### Course Allocation Solutions for Sciences Po\*

Eduardo Perez-Richet Edoardo Ciscato Alexandre Bauer
Oliver Cassagneau Pauline Corblet Arnaud Ors
Mattijs Van Miert Nicolas Vogtenberger Alice Winograd

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#### Abstract

Students at Sciences Po have expressed concern with the current registration system, which allocates courses in real time on a first-come, first-served basis. Course allocation is a well known and difficult problem in the area of market design, a branch of modern microeconomics. Recent research has made considerable progress in understanding this type of allocation problems. In this report, we present the criteria that the market design literature has used to assess course allocation mechanisms, we evaluate the Sciences Po mechanism according to these criteria, and present some alternatives by reviewing the three main classes of mechanisms that have been proposed in theory and implemented in practice. We discuss how these mechanisms perform, and whether they are easy to adapt at Sciences Po given the specificities of the institution.

<sup>\*</sup>This report is the fruit of a Market Design workshop specifically created during the Winter-Spring semester of 2017 to propose solutions for course allocation at Sciences Po. The workshop was run by Eduardo Perez-Richet, professor, and Edoardo Ciscato, Ph.D. student, both at the Department of Economics at Sciences Po. The students who participated in the workshop ranged from undergraduate students, to masters students in different specialties. We thank the administration at Sciences Po for its support, and in particular for helping us run a survey of Sciences Po students.

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#### 1 Introduction

Every semester, at Sciences Po as in universities worldwide, students need to choose the courses they will attend. Because of capacity limits, seats in some courses are scarce resources. This makes the course allocation problem a typical object of study for economics. The branch of economics that studies the design of market solutions to specific resource allocation problems is called *market design*.

Course allocation is one of numerous allocation problems for which market solutions with money are not permitted. Other examples include the assignment of students to public schools or universities, medical residents to hospitals, ENA graduates to their first position, or donors to patients in need of a kidney transplant. Contrary to all these examples, the course allocation problem is combinatorial meaning that each student requires a bundle of courses rather than a single course. Other examples of combinatorial allocation problems without monetary transfers include workforce scheduling (e.g. the allocation of working shifts to nurses in hospitals), the allocation of players to sport teams, and the allocation of takeoff-and-landing slots to airlines. These problems are very challenging for both theoretical and practical reasons, and the look for satisfactory solutions is an active research topic. For course allocation mechanisms, the goal is to maximize student satisfaction, which is generally understood as achieving fair and efficient mechanisms. Most mechanisms used in practice fail on at least one of these criteria, and sometimes on both.

Course Allocation at Sciences Po. The current course allocation mechanism at Sciences Po is a real-time, first-come first-served system. Enrollment takes place over a limited window of time in a given day. Each course has a given capacity, and students can enroll in a course as long as there remain available seats, on a first-come first-served basis. As a result, students rush to enroll in their favorite courses and/or in those that they think will be most popular among other students. In fact, most of the enrollments happen in the first five minutes after registration is open (see Santoul, 2014).

The primary criterion for evaluating a course allocation mechanism is student satisfaction. As shown in Figure 1, from a survey of Sciences Po students we conducted (see Section 3), 71.6% of the surveyed students are quite unsatisfied or very unsatisfied with the current system, which leaves lots of room for improvement. Students perceive the Sciences Po as unfair and stressful. The real time nature of the process contributes greatly to this perception. The need to rush is stressful for the students. Furthermore, there is a risk of being logged out of the system, and therefore losing any chance at getting a seat in the most popular courses. In addition, the students must not only think about the courses they would like to take, but also about which of those they think other students will also like, to figure out their priorities during the registration process.

## For you the current system (real time allocation/first comes, first served) is: (1995 réponses)

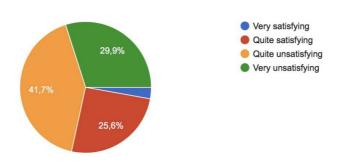


Figure 1: Student satisfaction with the current first-come-first-served system at Sciences Po.

Most existing course allocation mechanisms are not real-time mechanisms. In a first phase, students express their preferences over courses during a period that can last up to a few weeks. Then, in a second phase, an algorithm is run to obtain the final allocation of courses. Any of these systems is likely to be much less stressful for the students, as they avoid the rush over courses, and eliminate the risks associated with the possibility of being logged out of the system.

Ultimately, however, more important than the process itself is the final allocation. Figure 2 shows the satisfaction of Sciences Po students with their course schedules. Overall, 37.5% are quite unsatisfied or very unsatisfied (the figure would be 40.1% is we included PSIA students). While it shows that the lack of satisfaction is largely due to the process rather than the outcome itself, the figure is still quite high.

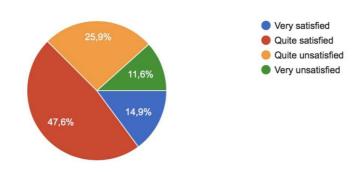
Evaluation Criteria. To maximize student satisfaction with their final allocation, market designers focus on several criteria. The two main criteria are efficiency and fairness. A course allocation is efficient if there is no way to improve the course schedule of any student without harming other students. This is the classical notion of (Pareto) efficiency in economics. In the context of course allocation, schools generally want all students to be entitled to the same level of satisfaction. For example, an allocation in which some students would get to pick all their preferred courses, while some other students would get none of their preferred courses would be unfair. While fairness can be defined in several ways, we will for this report retain a criterion of envy. A course allocation will be considered fair if it does not lead any student to greatly prefer the schedule of some other student to her own.

Another important criterion is *strategy proofness*. Economists generally try to design mechanisms that participants have no incentive to manipulate by misreporting their preferences. In the case of course allocation, having to figure out their true preferences is complicated enough

for students, that schools do not want to overburden them with having to think about how to strategically report their preferences. In the words of Al Roth, who has been awarded the Nobel prize in economics for his work on market design, strategy-proof mechanisms avoid participants "having to engage in costly and risky strategic behavior" (Roth, 2008)

Finally, there are two additional criteria that matter in the practical design of mechanisms: ease of use, and computation time of the algorithm. It must be sufficiently easy for students to participate in the mechanism, and specifically to report their preferences over course schedules, and the algorithm must be able to find a course allocation in a reasonable amount of time.

# Regarding the outcome of your "Inscriptions Pédagogiques", you were: (1535 réponses)



**Figure 2:** Student satisfaction with the outcome of course allocation at Sciences Po (excluding PSIA).

State of the Art. All existing course allocation mechanisms fail on at least one of the two main criteria. For example, we show in this report that the current mechanism at Sciences Po, while hard to fully model theoretically, is both inefficient, unfair and manipulable (see Section 3). A classical solution for single-item allocation problem is the serial dictatorship mechanism. It is the mechanism used to assign ENA graduates to their first position. In the case of course allocation, it would have students take turns to select their entire course schedule. While such a mechanism is strategy-proof and efficient, it leads to widely unfair outcomes. Other (more reasonable) theoretical and actually implemented solutions fall into three categories. *Draft Systems* are a variation over serial dictatorship in which students take turn to choose courses one-at-a-turn. They improve on the fairness dimension, but lead to a loss of strategy-proofness and efficiency. Such a system is used at the Harvard Business School for example. *Auctions with fake money* or *bidding mechanisms* are used in many business schools. However, contrary to auctions with real money, such auctions are not strategy-proof, and lead to allocations that are neither efficient nor fair. More recently, Budish (2011) has proposed a new mechanism based on competitive equilibrium, which leads to approximately fair and efficient

outcomes. A variation of this mechanism has been implemented at the Wharton School at the University of Pennsylvania.

In what follows, we propose an assessment of Sciences Po's current course allocation system, and a survey of the main existing theoretical and practical solutions. We try to assess the adaptability of each of these mechanisms to the particular context of Sciences Po. Before this, it is important to define more precisely the criteria according to which course allocation mechanisms are usually evaluated.

### 2 Evaluating Course Allocation Mechanisms

In this section, we provide more details about the criteria market designers and practitioners use, or may want to use, to evaluate allocation mechanisms.

Efficiency. The notion of efficiency is the usual one in economics. A course allocation is efficient if there is no way to give some student a better course schedule without giving some other students a worse course schedule. Efficiency is desirable in the sense that any inefficient allocation can be (weakly) improved for all students. On the practical side, delivering efficient course allocation avoids the creation of side markets and course switching between students. This is because an inefficient allocation is one that leaves trading opportunities to the students: for example, student  $s_1$  is registered in course  $c_1$ , but would prefer course  $c_2$ , whereas student  $s_2$  is registered in  $c_2$  and would prefer  $c_1$ .

Fairness. Notions of fairness are meant to capture the idea that students should have equal access to courses. In this report we will focus on a notion related to. A course allocation is *envy-free* if no student prefers the course schedule of another student. However, it is mathematically impossible to design a mechanism that is totally free of envy. A weaker notion is *envy bounded* by a single good. A course allocation satisfied this notion if, whenever a student  $s_1$  prefers the course schedule of another student  $s_2$  to his own, it would be sufficient to remove one course from  $s_2$ 's schedule to eliminate  $s_1$ 's envy for  $s_2$ 's schedule. Hence, this notion limits the degree to which a student may envy another student.

**Strategy-Proofness.** Strategy-proofness is a property of the mechanism itself, that is the rules according to which courses are allocated, rather than of the final allocation. A course allocation mechanism is strategy-proof if it is a dominant strategy for all students to report their preferences truthfully. That is, the best a student can do, regardless of other students preferences and reports, is to report her own true preferences. Mechanisms that are not strategy-proof are said to be *manipulable*. Because strategy-proofness may be difficult to satisfy along with other criteria, we will generally consider a weaker notion. A mechanism is *strategy-proof* 

in the large, if, in the limit, it becomes strategy-proof as the number of participants becomes large. This notion is appropriate in practice, as course allocation mechanisms typically involve a large number of students.

Strategy-proofness is not a performance criterion itself, but has some important practical implications, including for student satisfaction. Indeed, engaging in strategic thinking for the students may be costly and stressful. In addition, students may think that a manipulable mechanism gives an unfair advantage to students who are better at gaming the system. Just having to report their true preferences without worrying about how to game the system, or whether others are doing it, makes the task of students simpler. A side benefit of strategy-proof mechanism is that they allow the institution to get accurate data on the preferences of students over courses. This data may be used by the school to adjust its supply of courses, or simply to learn about the preferences of its students, or by researchers.

Bringing the Criteria Together. One of the difficulties of the course allocation problem is highlighted by several theoretical impossibility results. A common solution to allocation problems is *serial dictatorship*: students come in a given order, and sequentially choose their whole course schedule. The calling order may be determined randomly or based on some other criterion. Serial dictatorship is both efficient and strategy-proof. If the calling order is random, one could argue that it is procedurally fair, in the sense that students have an equal chance of getting a good draw. However, it does not satisfy any reasonable notion of allocation fairness. In fact, it is generally impossible to satisfy all criteria at the same time as the following results illustrate.

**Theorem 1** (Budish, 2011). There is no course allocation mechanism that is strategy-proof, efficient, and satisfies envy bounded by a single good.

**Theorem 2** (Hatfield, 2009). A course allocation mechanism is strategy-proof, efficient, non-bossy and neutral<sup>1</sup> if and only if it is a serial dictatorship

**Preferences.** Proposed mechanisms differ in the kind of student preferences they can handle. An issue with course allocation is that a student's preferences may exhibit strong substitutabilities and complementarities. For example, a student may prefer the bundle {political science, sociology} to {political science, economics}, but prefer the bundle {finance, economics} to the bundle {finance, sociology}. Some of the mechanisms we discuss work with such preferences, while others do not. Whether student preferences exhibit strong complementarities and substitutes may depend in practice.

 $<sup>^{1}</sup>$ A mechanism is non-bossy if a student cannot change the allocation of other students without changing her own allocation. It is neutral if the identity of students is irrelevant for the mechanism.

Computability. When actually implementing a course allocation mechanism, computation time becomes an important consideration. Typically, once the registration period is over, it should not take more than a few days with reasonable computing power to output the final allocation.

Preference Language. Relative ease of use of the mechanism is important. One important aspect of this is the simplicity of the language with which students can express their preferences. In theory, a preference is an ordering of all possible course schedules. In practice, this means ranking an unreasonably high number of possible schedules. Therefore a mechanism must use a simpler language to express preferences. If courses are homogeneous, a simple language is a ranking over all single courses. A problem with such a language is that it rules out preferences with complementarities and substitutes. In addition, courses are typically not homogeneous at Sciences Po, and it would not work to put fundamental courses and optional ones in the same ranking. Another possibility is to use a cardinal language, that is one that puts numbers interpreted as utilities on single courses, and possibly on pairs of courses as well, so as to capture possible complementarities. Yet another possibility is to let students bid over courses, in which case bids are the expression of preferences.

#### 3 Course Allocation at Sciences Po

The Current Real Time First-Come, First-Served Registration System. All Sciences Po students register for classes through the same online platform. Enrollment takes place on a dedicated day before the beginning of the semester, which may differ by study program. Students need to sign in to their Sciences Po account, where they can check their respective course requirements and have access to the full list of courses, divided into categories. Students interested in a specific course must open the course page to check availability. The principle of the first-come, first-served system is that, as long as seats are still available, the student can register for the course. Hence, a course is complete when the number of enrolled students reaches the preallocated capacity. During the registration day, the student can also decide to go back to the same page and unsubscribe from a course. For further details about the general functioning of the current system, see Santoul (2014).

Issues and Complaints There is much evidence about students' dissatisfaction with the system. This is confirmed by our own survey (see Figure 1 and Figure 2). Looking at the log-file of Sciences Po server, Santoul (2014) documents that most students rush to enroll in the very first minutes after the opening of the registration window. He also notes that trying to register for a course is time-consuming, as students need a few crucial seconds to check

the availability and make a demand. Surveyed about this issue, students confirmed that they behave strategically and try to enroll in the courses they believe to be the most popular (see Figure 10).

PSIA students have complained about the registration system in an open letter to the Dean of the school, Enrico Letta, and the Vice-Dean, Vanessa Scherrer. They write that the registration day "involves a certain amount of stress as they have to be faster than their peers", and report "frustration, disappointment and sometimes anger", as some of them fail to obtain the courses they want because of arbitrary circumstances (e.g., poor Internet connection). PSIA students also remark that they lack quality information about the courses (e.g., no access to students' feedback from past years) and that the registration day is followed by a chaotic period of informal add-and-drop process where students swap courses and complain with their academic advisors and professors, in the hope of being allowed into the courses they could not register for.

Evaluating the Sciences-Po Mechanism: Some Theory. While developing a fully fledged model of the registration process at Sciences Po would be difficult and is out of the scope of this report, the analysis of small theoretical examples shows that the Sciences Po system is neither strategy-proof, fair nor efficient. If we interpret truthful reporting as a strategy that would consist in registering for courses in the decreasing order of preferences<sup>2</sup>, it is clear that it is not a dominant strategy. Indeed, suppose a student's most preferred course is an unpopular course that never reaches its capacity limit, whereas his second most preferred course is a very popular course that is certain to reach its capacity limit. Then this student will be better of trying to register first in the popular course, and then only in his most preferred course.

To show that the registration system is not fair, we consider a simple example. Suppose that there are two students  $s_1$  and  $s_2$ , each of which can take only two courses. There are four courses, each with a capacity limit of one student. Courses  $c_1$  and  $c_2$  are star courses, whereas courses  $c_3$  and  $c_4$  are unpopular. The two students have the same preferences given by  $u(c_1) = 8$ ,  $u(c_2) = 6$ ,  $u(c_3) = 2$  and  $u(c_4) = 1$ . Their preferences are additive so that the value of a schedule is just the sum of the values of courses in the schedule. A strategy is just the order in which the students will register. If they try to register in the same course at the same time, each gets the course with probability one half. Hence, if student  $s_1$  and  $s_2$  first try to register in course 1, each of them is equally likely to be faster. But this advantage cumulates. Therefore if student  $s_1$  wins the race to course 1, and the two students then both try course 2, student  $s_2$  will have a time advantage over  $s_2$  and get course 2 with probability 1. Then we can show that the unique Nash equilibrium in pure strategy is to choose the order  $(c_1, c_2, c_3, c_4)$ . This equilibrium leads to an allocation in which one of the students gets  $\{c_1, c_2\}$ ,

<sup>&</sup>lt;sup>2</sup>This assumes preferences without strong complementarities or substitutes

and the other student gets  $\{c_3, c_4\}$ . While this outcome is efficient, it is unfair, whereas the allocation  $(\{c_1, c_3\}, \{c_2, c_4\})$  is fair and efficient. More generally, this example suggests that in an environment with homogeneous preferences, the registration system will behave as a serial dictatorship.

To show that the registration system is not efficient, we still assume that winning the first race gives a time advantage in the second race, but that this time advantage is not definitive. For example suppose that if two students compete in every race, and the first student wins the first race, she wins the second race with probability 90%. This could be due to the risk of being logged out of the system for example. Then consider the following example. There are three students  $s_1, s_2$  and  $s_3$  who have additive preferences and must take exactly two courses. There are four courses  $c_1, \ldots, c_4$ . Capacities and student preferences are given in the following table.

Course	Capacity	Payoffs			
Course		$s_1$	$s_2$	$s_3$	
$c_1$	1	10	8	0	
$c_2$	2	8	10	10	
$c_3$	3	3	3	3	
$c_4$	3	3	3	3	

Then, it is a Nash equilibrium<sup>3</sup> for  $s_1$  and  $s_2$  to use strategy:  $c_2, c_1, c_3, c_4$ , and for  $s_3$ , strategy:  $c_2, c_3, c_4, c_1$ . As a consequence it is possible for  $s_1$  to get the course schedule  $\{c_2, c_3\}$ , and for  $s_2$  to get the course schedule  $\{c_1, c_3\}$ . This happens if  $s_1$  and  $s_3$  get the two seats in  $c_2$ , and then  $s_1$  loses the race with  $s_2$  for a seat in  $c_1$  (for example because he gets logged out). The resulting allocation is inefficient<sup>4</sup> in the sense that  $s_1$  and  $s_2$  would be happy to trade their seats in  $c_2$  and  $c_1$ .

Evaluating the Sciences-Po Mechanism: Anecdotal Evidence. There is anecdotal evidence that the Sciences Po mechanism is not efficient. In their letter to Dean Letta, PSIA students write that "an idiosyncratic souk suddenly starts on Facebook where people trade and bargain classes". While post registration trade could happen as a result of changes of preferences of the students, it is mostly likely due to unexploited trade opportunities, implying that the initial allocation is inefficient. Figure 3 shows a screenshot of such a black market on Facebook.

<sup>&</sup>lt;sup>3</sup>Indeed, the only unilateral deviation that makes sense is for  $s_1$  or  $s_2$  to play  $c_1, c_2, c_3, c_4$ , and a straightforward computation shows that it yields a lower expected payoff for both players.

<sup>&</sup>lt;sup>4</sup>Note that this inefficient allocation occurs with probability 3.33% under our assumptions, but if instead we assume that the population of students is large and consists of a third of students with each  $s_i$ 's preferences, then this means that 2.22% of the student population could get a better outcome by performing mutually beneficial trades with other students.



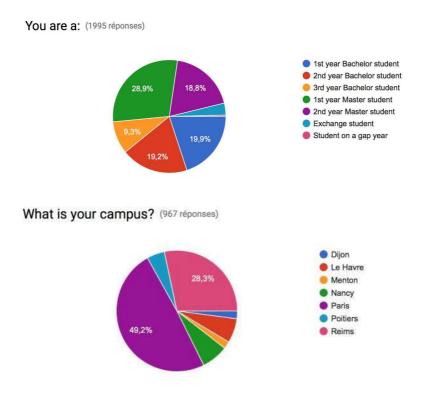
Figure 3: A black market for courses on Facebook.

Evaluating the Sciences-Po Mechanism: Survey. While the best way of evaluating the Sciences Po mechanism would be by using preference and allocation data, such data was not available to us and could not be obtained in a sufficiently short delay. Therefore we used a survey with targeted questions to get a better sense of how the system performs according to the different criteria.

We submitted a poll to the whole Science Po student population (12,500 individuals) and collected a total of 1,998 answers. The poll is composed of nine sets of questions:

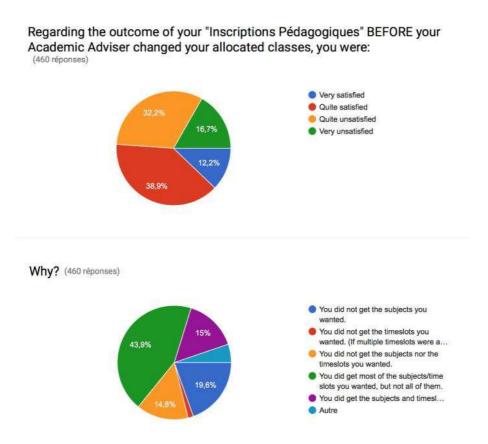
- Three of them aim at classifying the population: year at Sciences Po, campus for Bachelor students and school for master students;
- Two of them aim at evaluating students' satisfaction: a special section is for PSIA students, because we wanted to focus on the course switching possibilities in this school, the other is for the rest of the student population;
- One evaluates the student strategy related to picking classes;

- One evaluates the proportion of the population that tried to switch classes and has been successful at that;
- One aims at collecting the opinion of the students concerning the ideal system in their opinion;
- The last section is just a free speech section to get additional feedback.



**Figure 4:** Composition of participants in the survey.

We first focus on answers given by PSIA students. The first question (Figure 5) is about their satisfaction with the registration outcome right after the registration day, before any administrative change. Only half of the population (51.1%) is either very satisfied or quite satisfied. Meanwhile, more people had all the subjects and time slots they wanted (15,0%) than people said they were very satisfied (12,2%). The second question (Figure 6) asks PSIA students about their satisfaction after discussing course swaps with their academic advisors. The number of satisfied students increased (66.1% of the students are at least quite satisfied) and the number of very unsatisfied students has been divided by two (from 16.7% to 8.0%), showing that the intervention of the administration leads to some improvements. However, while the number of people having both their wished subjects and time slots slightly increased



**Figure 5:** PSIA: satisfaction before intervention.

(from 15.0% to 17.2%), the number of "other" answers greatly increased (from 5.2% to 19.1%), requiring further analysis of the free-speech answers.

We then aim to assess whether the system is fair with the notion of envy-freeness in mind(Figure 7. We also want to know how other elements (logouts, user-friendliness) could affect the user experience. Note that roughly half the respondents (54.6%) envy the schedule of a classmate. Less than a third of the students qualified the system as fair (28.7%).

Students from schools other than PSIA typically do not have an academic advisor and it is thus less common to ask for course swaps. Interestingly, the answers about general satisfaction (Figure 8) after the final outcome are quite close to those of PSIA students when asked about the course allocation *after* academic advisor's correction. Two thirds of the students (62.5%) are either very satisfied or quite satisfied with the outcome. We also have far fewer "other" answers when asking about the reasons explaining their satisfaction level (3.9%).

The share of students who envy a classmate's schedule increased for schools out of PSIA (66.4%), and again less than a third of the students qualified the system as fair (29.4%).

We asked all students how they decide the order in which they register for courses in order to know whether the system is strategy-proof (Figure 10). As expected, it is not, as most students

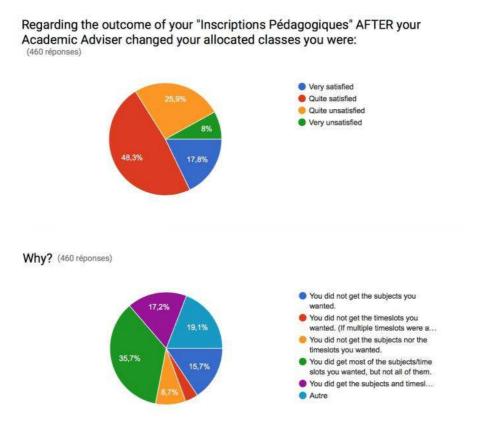


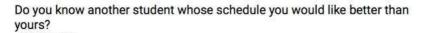
Figure 6: PSIA: satisfaction after intervention.

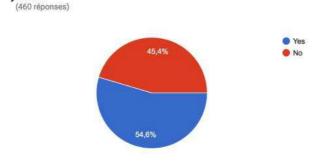
(54.0%) pick their courses beginning with those believed to be the most popular. About a third choose their most preferred class first (35.7%).

Only a small part of the population tried to swap courses (32.1% and 11.5%, respectively before and after the beginning of the semester). Among those who tried only a few have been successful (6.3% and 2.1%). While this is an indication of inefficiency, it is likely to underestimate the level of inefficiencies as the search for other students to swap with might be costly and difficult.

Only 27.7% of the population is satisfied with the current system (Figure 1). This is considerably lower than the number of people satisfied with the outcome of their registration process (roughly two thirds). This shows that other aspects account for students' satisfaction, possibly the stress and the uncertainty about the income implied by the current system.

Evaluating the Sciences-Po Mechanism: Measuring inefficiency through revealed preference data. Data generated by the current course allocation system is not informative about students' true preferences. We only know what courses a student registered for, but not how he ranks them with respect to each other and with respect to the courses he did not take. If we want to quantitatively assess the degree of efficiency and fairness of allocations output





#### Do you agree with the following statements?

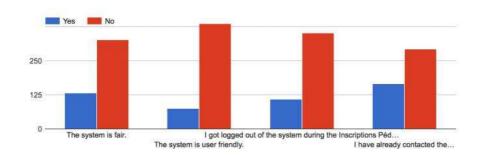


Figure 7: PSIA: fairness.

by the current system, we would need to first collect data about the students' true preferences through a survey before registration, then compare these preferences to the course schedules students obtain during registration.

Budish and Cantillon (2012) use a similar database for their analysis of the Harvard Business School registration system. They show that data on preferences can be used:

- 1. To make rigorous assessment about the current system, and in particular to measure efficiency losses due to students' strategic behavior;
- 2. To simulate the registration outcome under an alternative system, in order to compare the different allocations.

The best way to build such a dataset is to ask students to rank the courses they would like to enroll in before the registration day. Moreover, some registration systems, such as competitive equilibrium mechanisms, also require information about the intensity of the preferences. In other words, it might be necessary to ask students how much they like/dislike a certain course. In both cases, it is crucial to let the students understand that the preliminary survey has no effect on the actual registration process, and that they are expected to reveal their preferences

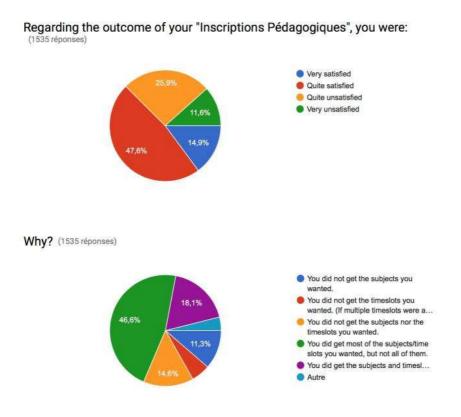
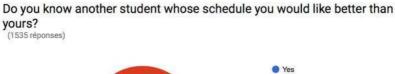


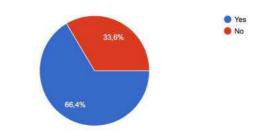
Figure 8: Other students: satisfaction.

truthfully. It is also important to survey all students, and thus possibly make the survey compulsory. This type of poll is also useful for at least other two reasons. First, such information can also be used to adjust course capacity and suppress courses that do not raise the students' interest in advance. Second, if a new registration system is implemented, it helps the students to familiarize with the interface.

A characteristic of the Sciences Po course offer is the high level of heterogeneity. Because of this, students might find difficult to state preference relations over very diverse courses, such as a compulsory law course, an artistic workshop and a language course. Moreover, Sciences Po students' schedule choice is subject to several restrictions and requirements. Sometimes, students are asked to choose at least one (or few) course(s) from a very large set. For instance, second-year undergraduate students must pick one (or two) elective(s) from a set of up to 121 courses. In other cases, students must choose the seminar associated with a compulsory lecture from a wide set of different time-and-teacher combinations.

It does not seem plausible to ask students to provide a complete preference relation over the whole set of courses. Lack of information about complete preferences might be problematic for the implementation of some allocation systems. However, it should be possible to work around this practical limitation, most likely by asking students their n most preferred courses





#### Do you agree with the following statements?

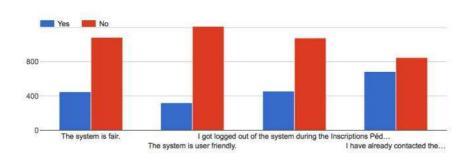


Figure 9: Other Students: fairness.

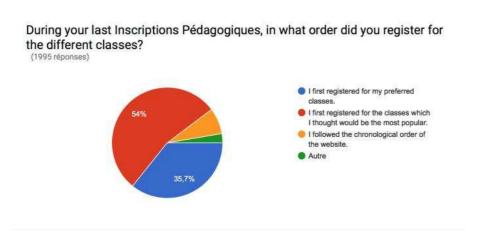


Figure 10: All students: strategy.

for each relevant category (e.g., languages, artistic workshops). The length of the ranking should be chosen taking into account the restrictions imposed by a student's curriculum, overlap in timetable, the students' marginal distribution of preferences, overlap of course offer across different study programs, and, last but not least, the registration system Sciences Po envisages Have you ever tried and been successful at switching courses with another student after the Inscriptions Pédagogiques, but before the beginning of the semester? (multiple answers possible) I have never tried I have tried, but didn't find anybody to switch with. I have tried and found someone, but the administration refused to switch our courses. I have successfully switched courses. Have you ever tried and been successful at switching courses with another student after the semester had already started? (multiple answers possible) I have never tried. I have tried, but didn't find anybody to switch with. I have tried and found someone, but the administration refused to switch our courses. I have successfully switched courses.

Figure 11: All students: course swapping.

implementing. Once a ranking over courses is provided, it is possible to build a student's most preferred schedules that are also feasible, i.e., that respect curriculum and timetable requirements.

Similarly, if information on the intensity of preferences is necessary, students might be asked to "grade" only a limited number of courses at their choice. In this case, it should be clarified that, by not grading a subset of the available courses, students implicitly express indifference among them. On the contrary, by assigning a grade to a course, students can signal that such course stands out with respect to the benchmark. It is also possible to ask students to signal courses that they dislike with respect to the same benchmark.

Specificities of Sciences-Po. Thinking about adopting an existing theoretical or implemented mechanism at Sciences Po implies that we should have in mind the specificities of the institution that will make these mechanisms more or less adaptable at Sciences Po. It is therefore useful to list some of these specificities.

• Sciences Po is composed of several schools, each of which imposes specific requirements on

students' curricula. In addition, certain schools offer different programs and ask students to pick concentrations and minors, which result in additional constraints (as an illustration, we documented the complexity of these constraints for PSIA in Appendix A). Only some types of classes can be considered as "scarce resources", and this highly depends on the student's curriculum. First-year undergraduate students compete over indivisible bundles of three compulsory supervised practical sessions in economics, history and politics, which differ in time and teachers ("triplettes"). Second-year undergraduate students compete over both main classes and electives. PSIA students compete over Core Curriculum and concentration classes.

- Some classes are made available to students from different programs and schools. Students with very different curricula apply to the same language and artistic classes, as well as the same master's Formation Commune classes. Hence, it is not straightforward to deal with different schools separately: this overlapping should be considered in the analysis.
- Classes greatly differ not only in terms of contents, but also in the relative importance they have in a student's curriculum. If a student fails to enroll in her favorite core classes, it might be hard to set up a compensation by assigning her to her favorite language class. Some classes seem hard to trade off because they are too diverse, and students might find difficult to express preference relations between different types of classes.

#### 4 Draft Mechanisms

General Description. Draft mechanisms are similar to serial dictatorship in the sense that students take turn to choose courses. Instead of choosing her whole schedule when turn comes, a student chooses one course at a time. In the first round, all students are called in a random order to choose their first course. In the second round, the choosing order reverses for the choice of the second course, and so on until all students complete their schedule. In practice, the students report their preferences over individual courses in the form of an ordered list, and then the computer chooses courses for them one at a time in random sequence that reverses at each round. The draft system has been used since the mid-1990s to allocate courses at Harvard Business School. Draft procedures are used in other contexts, such as the allocation of new players to professional sports teams in the NBA.

**Evaluation.** The HBS draft procedure has been studied extensively, both theoretically and empirically, in Budish and Cantillon (2012). Intuitively, alternating the order of choice should lead to fairer outcomes than serial dictatorship, but at a potential cost in terms of strategy-proofness and efficiency. Indeed, Budish and Cantillon (2012) show that the HBS draft system

is manipulable (non strategy-proof) in theory, and manipulated in practice. The draft system gives students an incentive to push popular classes up in their ranking, so as to get them in the early rounds, before they get filled up. For example, if student A's most preferred course  $c_1$  is relatively unpopular, so that she will get it for sure, but also likes course  $c_2$  which is very popular, she will rank  $c_2$  higher than  $c_1$ . Manipulations are costly, as the draft system would be efficient if students were to report their true preferences, but is no longer efficient if students behave strategically. Using preference data obtained through a survey, Budish and Cantillon (2012) evaluate a high magnitude of inefficiency as 64% of students could be involved in mutually beneficial trades. However, the HBS draft system performs well in terms of fairness, even when manipulated. In particular, it satisfies envy bounded by a single good. Budish and Cantillon (2012) show that, in spite of its manipulability and inefficiency, the HBS draft system is still preferable to serial dictatorship from an ex ante social welfare point of view.

Adaptability to Sciences Po. One problem with the draft system is its preference language. Students submit an ordered list of individual courses. This works well for homogeneous types of courses and homogeneous study programs. Science Po courses and study programs, by contrast, are very heterogeneous. There are many different study programs and courses with different status. Fundamental or mandatory types of courses can hardly be ranked together with optional language courses. Therefore adapting the draft system to Sciences Po would mean designating different lists of courses that the students need to order. When running the draft, we would need to choose the order in which different types of courses are allocated to different types of students. In addition, one course may appear in a different list for different types of students, and any choice of procedure might give advantages to students in a given study program over students in another study program in choosing a certain course. Furthermore, the amount of thought and reprogramming that would be needed would be considerable whenever the requirements of one study program change, or for any other curricular modification. While doable, a draft mechanism might therefore not be the best solution for Sciences Po.

### 5 Bidding Mechanisms

General Description. In bidding mechanisms, students are provided with tokens of fake money that they can use to bid in auctions for courses. These mechanisms are used in numerous schools such as the Booth Business School at University of Chicago, Columbia Business School, Haas School of Business at UC Berkeley, Kellogg Graduate School of Management at Northwestern, Princeton University, University of Michigan Business School, Wharton School at the University of Pennsylvania, and Yale School of Management. While not all schools use the same version, the following simplest version used at University of Michigan Business School cap-

tures the main features of these mechanisms. Its description is taken from Sönmez and Ünver (2010), which is the first market design paper to analyze the properties of these mechanisms:

- 1. Each student is given a bid endowment to allocate across courses.
- 2. All bids for all courses and from all students are ordered in a single list from highest to lowest and processed one at a time in a descending order. At each turn, a bid is *honored* if the bidding student has not filled his schedule, and there remain seats available in the corresponding course.

Evaluation. While used by many schools, bidding mechanisms have several issues. Auctions with real money are known to perform quite well in combinatorial allocation problems, and have been a fruitful area of research in market design with some widely recognized successes such as the design of spectrum auctions. Auctions with fake money, however, have very different properties (Sönmez and Unver, 2010). Intuitively, when losing a real auction, a bidder is left with something she values (money), whereas a losing bidder in an auction with fake money is left with useless tokens. This leads to incentive problems: bidding mechanisms do not satisfy strategy-proofness. If a student anticipates a course to be unpopular, that is to have a very low clearing price, she will place a low bid on that course even if she likes it, and save tokens for popular courses which she anticipates to have a high clearing price. Ultimately, this can lead to unfair and inefficient allocations. An additional issue with these mechanisms is that students may finish the registration period without being registered in a single course due to unsuccessful bids. At Wharton, where a bidding mechanism has recently been replaced by a market equilibrium mechanism (see Section 6), students reported low satisfaction with the bidding mechanism (Budish, Cachon, Kessler, and Othman, forthcoming; Budish and Kessler, 2016).

Adaptability to Sciences Po. Adapting a bidding mechanism to Sciences Po does not seem particularly problematic. However, bidding mechanisms are flawed in several ways, as we just mentioned, and are likely to be stressful to the students while not delivering fair and efficient allocations.

#### 6 Competitive Equilibrium Mechanisms

General Description. Budish (2011) proposes a novel mechanism which he terms *Approximate-Competitive Equilibrium from Equal Incomes* (A-CEEI), based on the theory of competitive equilibrium in markets. A variation on this mechanism has been implemented at Wharton to

replace the former bidding mechanism. This implementation, called *Course Match*<sup>5</sup> is described in Budish et al. (forthcoming).

The A-CEEI mechanism is based on the idea of competitive equilibrium from equal incomes (CEEI), which is an old idea from general equilibrium theory. CEEI consists in allocating goods to agents through a market mechanisms with competitive equilibrium prices and equal budgets for all agents. When the goods to be allocated are divisible, CEEI is known to provide an efficient and fair allocation (see Varian, 1974). Arnsperger (1994, 161) describes it as "essentially, to many economists, the description of perfect justice." The philosopher Ronald Dworkin uses it as the motivation for an important theory of fairness (see Budish, 2011).

Before describing the actual mechanisms, we describe the general idea of how a competitive equilibrium mechanism works. Before the beginning of the semester, students submit their preferences across possible schedules. Each student is given an endowment of fake money that is his budget. However, students do not have to think about how to allocate this budget as the algorithm will do it for them. When students have finished submitting their preferences, the mechanism looks for prices (that is a price for each course) that satisfy two requirements: first, prices clear the market in the sense that there is no seat in a course left to allocate that a student would be willing to take, and, second, at these prices each student receives the best course schedule he can afford given his budget. Loosely speaking, such prices can be found by a search algorithm that works as follows: start from arbitrary prices, give the students their best schedules given the prices, if the number of seats in a course exceeds capacity limits, change the price of this course. The algorithm keeps looking for prices until it stops, in which case the prices found by the algorithm are equilibrium prices.

If CEEI could be applied to the course allocation problem, it would therefore provide an attractive solution. However, seats in courses are indivisible, and preferences for courses might exhibit complementarities. Each of these features can prevent the existence of equilibrium prices (an algorithm searching for prices would never terminate). As a consequence A-CEEI introduces two modifications so as to recover existence:

- (i) Students are given approximately equal instead of exactly equal budgets.
- (ii) The final prices clear the market approximately instead of exactly, so that some capacity limits may be violated by a small amount

There are two important issues for implementation of A-CEEI in practice. First, a computational issue: is it possible to perform the price search in sufficient time for a real large scale problem? Second, the A-CEEI mechanism assumes that students can report their preferences accurately, that is their complete ordering of possible course schedules. This is impossible in

<sup>&</sup>lt;sup>5</sup>A description is available on the website https://mba-inside.wharton.upenn.edu/course-match/ with video tutorials.

### **COURSE MATCH**

#### UTILITY SELECTION

		Searc	h:	reset	▶ My Settings		
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ACCT611002	FINANCIAL ACCOUNTING ARMSTRONG C MW 10:30 AM - 12:00 PM (8/26/15 - 12/3/15) This course CANNOT be taken Pass/Fail.	1.00	S				
ACCT611003	FINANCIAL ACCOUNTING ARMSTRONG C MW 12:00 PM - 1:30 PM (8/26/15 - 12/3/15) This course CANNOT be taken Pass/Fail.	1.00	S				

Figure 12: The Course Match user interface displaying option to rank pairs of courses

practice, and the challenge is then to find a simplified preference language that is both sufficiently simple for students to use, and sufficiently rich so as to allow them to report preferences accurately enough that the algorithm actually solves the "right" problem. To achieve this, Course Match brings three important modifications. First, it implements a "software architecture that allows it to scale sufficiently to solve a Wharton-sized problem in a reasonable amount of time" (Budish et al., forthcoming). Second, it uses a rich preference language based on utility reports over individual courses, with modulations by pairs so that students can express complementarities or substitutes in their preferences (see Figure 12). Finally, it modifies the price finding algorithm of Othman, Sandholm, and Budish (2010) used to find approximate equilibrium prices as in Budish (2011) by adding two stages that reduce the amount of market clearing errors.

**Evaluation.** Budish (2011) shows that the A-CEEI mechanism is strategy-proof in the large, approximately efficient (that is up to the market clearing approximations), and fair in the sense that it satisfies envy bounded by a single good. Course Match has been used a Wharton from the Fall semester of 2013. Budish et al. (forthcoming) report excellent performances in terms of efficiency. As expected, some courses have exceed target capacity, but the excess is in the order of 2% of target capacity. They also compare the extent of inequality between the formerly used bidding system and Course Match, showing that the latter delivers fairer outcomes. In terms

of computation, they report using 7 compute servers from Amazon webservices. The algorithm runs over 48 hours in this setup.

Adaptability to Sciences Po. The Course Match system can be adapted to Sciences Po. It is actually a good fit for the specificities of Sciences Po, as the different curricular constraints would be easy to program within the user interface of the students. One way to take into account the different curricular requirements would be to allocate different budgets to different types of students, as is done in Wharton for students in different years. In terms of computing, it seems perfectly possible to run the Course Match algorithm in a reasonable time at the scale of Sciences Po. In fact, Budish et al. (forthcoming) mention that they expect the algorithm to be able to run for a University the size of Ohio State (60,000 students, and thousands of courses). However, implementing such a system is likely to require a significant investment of resources.

#### 7 Conclusion

Students at Sciences Po have expressed concern about the course allocation system. They question both the process itself, and the final allocation it delivers. As we show in this report, this concern is justified: the Sciences Po mechanism is manipulable, unfair and inefficient. While assessing the extent of these failures empirically would require more data than is available, our detailed survey supports these findings. Course allocation is a difficult problem, and research has shown that many of the mechanisms used in practice are flawed in some way. However, most of them seem less stressful than the real time process at Sciences Po, because they are run in two phases: a reporting phase, an an allocation finding phase. In practice, schools also run an organized add-and-drop phase following the first registration process that is likely to improve outcomes, as the preferences of students may change between registration and beginning of courses. More importantly, research has made significant progress in designing course allocation mechanisms, and the Course Match solution adopted at Wharton appears to be a good compromise on the different criteria. This solution also appears to be more adaptable to Sciences Po than draft mechanisms. Bidding mechanisms would be adaptable to Sciences Po, but they are not strategy proof, and may deliver inefficient and unfair outcomes.

#### References

- ARNSPERGER, C. (1994): "Envy-Freeness and Distributive Justice," *Journal of Economic Surveys*, 8, 155–186.
- Budish, E. (2011): "The Combinatorial Assignment Problem: Approximate Competitive Equilibrium from Equal Incomes," *Journal of Political Economy*, 119, 1061–1103.
- Budish, E., G. P. Cachon, J. Kessler, and A. Othman (forthcoming): "Course Match: A Large-Scale Implementation of Approximate Competitive Equilibrium from Equal Incomes for Combinatorial Allocation," *Operations Research*.
- Budish, E. and E. Cantillon (2012): "The Multi-Unit Assignment Problem: Theory and Evidence from Course Allocation at Harvard," *The American Economic Review*, 102, 2237–2271.
- Budish, E. and J. Kessler (2016): "Bringing Real Market Participants' Real Preferences into the Lab: An Experiment that Changed the Course Allocation Mechanism at Wharton," Working paper.
- HATFIELD, J. W. (2009): "Strategy-Proof, Efficient, and Nonbossy Quota Allocations," *Social Choice and Welfare*, 33, 505–515.
- Othman, A., T. Sandholm, and E. Budish (2010): "Finding approximate competitive equilibria: Efficient and fair course allocation," *Proceedings of the 9th International Conference on Autonomous Agents and Multiagent Systems*, 1, 873–880.
- ROTH, A. E. (2008): "What Have we Learned from Market Design," *Economic Journal*, 527, 285–310.
- Santoul, J. (2014): "The Fight for Courses: Documenting the Sciences-Po's Course Registration System," Master's Thesis, EPP, Sciences-Po.
- SÖNMEZ, T. AND M. U. ÜNVER (2010): "Course Bidding at Business Schools," *International Economic Review*, 51, 99–123.
- Varian, H. R. (1974): "Equity, Envy and Efficiency," Journal of Economic Theory, 9, 63–91.

#### **Appendix**

### A Registration at PSIA

The PSIA is Sciences Po's largest graduate school, offering nine different master programs<sup>6</sup> to more than 1,300 students, which represent around one tenth of Sciences Po's overall student body. Master programs typically last two years and vary in total number of students, International Security being the largest and Environmental Policy the smallest.

Masters in PSIA are all composed of four semesters in which every student takes up around 30 ECTS. The first semester offers general and introductory courses, whereas the 2nd and 4th semester have more specialized courses. In the Spring semesters (2nd and 4th), first and final year students have many classes together. This not only means that there are more students who might want to take a certain class, but also that 4th-year students have a second opportunity to enroll in classes that they did not obtain earlier. In the 3th semester, students either perform an internship (about 90%), write a thesis (about 8%), or go abroad for an exchange semester (about 2%). Depending from the semester, students take between 5 and 9 classes.

PSIA has an enormous and diverse course offering where students are rather free to build their own program. However, this also implies high competition over the individual courses. The stakes in the course allocation mechanism are quite important, since failing in the course registration might imply the student will not learn some valuable skills needed for the job he is aiming for. Master courses differ by content and relevance in the curriculum, and are categorized into different types:

- 1. "Formation Commune" (FC): required in the 1st and 2nd semester, all students have to take two FC classes in total. FC courses are shared with all other Sciences Po masters and can treat very different subjects. Students may thus have a clear preference regarding the content. In 2016, Sciences Po offered 18 (Fall) and 19 (Spring) FC classes.
- 2. Quantitative Skills (QS): all PSIA students have to take one QS class, either in the 1st or 2nd semester. Students can choose between two topics (statistical reasoning or quantitative tools), whether they want to follow it in English or French, the teacher and time slot. Students who fail to get their favorite QS subject in the first semester can try again in the 2nd, when there is less competition.
- 3. Core Curriculum (CC): CC classes cover a master program's key topics, and constitute the only element of distinction across programs. The program's choice is made when

<sup>&</sup>lt;sup>6</sup>The programs are: International Security, International Public Management, International Economic Policy, International Development, Human Rights and Humanitarian Action, Environmental Policy, International Energy, Journalism and International Affairs (Dual Degree), Master in Advanced Global Studies.

applying to Sciences Po and cannot be changed afterwards. For each program, a set of three CC classes needs to be picked each semester (excluding the 3rd). The choice for classes is restricted by module (i.e., topic) requirements which vary from one semester to another and from master to master. Some modules are only available to first years, others only available to final years, some others to both or even core for first years and optional for second years. Classes offered in the core curriculum of a certain master are often also offered in one or multiple concentrations. Sometimes one student can thus take the same class either as a CC class or through one of his two concentrations. There are also courses that feature in the CC offer of multiple PSIA masters. Since CC classes may greatly differ in contents even within the same program, preferences can be very outspoken and the stakes for this kind of classes are high in general.

- 4. Concentrations: PSIA students specialize in certain topics through minors, so-called "concentrations". Most students choose two concentrations. The choice for concentrations is made after being accepted at Sciences Po, but before the registration day. There is no limitation to the amount of students that can choose a certain concentration. PSIA offers 23 concentrations in total, which means that there are 506 different possible combinations. Within each concentration, students choose one class every semester. The amount of options varies per concentration and can range from 1 to 29. Just like the CC courses, concentration classes vary immensely in content and not having a certain class might imply that a student will not learn a valuable skill or some crucial knowledge he needs for a certain job. The stakes are thus high.
- 5. Language: all PSIA students are obliged to follow a modern language. The same language courses are also open to students from other masters and the undergraduate schools. SciencesPo offers 20 in-house taught languages and, through a cooperation with Inalco, students have access to even more languages. Each language is taught on different levels (A1 through C2). For popular languages, multiple lectures, differing by teacher and time slot, are organized per language level. Some advanced language classes for Spanish, Italian, German and Russian are less language focussed, but rather treat specific subjects in the foreign language (e.g., German history or Italian art).
- 6. Special features: PSIA also offers a set of optional special features, which are the same for all PSIA students. Students may choose up to one special feature every semester. This can either be a team project (first year only), a second modern language or one of the other offered courses. Some optional classes are also accessible through certain concentrations or the QS course set, open to students from other masters or are on admission (e.g., Business Plan). Some of those courses have proofed to be very popular (e.g., Public Speaking Workshop).

Where there is competition between seats in classes for both first and final year students, or for CC students vs. concentration students, the administration reserves a certain amount of seats for each group. The consequence is that fewer seats are available to each group. Nonetheless, if a certain group (e.g., final year concentration students) does not fill all their free sports, those will be opened to students from other groups (e.g., first year core curriculum) in a second round.

In fact, because of PSIA's complicated structure, registration system runs in two rounds. During the first round, students choose their FC, CC, QS and language classes. In the second round they choose their concentration and special feature classes. This generates a set of new problems. First, if a student does not get her desired classes in the first round, she might end up with a time schedule conflicting with her preferred concentration classes. Consequently, in her second round he will be forced again to take a class that is not his first choice. Because of this system, even an undesired time slot for the compulsory language class can be problematic. Second, students often have the possibility to choose a certain course both through their concentration and their core curriculum. Hence, additional strategic interplays between the first and the second round arise.

Finally, a peculiar feature of PSIA registration process is that students report trying to swap courses informally, negotiating on social networks and then proposing the swap to their academic advisors and professors. This suggests that the initial allocation is not efficient, and that Pareto-improvements are possible.

[image here]

#### Filling Rates and Competition across Programs at PSIA

We analyzed Fall 2016 data about filling rates of courses under the PSIA administration's responsibility. Filling rates are the ratios of the number of students who registered for a given course through the system over the maximum amount of students per course.

PSIA organised 136 own courses that semester, 45 of which were lectures and 91 were seminars. Lectures had an average fill rate of 98.3%, seminars of 95.2%. However, some courses have a low fill ratio, while others go far beyond their capacity, even up to 160%. Half of the classes are full or almost full (between 90% and 100%). About 1 in 4 classes faced an overcapacity right after the registration day, but for less than 1 in 10 this overcapacity exceeded the 110%. In total, 35 classes were congested before displacements were arranged by the academic advisors. For seminars, the extremes (lower or equal to 80% and more than 110%) take up a larger share than for lectures, which can probably be explained by the larger amount of course offerings and the lower average amount of available seats per course. About 45% of all classes were completely filled or had an overcapacity. 1 in 10 of the classes have less than 80% filling rate, and are thus at risk of being canceled.

	Lectures		Seminars		Total	
	#	%	#	%	#	%
$x \le 80\%$	2	4.4	13	14,3	15	11.0
$80\% < x \le 90\%$	8	17.8	10	11.0	18	13.3
90% < x < 100%	15	33.3	27	29.7	42	30.9
100%	8	17.8	18	19.8	26	19.1
$100\% < x \le 110\%$	9	20.0	15	16.5	24	17.6
110% < x	3	6.7	8	8.8	11	8.1

Classes acces-sible in	6 ways	5 ways	4 ways	3 ways	2 ways	1 way
#	1	7	3	13	38	67
%	0.1	5.4	2.3	10.1	29.5	51.9

A second dataset reflects all the 82 core curriculum and 99 concentration classes PSIA students can sign up for and which students can choose what classes through their master or concentration. This helps us to understand how many categories of students compete over a specific class. This data only includes the PSIA masters and concentrations that can elect a certain course, but it is possible that students from other master schools can also choose some of these classes.

In the Fall semester of 2015, half of the classes were accessible for only one category of student and almost one third for two students categories. In the last case, most classes (87%) are accessible through one master program and one concentration, while only two classes feature in the core curriculum of two distinct masters and three classes in two distinct concentration offerings. There are thus only 24 classes where more than 2 different kinds of PSIA students compete. Most of those classes are an option in the core curriculum of different masters and are accessible through one (and sometimes two) concentrations.