GUC
German University in Cairo

Department of Mathematics

(Math 204)

Linear Algebra

Lecture 5

Mathematics for Business Informatics II

Linear Dependency, Basis, Dimension & Finding Inverse Matrix

By
Prof.Dr. Fathy Helail

Contents

1. Linear Dependence & Independence of Vectors

Now, we shall study the case; if we have a set of vectors is linearly dependent (or not) if one of them can be written as a linear combination of the others. The formal definition is stated as follows:

Definition 1. **Let** $\underline{x}_1, \underline{x}_2, \ldots, \underline{x}_m \in R^n$ **, then:**

(a) The vectors $\underline{x}_1, \underline{x}_2, \ldots, \underline{x}_m$ **are called linearly dependent if:** **there are real numbers** $\underline{\lambda}_1, \underline{\lambda}_2, \ldots, \underline{\lambda}_m$ **with**

$$\lambda_1.\underline{x}_1 + \lambda_2.\underline{x}_2, \ldots + \lambda_m.\underline{x}_m = \underline{0}, \text{ and at least one } \lambda_i \neq 0.$$

(b) The vectors $\underline{x}_1, \underline{x}_2, \ldots, \underline{x}_m$ **are called linearly independent if:**

$$\lambda_1.\underline{x}_1 + \ldots + \lambda_m.\underline{x}_m = \underline{0}, \text{ induces } \lambda_1 = 0, \ \ldots \ , \lambda_m = 0.$$

To Check linearly Dependency by using the value of rank of a matrix, we give the following definition:

Definition 2. The m vectors $\underline{x}_1, \underline{x}_2, \ldots, \underline{x}_m$ are called linearly Independent if and only if the Rank of the matrix $(\underline{x}_1, \underline{x}_2, \ldots, \underline{x}_m)$ is equal to the number of vectors which equals to m.

From the above definitions we can present Theorem 1.

Theorem 1. The Vectors $\underline{x}_1, \underline{x}_2, \ldots, \underline{x}_m \in R^n$ are linearly Dependent if and only if at least one of the vectors x_j can be expressed as a linear combination of the other vectors.

Example 1: Decide whether the 3 vectors: $\begin{pmatrix}1\\0\\2\end{pmatrix}, \begin{pmatrix}2\\1\\0\end{pmatrix}$ & $\begin{pmatrix}3\\4\\2\end{pmatrix}$

are linearly dependent or independent?

Solution: By Applying the definition of dependency we get:

$$\lambda_1.\begin{pmatrix}1\\0\\2\end{pmatrix} + \lambda_2.\begin{pmatrix}2\\1\\0\end{pmatrix} + \lambda_3.\begin{pmatrix}3\\4\\2\end{pmatrix} = \underline{0}.$$

which is a system of linear equations in λ_1, λ_2 *and* λ_3 & then;

we solve it as follows:

$$\left(\begin{array}{ccc|c}1&2&3&0\\0&1&4&0\\2&0&2&0\end{array}\right) \xrightarrow{R_3 \leftrightarrow R_3 - 2\cdot R_1} \left(\begin{array}{ccc|c}1&2&3&0\\0&1&4&0\\0&-4&-4&0\end{array}\right)$$

Dividing Row 3 by -4

$$\left(\begin{array}{ccc|c}1&2&3&0\\0&1&4&0\\0&1&1&0\end{array}\right) \xrightarrow{R_3 \to R_3 - R_2} \left(\begin{array}{ccc|c}1&2&3&0\\0&1&4&0\\0&0&-3&0\end{array}\right)$$

This system have only the **trivial solution:** $\lambda_1 = 0, \lambda_2 = 0$ & $\lambda_3 = 0.$

Hence, the vectors are linearly independent.

Example 2. Decide whether the following vectors $\begin{pmatrix}2\\1\\2\end{pmatrix}, \begin{pmatrix}-1\\0\\1\end{pmatrix}$ & $\begin{pmatrix}3\\4\\2\end{pmatrix}$ are **linearly independent or not** ?

Solution: Firstly, find the rank of the matrix $\begin{pmatrix}2 & -1 & 3\\1 & 0 & 4\\2 & 1 & 2\end{pmatrix}$ as follows:

$$\begin{pmatrix}2 & -1 & 3\\1 & 0 & 1\\2 & 1 & 1\end{pmatrix} \xrightarrow[(1)\leftrightarrow(2)]{} \begin{pmatrix}1 & 0 & 1\\2 & -1 & 3\\2 & 1 & 1\end{pmatrix} \xrightarrow[\substack{(2)\to(2)-2\cdot(1)\\(3)\to(3)-2\cdot(1)}]{} \begin{pmatrix}1 & 0 & 1\\0 & -1 & 1\\0 & 1 & -1\end{pmatrix}$$

$$\xrightarrow[(3)\to(3)+(2)]{} \begin{pmatrix}1 & 0 & 1\\0 & -1 & 1\\0 & 0 & 0\end{pmatrix}$$; then rank of the matrix = 2.

Since the rank $r = 2$ & number of vectors = 3; then:

the given 3 vectors are not linearly independent.

2. Basis & Dimension of Subspaces

Some times we may found that two (or more) vectors spans its space and are linearly independent. This is the essence of a basis for a subspace.

Definition 3. The vectors $\underline{x}_1, \ldots, \underline{x}_m$ are called a Basis of a subspace $U \subseteq R^n$ if:

(a) U = span $(\underline{x}_1, \ldots, \underline{x}_m)$.

(b) The vectors $\underline{x}_1, \ldots, \underline{x}_m$ are linearly independent.

Definition 4. The Dimension of a subspace $U \subseteq R^n$ (dim U) is the number of vectors that forms a basis of U.

In other words, if the vectors $\underline{x}_1, \ldots, \underline{x}_m$ form a basis of the subspace $U \subseteq R^n$, then the dimension of U is the number m.

Recalling the definitions 3 & 4 , we give example 3:

Example 3. **Verify that the following vectors:** $\underline{e}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ : \\ 0 \end{pmatrix}, \ \underline{e}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ : \\ 0 \end{pmatrix}, \ ..., \ \underline{e}_n = \begin{pmatrix} 0 \\ 0 \\ : \\ 0 \\ 1 \end{pmatrix}$

are basis of R^n **and then find its dimension.**

Solution. **(*a*)** ***we apply the 1st condition of basis:***

$$For\ an\ arbitrary\ vector\ \underline{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ : \\ x_n \end{pmatrix} \in R^n\ we\ have:$$

$$\underline{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ : \\ x_n \end{pmatrix} = \begin{pmatrix} x_1 \\ 0 \\ 0 \\ : \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ x_2 \\ 0 \\ : \\ 0 \end{pmatrix} + \ldots + \begin{pmatrix} 0 \\ 0 \\ : \\ 0 \\ x_n \end{pmatrix} = \underline{x}_1 . \begin{pmatrix} 1 \\ 0 \\ 0 \\ : \\ 0 \end{pmatrix} + \ \underline{x}_2 . \begin{pmatrix} 0 \\ 1 \\ 0 \\ : \\ 0 \end{pmatrix} + \ldots + \underline{x}_n . \begin{pmatrix} 0 \\ 0 \\ : \\ 0 \\ 1 \end{pmatrix} = x_1 . \underline{e}_1 + x_2 . \underline{e}_2 + \ldots + x_n . \underline{e}_n .$$

$So,\ R^n = span\ (\underline{e}_1, \ldots, \underline{e}_n).$ **(1st *condition satisfied*)**

(*b*) For the 2^{nd} condition:

$$(\underline{e}_1 \ \ \underline{e}_2 \ \ \dots \ \ \underline{e}_n) = \begin{pmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{pmatrix}$$

clearly, the rank $= n$. Then the vectors $\underline{e}_1, \underline{e}_2, \dots, \underline{e}_n$ are linearly independent. (2^{nd} *condition satisfied*)

Therefore, from (*a*) & (*b*) the above unit vectors form a **Basis** for R^n, and called the **Standard Basis** for R^n.

Moreover, the rank $= n$, hence $\dim R^n = n$.

Example 4. **Determine the dimension of the following subspace:**

$$U=\left\{\begin{pmatrix}x\\y\end{pmatrix}\middle|\ x+2y=0\right\}$$

Solution.

The elements of U is the solution of the equation $x + 2y = 0$,

Let $y = r$, then we have $x = -2r$.

Now, we put U in the form: $U=\left\{r.\begin{pmatrix}-2\\1\end{pmatrix}\middle|\ r\in R\right\}=span\left(\begin{pmatrix}-2\\1\end{pmatrix}\right)$

Since we have the vector $\begin{pmatrix}-2\\1\end{pmatrix}$ is different from the zero vector

Then, the **dimension** of U is equal to $\mathbf{1}$.

Example 5. Show that the vectors $\begin{pmatrix} 2 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ form a Basis of R^2.

Solution.

1st condition: we have to prove that $R^2 = span(\begin{pmatrix} 2 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \end{pmatrix})$ as follows

i.e. we will solve the vector equation for a & b as follows:

$$a.\begin{pmatrix} 2 \\ -1 \end{pmatrix} + b.\begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

then, we get the equations:

$$\begin{aligned} 2a + b &= x_1 \\ -a + 3b &= x_2 \end{aligned}$$

We perform the solution for a and b in terms of x_1 & x_2 as follows:

Multiplying 2nd equation *by* 2 & *adding the* 2 equations we get:

$$7b = x_1 + 2x_2 \text{ , then } b = \frac{1}{7}(x_1 + 2x_2)$$

Similarly, we find the value of $a = \frac{1}{7}(3x_1 - x_2)$.

(Thus, the 1st condition for Basis is satisfied.)

2nd condition: The 2 vectors $\begin{pmatrix} 2 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ are linearly independent ?.

We construct the matrix of the 2 vectors & we get its rank= 2 *(verify)*;
then, vectors $\begin{pmatrix} 2 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ are linearly independent. **(2nd condition is satisfied)**

From above conditions 1 & 2; we conclude that:

The vectors $\begin{pmatrix} 2 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ form a Basis for R^2 .

Important Remark: A subspace can have more than one Basis.
e.g. Here, we see that, there are 2 Basis for R^2 as follows:

(a) $\left\{ \begin{pmatrix} 2 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \end{pmatrix} \right\}$ (b) Standard Basis $\left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}$.

Remark: However, it is proved that the number of vectors in a basis for a given subspace will always the same; dim $R^2 = 2$.

3. Special Types of Matrices.

We will present some types of matrices plays important roles in matrix theory:

(a) Transpose of a Matrix

The next definition has no such similarities to the operations on real numbers.

Definition 5. The transpose of an $m \times n$ matrix A is the $n \times m$ matrix A^T obtained by interchanging the rows and columns of A.

i.e. the ith column of A^T is the ith row of A for all i.

Example 6. Find the Transpose of the following matrices:

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}, \quad B = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \quad C = \begin{pmatrix} 5 & -4 & 6 \end{pmatrix}$$

Solution. The transposes are: $A^T = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}, \quad B^T = \begin{pmatrix} a & c \\ b & d \end{pmatrix}, \quad C^T = \begin{pmatrix} 5 \\ -4 \\ 6 \end{pmatrix}$

Important Remark. *We observe that the columns of A^T are the rows of A,* ***i.e. ifi we take the Transpose again, we get the original matrix A.***

Transpose Properties: Let A & B be matrices of suitable sizes. Then we have:

$$(1)\ (A^T)^T = A \qquad (2)\ (A+B)^T = A^T + B^T \qquad (3)\ (AB)^T = B^T A^T$$

(b) Identity Matrix

The Identity matrix is defined as follows:

It is an n x n **square matrix** of the form:

$$I_n = \begin{pmatrix} 1 & 0 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & 0 & \dots & 0 \\ 0 & 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 0 & 1 & \dots & 0 \\ & & & & \ddots & \\ 0 & 0 & 0 & 0 & \dots & 1 \end{pmatrix}.$$

Example 6. The Identity matrix **for** $n = 2$:

$$I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

And for $n = 3$:

$$I_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

4. Finding Inverse of Matrix Using Gauss-Jordan Method

Definition 6.(Inverse Matrix) Let A be an $(n \times n)$ matrix, then:

(1) The matrix A is called Invertible (or Regular) if there exist some

$(n \times n)$ matrix B such that: $A \cdot B = B \cdot A = I_n$.

The matrix B is called the Inverse of A or $B = A^{-1}$

(2) Otherwise, A is called not invertible (or Singular).

For finding the Inverse of Matrix, we use Gauss-Jordan method:

Theorem 2. Let A be a square matrix. If a sequence of *ero's* reduces A to I, then the same sequence of *ero's* transforms I into A^{-1}

i.e. $\boxed{\left[\begin{array}{c|c} A & I \end{array}\right] \to \left[\begin{array}{c|c} I & A^{-1} \end{array}\right]}$

Theorem 3.

(1) If A is an invertible matrix, then A^{-1} is invertible & $(A^{-1})^{-1} = A$

(2) If A & B are invertible matrices of the same size, then AB is invertible & $(AB)^{-1} = B^{-1}A^{-1}$

(3) Generally we have $(A_1 A_2 \ldots A_n)^{-1} = A_n^{-1} \ldots A_2^{-1} A_1^{-1}$ & $(A^T)^{-1} = (A^{-1})^T$

Example 7. Using **Gauss - Jordan** find **the inverse** (if exists) of the matrix: $A_1 = \begin{pmatrix} 2 & 1 \\ 5 & 3 \end{pmatrix}$

Solution:

$$\left(\begin{array}{cc|cc} 2 & 1 & 1 & 0 \\ 5 & 3 & 0 & 1 \end{array}\right) \xrightarrow{R_2 \to R_2 - 2R_1} \left(\begin{array}{cc|cc} 2 & 1 & 1 & 0 \\ 1 & 1 & -2 & 1 \end{array}\right) \xrightarrow{R_1 \leftrightarrow R_2} \left(\begin{array}{cc|cc} 1 & 1 & -2 & 1 \\ 2 & 1 & 1 & 0 \end{array}\right)$$

$$\xrightarrow{R_2 \to R_2 - 2R_1} \left(\begin{array}{cc|cc} 1 & 1 & -2 & 1 \\ 0 & -1 & 5 & -2 \end{array}\right) \xrightarrow{R_2 \to (-1)\cdot R_2} \left(\begin{array}{cc|cc} 1 & 1 & -2 & 1 \\ 0 & 1 & -5 & 2 \end{array}\right) \xrightarrow{R_1 \to R_1 - R_2} \left(\begin{array}{cc|cc} 1 & 0 & 3 & -1 \\ 0 & 1 & -5 & 2 \end{array}\right)$$

So, A_1 has an inverse & equals: $A_1^{-1} = \begin{pmatrix} 3 & -1 \\ -5 & 2 \end{pmatrix}$

Example 8. Using Gauss-Jordan find the inverse of matrix: $A_2 = \begin{pmatrix} 3 & 2 \\ 6 & 1 \end{pmatrix}$

Solution:

$$\left(\begin{array}{cc|cc} 3 & 2 & 1 & 0 \\ 6 & 4 & 0 & 1 \end{array}\right) \xrightarrow{R_2 \to R_2 - 2R_1} \left(\begin{array}{cc|cc} 3 & 2 & 1 & 0 \\ 0 & 0 & -2 & 1 \end{array}\right)$$

So, the matrix A_2 is not invertible. Also A_2 has rank $=1$.

4. Solved Examples

Example 9 By Gauss-Jordan find the inverse (if exists) of the following matrix:

$$B_2 = \begin{pmatrix} -1 & 3 & 7 \\ 2 & 1 & 0 \\ 3 & 5 & 7 \end{pmatrix}$$

Solution. Using *ero's* , we have:

$$\left(\begin{array}{ccc|ccc} -1 & 3 & 7 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 & 1 & 0 \\ 3 & 5 & 7 & 0 & 0 & 1 \end{array}\right) \xrightarrow[R_1 \to (-1)R_1]{} \left(\begin{array}{ccc|ccc} 1 & -3 & -7 & -1 & 0 & 0 \\ 2 & 1 & 0 & 0 & 1 & 0 \\ 3 & 5 & 7 & 0 & 0 & 1 \end{array}\right) \xrightarrow[R_2 \to R_2 - 2R_1]{}$$

$$\left(\begin{array}{ccc|ccc} 1 & -1 & -3 & -1 & 0 & 0 \\ 0 & 7 & 14 & 2 & 1 & 0 \\ 3 & 5 & 7 & 0 & 0 & 1 \end{array}\right) \xrightarrow[R_3 \to R_3 - 3R_1]{} \left(\begin{array}{ccc|ccc} 1 & -3 & -7 & -1 & 0 & 0 \\ 0 & 7 & 14 & 2 & 1 & 0 \\ 0 & 14 & 28 & 3 & 0 & 1 \end{array}\right) \xrightarrow[R_3 \to R_3 - 2R_2]{} \left(\begin{array}{ccc|ccc} 1 & -3 & -7 & -1 & 0 & 0 \\ 0 & 7 & 14 & 2 & 1 & 0 \\ 0 & 0 & 0 & -1 & -2 & 1 \end{array}\right)$$

We conclude that B_2 is **not invertible** & the matrix B_2 **has rank= 2.**

Example 10. By Gauss-Jordan find inverse (if exists) of matrix: $B_1 = \begin{pmatrix} 1 & -1 & -3 \\ 2 & 5 & 10 \\ 1 & 2 & 4 \end{pmatrix}$

Solution.

$$\left(\begin{array}{ccc|ccc} 1 & -1 & -3 & 1 & 0 & 0 \\ 2 & 5 & 10 & 0 & 1 & 0 \\ 1 & 2 & 4 & 0 & 0 & 1 \end{array}\right) \xrightarrow[R_2 \to R_2 - 2R_1]{} \left(\begin{array}{ccc|ccc} 1 & -1 & -3 & 1 & 0 & 0 \\ 0 & 7 & 16 & -2 & 1 & 0 \\ 1 & 2 & 4 & 0 & 0 & 1 \end{array}\right) \xrightarrow[R_3 \to R_3 - R_1]{}$$

$$\left(\begin{array}{ccc|ccc} 1 & -1 & -3 & 1 & 0 & 0 \\ 0 & 7 & 16 & -2 & 1 & 0 \\ 0 & 3 & 7 & -1 & 0 & 1 \end{array}\right) \xrightarrow[R_2 \to R_2 - 2R_3]{} \left(\begin{array}{ccc|ccc} 1 & -1 & -3 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & -2 \\ 0 & 3 & 7 & -1 & 0 & 1 \end{array}\right) \xrightarrow[R_3 \to R_3 - 3R_2]{}$$

$$\left(\begin{array}{ccc|ccc} 1 & -1 & -3 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & -2 \\ 0 & 0 & 1 & -1 & -3 & 7 \end{array}\right) \xrightarrow[R_2 \to R_2 - 2R_3]{} \left(\begin{array}{ccc|ccc} 1 & -1 & -3 & 1 & 0 & 0 \\ 0 & 1 & 0 & 2 & 7 & -16 \\ 0 & 0 & 1 & -1 & -3 & 7 \end{array}\right) \xrightarrow[R_1 \to R_1 + 3R_3]{}$$

$$\left(\begin{array}{ccc|ccc} 1 & -1 & 0 & -2 & -9 & 21 \\ 0 & 1 & 0 & 2 & 7 & -16 \\ 0 & 0 & 1 & -1 & -3 & 7 \end{array}\right) \xrightarrow[R_1 \to R_1 + R_2]{} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & -2 & 5 \\ 0 & 1 & 0 & 2 & 7 & -16 \\ 0 & 0 & 1 & -1 & -3 & 7 \end{array}\right)$$

So, the matrix B_1 is invertible & its inverse is: $B_1^{-1} = \begin{pmatrix} 0 & -2 & 5 \\ 2 & 7 & -16 \\ -1 & -3 & 7 \end{pmatrix}.$

Example 11. **Find the dimension of** $U = \text{span}\left(\begin{pmatrix}1\\2\\1\end{pmatrix}, \begin{pmatrix}2\\1\\3\end{pmatrix}, \begin{pmatrix}0\\3\\-1\end{pmatrix}\right)$

Solution: ***i.e.*** to determine the rank of the matrix: $\begin{pmatrix}1 & 2 & 0\\2 & 1 & 3\\1 & 3 & -1\end{pmatrix}$,

we have:

$$\begin{pmatrix}1 & 2 & 0\\2 & 1 & 3\\1 & 3 & -1\end{pmatrix} \xrightarrow[R_2 \to R_2 - 2\cdot R_1]{} \begin{pmatrix}1 & 2 & 0\\0 & -3 & 3\\1 & 3 & -1\end{pmatrix} \xrightarrow[R_3 \to R_3 - R_1]{} \begin{pmatrix}1 & 2 & 0\\0 & -3 & 3\\0 & 1 & -1\end{pmatrix}$$

$$\xrightarrow[R_2 \to \left(-\frac{1}{3}\right)\cdot R_2]{} \begin{pmatrix}1 & 2 & 0\\0 & 1 & -1\\0 & 1 & -1\end{pmatrix} \xrightarrow[R_3 \to R_3 - R_2]{} \begin{pmatrix}1 & 2 & 0\\0 & 1 & -1\\0 & 0 & 0\end{pmatrix} \xrightarrow[R_1 \to R_1 - 2\cdot R_2]{} \begin{pmatrix}1 & 0 & 2\\0 & 1 & -1\\0 & 0 & 0\end{pmatrix}$$

Thus the rank = 2 (*we have 2 nonzero rows*), **hence dim** $U = 2$.

Example 12 *Consider the 3 vectors* $\underline{e_1} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \underline{e_2} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, *and* $\underline{e_3} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$. *Can the three dimensional space* $\mathbb{R}^3$ *be spanned by the set* $U = \{\underline{e_1}, \underline{e_2}, \underline{e_3}\}$*? In other words, show whether* $span\left(\underline{e_1}, \underline{e_2}, \underline{e_3}\right) = \mathbb{R}^3$.

Solution. If we consider any three-dimensional vector $\underline{x} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \in \mathbb{R}^3$, we can write:

$$\underline{x} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} = a \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + b \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + c \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = a \cdot \underline{e_1} + b \cdot \underline{e_2} + c \cdot \underline{e_3}$$

Thus, any vector $\underline{x} \in \mathbb{R}^3$ is in span$(\underline{e_1}, \underline{e_2}, \underline{e_3})$, because $\underline{x}$ can be written as a linear combination of the 3 given vectors. Thus, the vectors $\underline{e_1}, \underline{e_2}, \underline{e_3}$ span $\mathbb{R}^3$.

Problem 1.

Use the Gauss- Jordan elimination method to find the inverse of the following matrices (if one exist). Verify the correctness of your answer :

(a) $A = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 3 & 1 \\ -1 & 0 & 2 \end{pmatrix}$

(b) $B = \begin{pmatrix} 1 & -1 & 3 \\ 2 & 4 & -1 \\ 3 & 3 & 2 \end{pmatrix}$

THANK YOU

Prof. Helail