1/A4

Solution — Homework 2

Date:

Q1) Mobility of electron in Si $= \mu_n = 1350 \frac{cm^2}{Vs} = 0.135 \frac{m^2}{Vs}$

$\mu_p =$ Mobility of ~~electron~~ holes in Si $= 480 \frac{cm^2}{Vs} = 0.048 \frac{m^2}{Vs}$

$$E = \frac{0.1\,V}{\mu m} = \frac{0.1}{1\times10^{-6}} = 1\times10^{5}\ V/m$$

$$\text{velocity of electrons} = \mu_n E = \left(\frac{0.135\,m^2}{Vs}\right)\left(\frac{1\times10^{5}\,V}{m}\right)$$

$$= 1.35\times10^{4}\ m/s$$

$$\text{velocity of holes} = \mu_p E = \left(\frac{0.048\,m^2}{Vs}\right)\left(\frac{1\times10^{5}\,V}{m}\right)$$

$$= 4.8\times10^{3}\ m/s$$

(b) As hole current is negligible

$$J_{Tot} = q[\mu_n n E + \mu_p p E] \approx q\mu_n n E \qquad J_{Tot} = 1mA/\mu m^2 = \frac{1\times10^{-3}}{1\times10^{-12}} = 1\times10^{9}$$

$$n = \frac{J_{Tot}}{q\mu_n E}$$

$$= \frac{1\times10^{9}\,A/m^2}{(1.6\times10^{-19})\left(0.135\frac{m^2}{Vs}\right)\left(1\times10^{5}\frac{V}{m}\right)} \qquad q = 1.6\times10^{-19}C$$

$$n = 4.6\times10^{23}\ \text{electrons}/m^3$$

or $n = 4.6\times10^{17}$ electrons/cm^3

Question 2) $L = 0.1\,\mu m = 0.1\times10^{-6} m$

$$A = (0.05\,\mu m)^2 = 2.5\times10^{-3}\times10^{-12}$$

$$= 2.5\times10^{-15}\ m^2$$

$$V = 1V$$

$$\mu_n = 0.135\ m^2/Vs$$

$$\mu_p = 0.048\ m^2/Vs$$

$$n = 10^{17}\ cm^{-3} = \frac{10^{17}}{10^{-6}} = 10^{23} m^{-3}$$

Date:

(a) $n_i(T=300K) = 5.2\times10^{15}(300)^{3/2}\ \exp\left[\frac{-1.12eV\times1.6\times10^{-19}}{2(1.38\times10^{-23}J/K)(300K)}\right]$

$= 1.08\times10^{10}\ cm^{-3}$

$= 1.08\times10^{16}\ m^{-3}$

$p = \frac{n_i^2}{n} = \frac{(1.08\times10^{16})^2}{10^{23}} = 1.17\times10^{9}\ m^{-3} = 1.17\times10^{3}\ cm^{-3}$

$E = V/L = 1/0.1\times10^{-6} = 1\times10^{7}\ V/m$

$I_{tot} = A\cdot J_{Tot}$

$= A\cdot q[\mu_n n + \mu_p p]E$

$= (2.5\times10^{-15}m^2)\cdot(1.6\times10^{-19}C)\left[0.135\frac{m^2}{Vs}\left(\frac{10^{23}}{m^3}\right) + 0.048\frac{m^2}{Vs}\left(\frac{1.17\times10^{9}}{m^3}\right)\right]\cdot\left(1\times10^{7}\frac{V}{m}\right)$

$I_{tot} = 5.4\times10^{-5} = 54\mu A$

(b) $n_i(T=400K) = 5.2\times10^{15}(400)^{3/2}\exp\left[\frac{-1.12eV\times1.6\times10^{-19}}{2(1.38\times10^{-23}J/K)(400K)}\right]$

$= 3.7\times10^{12}\ cm^{-3}$

$= 3.7\times10^{18}\ m^{-3}$

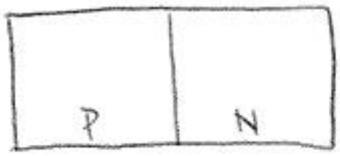
$p = \frac{n_i^2}{n} = \frac{(3.7\times10^{18})^2}{10^{23}} = 1.37\times10^{14}\ m^{-3}$

$E = 1\times10^{7}\ V/m$

$I_{Tot} = (2.5\times10^{-15}m^2)(1.6\times10^{-19}C)\left[0.135\frac{m^2}{Vs}\left(\frac{10^{23}}{m^3}\right) + 0.048\frac{m^2}{Vs}\left(\frac{1.37\times10^{14}}{m^3}\right)\right]\left(1\times10^{7}\frac{V}{m}\right)$

$I_{Tot} = 5.4\times10^{-5} = 54\mu A$

Q3 Given $N_D = 3 \cdot 10^{16}\ cm^{-3}$ $n_i = 1.08 \cdot 10^{10}\ cm^{-3}$



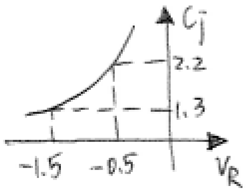
find V_0.

$$V_0 = \frac{kT}{q} \ln\left(\frac{N_D N_A}{n_i^2}\right) = \frac{kT}{q} \ln\left(\frac{N_D}{n_i}\right)$$

$$= \frac{(1.38 \cdot 10^{-23}\ J/K)(300K)}{1.6 \cdot 10^{-19} C} \ln\left(\frac{3 \cdot 10^{16}\ cm^{-3}}{1.08 \cdot 10^{10}\ cm^{-3}}\right)$$

$$= 0.384\ V$$

Q4



$$\frac{C_{j0}}{\sqrt{1+\frac{0.5}{V_0}}} = 2.2 \quad \text{——} \; ①$$

$$\frac{C_{j0}}{\sqrt{1+\frac{1.5}{V_0}}} = 1.3 \quad \text{——} \; ②$$

$① \div ②$: $$\frac{1+1.5/V_0}{1+0.5/V_0} = \left(\frac{2.2}{1.3}\right)^2 \Rightarrow V_0 = 0.0365\ V$$

Substitute V_0 into ①:

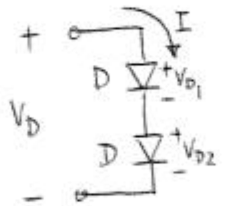
$$C_{j0} = 2.2\sqrt{1+\frac{0.5}{V_0}} \approx 8.43\ fF/\mu m^2$$

$$\Rightarrow \frac{N_A N_D}{N_A+N_D} = (C_{j0})^2 \cdot V_0 \cdot \frac{2}{\varepsilon_{si} q}$$
$$= \left(8.43\ \frac{fF}{\mu m^2}\right)^2 \times (0.0365V) \cdot \frac{2}{\varepsilon_{si} q} \approx 3.13 \cdot 10^{17}\ cm^{-3}$$

Fix a value for $N_A > \frac{N_A N_D}{N_A+N_D} \triangleq y$

$$N_A = 2 \cdot 10^{18}\ cm^{-3} \Rightarrow N_D = \frac{y N_A}{N_A - y}$$
$$= \frac{(3.13 \cdot 10^{17}\ cm^{-3})(2 \cdot 10^{18}\ cm^{-3})}{(2 \cdot 10^{18} - 3.13 \cdot 10^{17})\ cm^{-3}}$$
$$\approx 3.71 \cdot 10^{17}\ cm^{-3}$$

Q5 a)



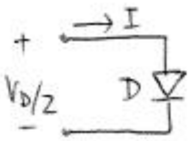
Suppose $I = I_s\left(e^{\frac{V_{D1}}{V_T}} - 1\right)$

$I_{D_2} = I_s\left(e^{\frac{V_{D2}}{V_T}} - 1\right)$

By KCL, $I_{D_1} = I_{D_2} = I$

$$\Rightarrow \left(e^{V_{D1}/V_T} - 1\right) = \left(e^{V_{D2}/V_T} - 1\right) \Rightarrow V_{D_1} = V_{D2} = \frac{V_D}{2}$$

$$\therefore\ I = I_s\left(e^{(V_D/2)/V_T} - 1\right)$$



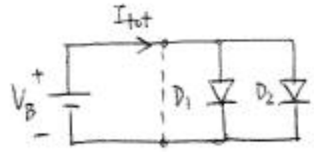
Therefore, a series combination can be viewed as a single two-terminal device with exponential characteristics.

(b) Suppose V_i = initial V_D. Need 10x
V_f = final V_D increase in I.

$$\Rightarrow 10 = \frac{I_s\left(e^{V_f/V_T} - 1\right)}{I_s\left(e^{V_i/V_T} - 1\right)} \approx e^{\frac{V_f - V_i}{V_T}}$$

$$\therefore\ \Delta V = V_f - V_i = V_T \ln(10) = (26\,mV)\ln(10)$$
$$\approx 60.\ mV.$$

Q6



$$I_{tot} = I_{D_1} + I_{D_2} = I_{S_1}(e^{V_B/V_T} - 1) + I_{S_2}(e^{V_B/V_T} - 1)$$

$$= (I_{S_1} + I_{S_2})(e^{V_B/V_T} - 1)$$

Therefore, the parallel combination operates as an exponential device, with an equivalent saturation current of $I_{S_1} + I_{S_2}$.

(b) By KVL, $V_{D_1} = V_{D_2}$

$$\Rightarrow V_T \ln\left(\frac{I_{D_1}}{I_{S_1}}\right) = V_T \ln\left(\frac{I_{D_2}}{I_{S_2}}\right)$$

Also, $I_{tot} = I_{D_1} + I_{D_2} \quad \Rightarrow I_{D_2} = I_{tot} - I_{D_1}$

$$\therefore V_T \ln\left(\frac{I_{D_1}}{I_{S_1}}\right) = V_T \ln\left(\frac{I_{tot} - I_{D_1}}{I_{S_2}}\right)$$

$$\Rightarrow I_{D_1} = I_{tot}\left(\frac{I_{S_1}}{I_{S_1} + I_{S_2}}\right)$$

$$\Rightarrow I_{D_2} = I_{tot}\left(\frac{I_{S_2}}{I_{S_1} + I_{S_2}}\right)$$