MOHAMMAD BASIL ALI KHAN
20K-0477.

EXERCISE #1.2.

Date

Q#1

$y(0) = -\frac{1}{3}$

$y = \dfrac{1}{(1+c_1e^{-x})}$ — ①

$\dfrac{-1}{3} = \dfrac{1}{1+c_1e^{0}}$

$\dfrac{-1}{3} = \dfrac{1}{1+c_1}$

$-(1+c_1) = 3$

$-1-c_1 = 3$

$c_1 = -1-3$

$c_1 = -4$

① ⇒ $y = \dfrac{1}{(1-4e^{-x})}$

Q#2

$y(-1) = 2$

$y = \dfrac{1}{(1+c_1e^{-x})}$ — ①

$2 = \dfrac{1}{1+c_1e^{1}}$

$1+c_1e = \dfrac{1}{2}$

$c_1e = -\dfrac{1}{2}$

$c_1 = -\frac{1}{2e}$

① ⇒ $y = \dfrac{1}{1-\frac{1}{2}e^{-x}}$

$y = \dfrac{1}{2-e^{-x-1}}$

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Q#3

$y(2) = 1/3$

$y = \frac{1}{x^2+c}$ — ①

$\frac{1}{3} = \frac{1}{2^2+c}$

$4+c = 3$

$c = -1$

① ⇒ $y = \frac{1}{x^2-1}$

the largest interval ⇒ $x \in (1, \infty)$

Q#4

$y(-2) = 1/2$

$y = \frac{1}{x^2+c}$ — ①

$\frac{1}{2} = \frac{1}{(-2)^2+c}$

$\frac{1}{2} = \frac{1}{4+c}$

$4+c = 2$

$c = -2$

① ⇒ $y = \frac{1}{x^2-2}$

This is not defined at $x = \sqrt{2}$ & $x = -\sqrt{2}$.
From initial condition we conclude it is defined at $x = -2$, therefore Interval at

$x \in (-\infty, -\sqrt{2})$

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Q#5

$y(0) = 1$

$y = \frac{1}{x^2+c}$ — ①

$1 = \frac{1}{0^2+c}$

$1 = \frac{1}{c}$

$c = 1$

① ⇒ $y = \frac{1}{x^2+1}$

$x \in (-\infty, \infty)$

Q#6

$y(1/2) = -4$

$y = 1$

$y' = \frac{1}{x^2+c}$

$-4 = \frac{1}{(1/2)^2+c}$

$-4 = \frac{1}{1/4+c}$

$\frac{1}{4}+c = -1/4$

$4+16c = -4$

$c = -8/16$

$c = -1/2$.

$y = \frac{1}{x^2-1/2}$.

The largest interval ⇒ $x \in (-1/\sqrt{2}, 1/\sqrt{2})$

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Q#7

$x(0) = -1 \quad , x'(0) = 8$

$x = c_1 \cos t + c_2 \sin t$

$x' = -c_1 \sin t + c_2 \cos t$

$x'(0) = 8$

$8 = -c_1 \sin 0 + c_2 \cos 0$

$8 = c_2$

$c_2 = 8$

$x(0) = -1$

$x = c_1 \cos t + c_2 \sin t$

$-1 = c_1 \cos 0 + c_2 \sin 0$

$-1 = c_1$

$c_1 = -1$

So solution of IVP is :

$x = -\cos t + 8 \sin t$

Q#8

$x(\pi/2) = 0 \, , \, x'(\pi/2) = 1$

$x = c_1 \cos t + c_2 \sin t$

$x' = -c_1 \sin t + c_2 \cos t$

$x'(\pi/2) = 1$

$1 = -c_1 \sin(\pi/2) + c_2 \cos(\pi/2)$

$1 = -c_1 (1) + 0$

$c_1 = -1$

$x(\pi/2) = 0$

$0 = c_1 \cos(\pi/2) + c_2 \sin(\pi/2)$

$0 = 0 + c_2(1)$

$c_2 = 0$

So solution of IVP is :

$x = -\cos t$

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EXERCISE # 1.2

Q# 9

$x(\pi/6) = 1/2, \quad x'(\pi/6) = 0$

$x = c_1 \cos t + c_2 \sin t$

$x' = -c_1 \sin t + c_2 \cos t$

$x'(\pi/6) = 0$

$0 = -c_1 \sin(\pi/6) + c_2 \cos(\pi/6)$

$-\frac{1}{2} c_1 + c_2 \frac{\sqrt{3}}{2} = 0$

$-c_1 + c_2\sqrt{3} = 0 \quad — (1)$

$x = c_1 \cos t + c_2 \sin t$

$1/2 = c_1 \cos(\pi/6) + c_2 \sin(\pi/6)$

$1/2 = c_1 \frac{\sqrt{3}}{2} + c_2 \frac{1}{2}$

$c_1\sqrt{3} + c_2 = 1 \quad — (2)$

(x) eq (1) by $\sqrt{3}$

$(1) \Rightarrow \sqrt{3}(-c_1 + c_2\sqrt{3} = 0)$

$(2) \Rightarrow -\sqrt{3}c_1 + 3c_2 = 0$

$\underline{\sqrt{3}c_1 + c_2 = 1}$

$4c_2 = 1$

$c_2 = 1/4$

$(1) \Rightarrow -c_1 + 1/4(\sqrt{3}) = 0$

$c_1 = \sqrt{3}/4$

So solution of IVP is

$x = \sqrt{3}/4 \sin t + 1/4 \cos t$

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Q#10

$x(\pi/4) = \sqrt{2}$; $x'(\pi/4) = 2\sqrt{2}$.

$x = c_1 \cos t + c_1 \sin t$

$\sqrt{2} = c_1 \cos(\pi/4) + c_2 \sin(\pi/4)$

$\sqrt{2} = c_1 \sqrt{2}/2 + c_2 (\sqrt{2}/2)$

$c_1 + c_2 = 2$. — ①

$x' = -c_1 \sin t + c_2 \cos t$

$2\sqrt{2} = -c_1 \sin(\pi/4) + c_2 \cos(\pi/4)$

$2\sqrt{2} = -c_1(\sqrt{2}/2) + c_2(\sqrt{2}/2)$

$2 = -\frac{1}{2} c_1 + \frac{1}{2} c_2$.

$-c_1 + c_2 = 4$. — ②

① ⇒ $\cancel{c_1} + c_2 = 2$

② ⇒ $\cancel{-c_1} + c_2 = 4$

$2c_2 = 6$

$c_2 = 6/2$

$c_2 = 3$

② ⇒ $-c_1 + 3 = 4$

$c_1 = -1$

so solution of IVP is.

$x = -\cos t + 3\sin t$.

Q#11

$y(0) = 1$; $y'(0) = 2$

$y = c_1 e^{x} + c_2 e^{-x}$

$1 = c_1 e^{0} + c_2 e^{-0}$

$c_1 + c_2 = 1$ — ①

$y' = c_1 e^{x} - c_2 e^{-x}$.

$2 = c_1 e^{0} - c_2 e^{0}$

$c_1 - c_2 = 2$ — ②

① ⇒ $c_1 + \cancel{c_2} = 1$

$c_1 - \cancel{c_2} = 2$

$2c_1 = 3$

$c_1 = 3/2$.

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① => $c\,^{3}/_{2} + c_2 = 1$

$c_2 = 1 - 3/2$

$c_2 = -1/2$

so solution of IVP is .

$y = \frac{3}{2} e^{x} - \frac{1}{2} e^{-x}$.

Q#12

$y(1) = 0 \quad ; \quad y'(1) = e$

$y = c_1 e^{x} + c_2 e^{-x}$

$0 = c_1 e^{1} + c_2 e^{-1}$

$0 = c_1 e + c_2 (1/e)$ — ①

$y' = c_1 e^{x} - c_2 e^{-x}$

$e = c_1 e^{1} - c_2 e^{-1}$

$e = c_1 e - c_2 (1/e)$

$e = c_1 e - c_2/e$ — ②

① => $c_1 e + \cancel{c_2 (1/e)} = 0$

② => $c_1 e - \cancel{c_2 (1/e)} = e$

$2c_1 e = e$

$c_1 = 1/2$

① => $c_1 e + c_2 (1/e) = 0$

$\frac{1}{2} e + c_2/e = 0$

$c_2 (1/e) = -\frac{1}{2} e$.

$c_2 = -\frac{1}{2} e^{2}$

so solution of IVP is .

$y = \frac{1}{2} e^{x} - \frac{1}{2} e^{2} e^{-x}$

$y = \frac{1}{2} e^{x} - \frac{1}{2} e^{2-x}$

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Q#13

$y(-1) = 5 \; ; \; y'(-1) = -5$

$y = c_1 e^{x} + c_2 e^{-x}$

$5 = c_1 e^{-1} + c_2 e^{1}$

$c_1 (1/e) + c_2 e = 5 \quad — ①$

$y' = c_1 e^{x} - c_2 e^{-x}$

$-5 = c_1 e^{-1} - c_2 e^{1}$

$-5 = c_1 (1/e) - c_2 e \quad — ②$

$$\begin{array}{r} c_1 (1/e) + \cancel{c_2 e} = 5 \\ c_1 (1/e) - \cancel{c_2 e} = -5 \\ \hline c_1 \qquad = 0 \end{array}$$

$① \Rightarrow \quad 0(1/e) + c_2 e = 5$

$c_2 e = 5$

$c_2 = 5/e$

so solution of IVP is.

$y = 5/e \, e^{-x}$

$y = 5e^{-x-1}$

Q#14

$y(0) = 0 \; ; \; y'(0) = 0$

$y = c_1 e^{x} + c_2 e^{-x}$

$0 = c_1 e^{0} + c_2 e^{0}$

$0 = c_1 + c_2 \quad — ①$

$y' = c_1 e^{x} - c_2 e^{-x}$

$0 = c_1 e^{0} - c_2 e^{0}$

$0 = c_1 - c_2 \quad — ②$

$$\begin{array}{r} c_1 + \cancel{c_2} = 0 \\ c_1 - \cancel{c_2} = 0 \\ \hline 2c_1 = 0 \end{array}$$

$c_1 = 0$

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(1) $\Rightarrow$ $0 + c_2 = 0$

$c_2 = 0.$

So solution of IVP is

$y = 0$

Q#31

(a)

$$y = -\frac{1}{x+c}$$

$y' = y^2.$

L.H.S.

$$y = -\frac{1}{x+c}$$

$$y' = \frac{d}{dx}\left[-\frac{1}{x+c}\right]$$

$$y' = \frac{1}{(x+c)^2}.$$

R.H.S

$$y = -\frac{1}{x+c}$$

$$y^2 = \frac{1}{(x+c)^2}.$$

Two sides are equal so $y = -\frac{1}{x+c}$ is one parameter family of solution to DE.

(b)

$y(0) = 1$

$$y = -\frac{1}{x+c}.$$

$$1 = -\frac{1}{c}$$

$c = -1$

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$$y = -\frac{1}{x-1}$$

$$y = \frac{1}{1-x}.$$

It is discontinuous at $x=1$
So largest interval of for this IVP is $(-\infty, 1)$.

$y(0) = -1$
$-1 = -1/c$

$c = 1$

$$y = -\frac{1}{x+1}$$

it is discontinuous at $x=-1$
So largest interval for this IVP is $(-1, \infty)$

Q#32

(a)

$y(0) = y_0$

$$y = -\frac{1}{x+c}$$

$y_0 = -1/c$.

$c = -1/y_0$.

$$y = -\frac{1}{1/y_0 - x}.$$

It is discontinuous at $x = 1/y_0$.
So largest interval for this IVP is $(-\infty, 1/y_0)$
when $y_0 > 0$

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or

$(1/y_0 ; \infty]$

when $y < 0$.

(b)

$y(0) = 0$

$0 = -1/c$

$c = 0$.

$y = 0$

interval $\Rightarrow (-\infty, \infty)$.

Q#33

(a)

$3x^2 - y^2 = c \quad ; \quad y\,dy/dx = 3x \quad -(1)$

$\cancel{3x + y\,dy/dx = 0}$

$\cancel{3x = y\,dy/dx}$

$6x - 2y\,dy/dx = 0$

$dy/dx = -6x/-2y$

$dy/dx = 3x/y$.

$(1) \Rightarrow \cancel{y}(3x/\cancel{y}) = 3x$

$3x = 3x$.

Proved.

$3x^2 - y^2 = c$ is one parameter family of solution to the DE.

(b)

$$3x^2 - y^2 = 3.$$
$$y^2 = 3x^2 - 3.$$
$$y^2 = 3(x^2 - 1)$$
$$y = \pm\sqrt{3(x^2-1)}$$

discontinuous at $x = \pm 1$

$(-\infty, -1) \cup (1, \infty)$