

1 Introduction

Compiler

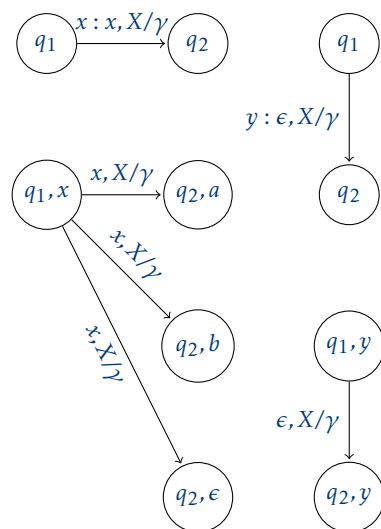
Front end: Translates src program into intermediate representation (lexical analysis, scanning; syntactic analysis parsing (type errors reported)). **Backend:** Translates the intermediate representation into the target program (synthesis: translates the abstract representation of the program into target language, target specific optimizations). **Optimizer:** transforms the abstract representation of the code to improve [optional].

Lexical units, lexemes, tokens

Lexical unit: sub-strings and groups created by the scanner (e.g. identifiers, keywords). **Lexeme:** Element of a lexical unit. **Token:** Pair (id=identifier, att=attribute), attribute = extra info.

2 Top-down parsing and parser generators

LPDA



Prediction

k-look-ahead PDA: A k-LPDA is a tuple $(Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$ different: $\delta: Q \times (\Sigma \cup \{\epsilon\}) \times \Gamma \times \Sigma^{\leq k} \rightarrow 2^{Q \times \Gamma^*}$. **Configuration change LPDA** The LPDA can move from $(q, auv, X\beta)$ to $(q', uv, \alpha\beta)$ if there is $(q', \alpha) \in \delta(q, a, X, au)$. $X \in \Gamma$; $a \in \Sigma \cup \{\epsilon\}$; $u \in \Sigma^{\leq k-1}$; $v \in \Sigma^*$; and if $|auv| \geq k$ then $|au| = k$, else $v = \epsilon$.

First & Follow

First: $First^k(\alpha) = \{w \in T^* : \alpha \Rightarrow^* wx \wedge (|w| = k \text{ or } |w| < k \wedge x = \epsilon)\}$; **Follow:** $Follow^k(\alpha) = \{w \in T^* : \exists \beta, \gamma : S \Rightarrow^* \beta \alpha \gamma \wedge w \in First^k(\gamma)\}$. **First/First:** Intersecting First sets. **First/Follow:** First and Follow sets of variable intersect. **Solutions:** Left factoring First/First; Substitution First/Follow, introduces First/First; Left recursion removal.

LL(k) grammar

A CFG is LL(K) iff for all pairs of derivations:
 $S \Rightarrow^* w\alpha\gamma \Rightarrow^* w\alpha_1\gamma \Rightarrow^* wx_1$
 $S \Rightarrow^* w\alpha\gamma \Rightarrow^* w\alpha_2\gamma \Rightarrow^* wx_2$
 with $w, x_1, x_2 \in T^*$, $A \in V$, and $\gamma \in (V \cup T)^*$, if $First^k(x_1) = First^k(x_2)$ then $\alpha_1 = \alpha_2$.
Strongly LL(K): iff for all pairs of rules $A \rightarrow \alpha_1$ and $A \rightarrow \alpha_2$ with $\alpha_1 \neq \alpha_2$: $First^k(\alpha_1 \cdot Follow^k(A)) \cap First^k(\alpha_2 \cdot Follow^k(A)) = \emptyset$. every strong LL(k) grammar is LL(k). There is a strict hierarchy.

Recursive-descent parsers

Instead of pushing sentential forms into a stack, we can make recursive calls to functions handling nonterminals. Consider, e.g., a grammar with rules $S \rightarrow Ab$ and $S \rightarrow Bc...$

Attribute grammar

S-attributed no inherited attributes. **L-attributed** they allow the attributes to be evaluated in 1 DFS Left-to-right traversal of the derivation tree; $S - att \subseteq L - att$. **L-attribute practice:** An attribute grammar is L-attributed if for all rules $A \rightarrow X_1...X_n$, for all inherited attributes α of X_j we have that the corresponding semantic rule depends only on: Attributes of $A \rightarrow X_1...X_n$ and inherited attributes of A.

3 Scanning with Regular Languages & Parsers with CFG

Scanner

Splits input in sub-strings and groups in lexical units. Feeds tokenized version of input to parser. **hand-built scanners:** Beyond regular languages; Easier to debug; More efficient.

Automata

NFA $(Q, \Sigma, \delta, q_0, F)$ $Q \times (\Sigma \cup \{\epsilon\}) \rightarrow 2^Q$ transition func. **DFA** NFA deterministic if $\delta(q, \epsilon) = \emptyset \forall q \in Q \wedge |\delta(q, a)| = 1 \forall (q, a) \in Q \times \Sigma$. For DFAs, δ form $Q \times \Sigma \rightarrow Q$. Can detect word w, in $O(|w|)$ from regex constructed in P-time.

Myhill-Nerode theorem

Equivalence classes: We say x, y are equivalent with respect to L, written $x \sim_L y$ if $xz \in L \iff yz \in L \forall z \in \Sigma^*$ **theorem:** Language L is regular iff L has finite number of equivalence classes. Number of equivalence classes of L is number of states of smallest DFA.

recognizing v scanning

Recognizing: Given string, simulate DFA and get boolean answer: (not) lexeme recognized. **Scanning:** Given string, scanner returns sequence tokens. Token is longest match (maximal much).

Longest matches

Find longest prefix of remaining string which is lexical unit. **2 or more matches:** 1. First will be chosen; 2. Disallow, DFAs cant accept same input. Doing: 1. DFAs ordered; 2. Language checking when generating scanner. For all pairs of EREs with L_1, L_2 , we should check: $L_1 \cap L_2 = \emptyset$.

Grammar

$G = (V, T, P, S)$ $P: A \rightarrow \beta$ with $\alpha \in (V \cup T)^* V (V \cup T)^*$ and $\beta \in (V \cup T)^*$. **Derivation** Let $\gamma \in (V \cup T)^* V (V \cup T)^*$ and $\delta \in (V \cup T)^*$. We say δ can be derived from γ iff there are $\gamma_1, \gamma_2 \in (V \cup T)^*$ and a rule $\alpha \rightarrow \beta \in P$ st $\gamma = \gamma_1 \cdot \alpha \cdot \gamma_2$ and $\delta = \gamma_1 \cdot \beta \cdot \gamma_2$.

Grammar classes

For all rules $\alpha \rightarrow \beta$: **class 0:** All grammars no restriction; **class 1:** Context-sensitive grammars: either $\alpha = S$ and $\beta = \epsilon$ or $|\alpha| \leq |\beta|$ and S does not appear in β ; **class 2:** Context-free grammars: $\alpha \in V$; **class 3:** $\alpha \in V$ and left-regular = $\beta \in T^* \cup (V \cdot T^*)$. right-regular = $\beta \in T^* \cup (T^* \cdot V)$. **Chomsky's hierarchy:** Languages: $Reg \subset CFL \subset CSL \subset RE$.

Universality of CNF

CFG to CNF: Eliminate: 1. Start symbol from RHS; 2. Rules with non-solitary terminals. 3. RHS with more than 2 nonterminals. 4. ϵ productions; 5. Unit rules. **splits:** For grammars in CNF, A can generate w if we can find a rule $A \rightarrow BC$ and we can split w in two non-empty words $w = u \cdot v$ st: $B \rightarrow u$ and $C \rightarrow v$. It does detect w in polynomial time $O(|w|^3)$

Attributes (semantics)

We associate to every nonterminal variable V of the grammar a finite set $A(V)$ of attributes partitioned into synthesized attributes $A_s(V)$ and inherited attributes $A_i(V)$. Each attribute $\alpha \in A(V)$ has a (potentially infinite) set of possible values. The actual value

will be selected based on the appearance of V in the derivation tree. **Attribute mapping functions:** For each rule $\alpha \rightarrow \beta_1... \beta_k$ we have a semantic rule functions $f_{\alpha, j}$ mapping values of certain attributes of $\beta_1... \beta_k$ to values of $\alpha \in A(\beta_j)$. **welldefinedness:** Semantic rule should not be recursive. They are formulated in a way st all attributes are always defined at all nodes in all possible derivation trees. **synthesized/inherited:** synthesized if its value depends on the values of its children in some derivation tree, if not the inherited.

(Non)deterministic pushdown automata

A is a tuple $(Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$ $\delta: Q \times (\Sigma \cup \{\epsilon\}) \times \Gamma \rightarrow 2^{Q \times \Gamma^*}$ transition func. **Deterministic:** 1. $\forall q \in Q$, all $a \in \Sigma \cup \epsilon$, and all $\gamma \in \Gamma$: $\delta(q, a, \gamma)$ has at most 1 element. 2. $\forall q \in Q$, and all $\gamma \in \Gamma$: if $\delta(q, \epsilon, \gamma) \neq \emptyset$ then $\delta(q, a, \gamma) = \emptyset$ for all $a \in \Sigma$, but does not accept CFL L_{pal}

Simplifying grammar

Factoring: $A \rightarrow wB$ and $A \rightarrow wC$, split A into $A_1 \rightarrow wA_2$ and $A_2 \rightarrow B, A_2 \rightarrow C$. **Indirect left recursion:** \forall pairs of rules $A \rightarrow Bw$ and $B \rightarrow \beta_1$ we replace the first rule by $A \rightarrow \beta_1 w$. Repeated. **Direct left recursion:** If we have rules $(V \rightarrow V\alpha_i)_{i \in I}$ and $(V \rightarrow \beta_j)_{j \in J}$, we can replace them by $(V' \rightarrow \alpha_i)_{i \in I}$, $(V \rightarrow \beta_j V')_{j \in J}$, and $V' \rightarrow \epsilon$. **unproductive:** A unproductive if there is no word w that can be derived from A. **unreachable:** $X \in T \cup V$ is unreachable if no sentential form contains X.

4 MIPS reference

Arithmetic: `<op> $d, $s, $t: <op> = [add(u)| and| nor| or| sub(u)| xor]; <op> $s, $t, i: <op> = [add((u)i)| andi| ori| xori]; <op> $s, $t: <op> = [div(u)| mult(u)] where result is in hi for div and mod in lo and result for mult in hi and lo; Branch: b[eq|ne] $s, $t, label; b[le|gtz] $s, label. Jump: j[al] label; j[al]r $s Load l l[b(u)|h(u)|w] $t, i($s) datamove- m[f|t][hi|lo] $[d|t]`

call codes

print 1:int, 2:fl, 3:dbl, 4:str, 11:char; **read** same as before + 1; 10: exit

stackframe

building `sw $fp, 0($sp); move $fp, $sp; subu $sp, $sp, <size> + 4; sw $ra, -4($fp); sw <reg_to_store>, -offset($fp)`

destructing `lw <reg_to_load>, -<size> + 4($fp); lw $ra, -4($fp); move $sp, $fp; lw $fp, 0($sp); j $ra`

5 LLVM Reference

Type

i[1-32]: int; float, double, fp128: float; <type>*: pointers

Function

`define <type> @<f_name>(<type> %<name>, ...) { ...instructions... } call <type> @<f_name>(<type0> <arg1>, ...)`

Labels

`<label>: ...instructions... br label %<label> br i1 <cond>, label <true>, label <false>`

Comparisons

`{i,f}cmp <policy> <type> <arg1>, <arg2>;` i-policies: `eq, neq, (u,s)gt, (u,s)lt, (u,s)le, (u,s)ge` [u=unsigned, s=signed]; f-policies: `oeq, ogt, ege, olt, ole, one, ord, ueq, ugt` etc[o=ordered, u=unordered]

Phi

`phi <type> [<val0>, <label0>], ...;` based on predecessor block

Arithmetic

`<op> <type> <arg1>, <arg2>: {f}add, {f}sub, {f}mul, {u,s,f}div, and, or, xor; {f,s,u}rem, ashl, lshr, ashr`

Load/Store

`alloca <type>: allocate space on the stack, ret=ptr; store <val_type> <val>, <ptr_type> <ptr>, ret=void; load <val_type>, <ptr_type> <ptr>, ret=val`

Conversion/Cast

`{trunc, zext, sext, fptrunc, fpext} <in_type> <in> to <out_type>: ret=out`

Arrays

`type = [<size> x <type>]; alloca; getelementptr <arr_elem_type>, <arr_type_ptr> <ptr_to_array>, <idx_type> 0, <idx_type> <idx> (arrprtr[0][idx] == arr[idx])`

Strictly dominates & dominance frontier

6 Good code generation

Usage and liveness

Usage: Let $i < j$. The value of x computed at i is used at j. **Liveness:** Let $i < k$. If value of x computed at i is used at j then x is live at all $i \leq j < k$. **Algorithm:** Instructions block from last to first. \forall instructions $i: x = yopz$: 1. Attach to instruction i the usage and

liveness information of x, y, z. 2. Set x to not live and no next use. 3. Set y, z to live and the next uses of y, z to i.

Datastructure: Contains additional information in the symbol table, for the algorithm. Or table per basic block. Usage and Liveness per instruction.

Minimizing load and store

Register descriptors: Variable names whose current value is in a register (per register). **Address descriptors:** All locations where the current value of a variable is stored.

Algorithm: \forall instruction $i : x = yopz$ we do the following. 1. Select registers R_x, R_y, R_z for the var using $getReg(x = yopz)$. 2. If y is not loaded in R_y then issue instruction. 3. If z is not loaded in R_z then issue instruction. 3. Issue instruction $R_x = R_y op R_z$ at end of block all marked live \rightarrow store instruction. Copy no machine-instruction output if $R_x == R_y$.

Algorithm reg for operands: R_y can be chosen: 1. If y in reg r then $R_y = r$. 2. If y is not in reg but r is empty then $R_y = r$. 3. r is candidate: If all var descriptor says their value is in r also have another location then return r; If only var whose descriptor says value is in r is x and $x \notin \{y, z\}$ then return r; If all var not used ofeter instruction return r; Spill the vars into memory. **result reg:** reg holding r or those of x, y if not live hereafter.

Peephole

Sliding window, replaces instructions. Optimizes: redundant-instructions; Flow-of-control; Algebraic; Machine idioms (auto-increments for addresses).

Systematic spilling

Infinite registers generate its code; Construct a register-interference graph (=k-colouring of graph). Where nodes are registers, vertices are nodes alive together.

Sethi-Ullmann

equal-children: recursively gen code right child base=b+1, result in R_{b+k-1} ; do the same for left child with base b result in register one lower; result in R_{b+k-1} .

different-children (m<k): big child same; result small child in R_{b+m-1} ; Answer in big child register.

With spilling: Big child chosen; Re-

cursive, use b=1, result in R_r then in memory; generating for small child; If label >r use base 1 else b=r-j. Recursive, result in R_r ; Big child mem in R_{r-1} ; compute in R_r .

Pipelining

1. Instruction fetch 2. Instruction decode and register fetch 3. Execute 4. Memory access 5. Register write back

7 Bottom-up parsers

Shift & Reduce

It builds a right-most derivation in reverse order. **Reduce/Reduce:** Top corresponds to handle for 2 different rules. **Shift/Reduce:** Top corresponds to handle, but parser can shift too.

PDA bottom-up

Shift: For all terminals a, the PDA has a transition that reads a and pushes a into the stack. Reduce: For every rule $A \rightarrow \alpha_1 \dots \alpha_n$ the PDA has: states $A, \alpha_1, \dots, \alpha_i \forall 1 \leq i < n$ and (A, ϵ) ; transition pops letters from $alpha^R$ from the stack without reading from the input. A transition from (A, ϵ) to push A without reading from input.

Sentential-form prefixes on the stack

$(q, w, \gamma Z_0)$ with q not of form (A, \dots) then: $S \Rightarrow^* \gamma^R \cdot w \cdot \gamma^R$ happens to be a **viable prefix**. All the viable prefixes are recognized by the CFSM.

Generalized algorithm action table:

Same CFSM for LR(0), SLR(K), LALR(k) **LR(0)** 1. Push the initial state 0 of the CFSM into the stack S. 2. As long as we can't accept or get an error: 2.1. If T (top(S)) is Shift, then put the next symbol in variable c. 2.2. Otherwise, if T(top(S)) is $\{j\}$. 2.2.1. pop $|a|$ times from S, where $A \rightarrow \alpha$ is the j-th rule. 2.2.2. and put A in c. 2.3. Let q' be the next state $\delta(\text{top}(S), x)$ of the CFSM after reading c. 2.4. Push q' into S.

SLR(k) 2.1. append "and the look-ahead starts with a, c=a". 2.2. Append "A \rightarrow is the j-th rule, and the look=ahead l is in $Follow^k(A)$ ", look-ahead combos from those sets become columns. **LALR(k)** same as SLR(k). But uses ItemFollow.

LALR(1) propagation graph (V, E, I) V contains all state-item-pairs (s, u) st $u \in s$; E contains $(\langle s, u \rangle, \langle s, u \rangle)$ iff 1. t is the closure of the a-successor of s, $u = A \rightarrow \alpha_1 \dots \alpha_i$, and $v = u = A \rightarrow \alpha_1 \dots \alpha_i$. 2. $s = t$ and $u = A \rightarrow \alpha_1 \dots \alpha_i, v = B \rightarrow \beta, \alpha_i$ can produce ϵ ; The labelling function $l : V \rightarrow 2^{\Sigma^{\leq 1}}$ is st: $l(\langle (s, B \rightarrow \beta) \rangle) =$

$\cup \{First^1(\alpha_2)\} \langle s, A \rightarrow \alpha_1 \dots \alpha_i \rangle \in V\}$ and all other vertices are mapped to the empty set.

ItemFollow: For a vertex v, $ItemFollow^1(v)$ is the set $\cup \{l(u) \mid u \text{ has a path to } v\}$ so this tells us whether we can apply reduction rule $A \rightarrow \beta$ from a CFSM state s based on the $ItemFollow^1(v)$ of $v = (s, A \rightarrow \beta)$.

LR(k)-CFSM

LR(k)-items: $(A \rightarrow \alpha_1 \dots \alpha_i, w)$ where $w \in \Sigma^{\leq k}$

LR(k)-closure: The closure of $(A \rightarrow \alpha_1 \dots \alpha_i, w)$ is the set of all LR(k)-items $(B \rightarrow \beta, y)$ where: 1. $B \rightarrow \beta$ grammar rule 2. $y \in First^k(\alpha_i w)$; It includes I and is minimal set. **CFSM states:** states of the CFSM are now subsets of LR(k)-items. The a-successor of a state I, for $a \in T \cup V$, is the closure of the set $\{(A \rightarrow \alpha_1 \dots \alpha_i, w) \mid (A \rightarrow \alpha_1 \dots \alpha_i, w) \in I\}$ **LR(k)-CFSM:** q_0 is the closure of $\{(A \rightarrow \alpha_1 \dots \alpha_i, w) \mid (A \rightarrow \alpha_1 \dots \alpha_i, w) \in I\}$

Action tables

LR(0) Index grammar rules $1 \leq j \leq n$ and states from the CFSM $0 \leq i \leq k$ The table T maps each i to a set of actions: 1. $T(i)$ contains (a Reduce) j if state i has the item $A \rightarrow \alpha \cdot$ with $A \rightarrow \alpha$ the j-th rule. 2. $T(i)$ contains Shift if state i has an item $A \rightarrow \alpha_1 \dots \alpha_i$. 3. $T(i)$ contains Accept if state i has an item $S \rightarrow \alpha \cdot$. 4. $T(\phi)$ contains an error action only.

SLR replace (a Reduce) by *nothing*, replace *Shift* by (a *Shift*) a and prepend an a to α_2

LR(k): add and look-ahead l to a set of actions to the 1st line; change to on 1 to $A \rightarrow \alpha \cdot, l$; change to on line 2 $(A \rightarrow \alpha_1 \dots \alpha_i, y)$ and $l \in First^k(\alpha_i y)$.

Conclusions

$\forall k \geq 1$, the LR(k) languages, LR(1) languages, and languages recognizable by DPDA are all the same. (Most languages use LALR(1) grammar)

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9 Type checking and other semantic analyses

Type checking

Assigning a type t to a variable x is, in essence, an invariant. Because languages are turing complete undecidable. Conservative if static, applies inference rules to deduce types of expression (without caring about reachability). We can use implicit casting if we have expressions of the type 4 + 1.0. The hierarchy is saved in memory. **Typechecker:** Verify the validity of the expression given the types of the descendants. Derive the type of parent expressions (propagation). Via bottom-up traversal of the AST using the symbol table. Statements have no type, so they start the recursive traversal of their

General reachability

Undecidable, halting problem. Conservative approach. Focus list of statements, reachable and terminate normally, ie control flow "falls through".

Algorithm: Consider a statement list $S = s_1, \dots, s_n$; 1. If S is reachable then s_1 is reachable. 2. If s_n terminates abnormally then so does S. 3. If S is the body of a function or procedure then S is reachable. 4. Local variable declarations, assignments, function calls, memory allocation, increment, decrement instructions all terminate normally. 5. A null statement or statement list does not generate error messages if it is not reachable. However, they propagate their non-reachability status to their successors. 6. If a statement has a predecessor then it is reachable if its predecessor terminates normally. For reachability analysis we assume the condition can be both true and false. *while loop* If it evaluates constant true \rightarrow the while-statement is marked as abnormally terminating — since it is an infinite loop. false \rightarrow unreachable; If body contains break then normal terminating (when was abnormal). If body terminates normally, then the while statement terminates normally (unless infinite with no break).