

Date: 23/1/24

ΔLP

⇒ Machine Learning ~~Machine Learning~~

→ Supervised: model training with labelled data

eg: Regression (LR), classification (KNN)

→ Unsupervised: model training with unlabelled data

eg: clustering (Kmeans)

→ Reinforcement: Model takes actions in the environment then received state updates and feedbacks

* if feature extraction (data preprocessing) is done by humans → it is Machine Learning

- Feature extraction + model = Deep Learning

↳ prediction on clean data

DL does feature engineering + model training

⇒ Applications of DL:

• GAN → deep fake, medical domain, receptionist avatar

↳ generating medical related images

• Vision Transformers → chatGPT

• CNN → news

• Face detection vs Face Recognition

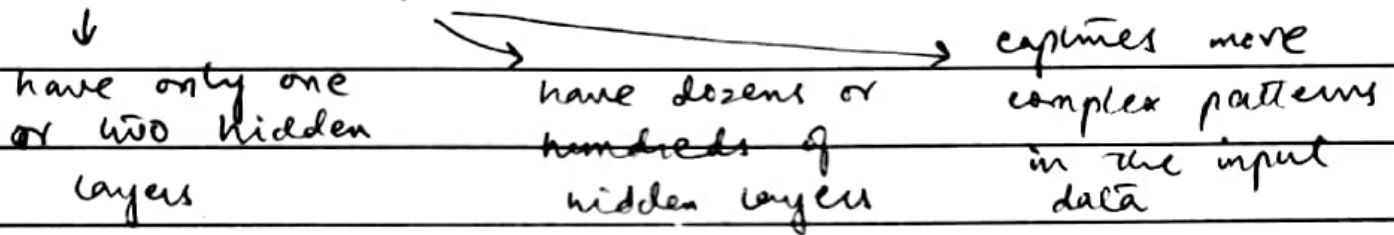
↳ either person or no

↳ specific person

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- Anomaly detection → surveillance videos
- RNN → predicts next word like in keyboard

→ Shallow vs Deep Neural Network:



→ Hyperparameters vs parameters:

- | | |
|--|--|
| <p>↓</p> <ul style="list-style-type: none">- allow the model to learn the rules from the data- estimated during training with historical data- eg: # of layers, depth, iterations, epoch, neuron in each layer | <p>↓</p> <ul style="list-style-type: none">- control how the model is training- values are set beforehand- eg: w, b |
|--|--|

→ Epoch:

Divide data into batches, each batch could be run a certain no. of times

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→ Cross validation : Divide data into ratios
no. of pairs we take might be biased

* K-fold cross validation → dataset is divided into k subsets/folds, model is evaluated and trained k times using a different fold as the validation set each time.

→ Overfitting vs Underfitting



- good accuracy on training data but bad on testing

- variance in data ↑



- bad accuracy on both training and testing data

- biasness ↑

Train	Test
-------	------

0.9	0.1
-----	-----

0.4	0.6
-----	-----

→ overfitting

0.3	0.7
-----	-----

0.3	0.7
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→ underfitting → parameters are not properly tuned

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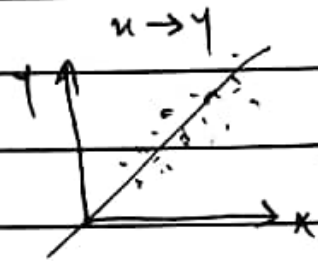
⇒ Linear Regression:

$x \rightarrow$ independent variable

$y \rightarrow$ dependent variable

$$y = mx + c$$

$$y = b_1 x + b_0 \quad | \quad y = w_1 x + w_0$$

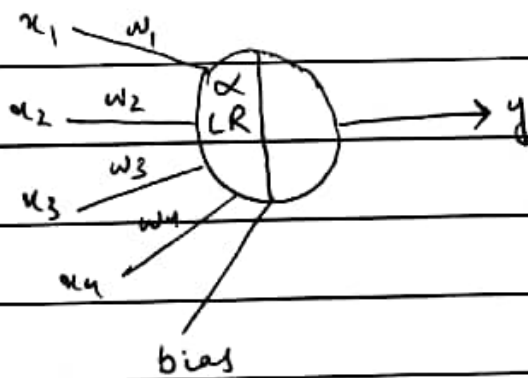


Simple LR : $\hat{y} = b_1 x + a$

Multiple LR : $\hat{y} = b_1 x_1 + b_2 x_2 + \dots + b_k x_k + a$

* Objective is to find weights

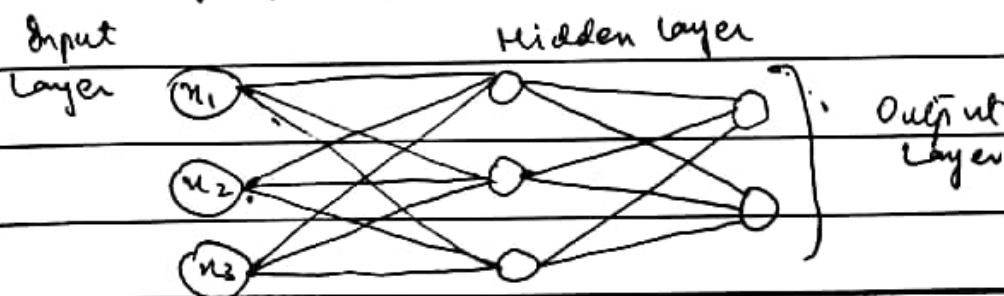
⇒ The Perceptron: Forward Propagation



$$y = w_0 + \sum_{i=1}^n w_i x_i$$

LR equation

* range for $y, x, w \rightarrow -\infty$ to $+\infty$



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* No cyclic graph, moving forward

How many hidden layers?

↳ could vary based on kisse result accha araha hai

+ weights tuning in back propagation — to minimize error by optimizing weights
parameters are tuned in back propagation

↳ weights, bias

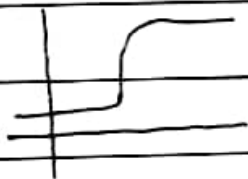
→ ReLU, Sigmoid, Softmax, Tanh

→ Activation Functions — adds non-linearity, minimizes ranges

A neuron is LR + Activation Function

↳ $-\infty$ to $+\infty$

• Sigmoid



$$\text{Sigmoid} = \frac{1}{1 + e^{-z}}$$

0 to 1

$$\frac{1}{1 + e^{-(-\infty)}} = \frac{1}{1 + e^{\infty}} = \frac{1}{\infty} = 0$$

$$\frac{1}{1 + e^{-(\infty)}} = \frac{1}{1 + e^{-\infty}} = \frac{1}{1 + \frac{1}{e^{\infty}}} = 1$$

$$g(z) = \frac{1}{1 + e^{-z}}$$

$$g'(z) = g(z)(1 - g(z))$$

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• Hyperbolic Tangent

$$g(z) = \frac{e^z - e^{-z}}{e^z + e^{-z}}$$

$$g'(z) = 1 - g(z)^2$$

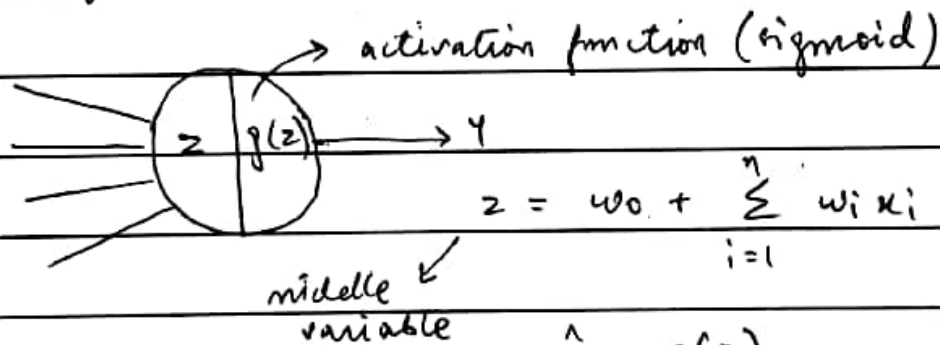
• ReLU

$$g(z) = \max(0, z)$$

$$g'(z) = \begin{cases} 1, & z > 0 \\ 0, & \text{otherwise} \end{cases}$$

LOGISTIC REGRESSION

- Single neuron is equivalent to Logistic Regression
- Classes can only be two
- Linear Regression + Activation Function = Logistic Regression



$$z = w_0 + \sum_{i=1}^n w_i x_i \quad \Bigg| \quad wx + b$$

$$\hat{y} = g(z)$$

$$g(z) = \frac{1}{1 + e^{-z}}$$

- values in terms of logs
- probability of event occurring vs the ratio of not occurring

$$\ln \left(\frac{p}{1-p} \right)$$

↳ odds

- Instead of modeling the outcome, y , directly, the method models the log odds (y) using the logistic function

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$$\text{logit}(p) = \ln\left(\frac{p}{1-p}\right) = z \Leftrightarrow p = \frac{\exp(z)}{1 + \exp(z)}$$

- sigmoid \Rightarrow maps $-\infty / +\infty$ to $0/1$
- Hyperbolic Tangent \Rightarrow maps variables to -1 to $+1$
- ReLU \Rightarrow negative values are clipped off
positive values remain same
- activation function adds non-linearity (curvy line)

$$z = \left(\sum_{i=1}^n w_i x_i \right) + b$$

$$y = \sigma(z) = \frac{1}{1 + e^{-z}}$$

$$\begin{aligned} P(y=1) &= \sigma(wx+b) \\ &= \frac{1}{1 + e^{-(wx+b)}} \end{aligned} \quad \left. \vphantom{\begin{aligned} P(y=1) &= \sigma(wx+b) \\ &= \frac{1}{1 + e^{-(wx+b)}} \end{aligned}} \right\} \begin{array}{l} \text{probability of event occurring} \\ \text{(class 1)} \end{array}$$

$$\begin{aligned} P(y=0) &= 1 - \sigma(wx+b) \\ &= 1 - \frac{1}{1 + e^{-(wx+b)}} \\ &= \frac{e^{-(wx+b)}}{1 + e^{-(wx+b)}} \end{aligned} \quad \left. \vphantom{\begin{aligned} P(y=0) &= 1 - \sigma(wx+b) \\ &= 1 - \frac{1}{1 + e^{-(wx+b)}} \\ &= \frac{e^{-(wx+b)}}{1 + e^{-(wx+b)}} \end{aligned}} \right\} \begin{array}{l} \text{probability of event not occurring} \\ \text{(class 2)} \\ \text{mostly not used} \end{array}$$

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⇒ Learning in Logistic Regression:

- 1) Loss Function / Cost Function: how close the current label (\hat{y}) is to the true label y .
- 2) Optimization algorithm: for iteratively updating weights

⇒ Loss / Cost Function

Cross entropy loss

$$L(\hat{y}, y)$$

predicted \nwarrow \nearrow actual

maximum likelihood
↳ maximize probability

- Conditional max likelihood estimation prefers the correct class labels of the training examples to be more likely.
- We choose w, b that maximize the log probability of the true y labels in the training data given the observations x .

→ Max likelihood Estimation

- For samples labelled as 1 : $\pi^{\hat{y}}$
↳ \hat{y} is as close to 1 as possible $\text{ s.t. } y_i = 1$

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- For samples labelled as 0: $\prod (1 - \hat{y})$

↳ $(1 - \hat{y})$ should be $\text{sim } y_i = 0$

as close to 1 as possible

* On combining both, $(w$ and $b)$ should be such that the product of both of these products is max over all the elements of the dataset.

$$L(\beta) = \prod_{\text{sim } y_i = 1} \hat{y} * \prod_{\text{sim } y_i = 0} (1 - \hat{y})$$

This function is the one we need to optimize and is called the 'likelihood function'

→ Loss Function:

$$\text{if } y = 1 : p(y|x) = \hat{y}$$

$$\text{if } y = 0 : p(y|x) = 1 - \hat{y}$$

$$p(y|x) = \hat{y}^y (1 - \hat{y})^{(1-y)}$$

→ Log likelihood:

this is a Bernoulli distribution since only two outcomes $(0, 1)$

$$p(y|x) = \hat{y}^y (1 - \hat{y})^{1-y}$$

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$$\begin{aligned}\log p(y|x) &= \log [\hat{y}^y (1-\hat{y})^{1-y}] \\ &= y \log \hat{y} + (1-y) \log (1-\hat{y})\end{aligned}$$

whichever values maximize a prob will also maximize the log of the probability

eg:

For class 1:

$$\begin{aligned}P(y|x) &= \hat{y}^y (1-\hat{y})^{1-y} \\ &= (0.7)^1 (1-0.7)^{1-1}\end{aligned}$$

For class 0:

$$P(y|x) = (0.7)^0 (1-0.7)^{1-0}$$

- likelihood should be maximized
- loss should be minimized

→ Cross Entropy:

negative log likelihood loss = cross-entropy loss

$$L_{CE}(\hat{y}, y) = -\log p(y|x) = -[y \log \hat{y} + (1-y) \log (1-\hat{y})]$$

$$L_{CE}(w, b) = -[y \log \sigma(wx+b) + (1-y) \log (1-\sigma(wx+b))]$$

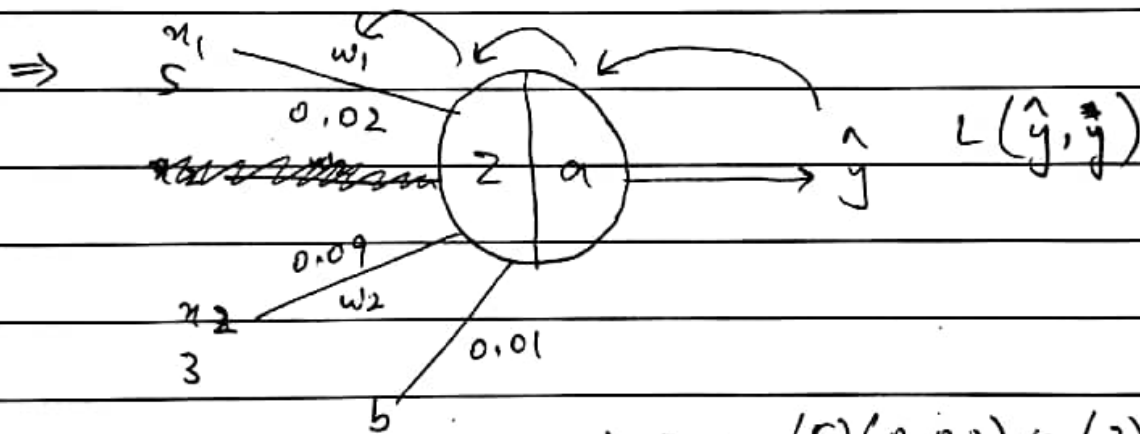
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→ Binary cross entropy loss:

↳ working on two classes

↳ calculated as the average cross-entropy across all data examples

$$L = -\frac{1}{N} \left[\sum_{j=1}^N [y \log \hat{y} + (1-y) \log (1-\hat{y})] \right]$$



$$\begin{aligned} z &= (5)(0.02) + (3)(0.09) + 0.01 \\ &= 0.38 \end{aligned}$$

$$\hat{y} = a = \frac{1}{1 + e^{-0.38}} = 0.5939$$

→ Cross entropy loss:

$$\begin{aligned} L(\hat{y}, y) &= -y \ln(\sigma(w \cdot x + b)) \\ &= -y \ln(0.5939) \\ &= 0.5210 \end{aligned}$$

$$* \log a = \frac{1}{a}$$

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→ Backpropagation:

$$\frac{dL}{dw_1} = \frac{dL}{da} \times \frac{da}{dz} \times \frac{dz}{dw_1}$$

$$L = [-y \log(\hat{y}) - (1-y) \log(1-\hat{y})]$$
$$= [-y \log a - (1-y) \log(1-a)]$$

$$\frac{dL}{da} = -y \cdot \frac{1}{a} - (1-y) \cdot \frac{1}{a} \cdot (-1)$$
$$= \frac{-y}{a} + \frac{(1-y)}{(1-a)}$$

$$a = \frac{1}{1+e^{-z}} \quad \frac{da}{dz} = a(1-a)$$

$$z = w_1 x_1 + w_2 x_2 + b$$

$$\frac{dz}{dw_1} = x_1$$

$$dw_1$$

$$\frac{dL}{dw_1} = \left[\frac{-y}{a} + \frac{(1-y)}{(1-a)} \right] \times [a(1-a)] \times [x_1]$$

$$= \frac{(1-a)(-y) + a(1-y)}{\cancel{a(1-a)}} \times \cancel{a(1-a)} \times x_1$$

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$$= (-y + ay + a - ay) x_1$$

$$= (-y + a) x_1$$

$a \Rightarrow$ predicted

$y \Rightarrow$ actual

$$\frac{\partial L}{\partial w_1} = ax_1 - yx_1 / (a - y) x_1$$

$$\frac{\partial L}{\partial w_2} = (a - y) x_2$$

$$\frac{\partial L}{\partial b} = (a - y)$$

$$\frac{\partial L}{\partial w_1} = (0.5939 - 1)(5) = -2.03$$

$$\frac{\partial L}{\partial w_2} = (0.5939 - 1)(3) = -1.218$$

$$\frac{\partial L}{\partial b} = (0.5939 - 1) = -0.4061$$

\Rightarrow update weights

$$w_1^{\text{new}} = w_1^{\text{old}} - \eta \frac{\partial L}{\partial w_1}$$

\rightarrow learning rate

$$\eta = 0.005$$

$$\begin{aligned} w_1^{\text{new}} &= 0.02 - 0.005(-2.03) \\ &= 0.03 \end{aligned}$$

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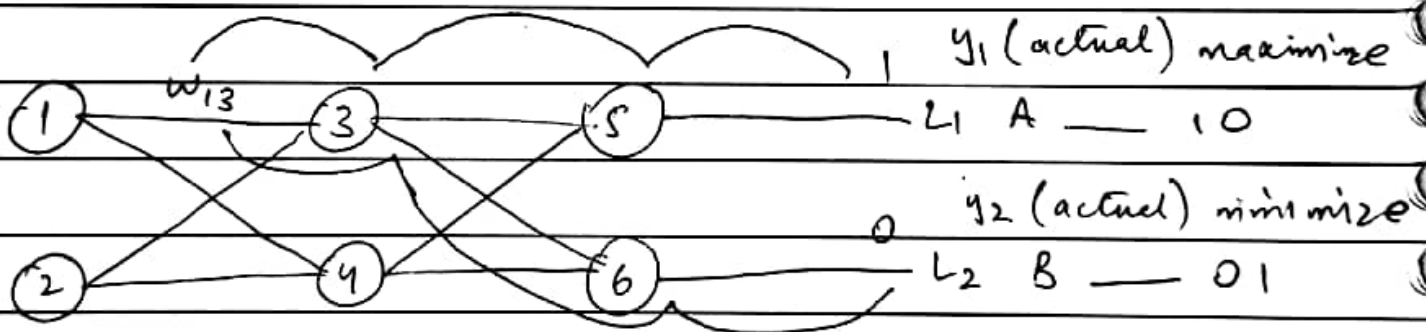
- for next iteration, use new w_1 as old w_1
- in each iteration, loss will be reduced

$$\frac{dL}{dw} = 0 \rightarrow \text{gradient descent } \text{at slope zero}$$

NEURAL NETWORKS

— from slides

⇒



$$\frac{dL}{dw_{13}} = \boxed{\frac{dL_1}{da_5}} \times \frac{da_5}{dz_5} \times \frac{dz_5}{da_3} \times \frac{da_3}{dz_3} \times \frac{dz_3}{dw_{13}}$$

$$+ \boxed{\frac{dL_2}{da_6}} \times \frac{da_6}{dz_6} \times \frac{dz_6}{da_3} \times \frac{da_3}{dz_3} \times \frac{dz_3}{dw_{13}}$$

One-hot encoding \rightarrow 4, 7, 2 — A (1, 0)
5, 6, 3 — B (0, 1)

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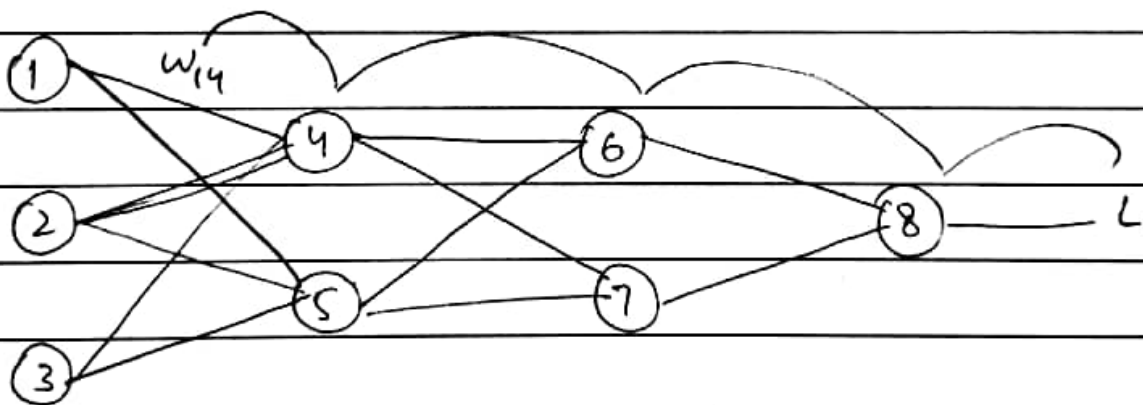
$$L = \frac{1}{2} (y - a)^2$$

$$= \frac{1}{2} (y_1 - a_5)^2 + \frac{1}{2} (y_2 - a_6)^2$$

$$= -(y_1 - a_5) \cdot a_5 (1 - a_5) a_3 + a_3 (1 - a_3) x_1 \\ - (y_2 - a_6) \cdot a_6 (1 - a_6) a_3 + a_3 (1 - a_3) x_1$$

$$\hookrightarrow \frac{\partial L}{\partial a_6}$$

\Rightarrow



$$\frac{\partial L}{\partial w_{14}} = \frac{\partial L}{\partial a_8} \times \frac{\partial a_8}{\partial z_8} \dots \frac{\partial a_4}{\partial z_4} \times \frac{\partial z_4}{\partial w_{14}}$$

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REGULARIZATION

overfitting — good accuracy on training, bad on testing

underfitting — bad accuracy on both

- NN are assumed to be overfitted models by default
- they are too complex
- training samples ko itne accuracy se learn karlete hain k wo unseen data pe perform nahi karpaata

$$w_1 x_1 + w_2 x_2$$
$$\rightarrow 0.1 x_1 + 0.3 x_2$$

more dominantly
affecting

there are so many variables
that we cannot know
their importance

⇒ Regularization makes slight modifications to the learning algorithm such that the model generalizes better and in turn improves the model's performance on the unseen data as well

- in ML, regularization penalizes the coefficients
- in DL, it penalizes the weight matrices of the nodes

Date:

- assume that the regularization coefficient is so high that some of the weight matrices are nearly equal to zero
- this will result in a much simpler linear network and slight underfitting of the training data
- a large value of the regularization coefficient is not that useful — we need to optimize the value of the coefficient

Training, testing pe sahi kaam karay — appropriate fitting to obtain a well-fitted model

⇒ L1 L2 Regularization :

L1 :

$$\text{Modified Loss Function} = \text{Loss Function} + \lambda \sum_{i=1}^n |w_i|$$

L2 :

$$\text{Modified Loss Function} = \text{Loss Function} + \lambda \sum_{i=1}^n w_i^2$$

$$0.001 [w_{14} + w_{15} + \dots]$$

When overfitting, weights values are higher like 45, 30..

↳ penalty to keep weights in range

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⇒ Dropout :

- randomly skip some neurons
- random sampling of neurons
- har layer pe probability set karake hain k in layer pe kitne neurons chahiye
- testing time pe neurons are set as it is
- eg: layer pe 50% ki prob set ki k in layer k 50% neurons chahiye
- Back propagation is done on reduced network
- clipped off weights are not updated in B.P
- Different no. of weights are updated on each iteration
- Full network on testing
- Random selection of neurons on each iteration
- Overall accuracy is improved — overfitting pe nahi jata

→ Method 1 :

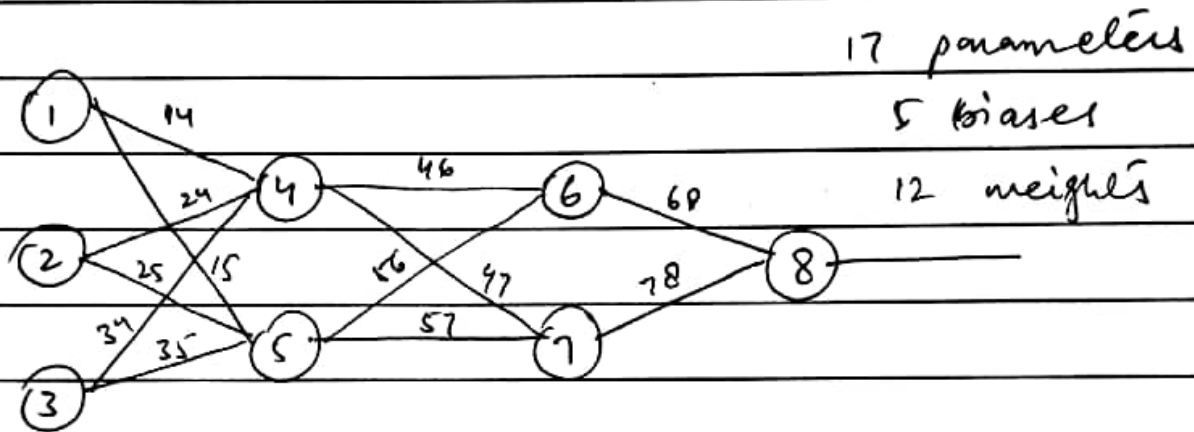
- Each unit is retained with a prob p . (train)
- weights multiplied with p (test)

→ Method 2 :

- weights multiplied with $\frac{1}{p}$ (train)
- no scaling (test)

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- Dropout main poor neuron delete hota hai —
uske ingoing and outgoing connections bhi



remove ⑤ — new parameters =

- On which layer to apply dropout?

↳ could be different combinations

CNN

↳ sharp edges, corners

- ideally $p = 0.5$

- 100 samples pe dropout ho tou error ziada ayega

↳ less data — model couldn't learn

- merge methods like dropout + L2