Midterm Exam I **Signals and Systems** **Spring 2021**

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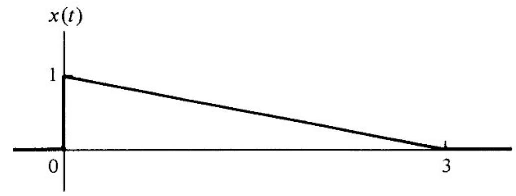
Question 1 [20]: For $x(t)$ indicated in Fig 1., determine and sketch the following by both methods:

(a) $x(2t+3)$
(b) $x(2-3t)$



Note: In method 1, you scale first and then shift. In method 2, you shift first and then scale.

Question 2 [15]: Show that:

$$\left(1-e^{j\alpha}\right)=2\sin\left(\frac{\alpha}{2}\right)e^{j(\alpha-\pi)/2}$$

Question 3 [25]:

With $x(t)=\cos(\omega_x(t+\tau_x)+\theta_x)$ and $y(t)=\sin(\omega_y(t+\tau_y)+\theta_y)$, determine for which of the following combinations $x(t)$ and $y(t)$ are identically equal for all t . Show all steps.

	ω_x	τ_x	θ_x	ω_y	τ_y	θ_y
(i)	$\pi/3$	0	2π	$\pi/3$	1	$-\pi/3$
(ii)	$3\pi/4$	$1/2$	$\pi/4$	$11\pi/4$	1	$3\pi/8$
(iii)	$3/4$	$1/2$	$1/4$	$3/4$	1	$3/8$

Question 4 [25]:

With $x[n]=\cos(\Omega_x(n+P_x)+\theta_x)$ and $y[n]=\cos(\Omega_y(n+P_y)+\theta_y)$, determine for which of the following combinations $x[n]$ and $y[n]$ are identically equal for all n. Show all steps.

	Ω_x	P_x	θ_x	Ω_y	P_y	θ_y
(i)	$\pi/3$	0	2π	$8\pi/3$	0	0
(ii)	$3\pi/4$	2	$\pi/4$	$3\pi/4$	1	$-\pi$
(iii)	$3/4$	1	$1/4$	$3/4$	0	1

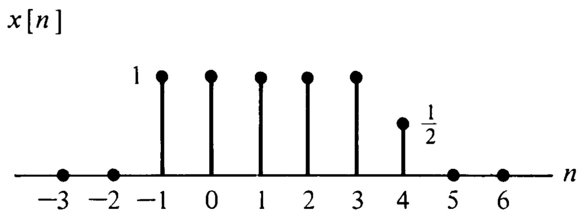
Question 5 [15]:

(a) Sketch $x[n]=\alpha^n$ for a typical α in the range $-1<\alpha<0$

(b) Assume that $\alpha=-e^{-1}$, and define $y(t)$ as $y(t)=e^{\beta t}$. Find a complex number β such that $y(t)$, when evaluated at t equal to an integer n, is described by $\left(-e^{-1}\right)^n$.
Hint: β is a complex number.

Question 6 [15]:

A discrete-time signal $x[n]$ is shown below. Sketch and carefully label each of the following signals:
(i) $x[4-n]$
(ii) $x[2n]$
(iii) $x[n/2]$



Question 7 [20]:

Consider the signals

$$x(t) = \cos\frac{2\pi t}{3} + 2\sin\frac{16\pi t}{3}$$
$$y(t) = \sin \pi t$$

Show that $z(t) = x(t)y(t)$ is periodic by determining its time period.
Hint: Find if it is possible to express $z(t)$ as a sum of several sinusoids, find the sinusoid term with the smallest frequency, call it f_0, and next show that the frequencies of all other sinusoid terms are integer multiple of f_0. The required time period is thus $1/f_0$.

Question 8 [15]:

Let $x_e(t)$ and $x_o(t)$ be the even and odd components of signal $x(t)$, i.e., $x(t) = x_e(t) + x_o(t)$.

(a) Show that $x_e(t)$ and $x_o(t)$ are orthogonal to each other

$$\int_{-\infty}^{+\infty} x_e(t)x_o(t)dt = 0$$

(b) Also show that

$$\int_{-\infty}^{+\infty} x^2(t)dt = \int_{-\infty}^{+\infty} x_e^2(t)dt + \int_{-\infty}^{+\infty} x_o^2(t)dt$$

and conclude your finding.

Question 9 [100]: 5 points for describing each required (unshaded) cell properly.

Table contains the input-output relations for several continuous-time and discrete-time systems, where $x(t)$ or $x[n]$ is the input. Indicate whether the property along the top row applies to each system by answering yes or no in the appropriate boxes, also show your work.
Do not mark the shaded boxes.

$y(t), y[n]$	Memoryless	Linear	Time-Invariant	Causal	Invertible	Stable
(a) $(2 + \sin t)x(t)$						
(b) $x(2t)$						
(c) $\sum_{k=-\infty}^{\infty} x[k]$						
(d) $\sum_{k=-\infty}^{n} x[k]$						
(e) $\frac{dx(t)}{dt}$						

Question 10 [25]:

Determine the continuous-time convolution of $x(t)$ and $h(t)$ for the following case:

$$h(t) = u(t + 2)$$

and

$$x(t) = e^{-t-2}u(t - 2)$$

Question 11 [25]:

Determine the discrete-time convolution of $x[n]$ and $h[n]$ for the following case:

$$h(t) = u[n]$$

and

$$x[n] = \frac{1}{3^n}u[n]$$