# The art of Stokes inversions

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#### Outline

#### PART 1: Theory

- 1. The RTE: formal solution & approximations
- 2. The Response Functions (RF)
- 3. What is an inversion technique? How does it work?
- 4. The SIR code.
- 6. How to choose an inversion technique?
- 7. Running SIR
  - Input files
  - Visualization of observations, fits, and model atmospheres
  - Tips and tricks

#### PART 2: SIR exercises

### Files

You can download the SIR code, and all the files to run it from:

https://github.com/BasilioRuiz/SIR-code

And the slides and exercises from

https://github.com/BasilioRuiz/SIR-course

RTE: 
$$\frac{dI_{v}}{ds} = -\chi_{v}I_{v} + \eta_{v}$$

Absorption coefficient: 
$$\chi_{v} = (\chi_{ff} + \chi_{bf}) + \chi_{bb} \phi(v)$$

Emission:  $\eta_{\nu}$ 

Optical depth:  $d\tau_{v} = -\chi_{v} ds$  (number of mean free path at v)

Continuum optical depth:  $d\tau_c = -\chi_c ds$  (number of mean free path at continuum)

Suppose no emission :  $\frac{dI_{\scriptscriptstyle V}}{ds} = -\chi_{\scriptscriptstyle V} I_{\scriptscriptstyle V} \longrightarrow I_{\scriptscriptstyle V} = I_{\scriptscriptstyle V}(\tau_0) e^{-\int_{\tau_0} \chi_{\scriptscriptstyle V} ds} = I_{\scriptscriptstyle V}(\tau_0) e^{-\Delta s/\bar{t}}$ 

RTE: 
$$\frac{dI_{v}}{ds} = -\chi_{v}I_{v} + \eta_{v}$$

Continuum optical depth:  $d\tau_c = -\chi_c ds$ 

$$\frac{dI_{v}}{d\tau_{c}} = \frac{\chi_{v}}{\chi_{c}} I_{v} - \frac{\eta_{v}}{\chi_{c}}$$

RTE: 
$$\frac{dI_{v}}{ds} = -\chi_{v}I_{v} + \eta_{v}$$

Continuum optical depth:  $d\tau_c = -\chi_c ds$ 

$$\frac{dI_{v}}{d\tau_{c}} = \frac{\chi_{v}}{\chi_{c}} I_{v} - \frac{\eta_{v}}{\chi_{c}} = \frac{\chi_{v}}{\chi_{c}} \left( I_{v} - \frac{\eta_{v}}{\chi_{v}} \right)$$

RTE: 
$$\frac{dI_{v}}{ds} = -\chi_{v}I_{v} + \eta_{v}$$

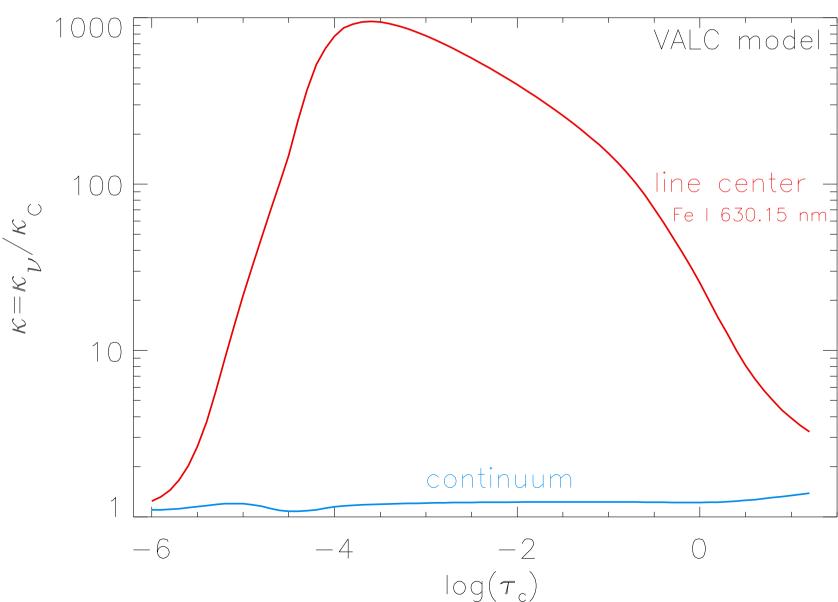
Continuum optical depth:  $d\tau_c = -\chi_c ds$ 

$$\frac{dI_{v}}{d\tau_{c}} = \frac{\chi_{v}}{\chi_{c}} I_{v} - \frac{\eta_{v}}{\chi_{c}} = \frac{\chi_{v}}{\chi_{c}} \left( I_{v} - \frac{\eta_{v}}{\chi_{v}} \right) = \kappa \left( I_{v} - S_{v} \right)$$

$$\frac{S_{v}}{S_{v}}$$

per cm per gram 
$$\chi_{v} = \kappa_{v} \rho \qquad \qquad \chi_{v} = \frac{\kappa_{v}}{\kappa_{c}} = \kappa$$
 
$$\chi_{c} = \kappa_{c} \rho \qquad \qquad \chi_{c} = \kappa_{c}$$

$$\frac{dI_{v}}{d\tau_{c}} = \kappa (I_{v} - S_{v})$$



$$\frac{dI_{v}}{d\tau_{c}} = \kappa (I_{v} - S_{v})$$

Formal solution: integration factor  $\, {\cal O} \,$ 

$$\frac{d(OI_{v})}{d\tau_{c}} = \frac{dO}{d\tau_{c}}I_{v} + O\frac{dI_{v}}{d\tau_{c}} = \frac{dO}{d\tau_{c}}I_{v} + O\kappa(I_{v} - S_{v})$$

$$\frac{dO}{d\tau_c} = -O\kappa S_v$$

$$\frac{d(OI_v)}{d\tau_c} = -O\kappa S_v$$

$$I_{v}(\tau_{1}) = O^{-1}(\tau_{1})O(\tau_{0})I_{v}(\tau_{0}) - \int_{\tau_{0}}^{\tau_{1}} O^{-1}(\tau_{1})O(\tau_{c}) \kappa S d\tau_{c}$$

$$\frac{dI_{v}}{d\tau_{c}} = \kappa (I_{v} - S_{v})$$

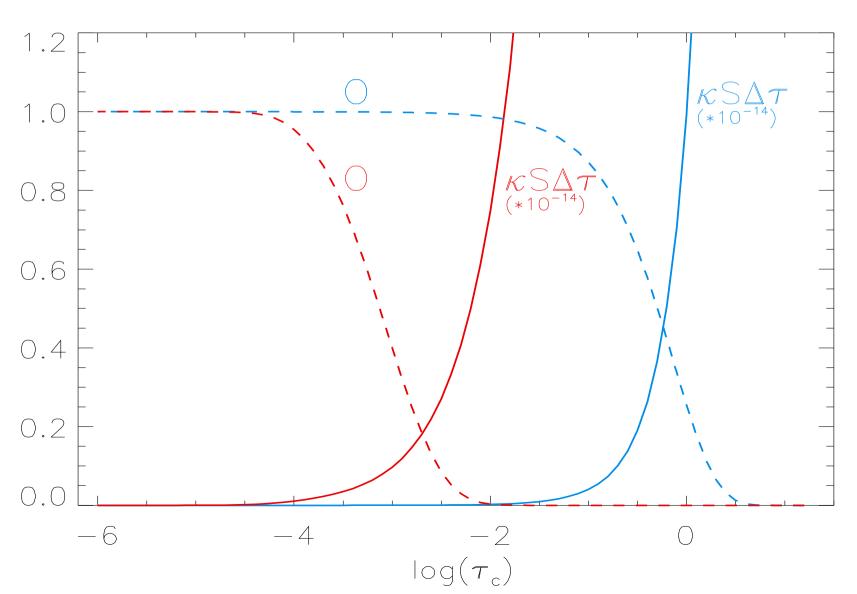
Formal solution: integration factor  $\, {\it O} \,$ 

$$\frac{d(OI_{v})}{d\tau_{c}} = \frac{dO}{d\tau_{c}}I_{v} + O\frac{dI_{v}}{d\tau_{c}} = \frac{dO}{d\tau_{c}}I_{v} + O\kappa(I_{v} - S_{v})$$

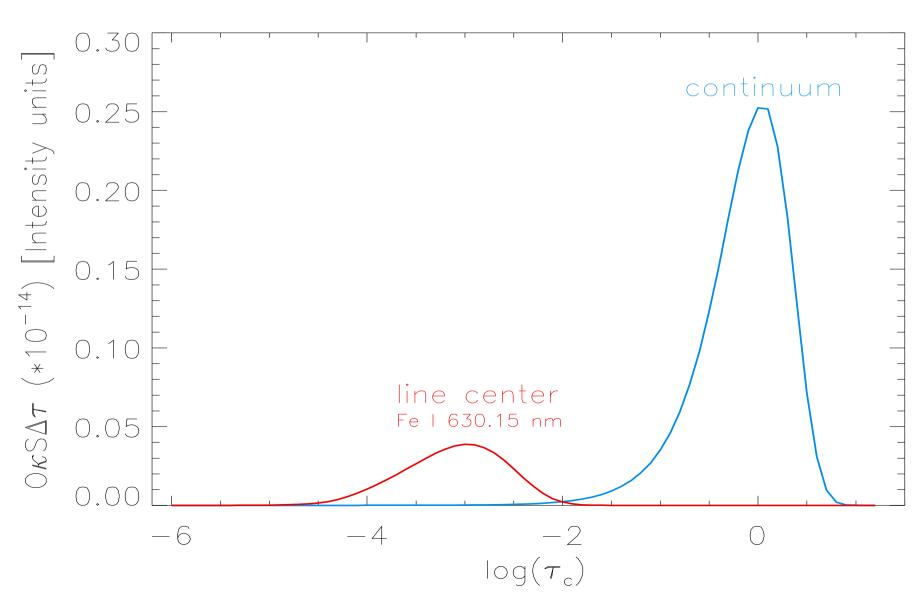
$$\frac{dO}{d\tau_c} = -O\kappa \qquad O(\tau) = e^{-\int_0^{\tau} \kappa d\tau} \qquad \frac{d(OI_v)}{d\tau_c} = -O\kappa S_v$$

$$I_{v} = O(\tau_{0})I_{v}(\tau_{0}) + \int_{0}^{\tau_{0}} O(\tau_{c}) \kappa S_{v} d\tau_{c}$$

$$I_{\nu} = O(\tau_0)I_{\nu}(\tau_0) + \int_0^{\tau_0} O(\tau_c) \kappa S d\tau_c$$



$$I_{\nu} = O(\tau_0)I_{\nu}(\tau_0) + \int_0^{\tau_0} O(\tau_c) \kappa S d\tau_c$$



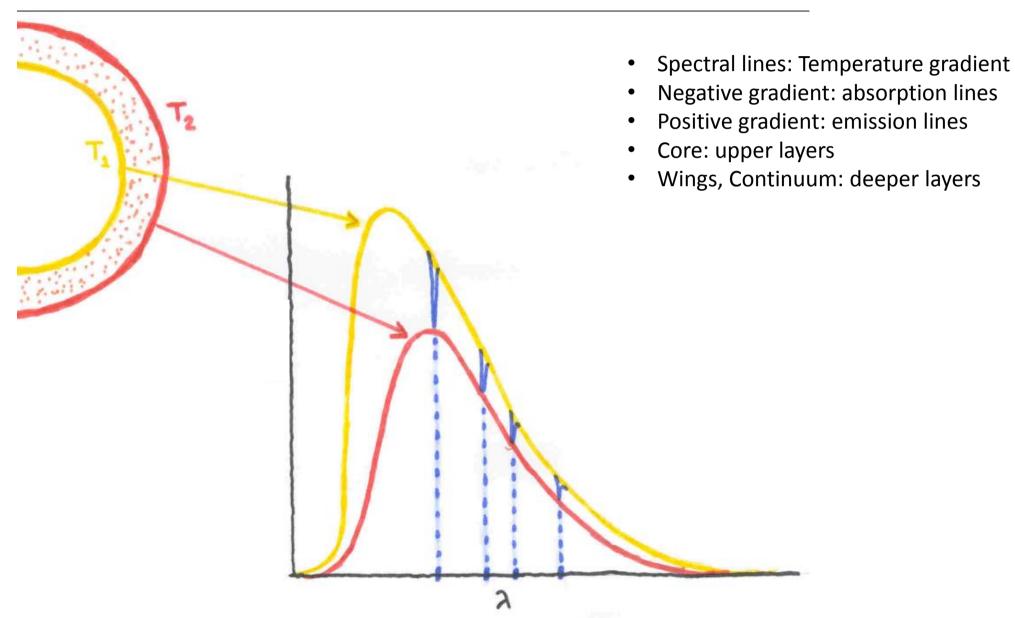
$$I_{\nu} = O(\tau_0)I_{\nu}(\tau_0) + \int_0^{\tau_0} O(\tau_c) \kappa S d\tau_c$$

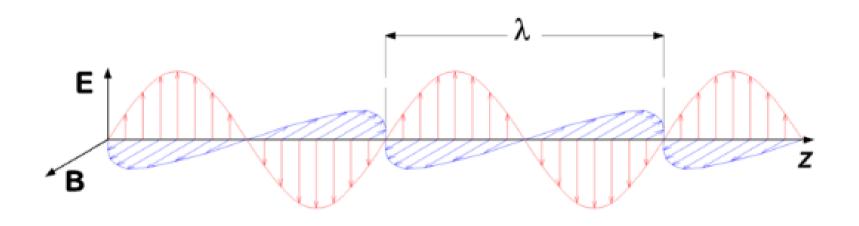
 $\mathcal{K}(\tau)$ ,  $S_{_{V}}(\tau)$  depend on the atomic populations n=n[T, P,  $I_{_{V}}(\tau)$ )]  $\longrightarrow$  SE equations Some possible approximations:

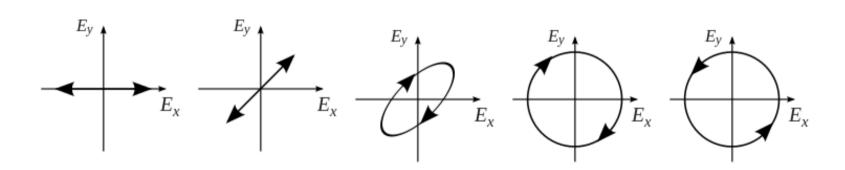
- LTE: n=n[T, P]  $\longrightarrow$   $\kappa[T(\tau),P(\tau)]$  &  $S_{_{V}}=B_{_{V}}[T(\tau)]$
- Milne- Eddington (ME) approximation: K( au) =cte,  $S_{_{V}}( au)=S_{_{0}}+S_{_{1}} au$

$$I_{v}(\tau_{v} = 0) = S_{v}(\tau_{v} = 1)$$

$$I_{\nu}(\tau_{\nu} = 0) = S_{\nu}(\tau_{\nu} = 1) \cong B_{\nu}[T(\tau_{\nu} = 1)]$$







$$I = \updownarrow + \leftrightarrow$$

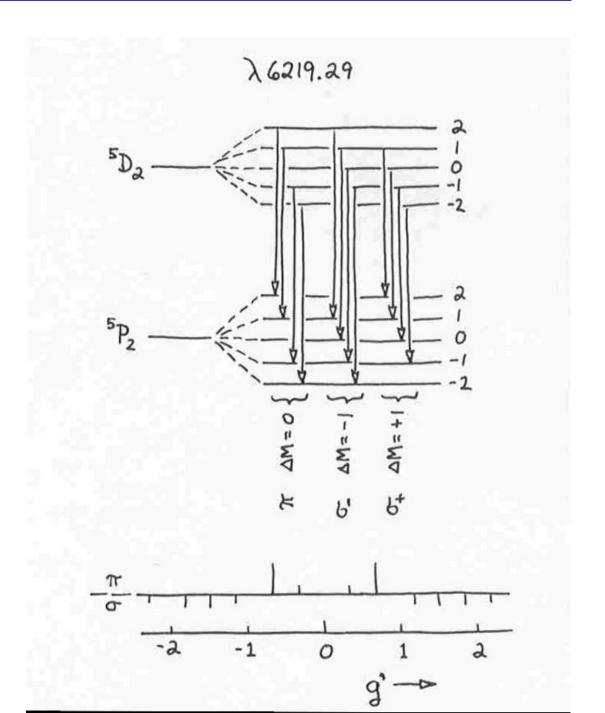
$$Q = \updownarrow - \leftrightarrow$$

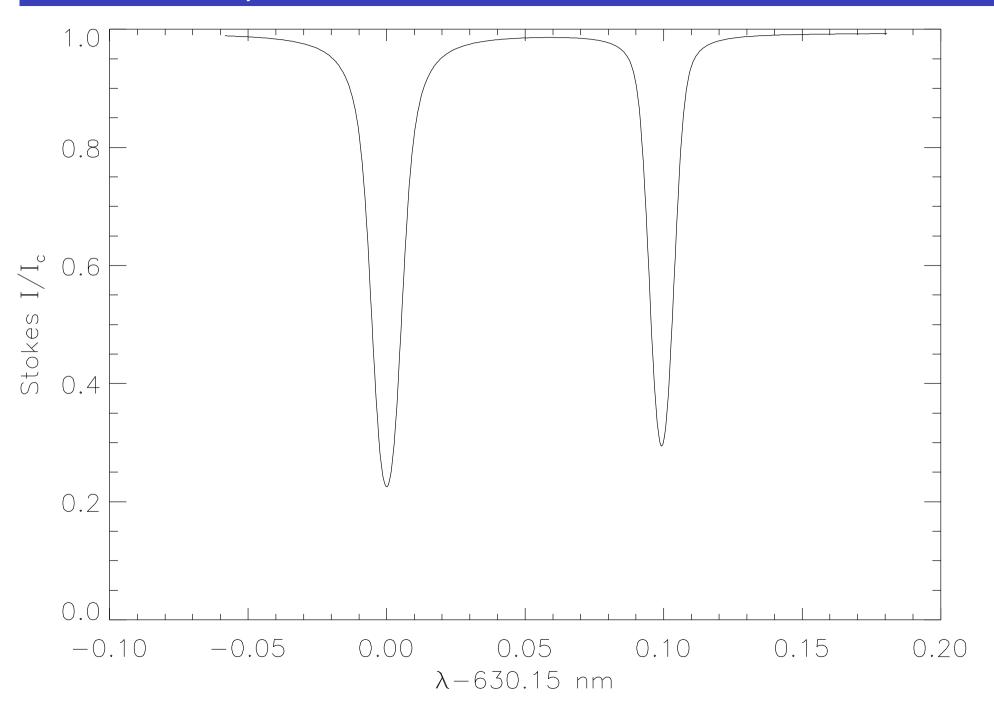
$$U = \nearrow - \nwarrow$$

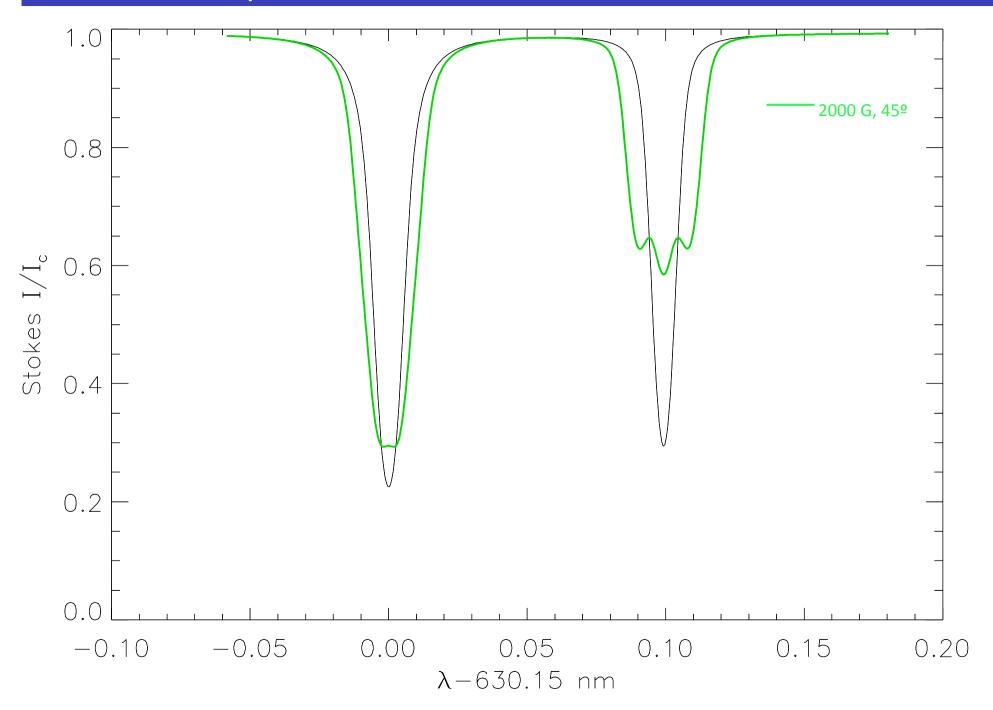
$$V = \circlearrowleft - \circlearrowright$$

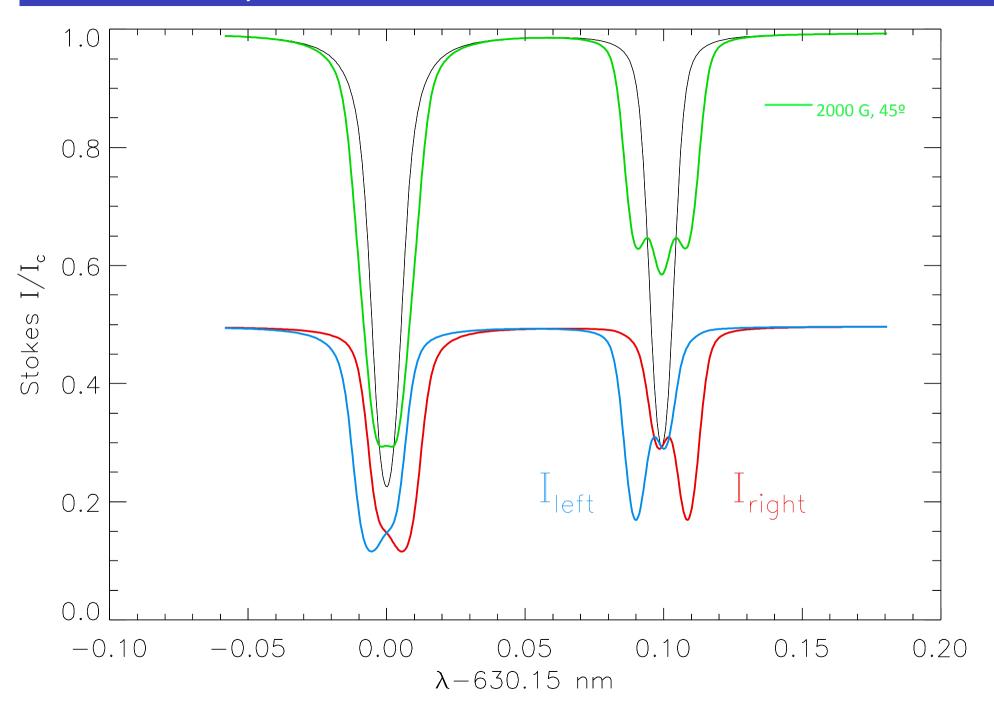


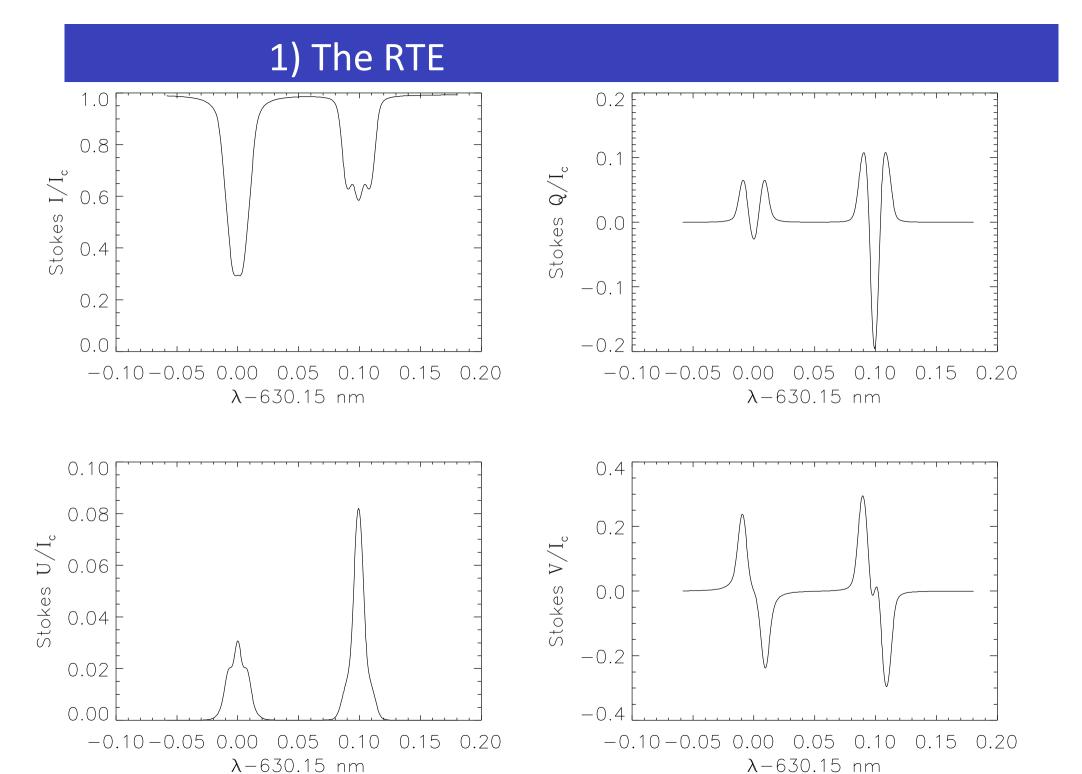
Pieter Zeeman (1865-1943)











$$\frac{dI_{v}}{d\tau_{c}} = \kappa (I_{v} - S_{v})$$

$$\frac{dI_{v}}{d\tau_{c}} = \kappa (I_{v} - S_{v})$$

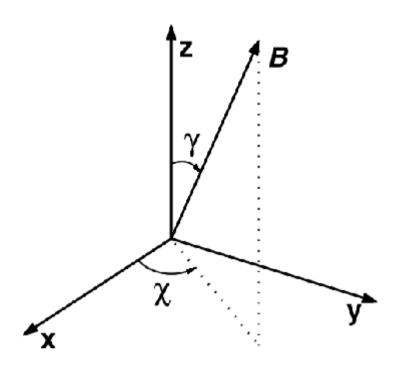
$$I_{v} \to \vec{I}_{v} = \begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix}$$

$$S_{v} 
ightarrow ec{S}_{v} = egin{bmatrix} S_{I} \ S_{Q} \ S_{U} \ S_{V} \ \end{bmatrix} egin{bmatrix} LTE \ D \ 0 \ 0 \ \end{bmatrix}$$

$$\kappa \to \overline{\overline{K}} = \begin{bmatrix} \eta_I & \eta_Q & \eta_U & \eta_V \\ \eta_Q & \eta_I & \rho_V & -\rho_U \\ \eta_U & -\rho_V & \eta_I & \rho_Q \\ \eta_V & \rho_U & -\rho_Q & \eta_I \end{bmatrix} \quad \text{Land}$$

Landi Degl'Innocenti & Landi Degl'Innocenti (1981)

### The RTE



$$\kappa \to \overline{K} = \begin{bmatrix} \eta_I & \eta_Q & \eta_U & \eta_V \\ \eta_Q & \eta_I & \rho_V & -\rho_U \\ \eta_U & -\rho_V & \eta_I & \rho_Q \\ \eta_V & \rho_U & -\rho_Q & \eta_I \end{bmatrix} \quad \begin{array}{c} \rho_V = \frac{1}{2} k_c (\rho_r - \rho_b) \cos \gamma \,. \\ \rho_V = \frac{1}{2} k_c (\rho_r - \rho_b) \cos \gamma \,. \\ \rho_V = \frac{1}{2} k_c (\rho_r - \rho_b) \cos \gamma \,. \end{array}$$

$$\begin{split} \eta_I &= k_c + \frac{1}{2} \, k_\ell \bigg[ \eta_p \, \sin^2 \gamma + \frac{\eta_b + \eta_r}{2} \, (1 + \cos^2 \gamma) \bigg] \,, \\ \eta_Q &= \frac{1}{2} \, k_\ell \bigg( \eta_p - \frac{\eta_b + \eta_r}{2} \bigg) \sin^2 \gamma \, \cos 2\chi \,, \\ \eta_U &= \frac{1}{2} \, k_\ell \bigg( \eta_p - \frac{\eta_b + \eta_r}{2} \bigg) \sin^2 \gamma \, \sin 2\chi \,, \\ \eta_V &= \frac{1}{2} \, k_\ell (\eta_r - \eta_b) \cos \gamma \,, \\ \rho_Q &= \frac{1}{2} \, k_\ell \bigg( \rho_p - \frac{\rho_b + \rho_r}{2} \bigg) \sin^2 \gamma \, \cos 2\chi \,, \\ \rho_U &= \frac{1}{2} \, k_\ell \bigg( \rho_p - \frac{\rho_b + \rho_r}{2} \bigg) \sin^2 \gamma \, \sin 2\chi \,, \\ \rho_V &= \frac{1}{2} \, k_\ell (\rho_r - \rho_b) \cos \gamma \,. \end{split}$$

Landi Degl'Innocenti & Landi Degl'Innocenti (1981)

$$\frac{d\vec{I}_{v}}{d\tau_{c}} = \overline{K} \left( \vec{I}_{v} - \vec{S}_{v} \right)$$

$$\frac{d\overline{O}}{d\tau_c} = -\overline{O}K$$

$$\overline{O}(\tau) = e^{-\int_0^{\tau} \overline{K} d\tau}$$

$$\frac{dI_{v}}{d\tau_{c}} = \kappa (I_{v} - S_{v})$$

$$\frac{dO}{d\tau_c} \equiv -O\kappa$$

$$O(\tau) = e^{-\int_0^\tau \kappa d\tau}$$

$$I_{v} = O(\tau_{0})I_{v}(\tau_{0}) + \int_{0}^{\tau_{0}} O(\tau_{c}) \kappa S_{v} d\tau_{c}$$

$$\vec{I}_{v} = \overline{\vec{O}}(\tau_{0})\vec{I}_{v}(\tau_{0}) + \int_{0}^{\tau_{0}} \overline{\vec{O}}(\tau_{c})\overline{\vec{K}}\vec{S}_{v}d\tau_{c}$$

$$\frac{d\vec{I}_{v}}{d\tau_{c}} = \overline{K} \left( \vec{I}_{v} - \vec{S}_{v} \right)$$

$$\overline{\overline{K}}$$
 &  $\vec{S}_v$  depend on atmosphere  $\vec{a}=(T(\tau),P(\tau),\vec{B}(\tau),v_{LoS}(\tau))$ 

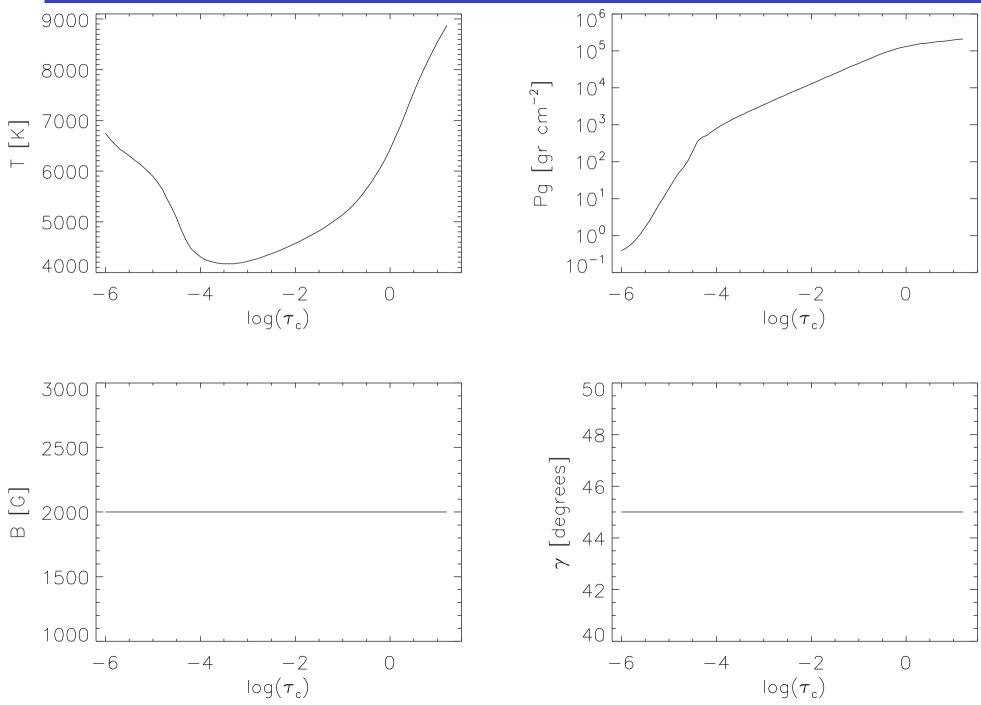
Can we evaluate, in first order, how much change  $I_{\nu}$  when we perturb  $x=a_i(\tau_i)$ ?

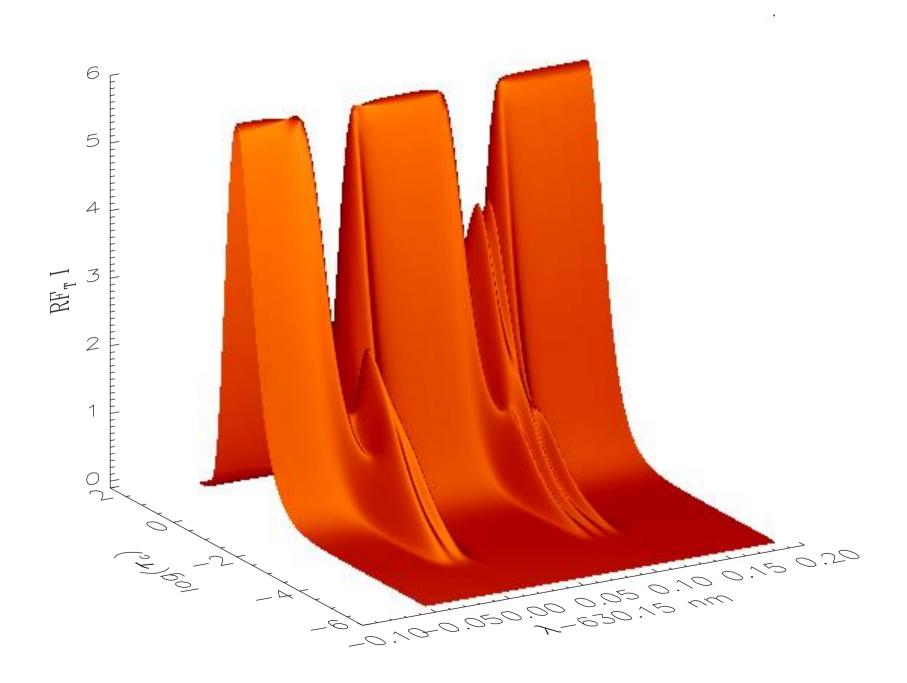
$$\frac{d(\vec{I}_{v} + \delta \vec{I}_{v})}{d\tau_{c}} = \overline{(K + \delta K)} \Big[ (\vec{I}_{v} + \delta \vec{I}_{v}) - (\vec{S}_{v} + \delta \vec{S}_{v}) \Big] \qquad \qquad \overline{\delta K} \approx \frac{\partial \vec{K}}{\partial x} \delta x \qquad \delta \vec{S} \approx \frac{\partial \vec{S}}{\partial x} \delta x$$

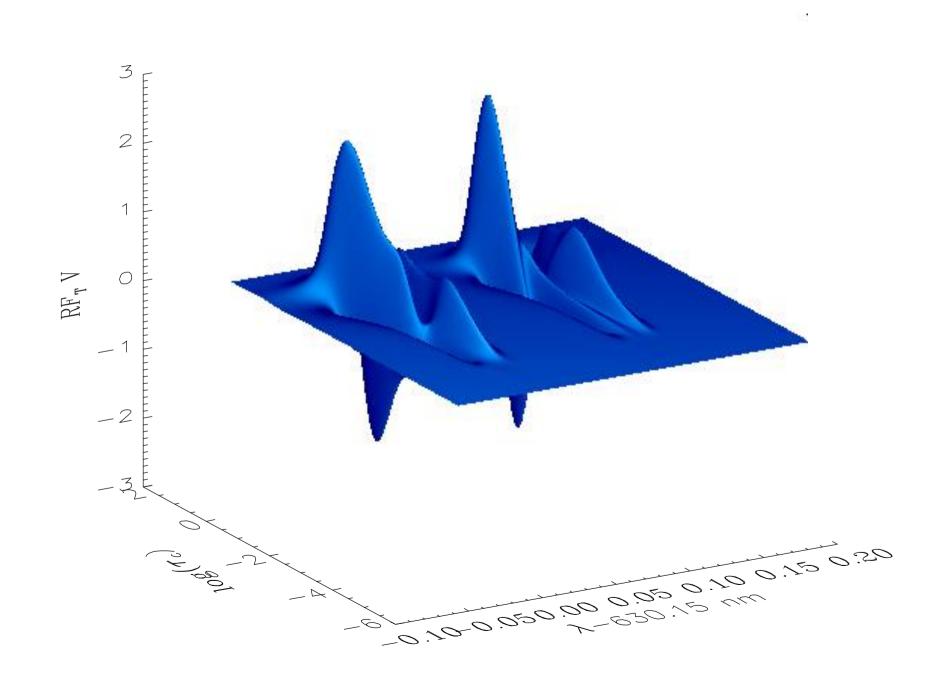
$$\frac{d(\delta \vec{I}_{v})}{d\tau_{c}} = \overline{K} \Big[ \delta \vec{I}_{v} - \overline{\hat{S}}_{v} \Big] \qquad \qquad \overline{\tilde{S}}_{v} = \overline{K}^{-1} \overline{\delta K} (\vec{I}_{v} - \vec{S}_{v}) - \delta \vec{S}_{v}$$

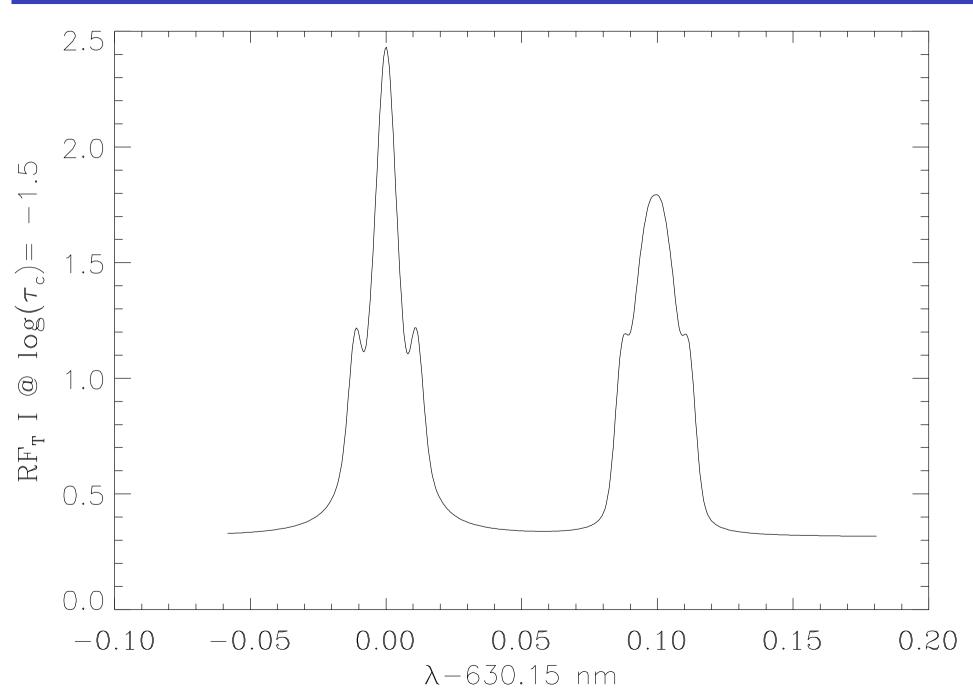
$$\delta \vec{I}_{v} = \int_{0}^{\tau_{c}} \vec{\overline{O}}(\tau_{c}) \vec{K} \vec{\tilde{S}} d\tau_{c} \equiv \int_{0}^{\tau_{c}} R_{x} \delta x d\tau_{c} \qquad R_{x} = \vec{\overline{O}} \left| \frac{\partial K}{\partial x} (\vec{I}_{v} - \vec{S}_{v}) - \vec{\overline{K}} \frac{\partial \vec{S}_{v}}{\partial x} \right|$$

$$R_{x} = \overline{\overline{O}} \left| \frac{\partial \overline{K}}{\partial x} (\vec{I}_{v} - \vec{S}_{v}) - \overline{\overline{K}} \frac{\partial \vec{S}_{v}}{\partial x} \right|$$









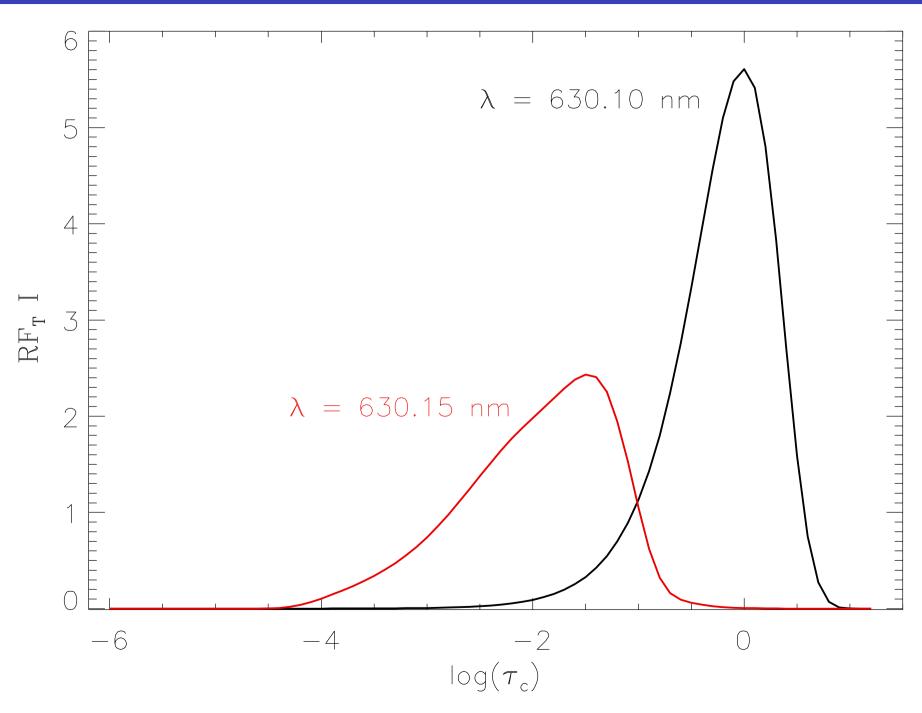


Table of physical magnitudes

$$\vec{a} = (T(\tau), P(\tau), \vec{B}(\tau), v_{LoS}(\tau))$$

- + atomic parameters
- + scenario
- + approximations
- + physical laws

Synthesis (solving RTE) 
$$\vec{I}_{\nu}$$
 &  $\vec{R}_{x}$ 

$$\delta I_{\lambda_i} = \int R_{\lambda_i,x} \, \delta x \, \mathrm{d} \, \tau$$

$$\left( egin{array}{c} \delta I_{\lambda_1} \ dots \ \delta I_{\lambda_n} \end{array} 
ight) = \left( egin{array}{ccc} R_{\lambda_1,x_1} & \cdots & R_{\lambda_1,x_m} \ & & & \ddots & & \ R_{\lambda_n,x_1} & \cdots & R_{\lambda_n,x_m} \end{array} 
ight) \left( egin{array}{c} \delta x_1 \ dots \ \delta x_m \end{array} 
ight)$$

- Any method used to infer the physical conditions of the atmosphere from the interpretation of Stokes profiles
  - Weak-field approximation, center-of-gravity method...
  - Forward modeling
  - PCA, artificial neural networks
  - Least-squares fitting

- What we want: vector magnetic field, gas temperature, gas velocity
- What to expect: a model atmosphere capable of reproducing the observations.... nothing else!

#### -What's an IT?

Fredholm's inhomogeneous ec. first kind:

$$g(t) = \int_a^b K(t,s) f(s) ds$$
  $g(t) = \int_a^b K(t,s) f(s) ds$   $g(t) = \int_a^b K(t,s) f(s) ds$   $f(s) = \int_a^b K(t,s) f(s) ds$ 

This equation, through a quadrature, becomes:

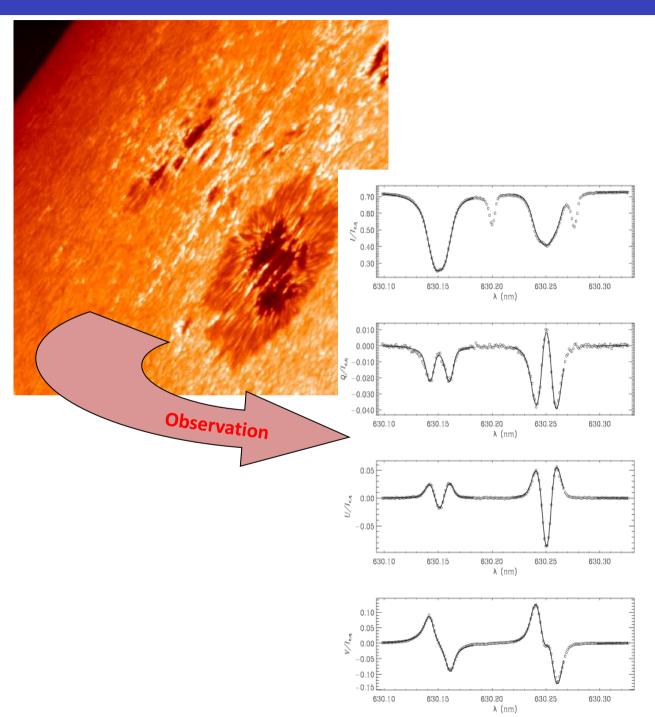
$$K \cdot f = g$$

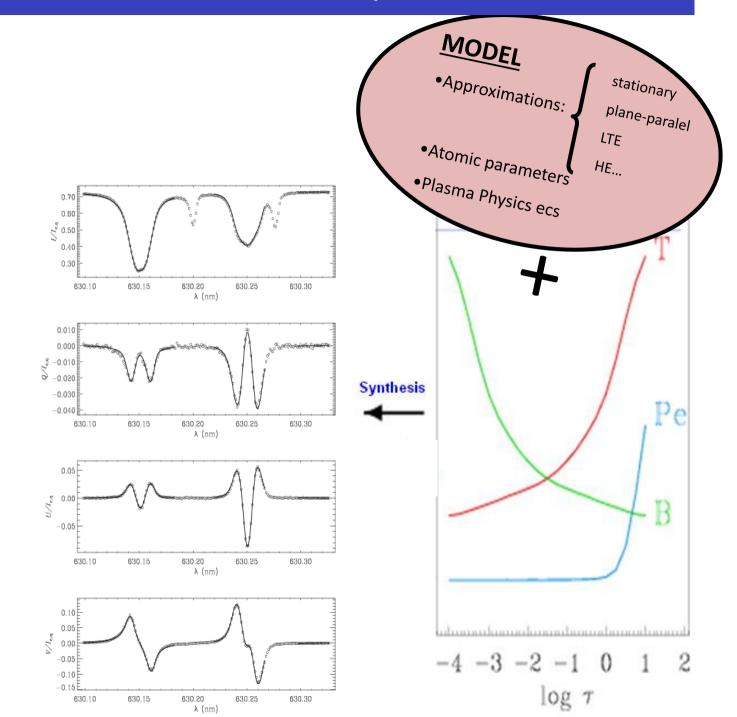
as g is no null if we could evaluate  $K^{-1}$ 

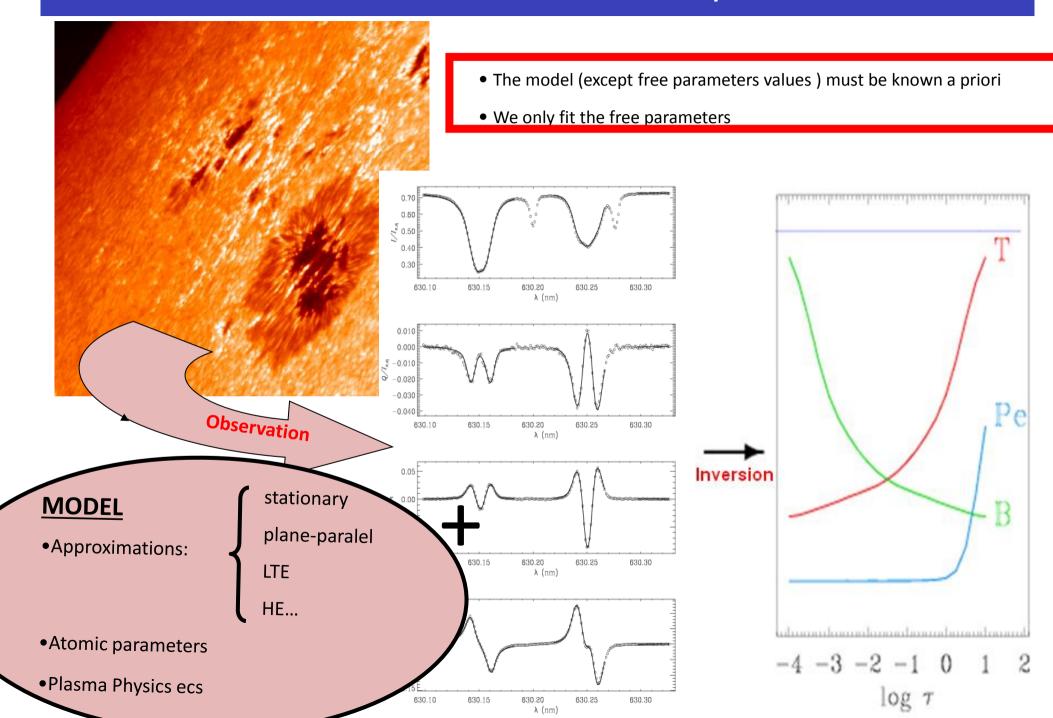
the solution:  $f = K^{-1} \cdot g$  but... this is not the general case

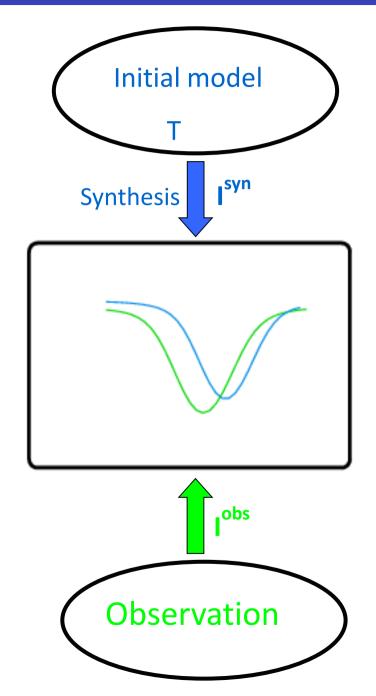
$$g(t) = \int_a^b K(t,s)f(s)ds$$

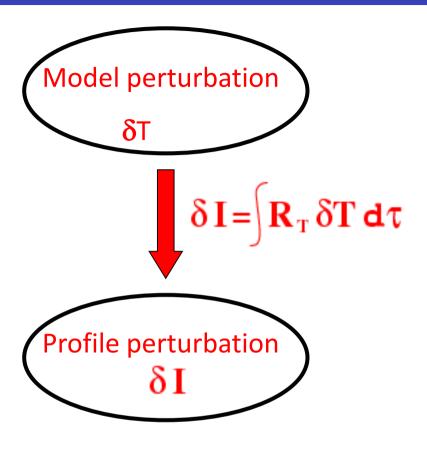
- f(s) become smoothed by the kernel K(t,s)
- 1) Small g(t) perturbations are compatible with huge changes in f(s)
- 2) Smoothing means lost information, then "recover" f(s) is imposible
- 3) We have a discrete set of g(t) values and we'd wish a function f(s)



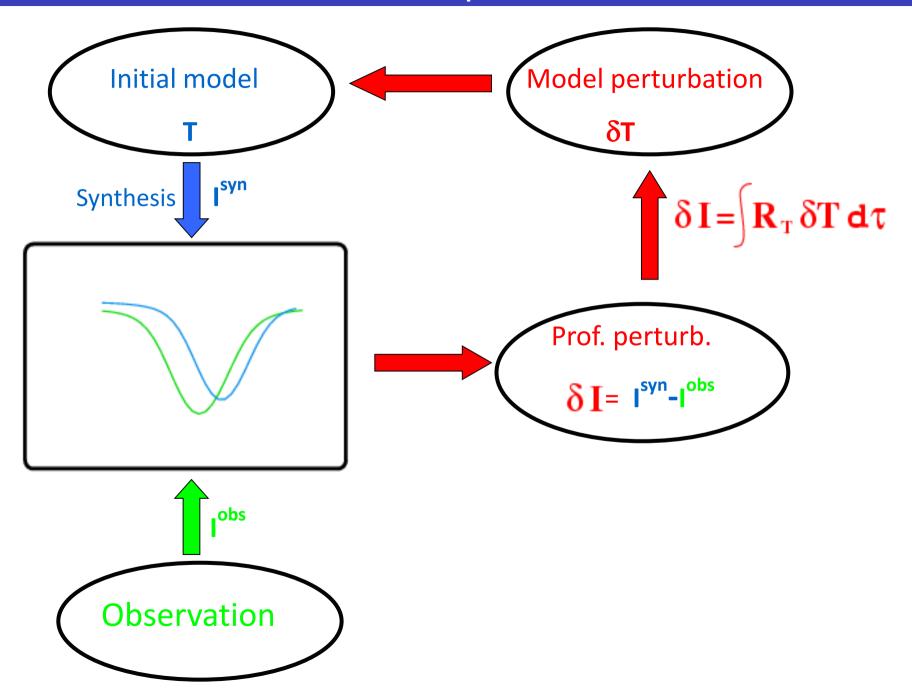








$$R_{T} = \frac{\delta I}{\delta T}$$



• Inversion driven by  $\chi^2$ -minimization

$$\chi^2(\mathbf{a}) = \frac{1}{N_{\text{free}}} \sum_{i=1}^4 \sum_{i=1}^{N_{\lambda}} \frac{w_{ij}^2}{\sigma_j^2} \left[ I_j^{\text{obs}}(\lambda_i) - I_j^{\text{syn}}(\lambda_i, \mathbf{a}) \right]^2$$

Minimization: 2nd order Levenberg-Marquardt algorithm

$$\nabla \chi^2(\mathbf{a}) + \mathbf{A}(\chi^2) \cdot \delta \mathbf{a} = 0$$

$$\chi^{2} \equiv \frac{1}{\nu} \sum_{i=1}^{M} \left[ \underline{I}^{\text{obs}}(\lambda_{i}) - \underline{I}^{\sin}(\lambda_{i}, \overline{\underline{a}}) \right]^{2}$$

Least squares

$$\vec{a} = (T_1, T_2, ... T_n, B_1, B_2, ... B_n, ...)$$

$$\nabla \chi^2(\vec{a}) = - [A] \delta \vec{a}$$

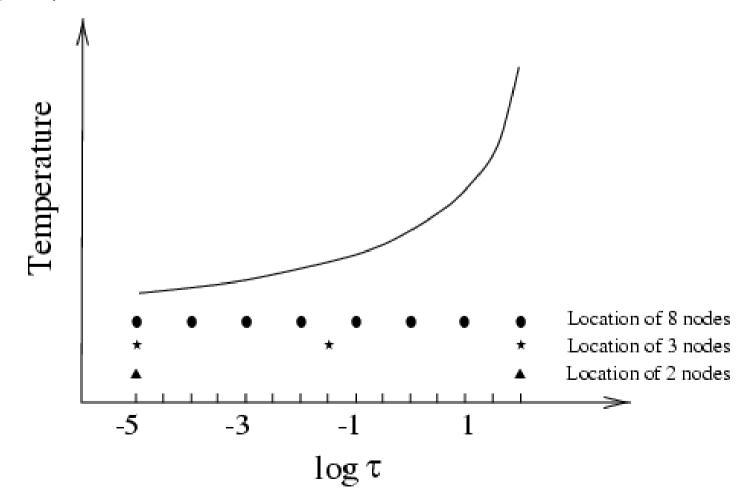
$$\delta \vec{a} = [A]^{-1} \cdot \nabla \chi^2(\vec{a})$$

$$\delta \vec{a} = [A]^{-1} \cdot \nabla \chi^2(\vec{a})$$
 inverting  $A$ 

Very large (1)

Singular (2)

1) Sequential increase of unknown number



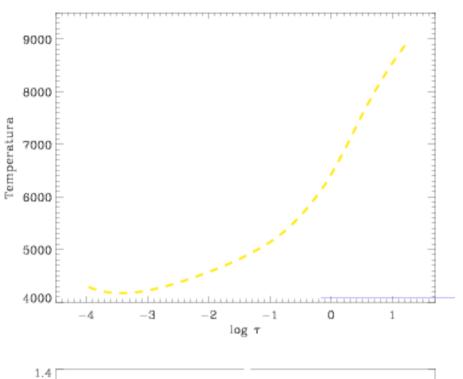
$$\delta \vec{a} = [A]^{-1} \cdot \nabla \chi^{2}(\vec{a}) \qquad \text{inverting } A$$

$$\begin{cases} \bullet \text{ Very large (1)} \\ \bullet \text{ Singular (2)} \end{cases}$$

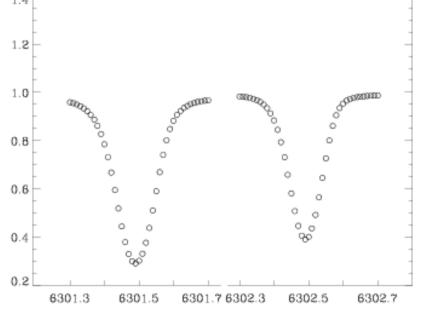
- 1) Sequential increase of unknown number
- 2) Singularities elimination through SVD (Singular Value Decomposition)

$$A = U W V^{T}$$
 with  $U \& V$  ortonormal matrix  
y  $W = diagonal(W_{jj})$ 

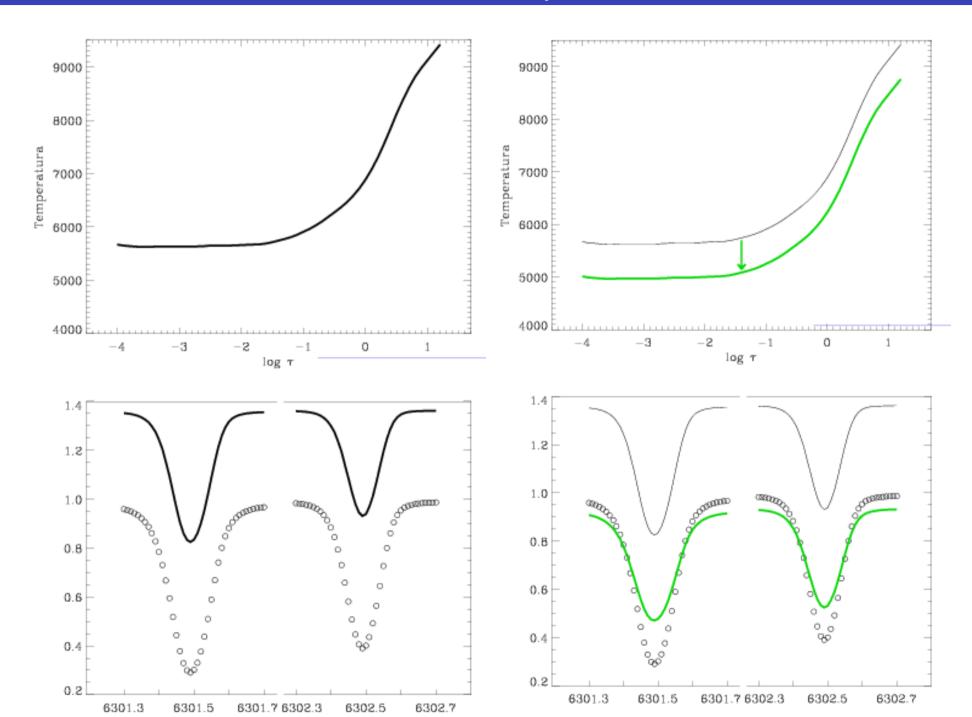
$$A^{-1} = V W^{-1} U^{T} \text{ with } W^{-1} = \text{diagonal}(1/W_{jj})$$
but doing  $(W^{-1})_{ij} = 0$  si  $W_{ij} \cong 0$ 

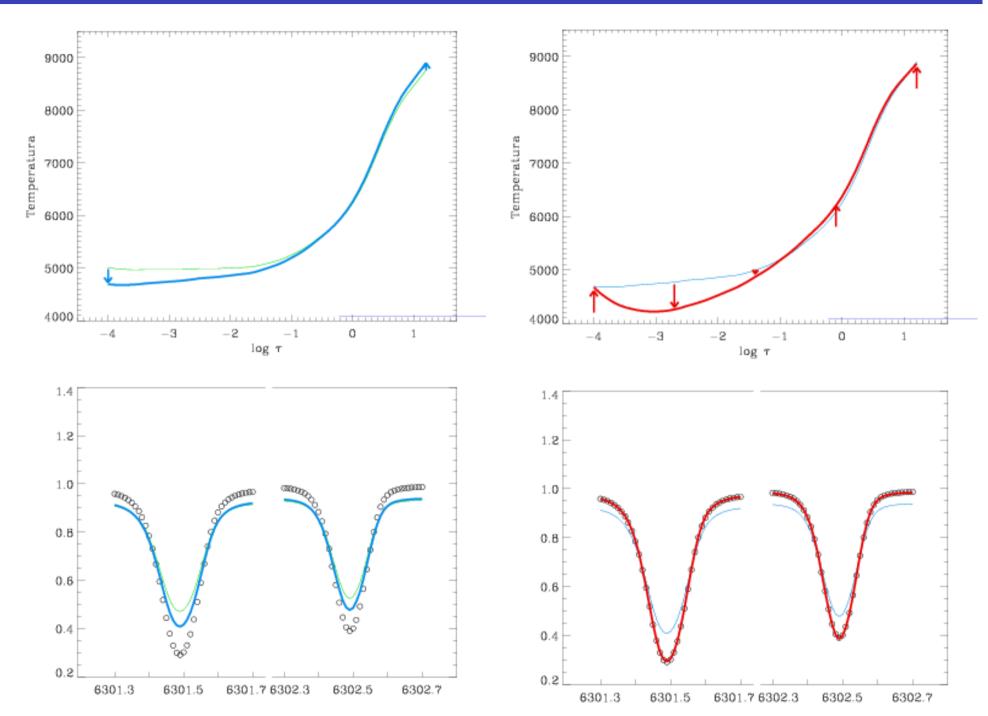


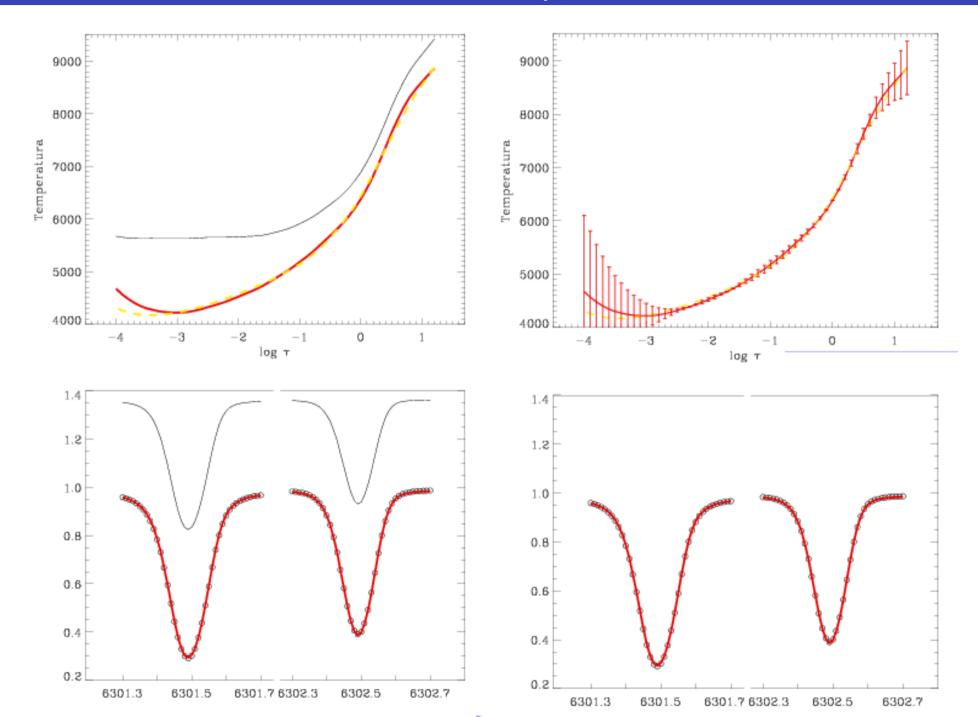
The "real" SUN

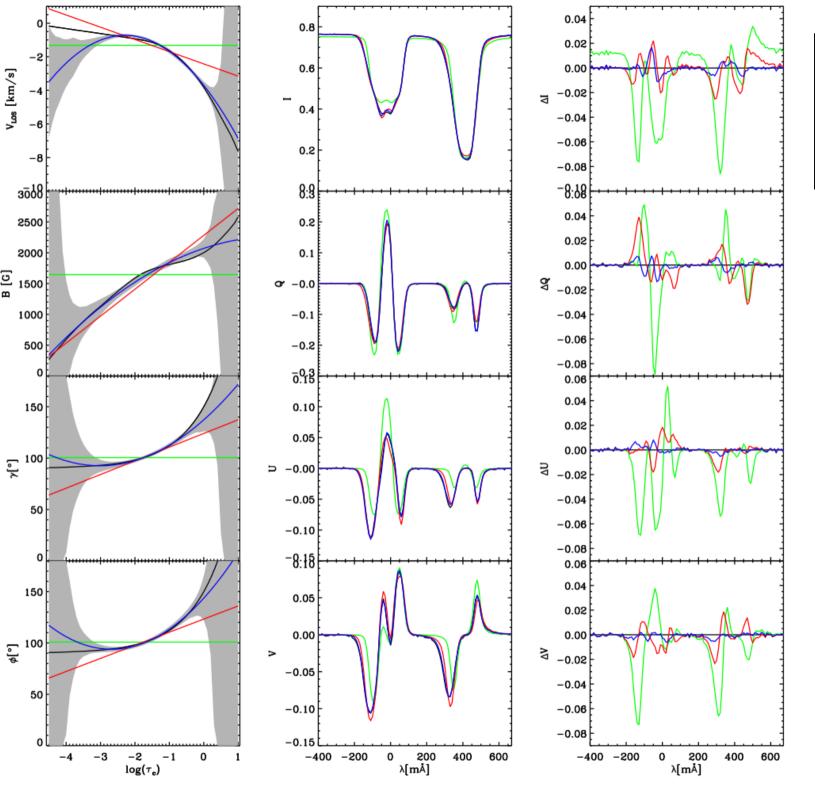


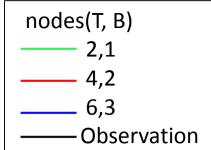
Synthetic "observations"

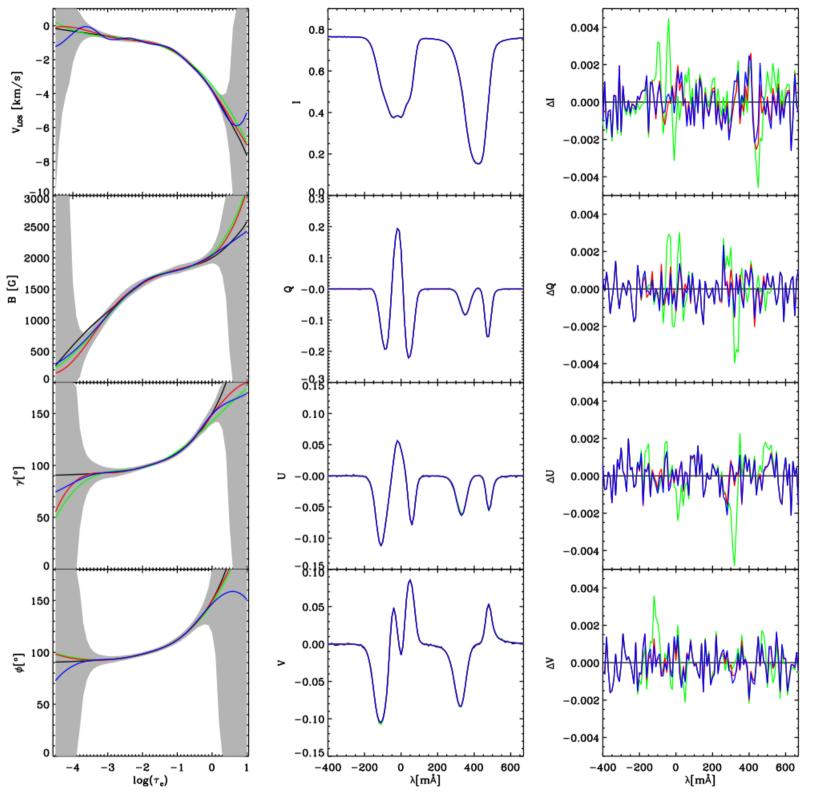












nodes(T, B)
—— 7,5
—— 9,7
—— Automatic
—— Observation

#### Some Reviews:

- Socas Navarro (2001) ASP Conf Series 236, 487
- del Toro Iniesta (2003) Astronomische Nachrichten 324, 383
- Bellot Rubio (2006) ASP Conf Series, 358, 107
- Ruiz Cobo (2007) 'Modern Solar Facilities', Göttingen
- Asensio Ramos et al (2012) ApJ 748, 83
- del Toro Iniesta & Ruiz Cobo (2016) Livings Reviews in Solar Physics, 13,4

Constant quantities			
KPNO	Harvey et al. (1972)	LM	
HAO-KPNO	Auer et al. (1977)	LM	
Florence	Landolfi et al. (1984)	LM	$\checkmark$
HAO-ASP	Skumanich et al. (1985)	LM	$\checkmark$
IAC MISMA	Sánchez Almeida (1997)	LM	$\checkmark$
CSIRO-Meudon	Rees et al. (2000)	PCA	$\checkmark$
HAO MELANIE	Socas-Navarro et al. (2001)	LM	$\checkmark$
HAO FATIMA	Socas-Navarro et al. (2001)	PCA	$\checkmark$
AIP ANN	Carroll and Staude (2001)	ANN	
HAO He I D <sub>3</sub>	López Ariste and Casini (2003)	PCA	$\checkmark$
HAO ANN	Socas-Navarro (2003)	ANN	
MPS HELIX	Lagg et al. (2004)	GA	$\checkmark$
IAC Molecular	Asensio Ramos (2004)	LM	
IAA MILOS	Orozco Suárez and del Toro Iniesta (2007)	LM	$\checkmark$
IAC HAZEL	Asensio Ramos et al. (2008)	LM	$\checkmark$
HAO VFISV	Borrero et al. (2011)	LM	$\checkmark$
IAC Sparse	Asensio Ramos and de la Cruz Rodríguez (2015)	GD	✓

LM Levenberg-Marquardt, ANN artificial neural networks, GA genetic algorithm, B Bayesian, GD gradient descent

Variable quantities			
ETH Flux tube	Keller et al. (1990)	LM	
IAC SIR	Ruiz Cobo and del Toro Iniesta (1992)	LM	✓
ETH IT	Solanki et al. (1992b)	LM	
IAC Flux tube	Bellot Rubio et al. (1997)	LM	✓
ETH SPINOR	Frutiger and Solanki (1998)	LM	$\checkmark$
IAC NLTE	Socas-Navarro et al. (2000)	LM	
HAO LILIA	Socas-Navarro (2001)	LM	✓
HAO-IAC NICOLE	Socas-Navarro (2001)	LM	$\checkmark$
KIS SIRGAUS	Bellot Rubio (2003)	LM	$\checkmark$
IAA SIRJUMP	Louis et al. (2009)	LM	$\checkmark$
IAC Bayes	Asensio Ramos et al. (2009)	В	$\checkmark$
MPS Spatially coupled	van Noort (2012)	LM	$\checkmark$
IAC Regularization	Ruiz Cobo and Asensio Ramos (2013)	LM	$\checkmark$

LM Levenberg-Marquardt, ANN artificial neural networks, GA genetic algorithm, B Bayesian, GD gradient descent

People usually select an inversion method for strange reasons: if the code is available, if it is easier or faster than others, (or was written for my boss), etc., instead of selecting the most convenient for each specific problem.

I classify available IT in 3 families:

- 1. ME must be used in case in which you do not know the physics controlling the line formation, or you have few wavelength points for each pixel.
- 2. PCA when you need <u>fast results</u>.
- 3. SIR (or similar) when you believe that your knowledge of the formation of the lines used is appropriate (and/or you be plenty of time or you be... a little crazy.)

#### 1. ME family

- Skumanich & Lites (1984, 1987); Landolfi (1984); Lagg et al (2004)
- Extensively used in HAO (ASP data).

Juanma Borrero told me that his ME code inverts a 1000x1000 Hinode data in just 3 min.

- Advantages: fast (a typical map takes ~ hr).
  - simple use & easy interpretation of results.
  - robust & reliable.
  - HE, LTE etc approximations are not needed.
- Shortcoming: unable to fit asymmetric Stokes profiles. (but: robust against noise)
  - no gradients.

(but: even theoreticians understand the results)

- no temperature, pressure & density information.

(but: no physics knowledge is required)

- the number of free parameter grows with the number of spectral lines

(but: no excuses!)

#### 2. SIR family (a lot of children): an incomplete list

- Ruiz Cobo & del Toro Iniesta (1992): SIR code
- Frutiger, Solanki et al (2000): SPINOR code [see also Bernasconi & Solanki (1996)]
- Bellot Rubio, Ruiz Cobo & Collados (1997): FT geometry [see also Frutiger & Solanki (1998)]
- Bellot Rubio (2003) [see also Borrero, Lagg, Solanki & Collados (2005)]:
   Uncombed penumbral model
- Socas Navarro, Trujillo Bueno & Ruiz Cobo (2000): SIR-NLTE code
- Sánchez Almeida et al (1996): MISMA code
- Allende Prieto, Ruiz Cobo & García López (1998): MISS code [see also Frutiger et al (2000), Rüedi et al (1997)]

#### 2. SIR family

- Advantages: Arbitrarily complex along line of sight (gradients, multicomponents, etc)
  - Thermodynamic information.
  - adaptation to many geometries & scenarios.
  - LTE/NLTE, HE, mass conservation etc.
  - uncertainties estimation.

- Shortcoming:- Slow (20-30 times slower than ME)
   (but: new SIR version –upcoming soon!!- will nearly reach ME rates)
  - Difficult use

(but: for simplified problems –like only linear stratifications-- is straightforward)

- Unicity (but: only if you are ambitious)

#### 3. PCA family

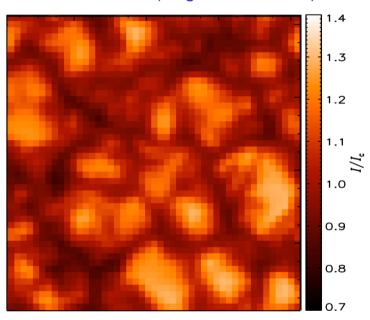
- Look-up-table inversion method, based in PCA
- Rees et al (2000); Socas Navarro et al (2001); López Ariste & Casini (2002)
- Since 2004 used in THEMIS for real-time inversions
- Advantages: extremely fast (one order of magnitude faster than ME)
  - simple use, robust & reliable
- Shortcoming: few free parameter determination (ME)
  - slightly more inaccurate that ME inversions

#### Conclusions

- ME must be used in case in which you do not know the physics controlling the line formation or few spectral points.
- 2. PCA when you need fast results.
- 3. SIR (or similar) when you believe that your knowledge of the formation of the lines used is appropriate.

# ME inversions of high-spatial resolution profiles

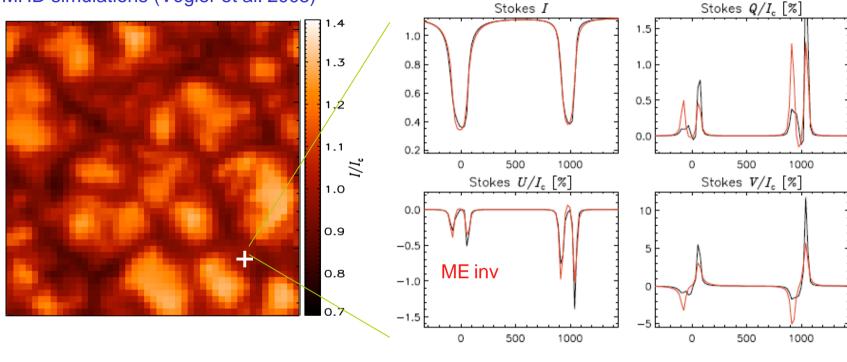
#### MHD simulations (Vögler et al. 2005)



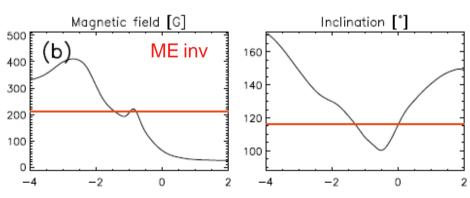
 Fe I 630.1 and 630.2 nm profiles degraded to Hinode/SP resolution and pixel size

# ME inversions of high-spatial resolution profiles



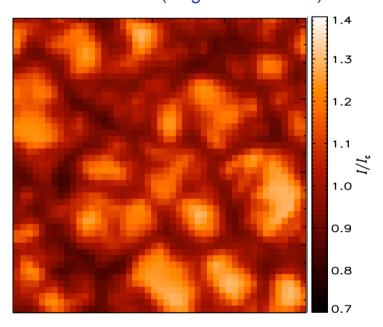


- Profiles reasonably well fitted
- ME results are some kind of "average" of physical parameters along the LOS

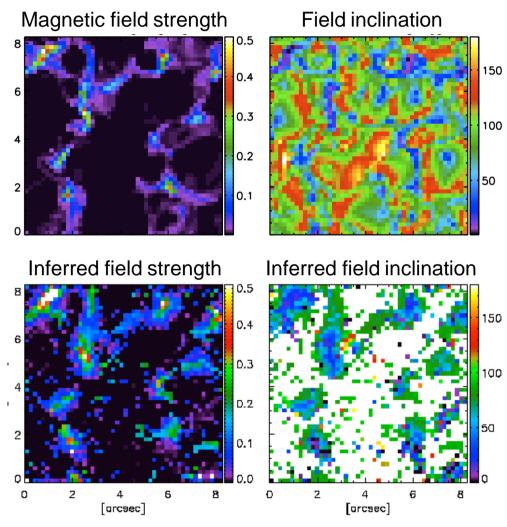


# ME inversions of high-spatial resolution profiles

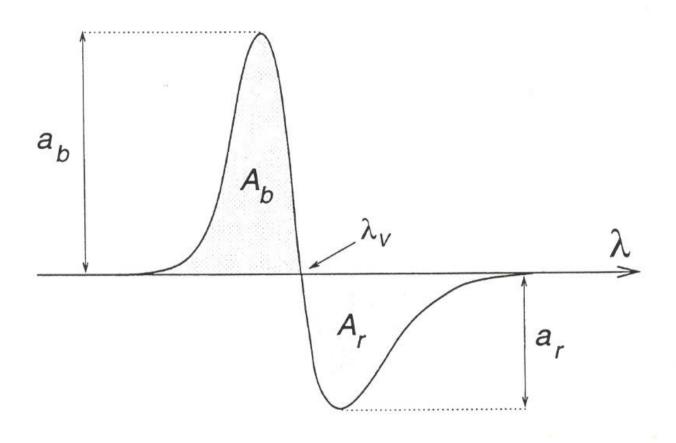
#### MHD simulations (Vögler et al. 2005)



- Atmospheric parameters from MHD simulation at log  $\tau$  = -2
- Maps of inferred B and γ similar to real ones!

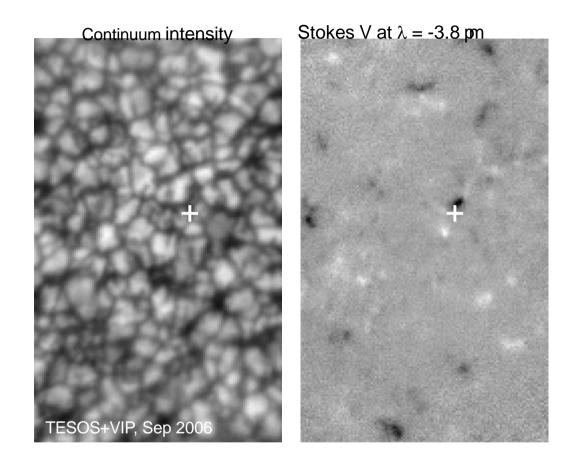


Orozco Suárez et al. 2007, ApJ, 662, L31



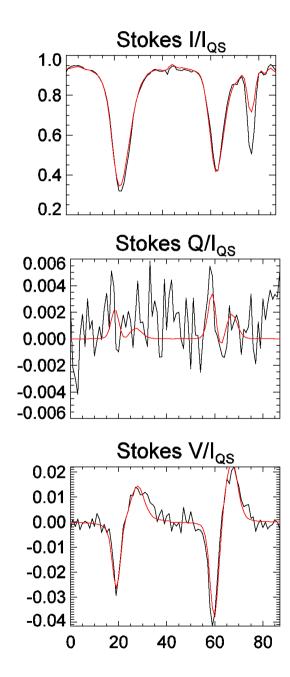
$$\delta a \equiv \frac{a_{\rm b} - a_{\rm r}}{a_{\rm b} + a_{\rm r}}, \quad \delta A \equiv \frac{A_{\rm b} - A_{\rm r}}{A_{\rm b} + A_{\rm r}},$$

# Asymmetric Stokes profiles





- VTT, Observatorio del Teide
- Spatial resolution: ~0.4"
- Quiet Sun at center, Fe I 630.15 and 630.25 nm



# The origin of asymmetries

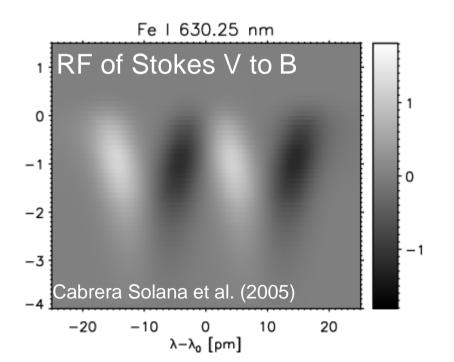
Amplitude asymmetry/
Multi-lobed Stokes profiles

Different magnetic atmospheres coexisting in resolution element

Area asymmetry

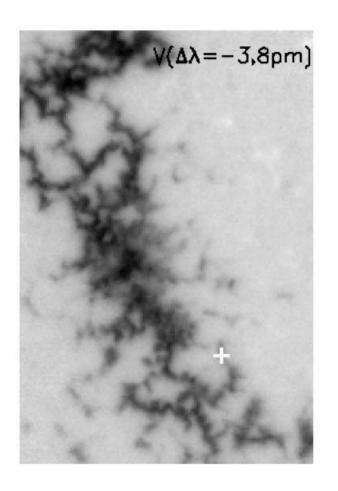
Gradients/discontinuities of B and v<sub>LOS</sub> along LOS

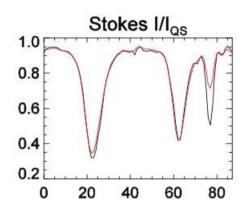
Auer & Heasley (1978)

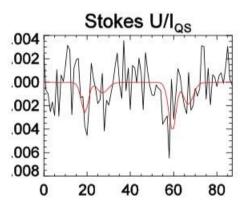


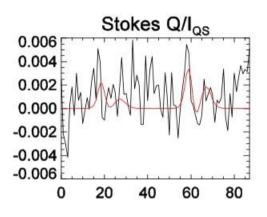
The area asymmetry gives information on the height variation of atmospheric parameters

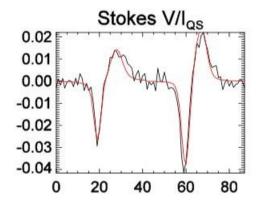
- Inversion codes capable of dealing with asymmetries
  - Are based on numerical solution of RTE
  - Provide reliable thermal information
  - Use less free parameters than ME codes
  - Infer stratifications of physical parameters with depth



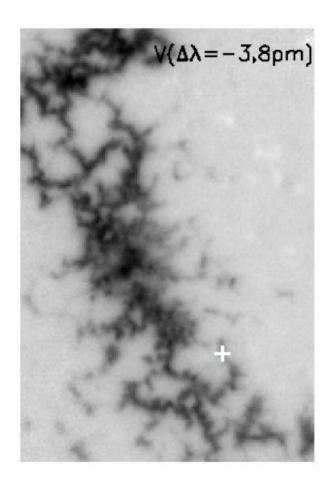


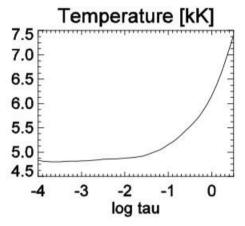


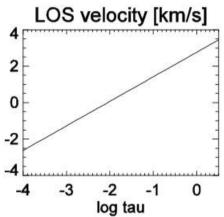


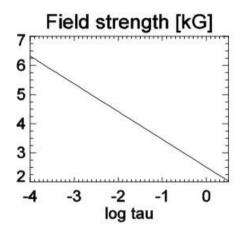


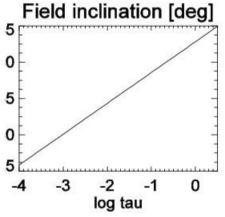
- VIP + TESOS + KAOS
- SIR with 10 free parameters
- Bellot Rubio et al. (2007)





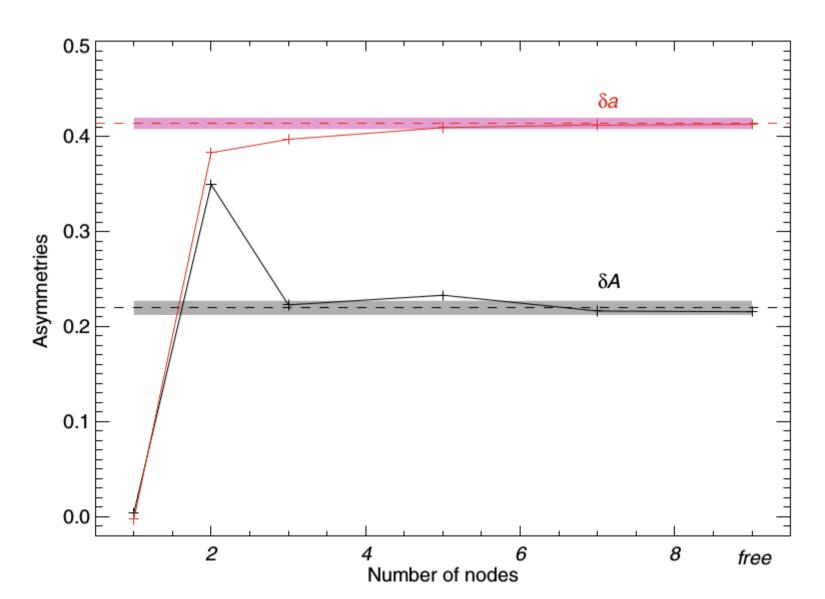






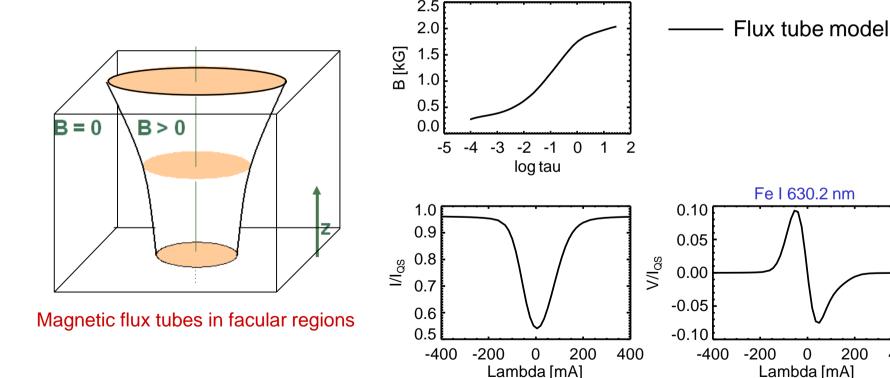
- VIP + TESOS + KAOS
- SIR with 10 free parameters
- Bellot Rubio et al. (2007)

$$\delta a \equiv \frac{a_{\rm b} - a_{\rm r}}{a_{\rm b} + a_{\rm r}}, \quad \delta A \equiv \frac{A_{\rm b} - A_{\rm r}}{A_{\rm b} + A_{\rm r}},$$



#### Be careful with the choice of atmospheric model!

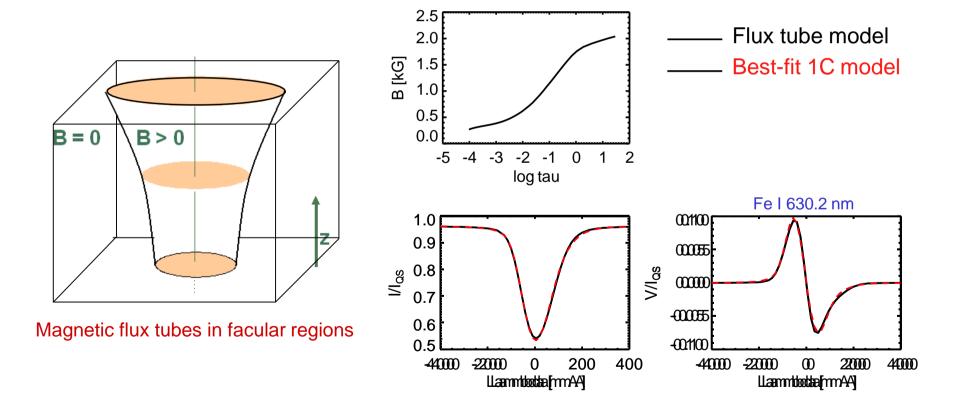
- The results change if the physical model is changed
  - Too simplistic models; often they cannot describe the real atmosphere
  - BUT: we get information about the magnetic structure of the atmosphere!



400

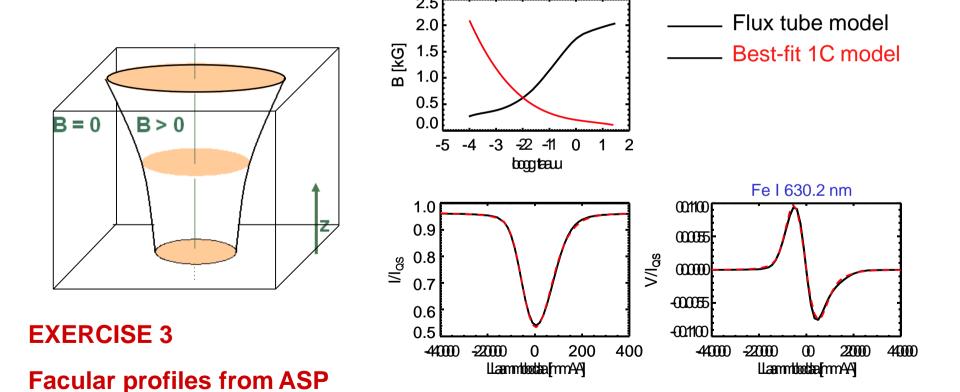
### Be careful with the choice of atmospheric model!

- The results change if the physical model is changed
  - Too simplistic models; often they cannot describe the real atmosphere
  - BUT: we get information about the magnetic structure of the atmosphere!



#### Be careful with the choice of atmospheric model!

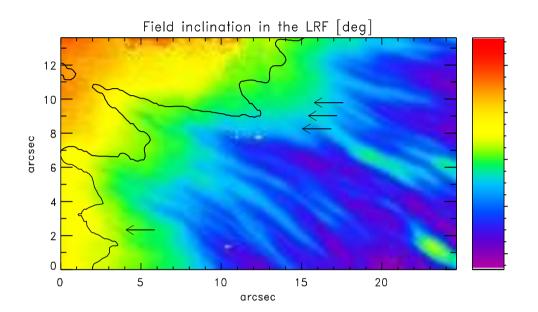
- The results change if the physical model is changed
  - Too simplistic models; often they cannot describe the real atmosphere
  - BUT: we get information about the magnetic structure of the atmosphere!



# Tips and tricks

- First of all, look at the profiles
- Try a ME-like inversion, it usually works
  - For example, use 1C SIR inversion with height-independent atmospheric parameters
  - If the V profiles are very asymmetric, fit only I, Q, and U
- Examine the fits: are they reasonably good?

### 1C SIR inversion of Hinode/SP data



Dark core 8.0 Stokes I/I<sub>c,qs</sub> 0.3 -0.5 0.0 0.0 0.5 1.5 -0.50.5 1.0 1.0 Stokes  $Q/I_{c,qs}\ [\%]$ 1.0 1.5 -0.5 0.0 0.5 -0.50.0 0.5 Stokes  $\mathrm{U/I_{c,qs}}$  [%] -0.50.0 0.5 1.0 -0.5 0.0 0.5 1.0 1.5 Stokes V/Ic.qs [%] -0.50.0 0.5 1.0 -0.50.0 0.5 1.0 1.5  $\Delta\lambda$  (nm)  $\Delta\lambda$  (nm)

**EXERCISE 2** 

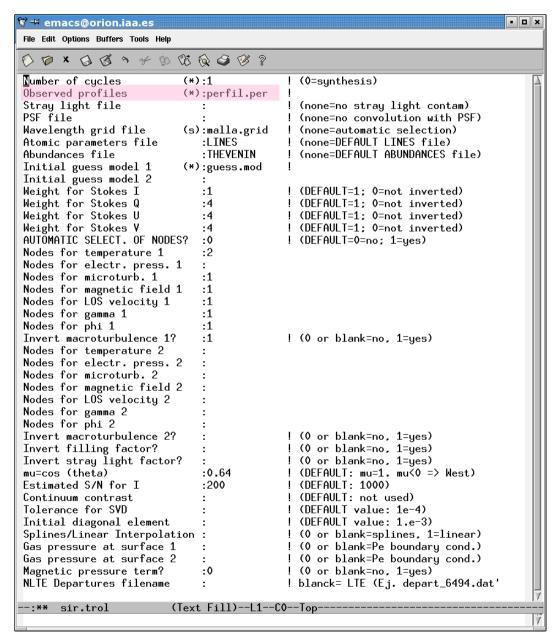
**Dark-cored penumbral filament** 

# Tips and tricks

- First of all, look at the profiles
- Try a ME-like inversion, it usually works
  - If the V profiles are very asymmetric, fit only I, Q, and U
- Examine the fits: are they reasonably good?
- Identify
  - Pixels with bad fits and/or large asymmetries
  - Regions where interesting physical processes occur
- Run more complex inversions on these pixels
  - Which model are you going to use?1C model, 2C model, flux tube model, uncombed model?

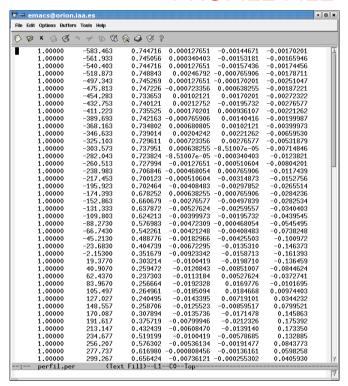
# Tips and tricks

- First of all, look at the profiles
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  - If the V profiles are very asymmetric, fit only I, Q, and U
- Examine the fits: are they reasonably good?
- Identify
  - Pixels with bad fits and/or large asymmetries
  - Regions where interesting physical processes occur
- Run more complex inversions on these pixels
  - Which model are you going to use?1C model, 2C model, flux tube model, uncombed model?
  - Use ME results as initialization
  - Give more weight to the strangest Stokes parameter
  - Keep it simple! See if linear stratifications (2 nodes) are sufficient
- Ask yourself if the retrieved model atmosphere makes sense!!
- Experts are always around: ask them for advice!



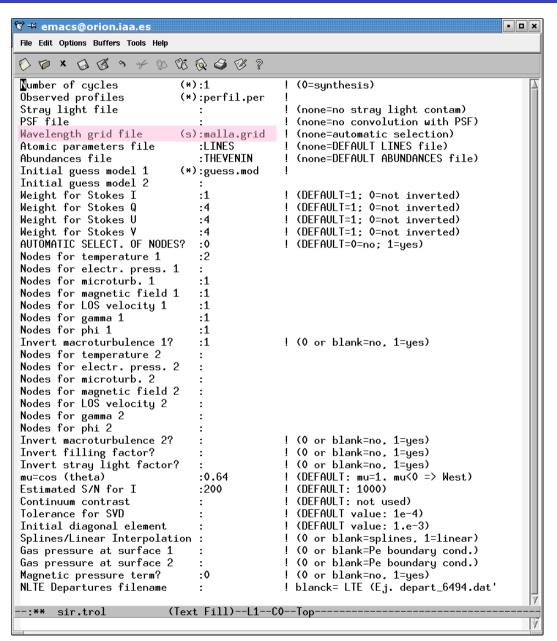
#### **CONTROL FILE**

#### **PROFILE FILE**

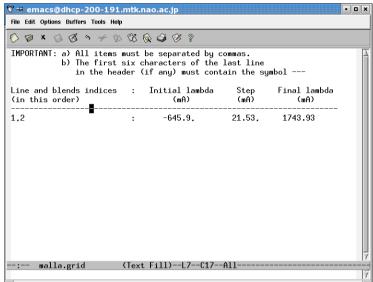


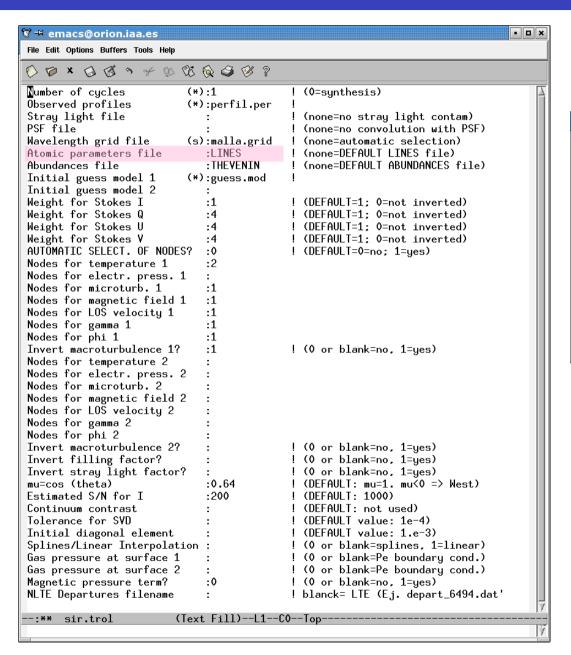
Line  $\Delta\lambda$  index [mA]  $I/I_{qs}$   $Q/I_{qs}$   $U/I_{qs}$   $V/I_{qs}$ 

read\_profiles.pro write\_profiles.pro

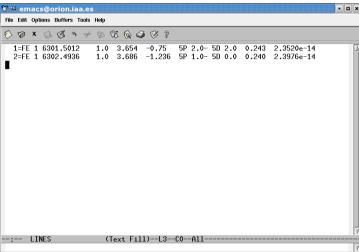


#### WAVELENGTH GRID FILE

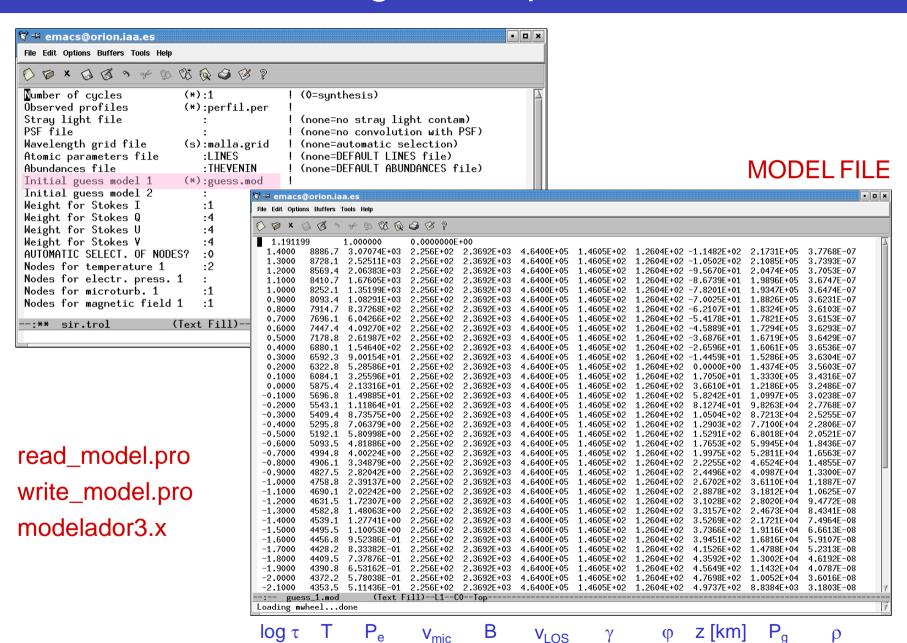




#### ATOMIC PARAMETER FILE

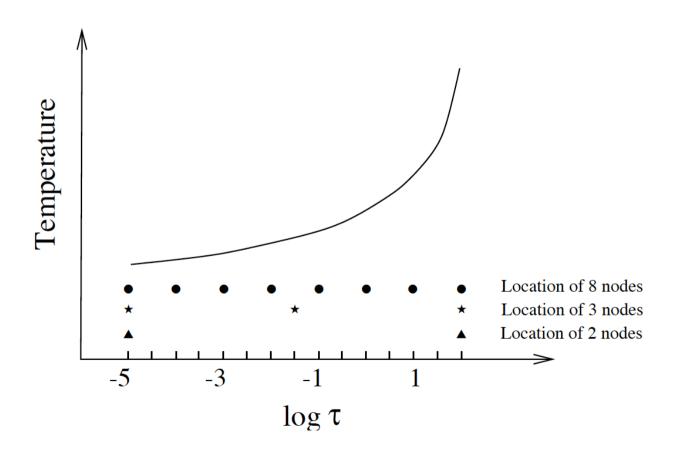


Line index Atom  $\lambda$  E  $\chi$  log gf transition



# Concept of nodes

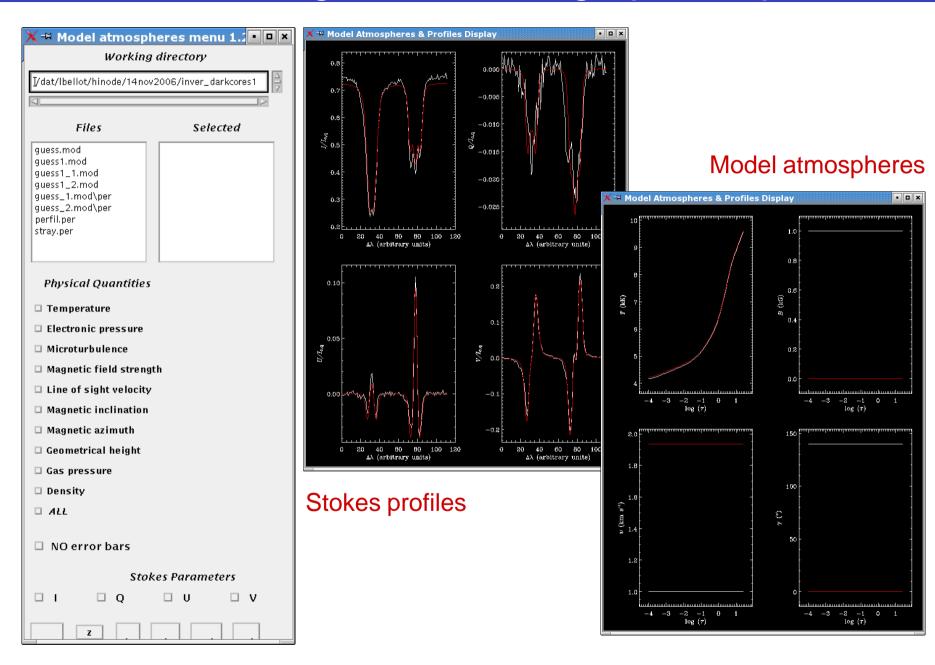
- Keeping the number of free parameters small:
  - Atmospheric parameters perturbed in coarse grid (nodes)
  - Full stratifications in finer grid by cubic spline interpolation



# Executing the inversion

echo sir.trol | sir.x

# Visualizing SIR results: graphics2.pro



### SIR exercises

Download exercises from

http:/SIR/SIR\_exercises.tar

### Spectral synthesis and inversion of synthetic profiles

Use HSRA model to synthesize Stokes profiles with

- 1. constant B, inclination and v<sub>LOS</sub> (e.g., 1 kG, 60°, 2 km/s)
- 2. constant v<sub>LOS</sub>, gradients of B and inclination
- 3. gradients of B, inclination and  $v_{LOS}$

Invert profiles from (3), starting from initial guess model with flat stratifications of B,  $v_{LOS}$ , and inclination (modify hsra.mod)

- 1 node in B, vlos, inclination
- 2 nodes in B, v<sub>LOS</sub> and inclination

read\_model,filename,tau,t,pe,mic,b,vel, gamma,phi,mac,filling,sl b=1000.+200\* tau write\_model,filename,tau,t,pe,mic,b,vel,gamma,phi,mac,filling,sl

### Inversion of profiles from dark-cored penumbral filament

Hinode/SP observations with SNR~1000, no telluric lines, two lines Fe I 630.1 and 630.2 nm
Strong, symmetric signals

- 1. What kind of model would you use to invert them?
- 2. Can the fit be improved with more nodes in T? (use 2 cycles!)
- 3. What happens with 2 nodes in B and  $v_{LOS}$ ?
- 4. What happens with 10 nodes in B and  $v_{LOS}$ ?

If no instrumental PSF is available, use macroturbulence to mimick its effect (i.e, invert v<sub>mac</sub>) Use more weight for Q, U and V to force better fits to those parameters A worse equivalent SNR does not necessarily mean a worse fit (i.e., a lower chi²) Beware of models with too much freedom!

### Inversion of facular profiles in quiet Sun

Advanced Stokes Polarimeter observations, averaged over facular region, SNR~10000, but poor spatial resolution
Two lines Fe I 630.1 and 630.2 nm (plus telluric lines!)
Strong signals, large Stokes V area and amplitude asymmetries

- 1. What kind of model would you try to invert them?
- 2. Use two cycles, increasing number of nodes in 2nd cycle
- 3. Invert stray-light fraction, micro- and macro-turbulence

We invert Stokes I and V only, so vertical fields should be assumed
Use large negative number (e.g., -2) in profiles to ignore blends in Stokes I during inversion
Use instrumental PSF and macroturbulence at the same time
Use stray light profile
Use weights of 10 and 100 for Stokes V

### Inversion of quiet-Sun internetwork profiles

Hinode/SP observations at disk center, integrated for 6 min, SNR~10<sup>5</sup>, still high spatial resolution

Two lines Fe I 630.1 and 630.2 nm

Extremely weak signals, but linear polarization clearly seen. Large asymmetries.

- 1. What kind of model would you try to invert them?
- 2. Use three cycles with increasing number of nodes
- 3. Invert stray-light fraction and microturbulence (flat stratification)
- 4. Interpret resulting model

No need for macroturbulence when high-resolution data are inverted using telescope PSF Use following weights: 1,4,4,4

### Inversion of sunspot penumbral profiles near PIL

Hinode/SP observations with SNR~1000, no telluric lines, two lines Fe I 630.1 and 630.2 nm
Strong signals, but Stokes V profile with three lobes......

- 1. What kind of model would you use to invert them?
- 2. One-component model with opposite magnetic polarities along LOS? Two-component model?
- 3. Try both!

Inversion of these profiles will not be easy. Do your best!

Give more weight to Stokes V to force better fits. Increase weight with cycle If everything fails, use superpowers...

### Internetwork profiles with very weak Q, U signals

Simulated Hinode obs, SNR~800, Fe I 630.1 and 630.2 nm

#### Synthesize Stokes profiles from 2 component model:

- 1. magnetic atmosphere with B=200 G,  $\gamma$  = 10°, filling factor=5%
- 2. non-magnetic atmosphere (hsra.mod), filling factor=95%
- 3. Save profiles. Then add noise at the level of 10<sup>-3</sup> using add\_noise, filename, 1.25e-3. Save the noisy profiles.

Invert noise-free, then noisy profiles. Use simple 2C model, freezing 2nd component to hsra.mod.

Interpret resulting field inclinations.

If you are curious (you should!), repeat exercise with only one line

Use 2C models for synthesis and inversion Beware of noisy linear polarization profiles, especially when only one line is available!

### Inversion of CRISP profiles from sunspot penumbrae

SST/CRISP observations with SNR~500, sequential spectral sampling of Fe I 617.3 nm (30 wavelengths in ~30 s) Strongly Doppler-shifted polarization profiles

- 1. What kind of model would you use to invert them?
- 2. Try stray-light contamination, 1C or 2C model
- 3. You are on your own! I have not inverted these profiles yet...

Example of Stokes profiles observed with a Fabry-Pérot interferometer Extremely high spatial resolution, but modest spectral resolution (~50 mA at 617 nm) Sequential sampling of line means first and last wavelengths are observed ~30 s apart

# Stray-light considerations

- Stray-light in 1C inversions:
  - $-I_{obs} = (1-\alpha)I_1 + \alpha I_{stray}$
  - Accounts for both stray light and/or magnetic filling factor
- Stray-light in 2C inversions:
  - It is NOT equivalent to a magnetic filling factor
  - SIR has two free parameters:  $\alpha$  and f
  - $I_{obs} = (1-\alpha) [f I_1 + (1-f) I_2] + \alpha I_{stray}$
- Global vs local stray-light profile
  - Classical treatment: global stray-light profile (average over FOV)
  - Orozco Suárez et al. (2007): local stray-light profile accounts for telescope diffraction