Equal

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Problem Submissions Leaderboard Discussions Editorial

Christy is interning at HackerRank. One day she has to distribute some chocolates to her colleagues. She is biased towards her friends and may have distributed the chocolates unequally. One of the program managers gets to know this and orders Christy to make sure everyone gets equal number of chocolates.

But to make things difficult for the intern, she is ordered to equalize the number of chocolates for every colleague in the following manner,

For every operation, she can choose one of her colleagues and can do one of the three things.

1. She can give one chocolate to every colleague other than chosen one.
2. She can give two chocolates to every colleague other than chosen one.
3. She can give five chocolates to every colleague other than chosen one.

Calculate minimum number of such operations needed to ensure that every colleague has the same number of chocolates.

Input Format

First line contains an integer T denoting the number of testcases. T testcases follow.
Each testcase has 2 lines. First line of each testcase contains an integer N denoting the number of co-interns. Second line contains N space separated integers denoting the current number of chocolates each colleague has.

Constraints

$1 <= T <= 100$
$1 <= N <= 10000$
Number of initial chocolates each colleague has < 1000

Output Format

T lines, each containing the minimum number of operations needed to make sure all colleagues have the same number of chocolates.

Sample Input

```
1
4
2 2 3 7
```

Sample Output

```
2
```

Explanation

1st operation: Christy increases all elements by 1 except 3rd one
2 2 3 7 -> 3 3 3 8
2nd operation: Christy increases all element by 5 except last one
3 3 3 8 -> 8 8 8 8

* Suppose we are given an initial distribution:

$v = [v_0 \; v_1 \; \dots \; v_{n-1}]$

* A goal is to equalize v by increasing all v_i except one by 1, 2 or 5
* This is dual to decreasing a single v_i by 1, 2, 5

To see that this is true consider:

$v = [1 \; 5 \; 5 \; 10 \; 10]$

Suppose we want to equalize by decreasing one at a time:

1	5	5	10	10
				-5
1	5	5	10	5
				-5
1	5	5	10	0
			-5	
1	5	5	5	0
			-5	
1	5	5	0	0

DUAL

1	5	5	10	10
+5	+5	+5	+5	
6	10	10	15	10
+5	+5	+5	+5	
11	15	15	20	10
+5	+5	+5		+5
16	20	20	20	15
+5	+5	+5		+5
21	25	25	20	20

1 5 5 0 0
−5
1 5 0 0 0
−5
1 0 0 0 0
−1
0 0 0 0 0

21 25 25 20 20
+5 +5 +5 +5
26 30 25 25 25
+5 +5 +5 +5
31 30 30 30 30
+1 +1 +1 +1
31 31 31 31 31

Number of operations is the same.

* It easier to decrease one at a time because we can state the target in advance.

* It is evident that the optimal target (one that produces the smallest # of steps) is :

$$0 \leq v^* \leq v_j = \min\{v_0, v_1, \dots v_{n-1}\}$$

* We show that v^* is no smaller than 0 or $v_j - 4$

* Consider target v_j

Each of $v_i - v_j$ falls into one of equivalence classes :

$C_0 = \{0, 5, 10, \cdots\}$

$C_1 = \{1, 6, 11, \cdots\}$

$C_2 = \{2, 7, 12, \cdots\}$

$C_3 = \{3, 8, 13, \cdots\}$

$C_4 = \{4, 9, 14, \cdots\}$

$v_i - v_j$ is represented as $a_i \times 5 + b_i \times 2 + c_i \times 1$

In C_0 all $v_i - v_j$ have $b_i = c_i = 0$

C_1 have $b_i = 0$ $c_i = 1$

C_2 have $b_i = 1$ $c_i = 0$

C_3 have $b_i = 1$ $c_i = 1$

C_4 have $b_i = 2$ $c_i = 0$

This arrangement guarantees minimum number of steps to reduce v_i to v_j

If $v_i - v_j \in C_0$ it requires a_i step.

$v_i - v_j \in C_1$ requires $a_i + 1$ steps

$v_i - v_j \in C_2$ requires $a_i + 1$ steps

$v_i - v_j \in C_3$ requires $a_i + 2$ steps

$v_i - v_j \in C_4$ requires $a_i + 2$ steps

Consider now $v' = v_j - 5$.

We have:

$$v_i - v' = v_i - v_j + 5$$

We see that $v_i - v'$ is the same class as $v_i - v_j$ except that number of a_i steps is increased by 1

This is true also for $v_i - (v_j - 1)$

and $v_i - v'' = v_i - (v_j - 6) =$

$= v_i - (v_j - 1) + 5$

etc.

Therefore:

$$v_j - 4 \leq v^* \leq v_j \quad , v^* \geq 0$$

NOTE:

If we imagine that $v_i - v^*$ is an amount and $\{1, 2, 5\}$ are coins, then coin change problem would be in how many ways $v_i - v^*$ could be represented with coins $\{1, 2, 5\}$.

This problem however requires that we pick representation with min # of coins. This could be figured out as follows:

$a_i^* = (v_i - v^*) / 5$

then: $b_i^* = ((v_i - v^*) \% 5) / 2$

then: $c_i^* = (((v_i - v^*) \% 5) \% 2)$

$$v_i - v^* = a_i^* \times 5 + b_i^* \times 2 + c_i^* \times 1$$

Total # of operations is:

$\sum a_i^* + \sum b_i^* + \sum c_i^*$

* For example:

2 5 5 5 5 5

$0 \leq v^* \leq 2$

1) Reduce to 2:

$v_i - 2$	0	3	3	3	3	3
a_i	0	0	0	0	0	0
b_i	0	1	1	1	1	1
c_i	0	1	1	1	1	1

# of ops = 10

2) Reduce to 1:

$v_i - 1$	1	4	4	4	4	4
a_i	0	0	0	0	0	0
b_i	0	2	2	2	2	2
c_i	1	0	0	0	0	0

# of ops = 11

3) 3) Reduce to 0 :

$v_i - 0$	2	5	5	5	5	5
a_i	0	1	1	1	1	1
b_i	1	0	0	0	0	0
c_i	0	0	0	0	0	0

# of ops = 6

Therefore $v^* = 0$ and min # of steps is 6.