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20K-0477

ASSIGNMENT # 02.

Q#1

$$y'' + 4y' + 3y = 0$$
$$m^2 + 4m + 3 = 0$$
$$m^2 + 3m + m + 3 = 0$$
$$m(m+3) + 1(m+3) = 0$$
$$(m+3)(m+1) = 0$$
$$m = -3 \quad m = -1$$
$$y = c_1 e^{-3x} + c_2 e^{-x}$$

Q#2

$$y''' - y'' + y' - y = 0$$
$$m^3 - m^2 + m - 1 = 0$$

using calculator

$$m = 1, \quad m = i, \quad m = -i$$
$$\alpha = 0, \quad \beta = 1$$
$$y = c_1 e^x + c_2 \cos x + c_3 \sin x$$

Q#3

$$2x^2 y'' + 3xy' - 15y = 0$$
$$y = x^m$$
$$y' = mx^{m-1}$$
$$y'' = m(m-1)x^{(m-2)}$$
$$2x^2(m^2-m)x^{m-2} + 3xm(x^{m-1}) - 15x^m = 0$$
$$(2m^2 - 2m)x^m + 3m(x^m) - 15x^m = 0$$
$$x^m(2m^2 - 2m + 3m - 15) = 0$$
$$2m^2 + m - 15 = 0$$
$$2m^2 + 6m - 5m - 15 = 0$$
$$2m(m+3) - 5(m+3) = 0$$
$$(m+3)(2m-5) = 0$$
$$m = -3, \quad m = 5/2$$
$$y = c_1 x^{-3} + c_2 x^{-3}$$

Q#4

$$y'' - 3y' + 2y = x^2 e^x$$

$$m^2 - 3m + 2 = 0$$
$$m^2 - 2m - m + 2 = 0$$
$$m(m-2) - 1(m-2) = 0$$
$$(m-2)(m-1) = 0$$
$$m = 2 , m = 1$$

$$y_c = c_1 e^x + c_2 e^{2x}$$
$$y_1 = e^x \quad y_2 = e^{2x}$$

$$W(y_1, y_2) = \begin{vmatrix} e^x & e^{2x} \\ e^x & 2e^{2x} \end{vmatrix}$$
$$= 2e^{3x} - e^{3x}$$
$$W(y_1, y_2) = e^{3x}$$
$$q(x) = x^2 e^x$$

$$u_1 = -\int \frac{e^{2x} \cdot x^2 e^x}{e^{3x}} \, dx$$
$$u_1 = -\int x^2$$
$$u_1 = -x^3/3$$

$$u_2 = \int \frac{e^x \cdot x^2 e^x}{e^{3x}}$$
$$u_2 = \int x^2 e^{-x}$$
$$u_2 = -x^2 e^{-x} - \int -2x e^{-x} \, dx \Rightarrow \text{ By parts.}$$
$$u_2 = -x^2 e^{-x} + 2(-x e^{-x} - e^{-x} \, dx)$$
$$u_2 = -x^2 e^{-x} + 2(-x e^{-x} - e^{-x})$$
$$u_2 = -x^2 e^{-x} + 2(-x e^{-x} - e^{-x})$$
$$u_2 = -x^2 e^{-x} - 2x e^{-x} - 2e^{-x}$$

$$y_p = y_1 u_1 + y_2 u_2$$
$$y_p = (-x^3/3) e^x + (-x^2 e^{-x} - 2x e^{-x} - 2e^{-x}) e^{2x}$$
$$y_p = c_1 e^x + c_2 e^{2x} - x^3 e^x/3 - x^2 e^{-x}$$

Q#5

$$y'' + 4y = xe^x + x\sin 2x$$

$$m^2 + 4 = 0$$

$$m^2 = -4$$

$$m = \pm 2i$$

$$y_c = C_1 \cos 2x + C_2 \sin 2x$$

$$y_{p_1} = (Ax + B)e^x$$

$$y_{p_1}' = (Ax + B)e^x + Ae^x$$

$$y_{p_1}'' = (Ax + b)e^x + 2Ae^x$$

$$(Ax + B)e^x + 2Ae^x + 4e^x(Ax + B) = xe^x$$

$$5(Ax + B)e^x + 2Ae^x = xe^x$$

$$5Ax = x \qquad 5B + 2A = 0$$

$$A = 1/5 \qquad B = -2/25$$

$$y_{p_1} = 1/5\,xe^x - 2/25\,xe^x$$

$$y_{p_2} = (Ax^2 + Bx)\cos 2x + (Cx^2 + Dx)\sin 2x$$

$$y_{p_2}' = 2Ax\cos 2x - 2Ax^2\sin 2x - 2B\cos 2x + B\cos 2x + 2Cx^2\cos 2x + 2Cx\sin 2x + 2D\cos 2x + D\sin 2x$$

$$y_{p_2}'' = -4Ax^2\cos 2x - 4Ax\sin 2x - 4Ax\sin 2x + 2A\cos 2x - 4Bx\cos 2x - 2B\sin 2x - 2B\sin 2x - 4Cx^2\sin 2x + 4x\cos 2x + 4Cx\cos 2x + 2C\sin 2x - 4Dx\sin 2x + 2D\cos 2x + 2D\cos 2x$$

$$y'' + 4y = x\sin 2x$$

$$-8Ax\sin 2x + 2A\cos 2x - 4B\sin 2x + 2C\sin 2x + 4D\cos 2x$$

$$8Cx\cos 2x = x\sin 2x$$

$$-9A = 1 \qquad\qquad 8C = 0$$

$$A = -1/8 \qquad\qquad C = 0$$

$$2A + 4D = 0$$

$$D = 1/16$$

$$-4B + 2C = 0$$

$$B = 0$$

$$y = c_1 \sin 2x + c_2 \cos 2x + \frac{1}{5} x e^x - \frac{2}{25} x e^x + \frac{1}{16}.$$
$$x \sin 2x - \frac{1}{8} x^2 \cos 2x.$$

Q#6

$$y'' - 2y' + y = x e^x \ln x.$$
$$m^2 - 2m + 1 = 0$$
$$(m - 1)^2 = 0$$
$$m - 1 = 0$$
$$m = 1$$
$$y_c = c_1 e^x + c_2 x e^x.$$
$$y_1 = e^x \qquad y_2 = x e^x.$$

$$W(y_1, y_2) = \begin{vmatrix} e^x & x e^x \\ e^x & e^x + x e^x \end{vmatrix}$$
$$= e^x (e^x + x e^x) - e^x x e^x.$$
$$= e^{2x} + x e^{2x} - x e^{2x}$$
$$= e^{2x}.$$

$$u_1 = - \int \frac{x e^x \cdot x e^x \ln x}{e^{2x}}$$
$$\qquad = - \int x^2 \ln x$$
Apply By Parts.

$$u_1 = x^3/9 - 3x^3 \ln x / 9.$$

$$u_2 = \int \frac{e^x \cdot e^x x \ln x}{e^{2x}}$$
$$\qquad = \int x \ln x$$
Apply By part
$$u_2 = 2x^2 \ln x - x^2/4 \Big/ 4.$$

$$y_p = \left(\frac{x^3}{9} - \frac{3x^3 \ln x}{9}\right) e^x + \left(\frac{2x^3 \ln x}{4} - \frac{x^2}{4}\right) x e^x$$

$$y_p = \left(\frac{\ln x \, x^3}{6} - \frac{5x^3}{36}\right) e^x$$

$$y = c_1 e^x + c_2 x e^x + \left(\frac{x^3 \ln x}{6} - \frac{5x^3}{36}\right) e^x$$

Q#8

$$y'' - 4y' - 12y = 2t^3 - t + 3$$

$$m^2 - 4m - 12 = 0$$
$$m^2 - 6m + 2m - 12 = 0$$
$$m(m-6) + 2(m-6) = 0$$
$$(m-6)(m+2) = 0$$
$$m_1 = -2 \quad, \quad m_2 = +6$$

~~Ex#9~~

$$y_c = c_1 e^{-2t} + c_2 e^{6t}$$
$$y_p = Ax^3 + Bx^2 + Cx + D$$
$$y_{p_1}' = 3Ax^2 + 2Bx + c$$
$$y_{p_2}'' = 6Ax + 2B$$

$$6Ax + 2B - 12Ax^2 - 8Bx - 4c - 12Ax - 12Ax^3 - 12Bx^2 - 12Cx$$
$$- 12D$$
$$= 2x^3 - x + 3$$

$$-12A = 2$$
$$A = -1/6$$
$$6A - 12C - 8B = -1$$
$$C = -1/9$$
$$-12A - 12B = 0$$
$$B = 1/6$$
$$3B - 4C - 12D = 3$$
$$D = -5/27$$

$$y = C_1 e^{-2t} + C_2 e^{6t} - \frac{1}{6}t^3 + \frac{1}{6}t^2 - \frac{1}{9}t - \frac{5}{27}$$

Q#7

$$y'' + 4y = \sec 2x$$
$$m^2 + 4 = 0$$
$$m^2 = -4$$
$$m = \pm 2i$$

$$y_c = C_1 \cos 2x + C_2 \sin 2x$$
$$y_1 = \cos 2x \qquad y_2 = \sin 2x$$

$$W(y_1, y_2) = \begin{vmatrix} \cos 2x & \sin 2x \\ -2\sin 2x & 2\cos 2x \end{vmatrix}$$
$$= 2\cos^2 2x - (-2\sin^2 2x)$$
$$= 2.$$

$$u_1 = -\int \frac{\sec 2x \, \sin 2x}{2}$$

$$u_1 = -\frac{1}{2}\int \tan 2x.$$

$$u_1 = \frac{-\ln|\sec 2x|}{4}$$

$$u_2 = \int \frac{\sec 2x \, \cos 2x}{2}$$

$$u = \int \frac{1}{2} \, dx$$

$$u = \frac{x}{2}$$

$$y_p = -\frac{\cos 2x \, \ln|\sec 2x|}{4} + \frac{x \sin 2x}{2}$$

$$y = C_1 \cos 2x + C_2 \sin 2x + \frac{x \sin 2x}{2} - \frac{\cos 2x \, \ln|\sec 2x|}{4}$$

Q#9

$$y'' + 5y' - 6y = 2x.$$
$$m^2 + 5m + 6 = 0$$

using calculator

$$m_1 = -2 \quad m_2 = -3.$$
$$y_c = c_1 e^{-2x} + c_2 e^{-3x}$$

$$y_p = Ax + B$$
$$y_p' = A$$
$$y_p'' = 0$$
$$5A + 6Ax + 6B = 2x.$$
$$6A = 2 \qquad 5A + 6B = 0$$
$$A = 1/3 \qquad B = -5/18.$$

$$y_p = x/3 - 5/18.$$

$$y = c_1 e^{-2x} + c_2 e^{-3x} - 05/18 + x/3.$$

Q#10

$$y'' + 5y' - 9y = e^{-2x} + 2 - x.$$
$$m^2 + 5m - 9 = 0$$
$$m_1 = 1.4 \ , \ m_2 = -6.4$$
$$y_c = c_1 e^{1.4x} + c_2 e^{-6.4x}$$

$$y_{p_1} = Ae^{-2x}$$
$$y_{p_1}' = -2Ae^{-2x}$$
$$y_{p_1}'' = -4Ae^{-2x}$$
$$4Ae^{-2x} - 10Ae^{-x} - 9Ae^{-2x} = e^{2x}$$
$$A = -1/15$$

$$y = C_1 e^{-2t} + C_2 e^{6t} - \tfrac{1}{6} t^3 + \tfrac{1}{6} t^2 - \tfrac{1}{9} t - \tfrac{5}{27}$$

Q#7

$$y'' + 4y = \sec 2x$$
$$m^2 + 4 = 0$$
$$m^2 = -4$$
$$m = \pm 2i$$

$$y_c = C_1 \cos 2x + C_2 \sin 2x$$
$$y_1 = \cos 2x \qquad y_2 = \sin 2x$$

$$W(y_1, y_2) = \begin{vmatrix} \cos 2x & \sin 2x \\ -2\sin 2x & 2\cos 2x \end{vmatrix}$$
$$= 2\cos^2 2x - (-2\sin^2 2x)$$
$$= 2$$

$$u_1 = -\int \frac{\sec 2x \, \sin 2x}{2}$$

$$u_1 = -\tfrac{1}{2} \int \tan 2x$$

$$u_1 = \frac{-\ln |\sec 2x|}{4}$$

$$u_2 = \int \frac{\sec 2x \, \cos 2x}{2}$$

$$u_2 = \int \tfrac{1}{2} \, dx$$

$$u_2 = \tfrac{x}{2}$$

$$y_p = -\cos 2x \, \frac{\ln |\sec 2x|}{4} + \frac{x \sin 2x}{2}$$

$$y = C_1 \cos 2x + C_2 \sin 2x + \frac{x \sin 2x}{2} - \frac{\cos 2x \, |\ln \sec 2x|}{4}$$

$$y_{P_2} = A\cos t + [Bt + C]\sin t.$$
$$y_{P_2}' = -A\sin t + \cos t (Bt + c) + \sin t (B).$$
$$y_{P_2}'' = -A\cos t + B\cos t - \sin t(Bt+c) + B\cos t + B\sin t$$

$$-A\cos t + B\cos t - \sin t[Bt+c] + B\cos t + \sin t = \cos t - t\sin t$$

$$-101A + 2B = 1 \qquad\qquad -101B = -1$$
$$A = -99/10201 \qquad\qquad B = 01/101$$

$$-2C = 0$$
$$C = 0.$$

$$y_{P_2} = -\frac{99\cos t}{10201} + \frac{t\sin t}{101}$$

$$y = C_1 e^{10t} + C_2 e^{-10t} + \frac{3t^3 e^{10t}}{20} - \frac{9t^2 e^{10t}}{400} +$$

$$\frac{9t e^{10t}}{4000} + \frac{t\sin t}{101} - \frac{99\cos t}{10201}$$

Q#12

$$y'' - 2y' + 2y = e^x \tan x$$
$$m^2 - 2m + 2 = 0$$
using calculator
$$m = 1 \pm i$$
$$y_c = e^x (C_1 \cos x + C_2 \sin x).$$

$$y_1 = \cos x e^x \qquad y_2 = e^x \sin x.$$

$$W(y_1, y_2) = \begin{vmatrix} e^x \cos x & e^x \sin x \\ e^x \cos x - e^x \sin x & e^x \sin x + e^x \cos x \end{vmatrix}$$

$$= e^{2x}$$

$$u_1 = -\int \frac{e^x \tan x \cdot e^x \sin x}{e^{2x}}$$

$$= -\int \frac{\sin^2 x}{\cos x} \, dx$$

$$u_1 = -\ln|\tan x + \sec x| + \sin x$$

$$u_2 = \int \frac{e^x \tan x \cdot e^x \cos x}{e^{2x}}$$

$$u_2 = \int \sin x \, dx$$

$$u_2 = -\cos x$$

$$y_p = -e^x \cos x \ln|\tan x + \sec x|$$

$$y = e^x(c_1 \cos x + c_2 \sin x) - e^x \cos x \ln|\sec x + \tan x|$$

Q#18

$$x^2 y'' - 4xy' + 6y = 2x^4 + x^2$$

$$m^2 - 4m + 6 = 0$$

$$am^2 + (b-a)m + c = 0$$

$$m^2 - 5m + 6 = 0$$

$$m^2 - 3m - 2m + 6 = 0$$

$$m(m-3) - 2(m-3) = 0$$

$$(m-3)(m-2) = 0$$

$$m_1 = 2, \quad m_2 = 3$$

$$y_c = c_1 x^3 + c_2 x^2$$

$$y_p = x^4(Ax^4 + Bx^3 + Cx^2 + Dx + E)$$

$$y_p' = Ax^8 + Bx^7 + Cx^6 + Dx^5 + Ex^4$$

$$y_p' = 8Ax^7 + 7Bx^6 + 6Cx^5 + 5Dx^4 + 4Ex^3$$

$$y_p'' = 56Ax^6 + 42Bx^5 + 30Cx^4 + 20Dx^3 + 12Ex^2$$

After.
Solving
$y = B$
$2E = 2$
$E = 1$

$$y = c_1 x^3 + c_2 x^2 + x^4 = x^0 \ln x$$

Q#14

$$x^2 y'' + 10 xy' + 8y = x^2$$
$$am^2 + (b-a)m + c = 0$$
$$m^2 + 9m + 8 = 0$$
$$m^2 + 8m + m + 8 = 0$$
$$m(m+8) + 1(m+8) = 0$$
$$(m+8)(m+1) = 0$$
$$m_1 = -1 \, , \, m_2 = -8$$

$$y_c = c_1 x^{-3} + c_2 x^{-1}$$
$$y_p = Ax^2 + Bx + C$$
$$y_p' = 2Ax + B$$
$$y_p'' = 2A$$

$$2Ax^2 + 20Ax^2 + 10Bx + 8Ax + 8Bx + 8C = x^2$$
$$30Ax^2 + 18Bx + 8C = x^2$$
$$A = 1/30 \, , \, B = 0 \, , \, C = 0.$$

$$y = c_1 x^{-3} + c_2 x^{-1} + 1/30 x^2.$$

Q #18

$x^2 y'' - 3xy' + 13y = 4 + 3x$

$m^2 - 4m + 13 = 0$

using calculator

$m = 2 \pm 3i$

$y_c = x^2 (C_1 \cos 3 \ln x + C_1 \sin 3 \ln x)$

$y_p = Ax + B$

$y_p' = A$

$y_p = 0$

$-3Ax + 13Ax + 13B = 4 + 3x$

$10A = 3$ $13B = 4$

$A = 3/10$ $B = 4/13$

$y_p = 3/10 x + 4/13$

$y = x^2 (C_1 \cos 3 \ln x + C_2 \sin 3 \ln x) + 3x/10 + 4/13$

Q#16

$$y'' - 2y' + y = \frac{1}{x}e^x$$

$$m^2 - 2m + 1 = 0$$

$$m^2 - m - m + 1 = 0$$

$$m(m-1) - 1(m-1) = 0$$

$$(m-1)^2 = 0$$

$$m = 1$$

$$y_c = c_1 e^x + c_2 x e^x$$

$$W = \begin{vmatrix} e^x & xe^x \\ e^x & xe^x + e^x \end{vmatrix} = xe^{2x} + e^{2x} - xe^{2x}e^x = e^{2x}$$

$$W_1 = \begin{vmatrix} 0 & xe^x \\ \frac{1}{x}e^x & xe^x + e^x \end{vmatrix} = -xe^x \cdot \frac{1}{x}e^x = -e^{2x} + \frac{1}{x}e^x$$

$$W_2 = \begin{vmatrix} e^x & 0 \\ e^x & \end{vmatrix}$$

$$W(y_1, y_2) = \begin{vmatrix} e^x & xe^x \\ e^x & xe^x + e^x \end{vmatrix}$$

$$= xe^{2x} + e^{2x} - xe^{2x}$$

$$= e^{2x}$$

$$u_1 = -\int \frac{xe^x \cdot e^x}{xe^{2x}}$$

$$= -x$$

$$u_2 = \int \frac{e^x e^x}{xe^{2x}} -$$

$$= \ln x$$

$$y_p = -x e^x + x e^x \ln x$$
$$y = c_1 e^x + c_2 e^x x - x e^x + x e^x \ln x$$
$$y' = c_1 e^x + c_2 e^x + x c_2 e^x - e^x - x e^x + x e^x \ln x +$$
$$e^x \ln x + e^x.$$

$$y(1) = 0$$
$$c_1 e + c_2 e - e = 0$$
$$c_1 = -c_2 + 1$$
$$y'(1) = 1$$
$$c_1 e + 2 c_2 e - e = 1$$
$$1/e = -c_2 + 1 + 2 c_2 - 1$$
$$c_2 = 1/e.$$

$$c_1 = -c_2 + 1$$
$$c_1 = e - 1/e$$

$$y = \left(\frac{e-1}{e}\right) e^x + 1/e \, e^x x - x e^x + x e^x \ln x$$
$$y = e^x \left(\frac{e-1}{e} + \frac{x}{e} - \frac{xe}{e}\right) + x e^x \ln x$$
$$y = e^{x-1}(e-1)(1-x) + x e^x \ln x$$

Q#17

$$y'' + 4y = \sin 2x$$
$$m^2 + 4 = 0$$
$$m = \pm 2i$$
$$y_c = c_1 \cos 2x + c_2 \sin 2x$$
$$y_1 = \cos 2x \quad y_2 \sin 2x$$

$$W(y_1, y_2) = \begin{vmatrix} \cos 2x & \sin 2x \\ -2\sin 2x & 2\cos 2x \end{vmatrix}$$

$$= 2$$

$$u_1 = -\int \frac{\sin^2 2x \cdot \sin 2x}{2}$$

$$u_1 = -\int \frac{\sin^3 2x}{2}$$

$$u_1 = \cos 2x/4 - \cos^3 2x/12$$

$$u_2 = \int \frac{\sin^2 2x \cdot \cos 2x}{2}$$

$$u_2 = \frac{1}{12} \sin^3 2x$$

$$y_p = \cos^3 2x/4 - \frac{1}{12} \cos^4 2x + \frac{1}{12} \sin^4 2x$$

$$y = c_1 \cos 2x + c_2 \sin 2x + \cos^3 2x/4 - \frac{1}{12} \cos^4 2x + \frac{1}{12} \sin^4 2x$$

$$y' = -2\sin 2x\, c_1 + 2\cos 2x\, c_2 + \frac{1}{2} \sin 2x \cos 2x - \frac{1}{6} \sin 2x$$

$$\cos^3 2x + \sin^3 2x \cos 2x/12$$

$$y(1) = 0$$
$$c_1 + 1/4 - 1/12 = 0$$

$$c_1 = -1/6$$
$$y'(0) = 0$$
$$2c_2 = 0$$
$$c_2 = 0$$

$$y = -\frac{1}{6} \cos 2x + \frac{1}{12} \sin^4 2x + \cos^3 2x/4 - \frac{1}{12} \cos^4 2x$$
$$y = -\frac{1}{6} \cos 2x + \frac{1}{6} \cos^2 2x - \frac{1}{12} \sin^3 2x$$

Q#18

$$y'' - 6y' - 7y = -9e^{-2x}$$

$$m^2 - 6m - 7 = 0$$

$$m^2 - 7m + m - 7 = 0$$

$$m(m-7) + 1(m-7) = 0$$

$$(m-7)(m+1) = 0$$

$$m = 7 \quad m = -1$$

$$y_c = c_1 e^{-x} + c_2 e^{7x}$$

$$y_p = A e^{-2x}$$

$$y_p' = -2A e^{-2x}$$

$$y_p'' = 4A e^{-2x}$$

$$4Ae^{-2x} + 12Ae^{-2x} - 2Ae^{-2x} = -9e^{-2x}$$

$$9e^{-2x} \cdot A = -9e^{-2x}$$

$$A = 1$$

$$y = c_1 e^{-x} + c_2 e^{7x} - e^{-2x}$$

$$y' = -c_1 e^{-x} + 7c_2 e^{7x} + 2e^{-2x}$$

$$y(0) = -2$$

$$c_1 + c_2 - 1 = -2$$

$$c_1 + c_2 = -1$$

$$y'(0) = -13$$

$$-c_1 + 7c_2 + 2 = -13$$

$$c_1 = 7c_2 + 15$$

Solving simultaneously

$$7c_2 + 15 + c_2 = -1$$

$$c_2 = -2$$

$$7(-2) + 15 = c_1$$

$$c_1 = 1$$

$$y = e^{-x} - 2e^{7x} - e^{-2x}$$

Q#19

$$y'' - 4y' + 4y = 2e^{2x} - 12\cos 3x - 5\sin 3x .$$

$$m^2 - 4m + 4 = 0$$

$$m^2 - 2m - 2m + 4 = 0$$

$$m(m-2) - 2(m-2) = 0$$

$$(m-2)^2 = 0$$

$$m - 2 = 0$$

$$m = 2.$$

$$y_c = C_1 e^{2x} + C_2 e^{2x}.$$

$$y_{p_1} = Ax^2 e^{2x}$$

$$y_{p_1}' = 2Ax e^{2x} + 2A x^2 e^{2x}$$

$$y_{p_1}'' = 2A e^{2x} + 4Ax e^{2x} + 4Ax e^{2x} + 4Ax^2 e^{2x}$$

$$2A e^{2x} + 8A x e^{2x} + 4Ax^2 e^{2x} - 8Ax e^{2x} - 8Ax^2 e^{2x} + 4Ax^2 e^{2x} = 2e^{2x}$$

$$2A = 2$$

$$A = 1$$

$$y_{p_1} = x^2 e^{2x}.$$

$$y_{p_2} = B\cos 3x + C\sin 3x$$

$$y_{p_2}' = -3B\sin 3x + 3C\cos 3x$$

$$y_{p_2}'' = -9B\cos 3x - 9C\sin 3x.$$

$$-9B\cos 3x + 9C\sin 3x + 12B\sin 3x - 12C\cos 3x + 4B\cos 3$$
$$+ 4C\sin 3x = -12\cos 3x - 5\sin 3x$$

$$5B - 12C = -12$$

$$-5C - 12B = -5$$

So Nirg simultne—

$$c = 1 \quad B = 0$$

$$y_{p_2} = \sin 3x.$$

$y_p = x^2 e^{2x} + \sin 3x$

$y = c_1 e^{2x} + c_2 x e^{2x} + \sin 3x + x^2 e^{2x}$

$y' = 2 c_1 e^{2x} + 2x c_2 e^{2x} + c_2 e^{2x} + 2x e^{2x} + 2x^2 e^{2x} +$ The answer + $3x^2 e^{2x}$ + $2x e^{2x}$

$3 \cos 3x$

$y(0) = -2$

$c_1 = -2.$

$y'(0) = 4.$

$2c_1 + c_2 + 3 = 4$

$c_2 = 5$

$y = -2e^{2x} + 5x e^{2x} + x^2 e^{2x} + \sin 3x$