ students.

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b) $P(171.7 < X < 182.25)$

$= P\left(\frac{171.25 - 174.5}{6.9} < \frac{X - 1745}{6.9} < \frac{18225 - 174.5}{6.9}\right)$

$= P(-0.4710 < Z < 1.1232)$

$= 0.8693 - 0.3188$

$= 0.5505$

$= 551$ students.

c) $P(174.75 < X < 175.25)$

$= P\left(\frac{174.75 - 174.5}{6.9} < \frac{X - 1745}{6.9} < \frac{175.25 - 1745}{6.9}\right)$

$= P[0.0362 < Z < 0.1087]$

$= 0.5433 - 0.5144$

$= 0.0289$

$= 29$ students

d) $P(X > 187.75$

$= P\left(\frac{X - 174.5}{6.9} > \frac{187.75 - 174.5}{6.9}\right)$

$= P(Z > 1.9203)$

$= \Phi(-1.9203)$

$= 0.0274$

$= 27$ Students.

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(6)

$n = 5000$
$\mu = 1250$
$\sigma = 250$

a) $P(X > 1500)$.

$z = \frac{1500 - 1250}{250} = 1$

$= 1 - \phi(1)$
$= 1 - 0.8413$
$= 0.1584$ $= 15.87$

b) $P(X < 750)$

$z = \frac{750 - 1250}{250} = -2$

$= P(z < -2)$
$= 0.0228$
$= 2.28$

c) $P(250 < X < 1500)$
$= P(X < 1500) - P(X \leq 250)$
$= 0.8413 - 0.028$
$= 0.8185$