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20K-0474

ASSIGNMENT #02

EXERCISE # 4.1

Q#2

$(u+v) = (u_1+v_1+1, u_2+v_2+1)$, $ku = (ku_1, ku_2)$

(a)

$u = (0,4)$ $\quad K=2$
$v = (1,-3)$

$u+v = (u_1+v_1+1, u_2+v_2+1)$
$= (0+1+1, 4+(-3)+1)$
$= (2, 2)$

$Ku = (Ku_1, Ku_2)$
$= (2(0), 2(4))$
$= (0, 8)$

(b)

Prove: $(0,0) \neq 0$

$(u_1,u_2) + (0,0) = (u_1, u_2)$

L.H.S

$(u_1, u_2) + (0,0) = (u_1+0+1, u_2+0+1)$
$= (u_1+1, u_2+1)$

$(u_1+1, u_2+1) \neq (u_1, u_2)$

so

$(0,0) \neq 0.$

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(c)

Proove: $(-1,-1) = 0$

$(u_1, u_2) + (-1,-1) = (u_1, u_2)$

L.H.S:

$(u_1, u_2) + (-1,-1) = (u_1 + (-1) + 1, u_2 + (-1) + 1)$

$= (u_1, u_2)$

$(u_1, u_2) + (-1,-1) = (u_1, u_2)$

So

$(-1,-1) = 0$

(d)

$u + (-u) = 0$

$(u_1, u_2) + (-u_1, -u_2) = 0$

L.H.S

$= (u_1, u_2) + (-u_1, -u_2)$

$= (u_1 - u_1 + 1, u_2 - u_2 + 1)$

$= (1, 1) \neq 0$

(e)

Axiom # 7:

$c(u+v) = c(u) + c(v)$

$= c(u_1 + v_1 + 1, u_2 + v_2 + 1)$

$= (cu_1 + cv_1 + c, cu_2 + cv_2 + c)$

R.H.S:

$= c(u_1, u_2) + c(v_1, v_2)$

$= (cu_1, cu_2) + (cv_1, cv_2)$

$= (cu_1 + cv_1 + 1, cu_2 + cv_2 + 1)$

L.H.S $\neq$ R.H.S.

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Axiom # 08

$(c+d)u = c(u) + d(u)$

L.H.S:

$= ((c+d)u_1 , (c+d)u_2)$

$= (cu_1 + du_1 , cu_2 + du_2)$

R.H.S:

$= c(u_1, u_2) + d(u_1, u_2)$

$= (cu_1 , cu_2) + (du_1 , du_2)$

$= (cu_1 + du_1 + 1 , cu_2 + du_2 + 1)$

L.H.S $\neq$ R.H.S.

Q#9

$u = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}, V = \begin{bmatrix} c & 0 \\ 0 & d \end{bmatrix}, w = \begin{bmatrix} e & 0 \\ 0 & f \end{bmatrix}$

Axiom#01:

$u + V$

$= \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} + \begin{bmatrix} c & 0 \\ 0 & d \end{bmatrix}$

$= \begin{bmatrix} a+c & 0 \\ 0 & b+d \end{bmatrix}$

Prove (resultant also diagnol)

Axiom # 02

$u+V = V+u.$

L.H.S: R.H.S

$= \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} + \begin{bmatrix} c & 0 \\ 0 & d \end{bmatrix}$ $= \begin{bmatrix} c & 0 \\ 0 & d \end{bmatrix} + \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$

$= \begin{pmatrix} a+c & 0 \\ 0 & b+d \end{pmatrix}$ $= \begin{pmatrix} a+c & 0 \\ 0 & b+d \end{pmatrix}$

L.H.S = R.H.S

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Axiom #03:

$u + (v+w) = (u+v) + w$

L.H.S:

$= \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} + \left(\begin{bmatrix} c & 0 \\ 0 & d \end{bmatrix} + \begin{bmatrix} e & 0 \\ 0 & f \end{bmatrix} \right)$

$= \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} + \begin{bmatrix} c+e & 0 \\ 0 & d+f \end{bmatrix}$

$= \begin{bmatrix} a+c+e & 0 \\ 0 & b+d+f \end{bmatrix}$

R.H.S:

$= \left(\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} + \begin{bmatrix} c & 0 \\ 0 & d \end{bmatrix} \right) + \begin{bmatrix} e & 0 \\ 0 & f \end{bmatrix}$

$= \begin{bmatrix} a+c & 0 \\ 0 & b+d \end{bmatrix} + \begin{bmatrix} e & 0 \\ 0 & f \end{bmatrix}$

$= \begin{bmatrix} a+c+e & 0 \\ 0 & b+d+f \end{bmatrix}$

L.H.S = R.H.S

Proved

Axiom # 04

$u + 0 = u$

L.H.S

$= \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$= \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$

Prove.

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Axiom #05

$u + (-u) = 0$

L.H.S:

$$= \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} + \begin{bmatrix} -a & 0 \\ 0 & -b \end{bmatrix}$$

$$= \begin{bmatrix} a-a & 0 \\ 0 & b-b \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Proved.

Axiom #06:

cu is in V

$$= c \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}.$$

$$= \begin{bmatrix} ca & 0 \\ 0 & cb \end{bmatrix}$$

Proved (Resutant also diagnol)

Axiom #07

$c(u+v) = c(u) + c(v)$

L.H.S

$$= c\left(\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} + \begin{bmatrix} c & 0 \\ 0 & d \end{bmatrix}\right)$$

$$= c \begin{bmatrix} a+c & 0 \\ 0 & b+d \end{bmatrix}$$

$$= \begin{bmatrix} c(a+c) & 0 \\ 0 & c(b+d) \end{bmatrix}$$

$$= \begin{bmatrix} ca+cc & 0 \\ 0 & cb+cd \end{bmatrix}.$$

R.H.S

$$= c\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} + c\begin{bmatrix} c & 0 \\ 0 & d \end{bmatrix}$$

$$= \begin{bmatrix} ca & 0 \\ 0 & cb \end{bmatrix} + \begin{bmatrix} cc & 0 \\ 0 & cd \end{bmatrix}$$

$$= \begin{bmatrix} ca+cc & 0 \\ 0 & cb+cd \end{bmatrix}$$

L.H.S = R.H.S

Proved.

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Axiom # 08:

$$(c+d)u = c(u) + d(u)$$

$$=(c+d)\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

$$= \begin{bmatrix} ca & 0 \\ 0 & cb \end{bmatrix} + \begin{bmatrix} da & 0 \\ 0 & db \end{bmatrix}$$

$$= \begin{bmatrix} ca+da & 0 \\ 0 & cb+db \end{bmatrix}$$

R.H.S

$$= c\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} + d\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

$$= \begin{bmatrix} ca & 0 \\ 0 & cb \end{bmatrix} + \begin{bmatrix} da & 0 \\ 0 & db \end{bmatrix}$$

$$= \begin{bmatrix} ca+da & 0 \\ 0 & cb+db \end{bmatrix}$$

L.H.S = R.H.S
Proved

Axiom # 09.

$$c(du) = cd(u)$$

L.H.S:

$$= c\left(d\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}\right)$$

$$= c\begin{bmatrix} da & 0 \\ 0 & db \end{bmatrix}$$

$$= \begin{bmatrix} cda & 0 \\ 0 & cdb \end{bmatrix}$$

R.H.S:

$$= cd\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

$$= \begin{bmatrix} cda & 0 \\ 0 & cdb \end{bmatrix}$$

L.H.S = R.H.S
Proved.

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Axiom # 10:

$$1u = u$$

$$= 1\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

$$= \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

Proved

All axioms hold
This is a vector space.

Q#11

$u = (1, y)$, $v = (1, y')$, $w = (1, y'')$

Axiom # 01:

$$u + v$$

$$= (1, y) + (1, y')$$

$$= (1, y + y')$$

Axiom # 02:

$$u + v = v + u$$

L.H.S

$$= (1, y) + (1, y')$$

$$= (1, y + y')$$

R.H.S

$$= (1, y') + (1, y)$$

$$= (1, y' + y)$$

L.H.S = R.H.S

Proved.

Axiom # 03:

$$u + (v + w) = (u + v) + w$$

L.H.S:

$$= (1, y) + ((1, y') + (1, y''))$$

$$= (1, y) + (1, y' + y'')$$

$$= (1, y + y' + y'')$$

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R.H.S:

$= ((1, y) + (1, y')) + (1, y'')$

$= (1, y + y') + (1, y'')$

$= (1, y + y' + y'')$

L.H.S = R.H.S

Proved

Axiom #04:

$u + 0 = u$

$= (1, y) + (1, 0)$

$= (1, y)$

Proved.

Axiom # 05:

$u + (-u) = 0$

$= (1, y) + (1, -y)$

$= (1, y - y)$

$= (1, 0)$

Proved

Axiom #06:

$Ku =$ is in V

$= K(1, y)$

$= (1, ky)$

Axiom #07:

$c(u+v) = c(u) + c(v)$

L.H.S:

$= k((1, y) + (1, y'))$

$= k(1, y + y')$

$= (1, ky + ky')$

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R.H.S

$= k(1,y) + k(1,y')$

$= (1, ky) + (1, ky')$

$= (1, ky + ky')$

L.H.S = R.H.S

Prove.

Axiom #08

$(c+d)u = c(u) + d(u)$.

L.H.S:

$= (c+d)(1,y)$

$= (1, (c+d)y)$

$= (1, cy + dy)$

R.H.S

$= c(1,y) + d(1,y)$

$= (1, cy) + (1, dy)$

$= (1, cy + dy)$

L.H.S = R.H.S

Proved.

Axiom #09

$c(du) = cd(u)$.

L.H.S

$= c(d(1,y))$

$= c(1, dy)$

$= (1, cdy)$

R.H.S

$= cd(1,y)$

$= (1, cdy)$

L.H.S = R.H.S

Proved.

Axiom #10

$1(u) = u$

$= 1(1,y)$

$= (1,y)$

Proved

All Axioms hold

This is a vector space.

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Q#12

$u = (a_0 + a_1x)$, $v = (b_0 + a_1x)$, $w = (c_0 + c_1x)$

Axiom #01:

$u + v$ is in V

$= (a_0 + a_1x) + (b_0 + b_1x)$

$= (a_0 + b_0) + (a_1 + b_1)x$

Axiom #02:

L.H.S $u + v = v + u$

$= (a_0 + a_1x) + (b_0 + b_1x)$

$= (a_0 + b_0) + (a_1 + b_1)x$

R.H.S

$= (b_0 + b_1x) + (a_0 + a_1x)$

$= (b_0 + a_0) + (b_1 + a_1)x$

$= (a_0 + b_0) + (a_1 + b_1)x$

L.H.S = R.H.S

Proved

Axiom #03:

$u + (v + w) = (u + v) + w$

L.H.S:

$= (a_0 + a_1x) + ((b_0 + b_1x) + (c_0 + c_1x))$

$= (a_0 + a_1x) + ((b_0 + c_0) + (b_1 + c_1)x)$

$= [a_0 + (b_0 + c_0)] + [a_1 + (b_1 + c_1)]x$

R.H.S

$= ((a_0 + a_1x) + (b_0 + b_1x)) + (c_0 + c_1x)$

$= ((a_0 + b_0) + (a_1 + b_1)x) + (c_0 + c_1x)$

$= ((a_0 + b_0) + c_0) + ((a_1 + b_1) + c_1)x$

L.H.S = R.H.S

Proved.

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Axiom # 04:

$$u + 0 = 0$$
$$= (a_0 + a_1 x) + (0 + 0)$$
$$= (a_0 + 0) + (a_1 + 0)x$$
$$= (a_0 + a_1 x)$$

Proved

Axiom #05:

$$u + (-u) = 0.$$
$$= (a_0 + a_1 x) + (-a_0 - a_1 x)$$
$$= (a_0 - a_0) + (a_1 - a_1)x$$
$$= (0 + 0)$$
$$= 0$$

Proved.

Axiom #06:

cu is in V

$$= c(a_0 + a_1 x)$$
$$= ca_0 + (ca_1)x$$

Axiom #07:

$$K(u+v) = K(u) + K(v).$$
$$= K((a_0 + a_1 x) + (b_0 + b_1 x))$$
$$= K((a_0 + b_0) + (a_1 + b_1)x)$$
$$= (Ka_0 + Kb_0) + (Ka_1 + Kb_1)x$$

R.H.S:

$$= K(a_0 + a_1 x) + K(b_0 + b_1 x)$$
$$= (Ka_0 + Ka_1 x) + (Kb_0 + Kb_1 x)$$
$$= (Ka_0 + Kb_0) + (Ka_1 + Kb_1)x.$$

$$L.H.S = R.H.S$$

Proved

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Axiom # 08:

$$(c+d)u = c(u) + d(u)$$

L.H.S:

$= (c+d)(a_0 + a_1 x)$

$= ((c+d)a_0) + ((c+d)a_1)x$

$= (ca_0 + da_0) + (ca_1 + da_1)x$

R.H.S:

$= c(a_0 + a_1 x) + d(a_0 + a_1 x)$

$= (ca_0 + ca_1 x) + (da_0 + da_1 x)$

$= (ca_0 + da_0) + (ca_1 + da_1)x$

L.H.S = R.H.S

Proved.

Axiom # 09.

$$c(du) = cd(u)$$

L.H.S:

$= c(d(a_0 + a_1 x))$

$= c(da_0 + da_1 x)$

$= cda_0 + cda_1 x$

R.H.S:

$= cd(a_0 + a_1 x)$

$= cda_0 + cda_1 x$

L.H.S = R.H.S

Proved.

Axiom # 10:

$$1(u) = u$$

$= 1(a_0 + a_1 x)$

$= a_0 + a_1 x$

Proved

All axioms hold

This is a Vector space

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EXERCISE # 4.2

Q#4

(a)

$A^T = -A$

$B^T = -B$.

Axiom # 01 :

$(A+B)^T = A^T + B^T$

$(A+B)^T = (-A) + (-B)$

$(A+B)^T = -(A+B)$

W is closed under addition.

Axiom # 06 :

$(KA)^T = KA^T$

$(KA)^T = K(-A)$

$(KA)^T = -(KA)$

W is closed under scaler multiplication.

Therefore W is subsace of M_{nn}.

(b)

$AX = 0$.

$A = \begin{bmatrix} 1 & -5 \\ 3 & -4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 5 \\ 3 & 4 \end{bmatrix}$

$A+B = \begin{bmatrix} 1 & -5 \\ 2 & -4 \end{bmatrix} + \begin{bmatrix} 1 & 5 \\ 3 & 4 \end{bmatrix}$

$A+B = \begin{bmatrix} 1 & 0 \\ 3 & 0 \end{bmatrix}$

$\det(A+B) = 0$

$AX = 0 \quad Bx = 0$

$\det(A) \neq 0 \quad \det(B) \neq 0$

$(A+B)x = 0$

$\det(A+B) = 0$

W is no closed under additio.

Therefore not a subspace of M_{nn}.

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(c).

$AB = BA$ Axiom #01 :

$A+B$. for some fixed matrix C.

$$(A+B)C = AC+BC$$
$$= CA+CB$$
$$(A+B)C = C(A+B)$$

W is closed under addition.

Axiom #06:

$$(KA)B = K(AB)$$
$$= K(BA)$$
$$(KA)B = B(KA)$$

W is closed under scalar multiplication

Therefore W is subspace of M_{nn}.

(d)

$$u = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}, \quad V = \begin{bmatrix} -1 & 0 \\ 0 & 3 \end{bmatrix}.$$

$$u + V = \begin{bmatrix} 0 & 0 \\ 0 & 6 \end{bmatrix}$$

Non invertible

W is not closed under addition.

Therefore W is not subspace of M_{nn}.

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Q#12

(a)

$$A\begin{bmatrix}1\\-1\end{bmatrix}=\begin{bmatrix}2\\0\end{bmatrix}$$

$$B\begin{bmatrix}1\\-1\end{bmatrix}=\begin{bmatrix}2\\0\end{bmatrix}.$$

$$A+B\begin{bmatrix}1\\-1\end{bmatrix}=A\begin{bmatrix}1\\-1\end{bmatrix}+B\begin{bmatrix}1\\-1\end{bmatrix}$$

$$=\begin{bmatrix}2\\0\end{bmatrix}+\begin{bmatrix}2\\0\end{bmatrix}$$

$$=\begin{bmatrix}4\\0\end{bmatrix}$$

W is not closed under addition

Therefore not a subspace of Mnn.

(b)

$$A\begin{bmatrix}0&2\\-2&1\end{bmatrix}=\begin{bmatrix}0&2\\-2&1\end{bmatrix}A$$

$$B\begin{bmatrix}0&2\\-2&1\end{bmatrix}=\begin{bmatrix}0&2\\-2&1\end{bmatrix}B.$$

$$(A+B)\begin{bmatrix}0&2\\-2&1\end{bmatrix}=\begin{bmatrix}0&2\\-2&1\end{bmatrix}+\begin{bmatrix}0&2\\-2&1\end{bmatrix}$$

$$=\begin{bmatrix}0&2\\-2&1\end{bmatrix}A+\begin{bmatrix}0&2\\-2&1\end{bmatrix}B$$

$$=\begin{bmatrix}0&2\\-2&1\end{bmatrix}(A+B)$$

W is closed under addition.

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$$(KA)\begin{bmatrix} 0 & 2 \\ -2 & 1 \end{bmatrix} = K\left(A\begin{bmatrix} 0 & 2 \\ -2 & 1 \end{bmatrix}\right)$$

$$= K\begin{bmatrix} 0 & 2 \\ -2 & 1 \end{bmatrix}A$$

$$= \begin{bmatrix} 0 & 2 \\ -2 & 1 \end{bmatrix}KA$$

W is close under scalar multiplication
Therefore W is subspace of M_{nn}

(c)

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 2 \\ 0 & 2 \end{bmatrix}$$

$\det(A) = 0$ $\det(B) = 0$

$$A + B = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$$

$\det(A+B) \neq 0$

W is not closed under addition.
Therefore not a subspace of M_{nn}.

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Q#14

(a)

$$Ax = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$A \begin{bmatrix} 0 & -1 & 0 & 2 \\ -1 & 1 & 0 & 1 \end{bmatrix}$$

$$Au = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$Av = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$A(u+v) = Au + Av$$

$$= \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$A(u+v). = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

W is not closed under additon
Therefor not a subspace of M_{nn}

(b)

$$Ax = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$Au = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$Av = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$A(u+v) = Aw + Av$$

$$= \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

W is not closed under addition.
Not a subspace of M_{nn}.

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~~Q#16~~

(a)

~~$a_0 = 2a_0 + 4x + 6x^2$~~

~~$\cancel{\emptyset} =$~~

Adding any 2 even numbers will result in even number so W is close under addition
Multiplying any scalar value with even number also results in even number
So therefore W is a subspace of ~~..~~ all pollynomials with even coefficients.

(b)

Let

$$u = a_0 + a_1 x + a_2 x^2 + a_3 x^3.$$
$$v = b_0 + b_1 x + b_2 x^2$$

$$a_0 + a_1 + a_2 + a_3 = 0$$
$$b_0 + b_1 + b_2 = 0$$

$$u + v = (a_0 + a_1 x + a_2 x^2 + a_3 x^3) + (b_0 + b_1 x + b_2 x^2)$$
$$= (a_0 + b_0) + (a_1 + b_1)x + (a_2 + b_2)x^2 + a_3 x^3$$

Evaluating all coefficients

$$= (a_0 + b_0) + (a_1 + b_1) + (a_2 + b_2) + a_3$$
$$= (a_0 + a_1 + a_2 + a_3) + (b_0 + b_1 + b_2 +)$$
$$= 0 + 0$$
$$= 0$$

W is closed under addition

$$K u = K(a_0 + a_1 x + a_2 x^2 + a_3 x^3)$$
$$= K a_0 + K a_1 x + K a_2 x^2 + K a_3 x^3$$

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Evaluating coefficient

$Ka_0 + Ka_1 + Ka_2 + Ka_3 = K(a_0 + a_1 + a_2 + a_3)$
$= K(0)$
$= 0$

W is closed under scaler multiplication.

Therefore W is subspace of ~~set polynomials~~ P_3.

(c)

Adding any two polynomials of even degree result in even degree Therefore W is closed under addition.

When scalar value is multiplied even degree polynomials remain even so Therefore W is closed under scaler multiplication.

Therefore W is subspace.

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EXERCISE #4.3.

Q#15

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

a) $u = (1, 0, -1, 0)$, $v = (0, 1, 0, -1)$

$R_2 - R_1$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -1 & 0 & -1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

$-R_2$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

$R_3 - R_2$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_1 - R_2$

$$= \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Therefore

$x + z = 0$

$y + w = 0$

Let $x = -s$ & $y = t$

$z = -s$

$w = -t$.

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$(x, y, z, w) = (s, t, -s, -t)$.
$= s(1, 0, -1, 0) + t(0, 1, 0, -1)$

set (u, v) spans W.

b) $u = (1, 0, -1, 0)$, $v = (1, 1, -1, -1)$
$$\left.\begin{matrix} v_1 (1, 0, -1, 0) \\ v_2 (0, 1, 0, -1) \end{matrix}\right\} \text{from previous RREF.}$$

$u = v_1 = (1, 0, -1, 0)$
$v = v_1 + v_2$
$v = (1, 0, -1, 0) + (0, 1, -0, -1) = (1, 1, -1, -1)$

set (u, v) spans W.

Q #16

$$A = \begin{bmatrix} 0 & 1 & -1 & 1 \\ 0 & 2 & -2 & 2 \\ 0 & 3 & -3 & 3 \end{bmatrix}$$

a) $u = (1, 1, 1, 0)$, $v = (0, -1, 0, 1)$
$R_2 - 2R_1$
$$= \begin{bmatrix} 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 3 & -3 & 3 \end{bmatrix}$$
$R_3 - 3R_1$
$$= \begin{bmatrix} 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

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$y - z + w = 0.$

$0 = 0$

$0 = 0.$

Let $z = t$, $w = s$, $x = k$

$y - (t) + s = 0$

$y = t - s.$

$(x, y, z, w) = (k, t-s, t, s)$

$= k(1, 0, 0, 0) + t(0, 1, 1, 0)$

$+ s(0, -1, 0, 1).$

$v_1 = (1, 0, 0, 0)$

$v_2 = (0, 1, 1, 0)$

$v_3 = (0, -1, 0, 1)$

$V = V_3 = (0, -1, 0, 1)$

$u = v_1 + v_2 = (1, 0, 0, 0) + (0, 1, 1, 0) = (1, 1, 1, 0)$

The set $\{u, v\}$ spans W.

b) $u = (0, 1, 1, 0)$, $v = (1, 0, 1, 1)$.

$y - z + w = 0$

Let $z = t$, $w = s$, $x = k$

$y - t + s = 0$

$y = t - s$

$(x, y, z, w) = (k, t-s, t, s)$

$= k(1, 0, 0, 0) + t(0, 1, 1, 0)$

$+ s(0, -1, 0, 1)$

Set $\{u, v\}$ doesnot span W

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Q#18

$T_A : R^3 \to R^2$

$u_1 = (0, 1, 1)$

$u_2 = (2, -1, 1)$

$u_3 = (1, 1, -2)$.

$\{T_A(u_1), T_A(u_2), T_A(u_3)\}$

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \end{bmatrix}$

$T_A(u_1) = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0+1+0 \\ 0+1-1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$T_A(u_2) = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2-1+0 \\ 0-1-1 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

$T_A(u_3) = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 1+1+0 \\ 0+1+2 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

$v_1 = (1, 0)$

$v_2 = (1, -2)$

$v_3 = (2, 3)$

let (a, b) be any arbitary point in R^2

$(a, b) = (a, 0) + (0, b)$

$= a(1, 0) + b(0, 1)$

$= a v_1 + b\left(\frac{v_3 - 2v_1}{3}\right)$

the vectors span R^3.

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b) $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & -3 \end{bmatrix}$

$T_A(u_1) = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0+1+0 \\ 0+1-3 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

$T_A(u_2) = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0-1+0 \\ 2-1-3 \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$

$T_A(u_3) = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 0+1+0 \\ 1+1+6 \end{bmatrix} = \begin{bmatrix} 1 \\ 8 \end{bmatrix}$

$v_1 = (1, -2)$
$v_2 = (-1, -2)$
$v_3 = (1, 8)$

The set $\{u, v\}$ doesnot span W.

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EXERCISE # 4.4

Q#11

$V_1 = (\lambda, -1/2, -1/2)$
$V_2 = (-1/2, \lambda, -1/2)$
$V_3 = (-1/2, -1/2, \lambda)$

$a(\lambda, -1/2, -1/2) + b(-1/2, \lambda, -1/2) + c(-1/2, -1/2, \lambda) = 0$

$a\lambda - 1/2\, b - 1/2\, c = 0$
$-1/2\, a + \lambda b - 1/2\, c = 0$
$-1/2\, a - 1/2\, b + \lambda c = 0$

$$\underbrace{\begin{bmatrix} \lambda & -1/2 & -1/2 \\ -1/2 & \lambda & -1/2 \\ -1/2 & -1/2 & \lambda \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} a \\ b \\ c \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}_{b}$$

$$|A| = \begin{vmatrix} \lambda & -1/2 & -1/2 \\ -1/2 & \lambda & -1/2 \\ -1/2 & -1/2 & \lambda \end{vmatrix}$$

$$= \lambda \begin{vmatrix} \lambda & -1/2 \\ -1/2 & \lambda \end{vmatrix} + \frac{1}{2} \begin{vmatrix} -1/2 & -1/2 \\ -1/2 & \lambda \end{vmatrix} - \frac{1}{2} \begin{vmatrix} -1/2 & \lambda \\ -1/2 & -1/2 \end{vmatrix}$$

$$= \lambda\left(\lambda^2 - 1/4\right) + \frac{1}{2}\left(-\frac{\lambda}{2} - 1/4\right) - \frac{1}{2}\left[1/4 + \frac{\lambda}{2}\right]$$

$$= \lambda^3 - \lambda/4 - \lambda - 1/8 - 1/8 - \lambda$$

$$= \lambda^3 - 3/4\,\lambda - 1/4$$

$\lambda^3 - 3/4\,\lambda - 1/4 = 0$
$4\lambda^3 - 3\lambda - 1 = 0.$

Using long division

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$$-(\lambda-1)\quad(2\lambda+1)^2=0$$

$$\lambda-1=0 \qquad 2\lambda+1=0$$

$$\lambda_1=1 \qquad \lambda_1=-1/2$$

Given vectors are linearly dependent if $\lambda=1$ or $\lambda=-1/2$.

Q#14

$u_1=(1,0,0)$ & $u_2=(2,-1,1)$; $u_3=(0,1,1)$

a) $A=\begin{bmatrix}1&1&2\\1&0&-3\\2&2&0\end{bmatrix}$

$$T_A(u_1)=\begin{bmatrix}1&1&2\\1&0&-3\\2&2&0\end{bmatrix}\begin{bmatrix}1\\0\\0\end{bmatrix}=\begin{bmatrix}1\\1\\2\end{bmatrix}$$

$$T_A(u_2)=\begin{bmatrix}1&1&2\\1&0&-3\\2&2&0\end{bmatrix}\begin{bmatrix}2\\-1\\1\end{bmatrix}=\begin{bmatrix}3\\-1\\2\end{bmatrix}$$

$$T_A(u_3)=\begin{bmatrix}1&1&2\\1&0&-3\\2&2&0\end{bmatrix}\begin{bmatrix}0\\1\\1\end{bmatrix}=\begin{bmatrix}3\\-3\\2\end{bmatrix}$$

$$a(1,1,2)+b(3,-1,2)+c(3,-3,2)=(0,0,0)$$

$$(a,a,2a)+(3b,-b,2b)+(3c,-3c,2c)=(0,0,0)$$

$$a+3b+3c=0$$

$$a-b-3c=0$$

$$2a+2b+2c=0$$

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$$\begin{vmatrix}1&3&3\\1&-1&-3\\2&2&2\end{vmatrix}$$

$$= 1\begin{vmatrix}-1&-3\\2&2\end{vmatrix} - 3\begin{vmatrix}1&-3\\2&2\end{vmatrix} + 3\begin{vmatrix}1&-1\\2&2\end{vmatrix}$$

$$= 1(-2+6) - 3(2+6) + 3(2+2)$$

$$= 1(+4) - 3(8) + 3(4)$$

$$= 4 - 24 + 12$$

$$= -8 \neq 0$$

$\{T_A(u_1), T_A(u_2), T_A(u_3)\}$ is a linearly independent set in $\mathbb{R}^3$.

b) $A = \begin{bmatrix}1&1&1\\1&1&-3\\2&2&0\end{bmatrix}$

$$T_A(u_1) = \begin{bmatrix}1&1&1\\1&1&-3\\2&2&0\end{bmatrix}\begin{bmatrix}1\\0\\0\end{bmatrix} = \begin{bmatrix}1\\1\\2\end{bmatrix}$$

$$T_A(u_2) = \begin{bmatrix}1&1&1\\1&1&-3\\2&2&0\end{bmatrix}\begin{bmatrix}2\\-1\\1\end{bmatrix} = \begin{bmatrix}2\\-2\\2\end{bmatrix}$$

$$T_A(u_3) = \begin{bmatrix}1&1&1\\1&1&-3\\2&2&0\end{bmatrix}\begin{bmatrix}0\\1\\1\end{bmatrix} = \begin{bmatrix}2\\-2\\2\end{bmatrix}$$

$$a(1,1,2) + b(2,-2,2) + c(2,-2,2) = (0,0,0)$$

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$(a, a, 2a) + (2b, -2b, 2b) + (2c, -2c, 2c) = (0, 0, 0)$

$a + 2b + 2c = 0$

$a - 2b - 2c = 0$

$2a + 2b + 2c = 0.$

$$\begin{vmatrix} 1 & 2 & 2 \\ 1 & -2 & -2 \\ 2 & 2 & 2 \end{vmatrix}$$

$$= 1 \begin{vmatrix} -2 & -2 \\ 2 & 2 \end{vmatrix} - 2 \begin{vmatrix} 1 & -2 \\ 2 & 2 \end{vmatrix} + 2 \begin{vmatrix} 1 & -2 \\ 2 & 2 \end{vmatrix}$$

$= 1(-4+4) - 2(2+4) + 2(2+4)$

$= 0 - 2(6) + 2(6)$

$= -12 + 12$

$= 0$

$\{T_A(u_1), T_A(u_2), T_A(u_3)\}$ is a linearly dependent set in R^3.