

GT Assignment #02

Date:

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20K-0477

Question # 01

(i)

Since tree is connected and every vertex has degree 1.

\therefore sum of all degrees of vertices = $2E$

In tree n vertices has $n-1$ edges.

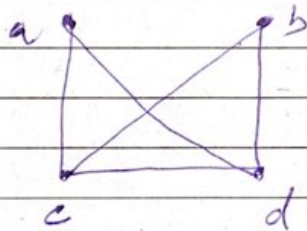
So

$$2E = 2(n-1) \\ = 2n-2$$

Proved.

(ii)

let $n=4$

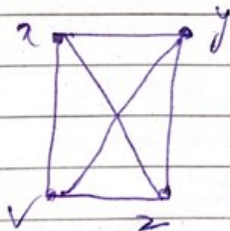


$$K(G) \geq 2$$

We need ~~minimum~~ exactly two vertex to disconnect a graph

$$\text{So } K(G) = 2 \text{ proved}$$

(iii)



$$d(x) = 1$$

$$d(x, y) = 1$$

$$d(x, v) = 1$$

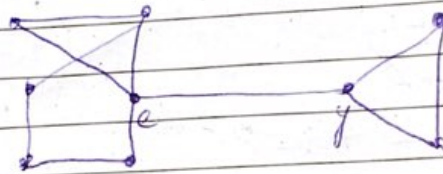
$$d(x, z) = 1$$

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$$\begin{aligned} \varepsilon(y) &= 1 \\ d(y, x) &= 1 \quad d(y, v) \neq 1 \quad d(y, z) = 1 \\ \varepsilon(x) &\leq \varepsilon(y) \end{aligned}$$

Question # 2

(a)



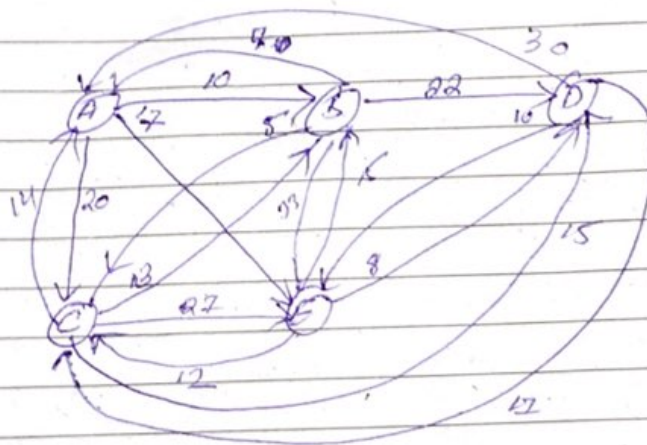
if graph has any bridge it is possible
that it may have cut vertex. The
above figure if we remove bridge of the
graph will be disconnected.

Q#3 After 4 5

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Question #04

	A	B	C	D	E
A	0	10	20	30	17
B	7	0	5	22	23
C	14	13	0	15	22
D	30	22	15	0	16
E	17	23	22	16	0



$1-2=10$	$2-3-4=20$	$4-5=17$
$1-3=20$	$3-1=14$	$4-5=10$
$1-2-4=32$	$3-2=13$	$4-5=10$
$1-5=17$	$3-4=15$	$5-2-1=22$
$2-1=7$	$3-4-5=25$	$5-2=15$
$2-3=5$	$4, 4=30$	$5-4=9$ least
	$4+1+2=12$	$5-3=12$

Q#5

$n=16$

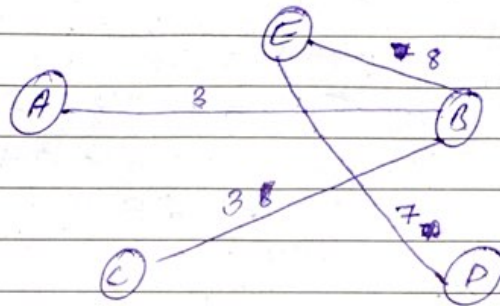
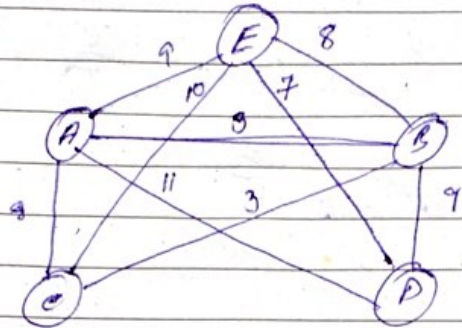
(a)

(b)

$n=24$

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Question # 03

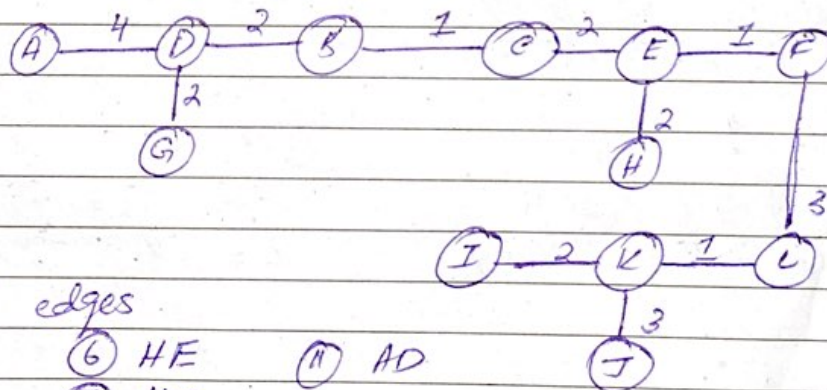


$$\text{Cost} = 3 + 3 + 8 + 7$$

$$= \boxed{21}$$

Question # 6

(a)



Order of edges

① BC

⑥ HE

⑪ AD

② EF

⑦ KI

③ KL

⑧ DG

④ DB

⑨ JK

⑤ CE

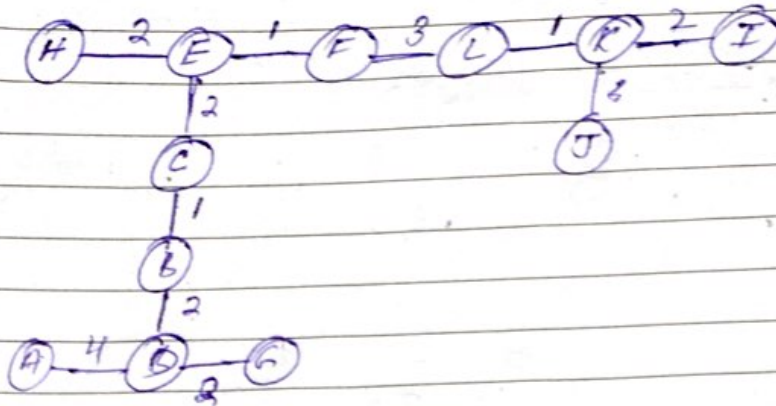
⑩ FL

$$\text{Cost} = 1 + 1 + 1 + 2 + 2 + 2$$

$$+ 2 + 2 + 3 + 4$$

$$= \boxed{28}$$

(5)



Order of edges:

- | | |
|------|------|
| ① HE | ⑦ FL |
| ② EF | ⑧ LK |
| ③ FC | ⑨ KI |
| ④ CB | ⑩ KJ |
| ⑤ BD | ⑪ DA |
| ⑥ DG | |

$$\begin{aligned} \text{Cost} &= 1+1+1+2+2+2+ \\ &\quad 2+2+0+3+4 \\ &= 23 \end{aligned}$$

Question # 07:

$$\begin{aligned} \text{i)} \quad S &= (2, 2, 3, 2, 2) & n-2 &= 6 \\ L &= (2, 3, 4, 5, 6, 7, 8) & n &= 8 \end{aligned}$$

$$\begin{aligned} \text{②} \Rightarrow & \quad \text{---}^1 \text{---}^3 \text{---} \\ S &= (2, 2, 3, 2, 2) \\ L &= (2, 3, 4, 5, 6, 7, 8) \end{aligned}$$

$$\begin{aligned} & \quad \text{---}^1 \text{---}^2 \text{---} \\ & \quad \text{---}^4 \text{---}^2 \text{---} \end{aligned}$$

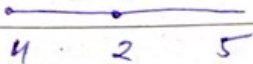
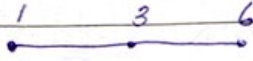
$$\begin{aligned} \text{③} \Rightarrow & \quad S = (2, 3, 2, 2) \\ & \quad L = (2, 3, 4, 5, 6, 7, 8) \end{aligned}$$

$$\begin{aligned} & \quad \text{---}^1 \text{---}^2 \text{---} \\ & \quad \text{---}^4 \text{---}^2 \text{---}^5 \end{aligned}$$

④)

$$S = (\overset{2}{\textcircled{2}}, \textcircled{2})$$

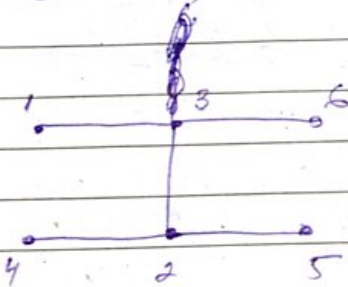
$$L = (2, 3, \textcircled{6}, 7, 8)$$



⑤)

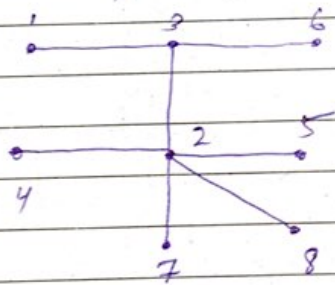
$$S = (\textcircled{3}, 2)$$

$$L = (2, \textcircled{3}, 7, 8)$$



$$S = (\textcircled{2})$$

$$L = (2, \textcircled{7}, 8)$$



①→

(ii')

$$S = (5, 3, 2, 3, 3, 1)$$

$$n-2 = 6$$

$$L = (1, 2, 3, 4, 5, 6, 7, 8)$$

$$n = 8$$

②→



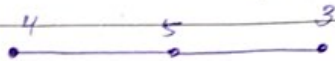
⑥→

$$S = (1)$$

$$L = (1, 3, 8)$$

$$S = (3, 2, 3, 3, 1)$$

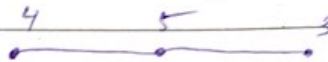
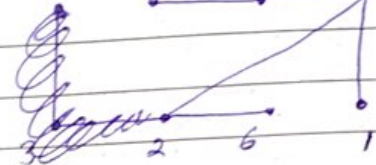
$$L = (1, 2, 3, 5, 6, 7, 8)$$



③→

$$S = (2, 3, 3, 1)$$

$$L = (1, 2, 3, 6, 7, 8)$$



⑦→

$$S = (1)$$

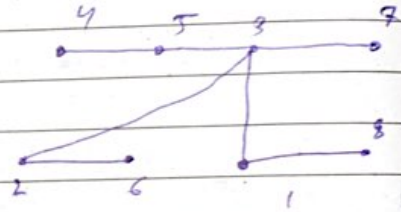
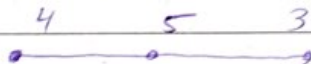
$$L = (1, 8)$$



④→

$$S = (3, 3, 1)$$

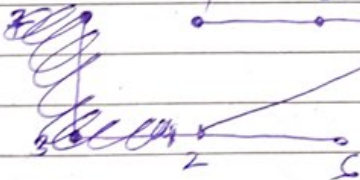
$$L = (1, 2, 3, 7, 8)$$



⑤→

$$S = (3, 1)$$

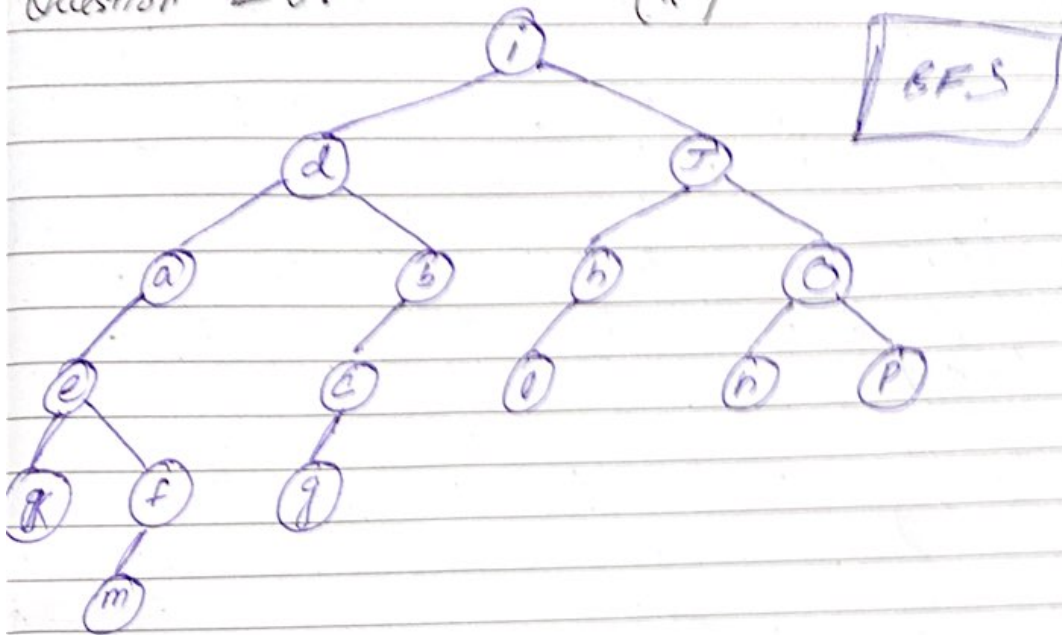
$$L = (1, 3, 7, 8)$$



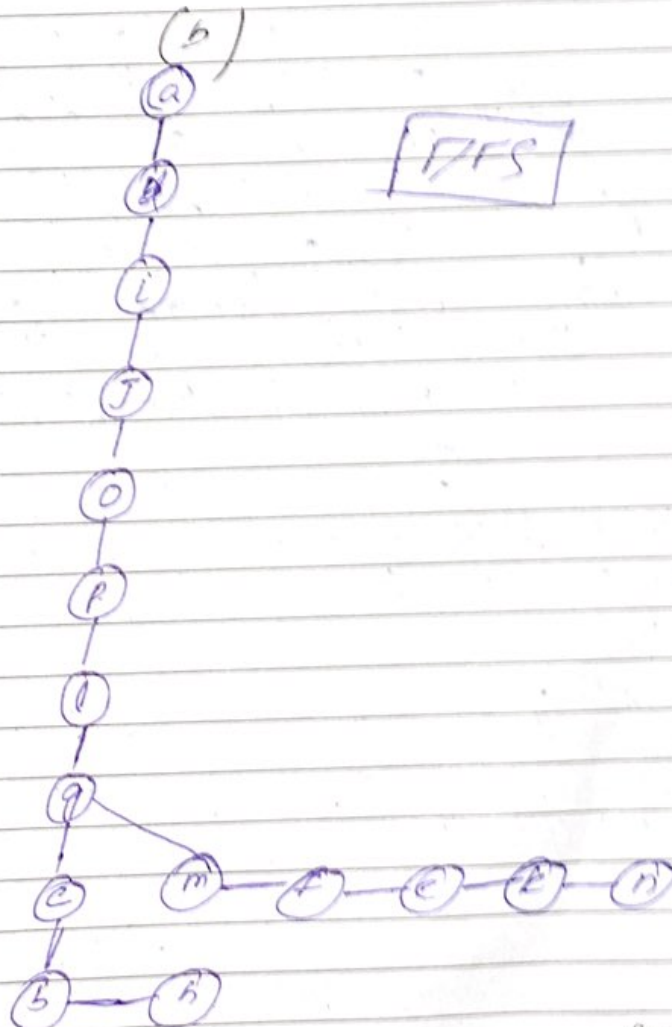
Question # 08

Date:

(a)



(b)



Date: _____

Question # 09

x

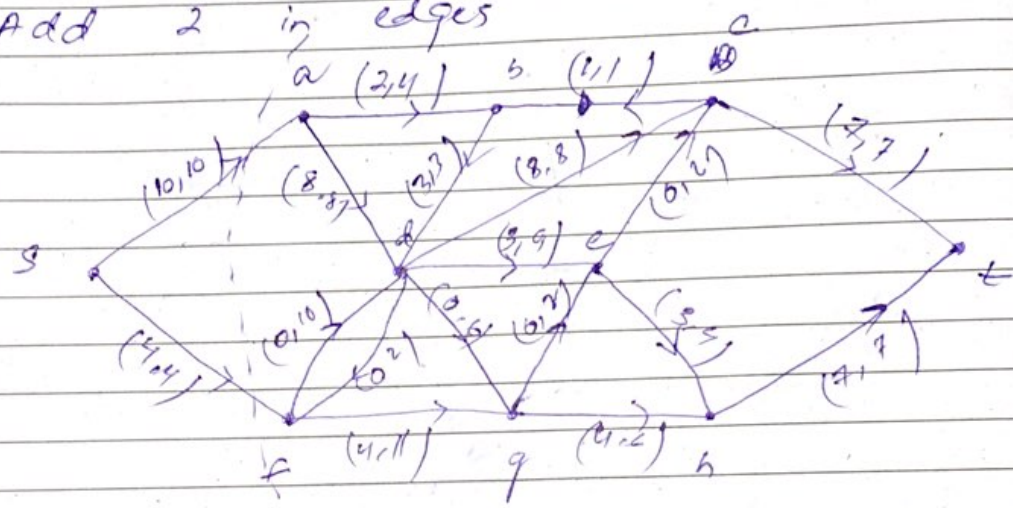
$s(-, \infty)$
 $a(s^+, 2)$
 $b(a^+, 2)$
 $d(b^+, 2)$
 $e(d^+, 2)$
 $c(e^+, 2)$
 $f(e^-, 0)$
 $h(e^+, 2)$
 $t(h^+, 2)$

y

$a \checkmark$
 $b \checkmark$
 $c \times$
 $d \checkmark$
 $e \checkmark$
 $f \times$
 $g \times$
 h

t

Add 2 in edges



$s a b d e h t$
 Max Flow $\Rightarrow 14$

P
 $\{s\}$
 $\{a, b, c, d, e, f, g, h, t\}$
 $\{s^w, s^f\}$
 MIN CUT $\Rightarrow 10 + 4$
 $\Rightarrow 14$