Homework III
Advanced Topics in Optimization

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1 Question 1

system description: $1|prec|h_{\max}$

jobs	1	2	3	4	5	6	7
p_j	4	8	12	7	6	9	9
$h_j(C_j)$	$3C_1$	77	C_3^2	$1.5C_4$	$70+\sqrt{C_5}$	$1.6C_6$	$1.4C_7$

$$1 \to 7 \to 6$$
$$5 \to 7$$
$$5 \to 4$$

Set $J=\emptyset,\ J^c=\{1,2,\cdots,7\},\ J'=\{2,3,4,6\}$

$$h_{j*}\left(\sum_{k\in J^c} p_k\right) = \min_{j\in J'}\left(h_j\left(\sum_{k\in J^c} p_k\right)\right)$$

$\sum_{k\in J^c} p_k = 55$

$$h_2\left(\sum_{k\in J^c} p_k\right) = 77$$
$$h_3\left(\sum_{k\in J^c} p_k\right) = (55)^2$$
$$= 3025$$
$$h_4\left(\sum_{k\in J^c} p_k\right) = 1.5\times 55$$
$$= 82.5$$
$$h_6\left(\sum_{k\in J^c} p_k\right) = 1.6\times 55$$
$$= 88$$

$\min_{j\in J'}\left(h_j\left(\sum_{k\in J^c} p_k\right)\right) = 77$

$J=\{2\},\ J^c=\{1,3,4,5,6,7\},\ J'=\{3,4,6\}$

$\sum_{k \in J^c} p_k = 47$

$$h_3\left(\sum_{k \in J^c} p_k\right) = (47)^2$$
$$= 2209$$
$$h_4\left(\sum_{k \in J^c} p_k\right) = 1.5 \times 47$$
$$= 70.5$$
$$h_6\left(\sum_{k \in J^c} p_k\right) = 1.6 \times 47$$
$$= 75.2$$

$\min_{j \in J'}\left(h_j\left(\sum_{k \in J^c} p_k\right)\right) = 70.5$

$J = \{2,4\},\ J^c = \{1,3,5,6,7\},\ J' = \{3,6\}$
$\sum_{k \in J^c} p_k = 40$

$$h_3\left(\sum_{k \in J^c} p_k\right) = (40)^2$$
$$= 1600$$
$$h_6\left(\sum_{k \in J^c} p_k\right) = 1.6 \times 40$$
$$= 64$$

$\min_{j \in J'}\left(h_j\left(\sum_{k \in J^c} p_k\right)\right) = 64$

$J = \{2,4,6\},\ J^c = \{1,3,5,7\},\ J' = \{3,7\}$
$\sum_{k \in J^c} p_k = 31$

$$h_3\left(\sum_{k \in J^c} p_k\right) = (31)^2$$
$$= 961$$
$$h_7\left(\sum_{k \in J^c} p_k\right) = 1.4 \times 31$$
$$= 43.4$$

$\min_{j \in J'}\left(h_j\left(\sum_{k \in J^c} p_k\right)\right) = 43.4$

$J = \{2,4,6,7\},\ J^c = \{1,3,5\},\ J' = \{1,3,5\}$
$\sum_{k \in J^c} p_k = 22$

$$h_1\left(\sum_{k\in J^c} p_k\right) = 3\times 22$$
$$= 66$$
$$h_3\left(\sum_{k\in J^c} p_k\right) = (22)^2$$
$$= 484$$
$$h_5\left(\sum_{k\in J^c} p_k\right) = 70+\sqrt{22}$$
$$= 74.69$$
$\min_{j\in J'}\left(h_j\left(\sum_{k\in J^c} p_k\right)\right) = 66$

$J = \{2,4,6,7,1\},\ J^c = \{3,5\},\ J' = \{3,5\}$
$\sum_{k\in J^c} p_k = 18$

$$h_3\left(\sum_{k\in J^c} p_k\right) = (18)^2$$
$$= 324$$
$$h_5\left(\sum_{k\in J^c} p_k\right) = 70+\sqrt{18}$$
$$= 74.24$$
$\min_{j\in J'}\left(h_j\left(\sum_{k\in J^c} p_k\right)\right) = 74.24$

$J = \{2,4,6,7,1,5,3\},\ J^c = \emptyset,\ J' = \emptyset$

2 Question 2

system description: $1||\sum T_j$

jobs	1	2	3	4	5	6	7	8
p_j	6	18	12	10	10	11	5	7
d_j	8	42	44	24	26	26	70	75

Lemma 3.4.1. If $p_j \le p_k$ and $d_j \le d_k$, then there exists an optimal sequence in which job j is scheduled before job k.
$1 \rightarrow 2,3,4,5,6,8\ \ (7)$
$2 \rightarrow\ \ (3,7,8)$
$3 \rightarrow\ \ (2,7,8)$ $4 \rightarrow 2,3,5,6\ \ (7,8)$
$5 \rightarrow 2,3,6\ \ (7,8)$
$6 \rightarrow 2,3\ \ (7,8)$
$7 \rightarrow 8\ \ (1,2,3,4,5,6)$
$8 \rightarrow\ \ (2,3,4,5,6)$

$V(\emptyset,t) = 0$
$V(\{j\},t) = \max(0, t+p_j-d_j)$

$V(J(j,l,k),t)$
$= \min_\delta \left(V(J(j,k'+\delta,k'),t) + \max(0, C_{k'}(\delta) - d_{k'}) + V(J(k'+\delta+1,l,k'),C_{k'}(\delta))\right)$
Where $p_{k'} = \max\left(p_{j'} | j' \in J(j,l,k1)\right)$
$V(\{1,\cdots,8\},0) =$

3 Question 3

Consider the problem $1|prec|\max(h_1(S_1),\cdots,h_n(S_n))$, where S_j denotes the starting time of job j. The cost function h_j , $j=1,\cdots,n$ is decreasing. Unforced idleness of the machine is not allowed.Describe a dynamic programming type algorithm for this problem similar to the one in Section 3.2. Why does one have to use here forward dynamic programming instead of backward dynamic programming?
placeholder

4 Question 4

system description: $P_m||C_{\max}$

$$C_{\max}(LS) \leq \left(2 - \frac{1}{m}\right) C^*_{\max}$$

Find an example of an ordering of jobs such that the inequality holds with equality.
Given m identical machines in parallel.
Consider $m-1$ job with arbitrarily long processing time, such that $p_i = C^*_{\max} \forall i$
Consider single jobs j and k with processing times $\frac{m-1}{m}C^*_{\max}$ and $\frac{1}{m}C^*_{\max}$ respectively.
Therefore, the trivial optimal makespan is $C^*_{\max}$
Now consider a list such that job j and k are not processed by the same machine under the LS algorithm.
The resulting makespan would be an equivalence of
$C^*_{\max} + \frac{m-1}{m}C^*_{\max} = \left(2 - \frac{1}{m}\right) C^*_{\max}$.

5 Question 5

(a)

(b)

(c)