Habib University; City Campus	**Date: May 26, 2021**	**Class-ID: EE 252-L1**	
Examination: Final	**Course: Signals and Systems**	**Term: SP-21**	
Instructor: Dr. Abrar, S.	**Electrical & Computer Engineering**	**Duration: 220 min**	**Marks: 250**

INSTRUCTIONS:

1. YOU ARE NOT ALLOWED TO DISCUSS WITH EACH OTHER.
2. DO NOT SHARE YOUR ROUGH/FAIR WORK WITH OR COPY/CHEAT FROM OTHERS; YOUR WORK MUST BE YOUR "INDEPENDENT" WORK.
3. NO PENCIL WORK, USE PROPER BLACK MARKER OR POINTER.
4. PROVIDE A NEAT WORK WHILE AVOIDING EXCESSIVE NUMBER OF PAGES.
5. ENCLOSE YOUR FINAL RESULTS IN RECTANGULAR BOXES.
6. SINGLE COLUMN ONLY IN PORTRAIT FORMAT. LEAVE SOME SPACES AS LEFT/RIGHT MARGINS.
7. IF REQUIRED, ADJUST CONTRAST LEVEL IN SCANS TO REMOVE DARK BACKGROUND.
8. IF REQUIRED, CROP YOUR IMAGES TO REMOVE UNNECESSARY SURROUNDINGS.

Question 1: [90 points]

Allah Bakhsh is a radio engineer, he needs to transmit the *message* signal $x(t) = A\cos(20\pi t)$. The available *local oscillator* (LO) can provide a single *tone* frequency of 200 Hz; in simple words, the *carrier frequency*, f_c, is 200 Hz, that is, $c(t) = A_c \cos(2\pi f_c t)$ is the *carrier signal*.

There are two possible ways, he may *modulate* $x(t)$ using $c(t)$, and transmit the *modulated signal*:

Method-1:

The modulated signal $x_m(t)$ is obtained by simply multiplying $x(t)$ and $c(t)$ by using a piece of hardware commonly known as *mixer*.

$$x_m(t) = x(t)\, c(t) \tag{1.1}$$

Method-2:

The modulated signal $x_m(t)$ is obtained by multiplying $x(t)$ and $c(t)$ with a certain gain g, and also adding $c(t)$.

$$x_m(t) = c(t) + g\, x(t)\, c(t) = \big(1 + g\, x(t)\big)\, c(t) \tag{1.2}$$

where g is positive and less than or equal to unity, $g \leq 1$.

(a) **[10 pt.]** Express the Fourier transform of the given modulating signal $x(t)$ and plot its amplitude spectrum. **Note:** detailed derivation of Fourier transform is not required.

(b) **[15 pt.]** Assume that Bakhsh uses **Method-1**, as shown in (1.1), to modulate. Find the expressions of $x_m(t)$ in time and frequency domains. Plot the amplitude spectrum of $x_m(t)$.

(c) **[15 pt.]** Assume that Bakhsh uses **Method-2**, as shown in (1.2), to modulate. Find the expressions of $x_m(t)$ in time and frequency domains. Plot the amplitude spectrum of $x_m(t)$. Assume that $g = 0.5$.

At the receiver end, Rukhsana Chandio is responsible to receive the modulated signal, demodulate it, and somehow extract the message signal $x(t)$. She also has an identical LO with frequency 200 Hz.

In order to demodulate, she uses a mixer and simply multiplies the received signal $x_m(t)$ with LO tone $c(t) = A_c \cos(2\pi f_c t)$. Here, it is assumed that the LO tones at the transmitting and receiving ends are fully synchronized; there is no phase, frequency, or amplitude errors. Further, irrespective of the method used for the modulation, the demodulation is done in the following manner:

$$x_d(t) = x_m(t)\, c(t) \tag{1.3}$$

where $x_d(t)$ represents the demodulated signal.

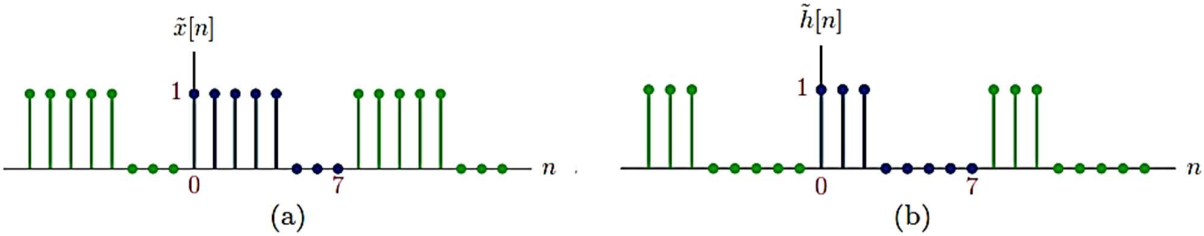
(d) **[15 pt.]** Assume that the **Method-1** was used for the modulation. Substituting the values of $x_m(t)$ and $c(t)$, expand and express $x_d(t)$ in both time [5 pt.] and frequency [5 pt.] domains. Plot the amplitude spectrum of $x_d(t)$ [5 pt.].

(e) **[15 pt.]** Assume that the **Method-2** was used for the modulation. Substituting the values of $x_m(t)$ and $c(t)$, expand and express $x_d(t)$ in both time [5 pt.] and frequency [5 pt.] domains. Plot the amplitude spectrum of $x_d(t)$ [5 pt.].

(f) **[20 pt.]** Discuss how the message signal $x(t)$ may be extracted form $x_d(t)$ for both modulation methods [5 pt.]. You have to describe properly if you need to use some recovery filter; discuss all parameters of the recovery filter like the gain [5 pt.], bandwidth [5 pt.] and the transfer function [5 pt.].

Question 2 [40]:

Consider the two periodic signals as shown below:



(a) Compute the periodic convolution

$$\tilde{y}[n] = \tilde{x}[n] \otimes \tilde{h}[n] \tag{2.1}$$

of the two signals.

(b) Determine the DTFS coefficients of signals $\tilde{x}[n], \tilde{h}[n]$, and $\tilde{y}[n]$. Verify that the periodic convolution property (as stated below) holds:

$$\tilde{x}[n] \overset{\text{DTFS}}{\longleftrightarrow} \tilde{c}_k \qquad \text{and} \qquad \tilde{h}[n] \overset{\text{DTFS}}{\longleftrightarrow} \tilde{d}_k$$

$$\tilde{x}[n] \otimes \tilde{h}[n] \overset{\text{DTFS}}{\longleftrightarrow} N\, \tilde{c}_k\, \tilde{d}_k \tag{2.2}$$

Question 3 [40]:

Find the DTFT of each signal given below. For each, sketch the magnitude and the phase of the transform using MATLAB. Provide your code and properly illustrated and labelled plots.

(a) $x[n] = \delta[n+1] + \delta[n] + \delta[n-1]$

(b) $x[n] = (0.5)^n u[n]$

Question 4 [80]:

Consider a signal $x(t) = \operatorname{sinc}^2\left(\frac{2}{3}t\right)$. Note: there is a square on the sinc finction.

Use zero-order hold sampler to sample this signal. Keep the duty cycle, $d = 1/3$.

(a) Properly sketch the signal $x(t)$. [10 pt.] The plot is not required to be to-the-scale, you may exaggerate some part of the plot to clarify the shape of the signal.

(b) Properly sketch the Fourier transform of $x(t)$. [10 pt.]

(c) Properly sketch the sampled signal $x_{\text{ZOH}}(t)$. [10 pt.] The plot is not required to be to-the-scale, you may exaggerate some part of the plot to clarify the shape of the signal.

Q5 [100 points] In zero-order hold (**ZOH**) sampling, a given sample of analog signal is hold for a duration of dT_s as shown in `Figure 6.22` in the book. When $dT_s = T_s/2$ (where $T_s = 1/f_s$ is sampling time), this is called return-to-zero (**RZ**) flat-top sampling; and when $dT_s = T_s$, non-return-to-zero (**NRZ**) flat-top sampling. The Fourier transform of ZOH sampled signal $x_{\text{ZOH}}(t)$ is expressed below

$$X_{\text{ZOH}}(f) = \frac{dT_s}{T_s}\,\text{sinc}(dT_s f)\,e^{-j2\pi f dT_s/2}\sum_{k=-\infty}^{\infty} X_a(f - kf_s)$$

where $X_a(f)$ is the Fourier transform of the analog sampling signal $x_a(t)$.

Consider an analog sampling signal $\boxed{x_a(t) = \text{sinc}(2t)}$ and assume that $f_s = 1/T_s = 3$ Hertz.

Task 1 Draw the sampling signal $[x_a(t)]$ together with ZOH sampled signal $[x_{\text{ZOH}}(t)]$ when **RZ** scheme is employed; show for the time interval $-2 \le t \le 2$.

Task 2 Draw the sampling signal $[x_a(t)]$ together with ZOH sampled signal $[x_{\text{ZOH}}(t)]$ when **NRZ** scheme is employed; show for the time interval $-2 \le t \le 2$.

Task 3 For **RZ** scheme, draw the amplitude spectrum $|X_{\text{ZOH}}(f)|$; show spectrum for the range $-6 \le f \le 6$.

Task 4. With the aid of diagram, explain if it is possible to reconstruct the original signal $x_a(t)$ using an ideal lowpass filter (LPF), if yes, find the bandwidth of the filter.

Task 5. Based on **Task 1.4**, Obtain and draw the amplitude spectrum of the reconstructed signal in the interval $-2 \le f \le 2$. Discuss if there is any distortion in the reconstructed spectrum.

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Task 6. For **NRZ** scheme, draw the amplitude spectrum $|X_{\text{ZOH}}(f)|$; show spectrum for the range $-6 \le f \le 6$.

Task 7. With the aid of diagram, explain if it is possible to reconstruct the original signal $x_a(t)$ using an ideal lowpass filter (LPF), if yes, find the bandwidth of the filter.

Task 8. Based on **Task 1.7**, obtain and draw the amplitude spectrum of the reconstructed signal in the interval $-2 \le f \le 2$. Discuss if there is any distortion in the reconstructed spectrum.

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