Lab session 5

On signal windowing and filtering

Exercice 1 *Signal windowing*

To consider a signal of finite length N corresponds to the multiplication of the signal with a rectangular window of length N, denoted in the sequel by w_r. In the Fourier domain, this corresponds to the convolution with the Fourier transform of the rectangular window.
Alternatively, one can consider the Hamming window of length N.

$$w_h(n) = \begin{cases} 0.54 - 0.46\cos(\frac{2\pi n}{N}) \text{ si } n \in \{0, \cdots, N-1\} \\ 0 \text{ otherwise} \end{cases}$$

As the Dirac distribution is the neutral operator for the convolution, smaller distortions are obtained when the Fourier transform of the window is closer to the Dirac distribution.

1. Assuming the number of frequency bins is L=1024 (look at the parameters of the fft function), and considering $N = 10$, $N = 100$, $N = 200$, write a program that plots the modulus of the Fourier transforms of the windows w_r and w_h at frequency locations $\frac{k}{L}$ for $-\frac{L}{2} \leq k \leq \frac{L}{2} - 1$(use fft for that purpose, normalize the windows so that there are with unit l^1 norm before computing the fft).

2. What is the effect of N on the frequency representation?

Exercice 2 *Low-pass signal filtering*

In this exercise, we would like to build a finite impulse response filter h whose Fourier transform approximates $\mathbf{1}_{[-f_0,f_0]}$, the indicator function of interval $[-f_0, f_0]$ for $f_0 < 1/2$.

1. Assuming $\mathbf{1}_{[-f_0,f_0]}$ is periodized into a 1-periodic function, compute the Fourier coefficients of the function, denoted by h_n.

2. As in the previous lab session, we truncate the h_n and keep only N coefficients, and we adopt the same rule as in the previous lab session to obtain $\tilde{h}_n$ when N is even.

3. Multiply h with the Hamming window of length N (translated by a factor $N/2$) to obtain g, this should be done by writing a function $g = FIR(f_0, N)$.

4. Compute the modulus of the DFT of h and g and conclude which filter is better namely h or g (numerical applications $N = 15$, $f_0 = 1/4$)

Exercice 3 *Band-pass signal filtering*

In this exercise, we would like to build a finite impulse response filter h whose Fourier transform approximates $\mathbf{1}_{[-f_0-f_1,f_0-f_1]} + \mathbf{1}_{[-f_0+f_1,f_0+f_1]}$ for some $f_0 < 1/2$ and $f_1 > f_0$ and $f_1 + f_2 < 1/2$.

1. If we denote by H the Fourier transform of a sequence h, show that the Fourier transform of the sequence $2h_n\cos(2\pi f_1 n)$ is $H(\xi - f_1) + H(\xi + f_1)$.

2. Modify the program of the previous question to obtain filters of size N with Fourier transform approximating $\mathbf{1}_{[-f_0-f_1,f_0-f_1]} + \mathbf{1}_{[-f_0+f_1,f_0+f_1]}$. Deal with the cases N odd and even, and plot the modulus of the corresponding discrete Fourier transform. Numerical application: $f_0 = 1/8$, $f_1 = 1/4$. What is a role of such a filter?