JASIL ALI KHAN (1)
20K-0474

Date

ASSIGNMENT # 01

Q#1

i) $\frac{dy}{dx} = x/y$

$\int y\,dy = \int x\,dx$

$\frac{y^2}{2} = \frac{x^2}{2} + c$

Q#2

ii) $x\frac{dy}{dx} + y = x^2y^2$

$\frac{dy}{dx} + y/x = xy^2$

$P(x) = 1/x$

$n = 2$

$Q(x) = x$

$y^{1-x} = \frac{1}{IF} \int (1-x)\,dx \cdot IF\,dx + c$

$IF = e^{\int -1/x\,dx} = e^{-\ln x} = 1/x$

$y^{-1} = x\left[-1(x)\,1/x\,dx\right] + c$

$x\left[-1\,dx\right] + c$

$1/y = -x^2 + c$

$y = \frac{1}{-x^2+c}$

Date

iii) $(x^2+y^2)\,dx + (xy)\,dy = 0$

$xy\,dy = -(x^2+y^2)\,dx$

$$\frac{dy}{dx} = \frac{-(x^2+y^2)}{xy}.$$

$y = vx$

$dy = x\,dv + v\,dx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$(x^2 + x^2v^2)\,dx + x^2v\,(x\,dv + v\,dx) = 0.$

$(x^2+x^2v^2)\,dx + x^3v\,dV + x^2v^2\,dx = 0.$

$$\frac{x^2 + 2x^2v^2\,dx + x^3v\,dx}{x^2} = 0$$

$(1+2v^2)\,dx + xv\,dV = 0$

$$\int\frac{dx}{x} + \int\frac{v\,dv}{1+2v^2} = 0$$

$$\int\frac{v}{1+2v^2} \Rightarrow \text{let } u = 1+2v^2$$

$du = 4v\,dV$

$$dV = \frac{du}{dx}$$

$$\int \frac{v\,du}{x}\cdot\frac{dx}{4xV}$$

$$\frac{1}{4}\int\frac{du}{u} = \frac{1}{4}\ln u + c$$

$$= \frac{1}{4}\ln(1+2v^2) + c$$

$\ln x + \frac{1}{4}\ln|1+2v^2| = c$

$\ln x + \ln(1+2v^2)^{1/4} = c.$

$\ln x^4 + \ln|1+2u^2| = c4.$

Date

$x^4(1+2v^2) = c^4.$

$y = vx \Rightarrow v = y/x$

$x^4\left(1+2\left(\frac{y^2}{x^2}\right)\right) = c$

$x^4\left(1+\frac{2y^2}{x^2}\right) = c$

$x^4 + 2y^2 x^2 = c$

$2x^2y^2 + x^4 = c.$

iv) $(x-y^2)\,dx + 2xy\,dy = 0$

$m = x - y^2 \qquad N = 2xy$

$\frac{dm}{dy} = 0 - 2y \qquad \frac{dN}{dx} = y(2)$

Not exact

$\frac{My - Nx}{N} = \frac{-2y-2y}{2xy} = \frac{-4y}{2xy} = \frac{-2}{x}$

$\frac{2}{x}(x-y^2)\,dx + (-2/x)\,2xy\,dy = 0$

$e^{\int -2/x} = e^{-2\ln x} = \ln x^{-2} = x^{-2} = 1/x^2$

$1/x^2\,(x-y^2)\,dx + 1/x^2\,(2xy)\,dy = 0$

$\left(1/x - y^2/x\right)dx + \frac{2y}{x}\,dy = 0$

Exact

$\int 1/x - y^2/x^2\,dx + \int (0)\,dy = 0$

(1)

Date

$$\ln x = \int y^2 \left(\frac{1}{x^2}\right) dx$$

$$\ln x + y^2\left(\frac{1}{x}\right) + 0 = c$$

$$\ln x + \frac{y^2}{x} = c.$$

v) $e^y\left(\frac{dy}{dx} - 1\right) = e^x$

$$e^y \frac{dy}{dx} - e^y = e^x$$

let $e^y = t$

$$e^y \frac{dy}{dx} = \frac{dt}{dx}$$

$$\frac{dt}{dx} - t = e^x$$

I.F: $e^{\int -dx}$

$$= e^{-x}.$$

$$t(e^{-x}) = \int e^x e^{-x}\, dx$$

$$e^y e^{-x} = x + c$$

$$e^{y-x} = x + c.$$

vi) $\sin y \frac{dy}{dx} = \cos x\,(2\cos y - \sin x)$

$M = \sin y \qquad N = -2\cos y \cos x + \cos x \sin x$

$\frac{dM}{dy} = \cos y \qquad \frac{dN}{dx} = -\cos y\,(-2\sin x) + \cos x(\cos x) + \sin x(-\sin x)$

Not exact.

$$M(y) = \cos y$$

$$N(x) = 2\sin x \cos y + \cos^2 x - \sin^2 x$$

$$\frac{\cos y - 2\sin x \cos y - \cos^2 x + \sin^2 x}{-2\cos x \cos y + \cos x \sin x}$$

Date

vii) $x(3x+2y^2)\,dx + 2y(1+x^2)\,dy = 0$

$(3x^2+2xy^2)\,dx + (2y+2x^2y)\,dy = 0$

$M = 3x^2+2xy^2$ $\quad$ $N = 2y+2x^2y$

$\frac{\partial M}{\partial y} = 0 + 2x(2y)$ $\quad$ $\frac{\partial N}{\partial x} = 0 + 2(2x)y$

$\frac{\partial M}{\partial y} = 4xy$ $\quad$ $\frac{\partial N}{\partial x} = 4xy$

exact

$\int 3x^2+2xy^2\,dx + \int 2y\,dy = 0$

$\frac{3x^3}{3} + \frac{y^2 2x^2}{2} + \frac{2y^2}{2} = 0$

$x^3 + x^2y^2 + y^2 = 0$

viii) $e^{-y}\sec^2 y\,dy = dx + x\,dy$

$e^{-y}\sec^2 y = \frac{dx + x\,dy}{dy}$

$e^{-y}\sec^2 y = \frac{dx}{dy} + x$

$\sec^2 y = e^y\,dx/dy + e^y x$

$\sec^2 y = \frac{d}{dy}(e^y x)$

$xe^y = \tan y + c$

$\tan y - xe^y + c = 0$

Date

ix) $(x^2+y^2)dx + (x^2-xy)dy = 0$.

$$\frac{dy}{dx} = -\frac{x^2+y^2}{x^2-xy}$$

$$= -\frac{1/x^2\,(x^2+y^2)}{1/x^2\,(x^2-xy)}$$

$$= -\frac{1+(y/x)^2}{1-y/x}$$

let

$y = vx$

$v = y/x$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = -\frac{(1+v^2)}{1-v}$$

$$= \frac{1+v^2}{1-v}$$

$$x\frac{dv}{dx} = \frac{1+v^2-v}{v-1}$$

$$x\frac{dv}{dx} = \frac{v+1}{v-1}$$

$$\frac{v\,\cancel{-1}}{v+1}\frac{dv}{dx} = 1/x$$

$$\int \frac{v-1}{v+1}\,dv = \int dx/x$$

$$\int \frac{v+1-2}{v+1}\,dv = \int dx/x$$

$$\int 1 - \frac{2}{v+1}\,dv = \int dx/x$$

$$v - 2\ln|v+1| = \ln x + c.$$

$$y/x - 2\ln(y/x + 1) = \ln x + C$$

Date

x) $(y-x)\frac{dy}{dx} = a\left(y^2+\frac{dy}{dx}\right)$

$y - x\frac{dy}{dx} = ay^2 + \frac{a\,dy}{dx}$

$y - ay^2 = (a+x)\,dy/dx$

$\frac{dx}{a+x} = \frac{1}{y-ay^2}\,dy$

$\frac{dx}{a+x} = \frac{1}{y(1-ay)}\,dy$

Partial fraction

$\int\frac{dx}{a+x} = \int\left[\frac{1}{y} + \frac{a}{1-ay}\right]dy$

$\ln|a+x| = \ln y + \frac{a\ln(1-ay)}{-a} + c.$

$\ln|a+x| = \ln y - \ln|1-ay| + c.$

$a+x = \left(\frac{y}{1-ay}\right)\cdot c.$

xi) $(x+1)\frac{dy}{dx} + 1 = 2e^{-y}.$

$(x+1)\frac{dy}{dx} = 2e^{-y} - 1$

$\int\frac{dy}{2e^{-y}-1} = \int\frac{1}{1+x}\,dx.$

$\int\frac{dy}{2/e^{y}-1} = \ln|1+x| + c.$

$\int\frac{dy}{\frac{2-e^{y}}{e^{y}}} = \ln|1+x| + c$

Date________

$$\int \frac{e^y}{2-e^y}\,dy = \ln|1+x| + c$$

$$2-e^y = t.$$
$$dt = -e^y\,dy$$
$$-dt = e^y\,dy$$
$$\int \frac{-dt}{t} = \ln|t|$$

$$-\ln(2-e^y) = \ln(x+1) + c.$$

xii) $x^2 \frac{dy}{dx} + y(x+y) = 0.$

$$y = vx$$
$$\frac{dy}{dx} = x + x\frac{dv}{dx}.$$
$$\frac{dy}{dx} = \frac{-y(x+y)}{x^2}.$$
$$v + x\frac{dv}{dx} = \frac{-vx(x+vx)}{x^2}$$
$$v + x\frac{dv}{dx} = -v - v^2.$$
$$x\frac{dv}{dx} = 2v - v^2.$$
$$\frac{dv}{v(2+v)} + \frac{dx}{dx} = 0$$
$$\frac{1}{2}\int \frac{1}{v} - \frac{1}{v+2}\,dv + \int \frac{dx}{x} = \ln c.$$

$$\frac{1}{2}\ln\left(\frac{v}{v+2}\right) + \ln x = \ln c$$
$$\ln\left(\frac{v}{v+2}\right) + \ln x^2 = \ln c$$

Date

$$\frac{y/x}{y/x + 2} \quad x^2 = c.$$

$$\frac{y}{y+2x} \cdot x^2 = c.$$

$$y + 2x = Cx^2 y.$$

xiii) $(\sec x \tan x \tan y - e^x)dx + \sec x \sec^2 y \, dy = 0$

$\sec x \tan x \tan y \, dx + \sec x \sec^2 y \, dy = e^x dx$

$d[\sec x \tan y] = \tan y \sec x \tan x \, dx + \sec x \sec^2 y \, dy$

$\int d(\sec x \tan y) = \int e^x dx$

$\sec x \tan y = e^x + c.$

$\tan y = \dfrac{e^x + c}{\sec x}$

$y = \tan^{-1}\left(\dfrac{e^x + c}{\sec x}\right)$

xiv) $x\cos x \dfrac{dy}{dx} + y(x\sin x + \cos x) = 1$

$\dfrac{dy}{dx} + \dfrac{y(x\sin x + \cos x)}{x\cos x} = \dfrac{1}{x\cos x}$

$\dfrac{dy}{dx} + \left(\dfrac{x\sin x}{x\cos x} + \dfrac{\cos x}{x\cos x}\right)y = \dfrac{1}{x\cos x}$

Data

$$\frac{dy}{dx} + \left(\tan x + \frac{1}{x}\right)y = \frac{1}{x\cos x}$$

$$P(x) = \tan x + \frac{1}{x}$$

$$e^{\int \tan x + 1/x} = e^{\ln\sec x + \ln x}$$
$$= e^{\ln x\sec x}$$
$$= x\sec x$$

$$y\,x\sec x = \int \frac{\sec x}{x} \cdot x \,(x\sec x$$

$$xy\sec x = \int \sec^2 x \, dx$$

$$xy\sec x = \tan x + c$$
$$y = \frac{\tan x + c}{x\sec x}$$

xv) $x\ln x \, dy/dx + y = 2\ln x$

$$\frac{dy}{dx} + \frac{1}{x\ln x}(y) = \frac{2\ln x}{x\ln x}$$
$$\frac{dy}{dx} + \left(\frac{1}{x\ln x}\right)y = 2/x$$

IF:

$$= e^{\int 1/x\ln x \, dx}$$
$$= e^{\int dt/t}$$
$$= e^{\ln t}$$
$$= t$$

$$y\ln x = c + \int \ln x \cdot 2/x \, dx.$$

$$\ln x = u$$
$$1/x \, dx = du$$

Date

$$y\ln x = c + \int 2u\,du$$
$$y\ln x = c + u^2$$
$$y\ln x = c + (\ln x)^2.$$
$$y = \frac{c + (\ln x)^2}{\ln x}.$$

xvi) $y' + \frac{4}{x}y = x^2y^2$

$P(x) = 4/x$

$Q(x) = x^2$

$n = 2$

$e^{\int -4/x} = e^{-4\ln x} = e^{\ln x^4} = x^4.$

$$y^{1-n} = \frac{1}{IF}\left[\int (1-n)\,dx \cdot I(x)\,dx + c\right]$$

$$= \frac{1}{x^4}\left[\int -x^3 \cdot x^4\,dx + c\right]$$

$$= \frac{1}{x^4}\left[\int -x^7\,dx + c\right]$$

$$= \frac{1}{x^4}\left[\frac{-x^8}{8}\right] + c$$

$$= \frac{-x^4}{8} + \frac{c}{x^4}$$

Date

Q#2

(i)

Intial = P_0
in 5 years = $2P_0$
time for $8P_0$ = ?
tim for $4P_0$ = ?

$P(t) = P_0 e^{kt}$
$P = P_0 e^{(0)}$
$P = P_0$
$P(t) = P_0 e$

$\ln 2 = 5K.$
$K = \frac{\ln 2}{5}$
$K = 0.138624.$
$\ln 3 = 0.138629\,t$
$= 7.92$

for quadraple

$4P_0 = P_0 e^{-0.135629t}$
$\frac{\ln 4}{0.13541} = t$
$t = 10.$

Date

(ii)

$A(t) = A_0 e^{kt}$

$\frac{1}{2} A_0 = A_0 e^{3.3k}$

$\ln \frac{1}{2} = 3.3k$

$k = -0.21004.$

$A_0 = 1$ gram.

90% decayed.

$A = (1-0.9) A_0$

$0.1 = 1e^{-0.21004t}$

$\frac{\ln 0.1}{-0.21004} = t$

$t = 10.96 \approx 11$ hours.

(iii)

$T = T_m + Ce^{kt}$

$T(0) = 70°F$

$70 = T_m + C$

$C = 70 - T_m$

110°F after ½ min

$110 = T_m + (70 - T_m) e^{\frac{1}{2}k}$ — ①

145°F after 1 min

$145 = T_m + (70 - T_m) e^{k}$ — ②

$\frac{e^{k}}{e^{0.5k}} = \frac{145 - T_m}{110 - T_m}.$

Date

$$\frac{145-T_m}{110-T_m} = \frac{110-T_m}{70-T_m}$$

$$(145-T_m)(70-T_m) = (110-T_m)^2$$

$$10150 - 215T_m + T_m^2 = 12100 - 220T_m + T_m^2$$

$$10150 - 215T_m = 12100 - 220T_m$$

$$220T_m - 215T_m = 12100 + 10150$$

$$5T_m = 1950$$

$$T_m = \cancel{\frac{1950}{5}}$$

$$T_m = 390°F$$

(iv)

$$L\frac{di}{dt} + Ri = E(t)$$

$L = 0.1 H$

$R = 50 \Omega$

$E = 30 V$

$$0.1\frac{di}{dt} + 50i = 30$$

$$\frac{di}{dt} + 500i = 300$$

IF:

$$= e^{\int 500 dt}$$

$$= e^{500t}$$

$$e^{500t}\frac{di}{dt} + 500e^{500t} = 300e^{500t}$$

$$\frac{d}{dt}[e^{500t}i] = 300e^{500t}$$

Date

$$e^{500t} i = \int 300 e^{500t} dt$$

$$e^{500t} i = \frac{300}{500} e^{500t} + C.$$

$i(0) = 0$

$0 = 3/5 + C$

$C = -3/5$

$$i(t) = 3/5 - 3/5 e^{500t}$$

At $t = \infty$

$$i(t) = \lim_{t \to \infty} \left(3/5 - 3/5 e^{500t} \right)$$

$$= 3/5 - 0$$

$$\lim_{t \to \infty} i(t) = 3/5$$