Homework III
Advanced Topics in Optimization

Basil R. Yap

2018 February 16

1 Question 1

system description: $1|prec|h_{\max}$

jobs	1	2	3	4	5	6	7
p_j	4	8	12	7	6	9	9
$h_j(C_j)$	$3C_1$	77	C_3^2	$1.5C_4$	$70+\sqrt{C_5}$	$1.6C_6$	$1.4C_7$

$$\begin{aligned} 1 \to 7 \to 6 \\ 5 \to 7 \\ 5 \to 4 \end{aligned}$$

Set $J = \emptyset,\ J^c = \{1, 2, \cdots, 7\},\ J' = \{2, 3, 4, 6\}$

$$h_{j*}\left(\sum_{k \in J^c} p_k\right) = \min_{j \in J'}\left(h_j\left(\sum_{k \in J^c} p_k\right)\right)$$

$\sum_{k \in J^c} p_k = 55$

$$\begin{aligned} h_2\left(\sum_{k \in J^c} p_k\right) &= 77 \\ h_3\left(\sum_{k \in J^c} p_k\right) &= (55)^2 \\ &= 3025 \\ h_4\left(\sum_{k \in J^c} p_k\right) &= 1.5 \times 55 \\ &= 82.5 \\ h_6\left(\sum_{k \in J^c} p_k\right) &= 1.6 \times 55 \\ &= 88 \end{aligned}$$

$\min_{j \in J'}\left(h_j\left(\sum_{k \in J^c} p_k\right)\right) = 77$

$J = \{2\},\ J^c = \{1, 3, 4, 5, 6, 7\},\ J' = \{3, 4, 6\}$

$\sum_{k \in J^c} p_k = 47$

$$h_3\left(\sum_{k \in J^c} p_k\right) = (47)^2$$
$$= 2209$$
$$h_4\left(\sum_{k \in J^c} p_k\right) = 1.5 \times 47$$
$$= 70.5$$
$$h_6\left(\sum_{k \in J^c} p_k\right) = 1.6 \times 47$$
$$= 75.2$$

$\min_{j \in J'} \left(h_j\left(\sum_{k \in J^c} p_k\right)\right) = 70.5$

$J = \{2, 4\},\ J^c = \{1, 3, 5, 6, 7\},\ J' = \{3, 6\}$
$\sum_{k \in J^c} p_k = 40$

$$h_3\left(\sum_{k \in J^c} p_k\right) = (40)^2$$
$$= 1600$$
$$h_6\left(\sum_{k \in J^c} p_k\right) = 1.6 \times 40$$
$$= 64$$

$\min_{j \in J'} \left(h_j\left(\sum_{k \in J^c} p_k\right)\right) = 64$

$J = \{2, 4, 6\},\ J^c = \{1, 3, 5, 7\},\ J' = \{3, 7\}$
$\sum_{k \in J^c} p_k = 31$

$$h_3\left(\sum_{k \in J^c} p_k\right) = (31)^2$$
$$= 961$$
$$h_7\left(\sum_{k \in J^c} p_k\right) = 1.4 \times 31$$
$$= 43.4$$

$\min_{j \in J'} \left(h_j\left(\sum_{k \in J^c} p_k\right)\right) = 43.4$

$J = \{2, 4, 6, 7\},\ J^c = \{1, 3, 5\},\ J' = \{1, 3, 5\}$
$\sum_{k \in J^c} p_k = 22$

$$h_1\left(\sum_{k\in J^c} p_k\right) = 3\times 22$$
$$= 66$$
$$h_3\left(\sum_{k\in J^c} p_k\right) = (22)^2$$
$$= 484$$
$$h_5\left(\sum_{k\in J^c} p_k\right) = 70+\sqrt{22}$$
$$= 74.69$$

$\min_{j\in J'}\left(h_j\left(\sum_{k\in J^c} p_k\right)\right) = 66$

$J = \{2,4,6,7,1\},\ J^c = \{3,5\},\ J' = \{3,5\}$
$\sum_{k\in J^c} p_k = 18$

$$h_3\left(\sum_{k\in J^c} p_k\right) = (18)^2$$
$$= 324$$
$$h_5\left(\sum_{k\in J^c} p_k\right) = 70+\sqrt{18}$$
$$= 74.24$$

$\min_{j\in J'}\left(h_j\left(\sum_{k\in J^c} p_k\right)\right) = 74.24$

$J = \{2,4,6,7,1,5,3\},\ J^c = \emptyset,\ J' = \emptyset$

2 Question 2

system description: $1||\sum T_j$

jobs	1	2	3	4	5	6	7	8
p_j	6	18	12	10	10	11	5	7
d_j	8	42	44	24	26	26	70	75

3 Question 3

Consider the problem $1|prec|\max(h_1(S_1),\cdots,h_n(S_n))$, where S_j denotes the starting time of job j. The cost function h_j , $j=1,\cdots,n$ is decreasing. Unforced idleness of the machine is not allowed.Describe a dynamic programming type algorithm for this problem similar to the one in Section 3.2. Why does one have to use here forward dynamic programming instead of backward dynamic programming?
placeholder

4 Question 4

5 Question 5

(a)

(b)

(c)

6 Question 6

(a)

(b)

(c)

(d)

(e)

(f)