HABIB UNIVERSITY

EE-252 Midterm-I Exam (Retake)
Signals and Systems
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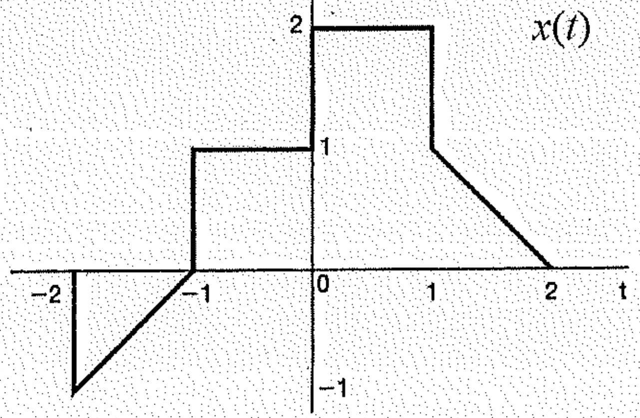
Duration: 60 min

Q1 [50 points]: If a periodic signal has the even-symmetry property $v(-t) = v(t)$, then the exponential Fourier series (analysis) may be written as

$$c_n = \frac{2}{T_0}\int_0^{T_0/2} v(t)\cos(2\pi nt/T_0)\,dt$$

Use this expression to find c_n when $v(t) = A - 2A|t|/T_0$ for $|t| < T_0/2$.
(A) Sketch the waveform $v(t)$, and **determine** c_0 directly from the average value of $v(t)$. **(B) Evaluate** c_n, **sketch** and **label** the spectrum.

Q2 [40 points]: A continuous-time signal $x(t)$ is shown in Figure 1.
(A) Express $x(t)$ mathematically in one expression.
(B) Sketch and **label** carefully each of the following signals:
- (a) $x(2-3t)$
- (b) $[x(t) + x(-t)]u(t)$

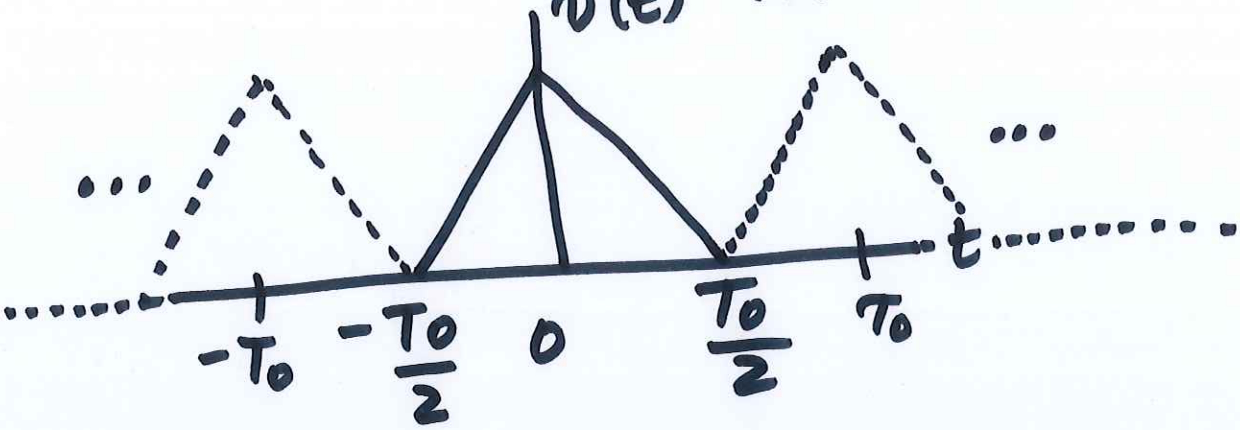


Q3 [40 points]: For each of the following input-output relationships, **determine** whether the corresponding system is linear, time invariant or both.
- (a) $y(t) = t^2 x(t-1)$
- (b) $y[n] = x^2[n-2]$

Q4 [40 points]: Let $x(t) = u(t-3) - u(t-5)$ and $h(t) = e^{-3t}u(t)$. **Compute** $y(t) = x(t) * h(t)$.

1. $\tilde{v}(t) = A\left(1 - \frac{2|t|}{T_0}\right)$

$|t| \leq T_0/2.$



$$c_0 = \langle \tilde{v}(t) \rangle = \frac{2}{T_0} * \frac{T_0}{2} * \frac{A}{2} = \frac{A}{2}$$

$$c_n = \frac{2}{T_0}\int_0^{T_0/2} A\left(1 - \frac{2t}{T_0}\right)\cos\left(\frac{2\pi n t}{T_0}\right)dt$$

$$= \underbrace{\frac{2A}{T_0}\sin\left(\frac{2\pi n t}{T_0}\right)\frac{T_0}{2\pi n}\Big|_0^{T_0/2}}_{(=0)} - \frac{4A}{T_0^2}\int_0^{T_0/2} t\cos\left(\frac{2\pi n t}{T_0}\right)dt$$

$$= -\frac{4A}{T_0^2}\,\frac{\left(\frac{2\pi n}{T_0}\right)t\sin\left(\frac{2\pi n t}{T_0}\right) + \cos\left(\frac{2\pi n t}{T_0}\right)}{\left(\frac{2\pi n}{T_0}\right)^2}\Bigg|_0^{T_0/2}$$

$$C_n = \frac{-4A}{T_0^2} \left. \frac{\cos(2\pi n t/T_0)}{(2\pi n/T_0)^2} \right|_0^{T_0/2}$$

$$= -\frac{A}{(\pi n)^2}\left(\cos(\pi n) - 1\right)$$

$$C_n = \frac{A}{\pi^2 n^2}\left(1 - \cos(\pi n)\right)$$

n	0	1	2	3	4	5	6
$\lvert C_n \rvert^2$	0.5A	0.2A	0	0.02A	0	0.01A	0
$\arg C_n$	0	0	0	0	0	0	0

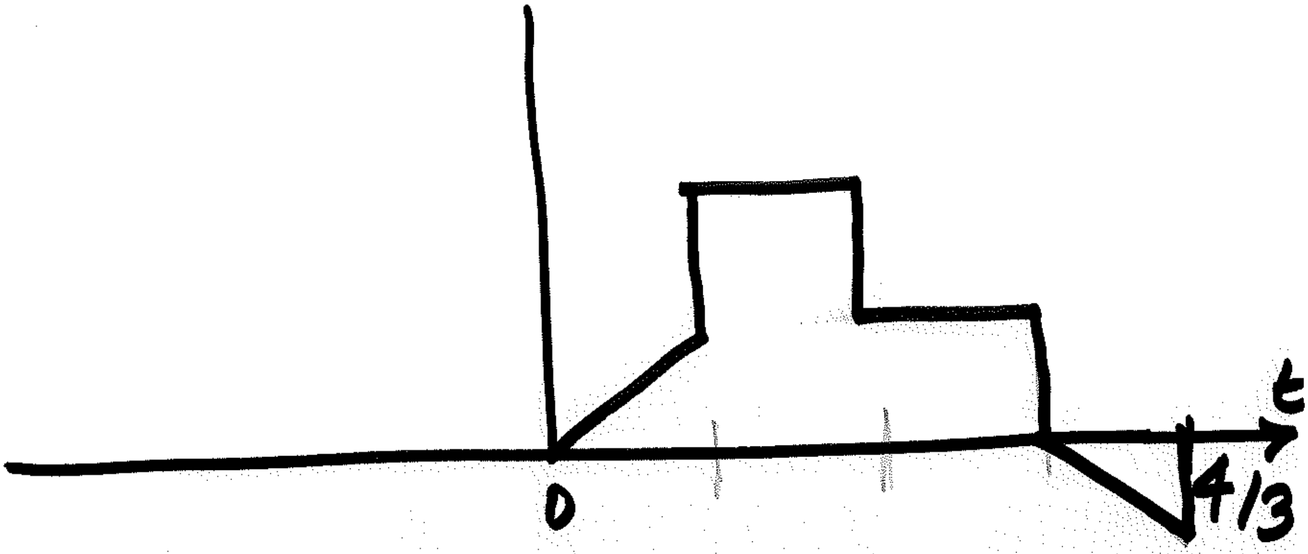
Q2. (A)

$$x(t) = \begin{cases} 0, & t \leq -2 \\ t+1 & -2 \leq t \leq -1 \\ 1 & -1 \leq t \leq 0 \\ 2 & 0 \leq t \leq 1 \\ 2-t & 1 \leq t \leq 2 \\ 0 & t \geq 2 \end{cases}$$

$$\begin{aligned} x(t) = {} & (t+1)\left[u(t+2) - u(t+1)\right] \\ & + \left[u(t+1) - u(t)\right] \\ & + 2\left[u(t) - u(t-1)\right] \\ & + (2-t)\left[u(t-1) - u(t-2)\right]. \end{aligned}$$

(B)
(a)

$x(2-3t)$



$2-3t = -2 =$ start time of $x(t)$

$3t = 4 \Rightarrow t = 4/3 =$ end time of $x(2-3t)$

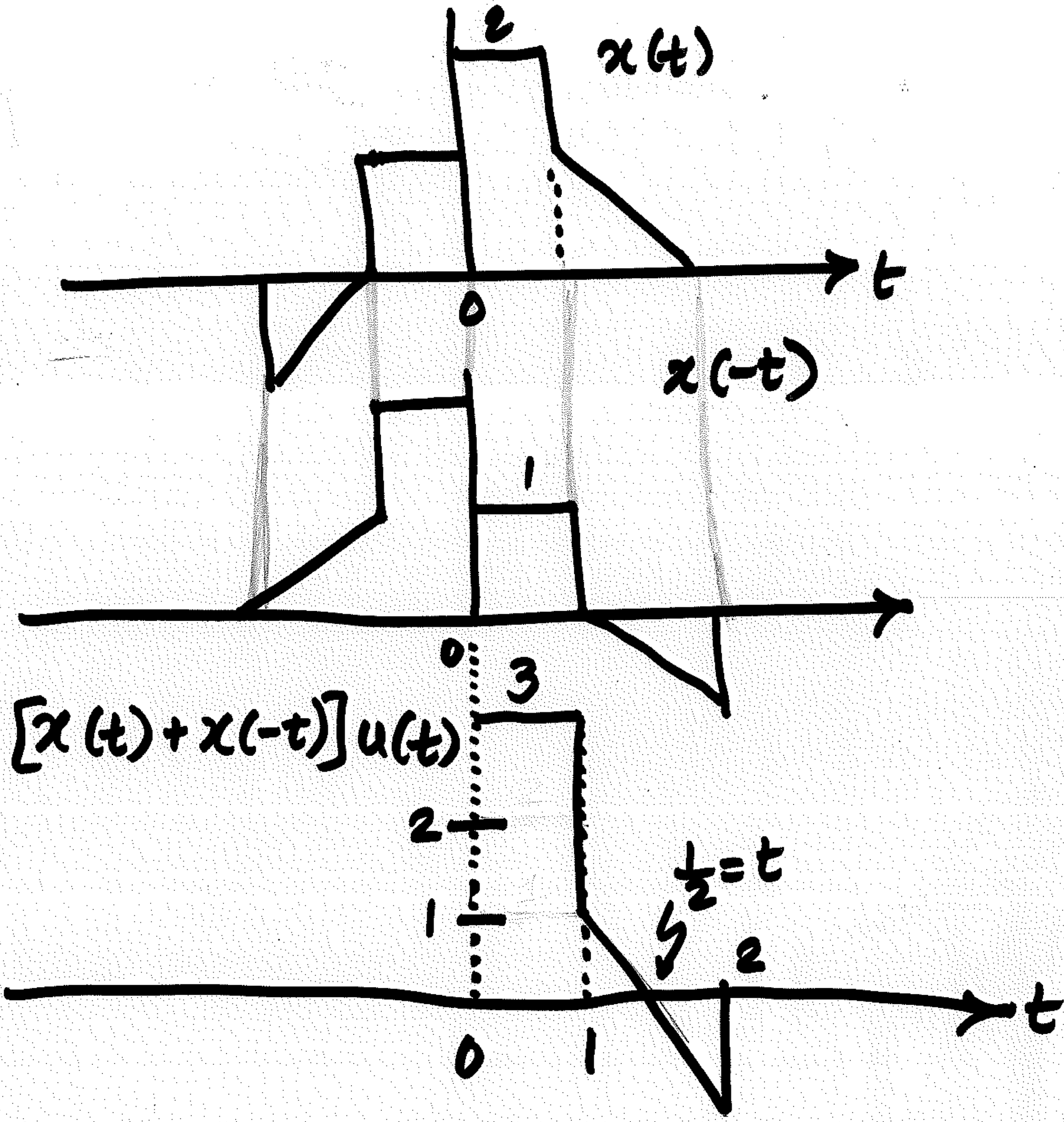
$2-3t = 2 =$ end time of $x(t)$

$3t = 0 \Rightarrow t = 0 =$ start time of $x(2-3t)$

The signal is compressed in time by three times.

(B)

(b) $[x(t) + x(-t)]\, u(-t)$



Q3. Consider two arbitrary inputs $x_1(t)$ & $x_2(t)$

$$x_1(t) \longrightarrow y_1(t) = t^2 x_1(t-1)$$

$$x_2(t) \longrightarrow y_2(t) = t^2 x_2(t-1)$$

Let $x_3(t) = \alpha x_1(t) + \beta x_2(t)$

$$x_3(t) \longrightarrow y_3(t) = t^2 x_3(t-1)$$

$$y_3(t) = t^2 (\alpha x_1 + \beta x_2 (t-1))$$

$$= \alpha t^2 x_1(t-1) + \beta t^2 x_2(t-1)$$

$$= \alpha y_1(t) + \beta y_2(t)$$

System is LINEAR.

Q3. (a) <u>first system, then delay</u>

$$g_1(t) = Sys\{x(t)\} = t^2 x(t-1)$$

$$y_1(t) = Delay\{g_1(t)\} = (t-t_0)^2 x(t-t_0-1)$$

<u>first delay and then system:</u>

$$g_2(t) = Dely\{x(t)\} = x(t-t_0)$$

$$y_2(t) = Sys\{g_2(t)\} = t^2 x(t-t_0-1)$$

Since $y_1(t) \neq y_2(t)$

sys is NOT TIME INVARIANT.

Q3 (b) $y[n] = x^2[n-2]$

$x_1(n) \longrightarrow y_1[n] = x_1^2[n-2]$

$x_2(n) \longrightarrow y_2[n] = x_2^2[n-2]$

Let $x_3[n] = \alpha\, x_1[n] + \beta\, x_2[n]$

$x_3[n] \longrightarrow y_3[n] = x_3^2[n-2]$

$= (\alpha\, x_1[n-2] + \beta\, x_2[n-2])^2$

$= \alpha^2 x_1^2[n-2] + \beta^2 x_2^2[n-2] + 2\alpha\beta\, x_1[n-2]\, x_2[n-2]$

$\neq \alpha\, y_1[n] + \beta\, y_2[n]$

System is not Linear.

Q3 (b) first system, then delay

$$g_1[n] = \text{sys}\{x[n]\} = x^2[n-2]$$

$$y_1[n] = \text{Delay}\{g_1[n]\} = x^2(n-2-n_0)$$

first delay, then system.

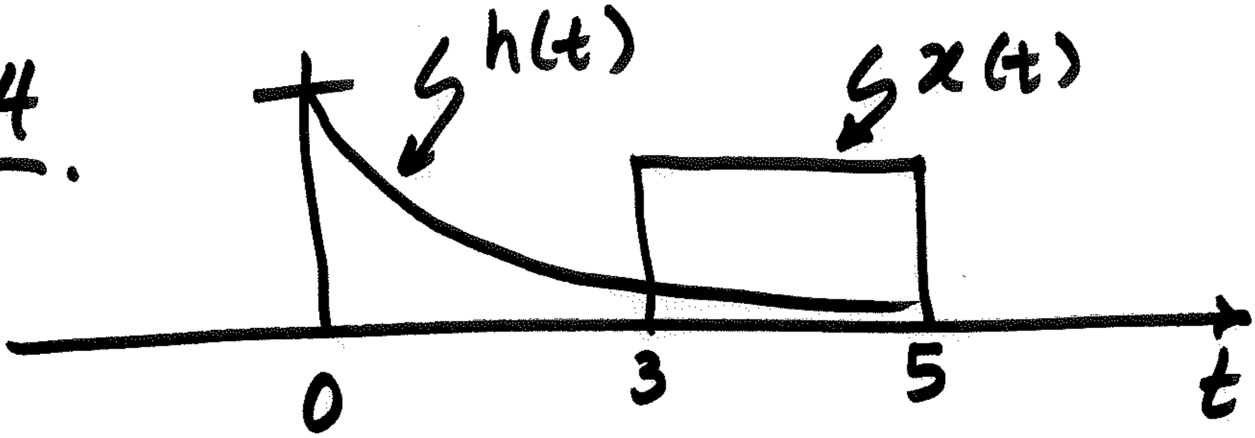
$$g_2[n] = \text{Delay}\{x[n]\} = x[n-n_0]$$

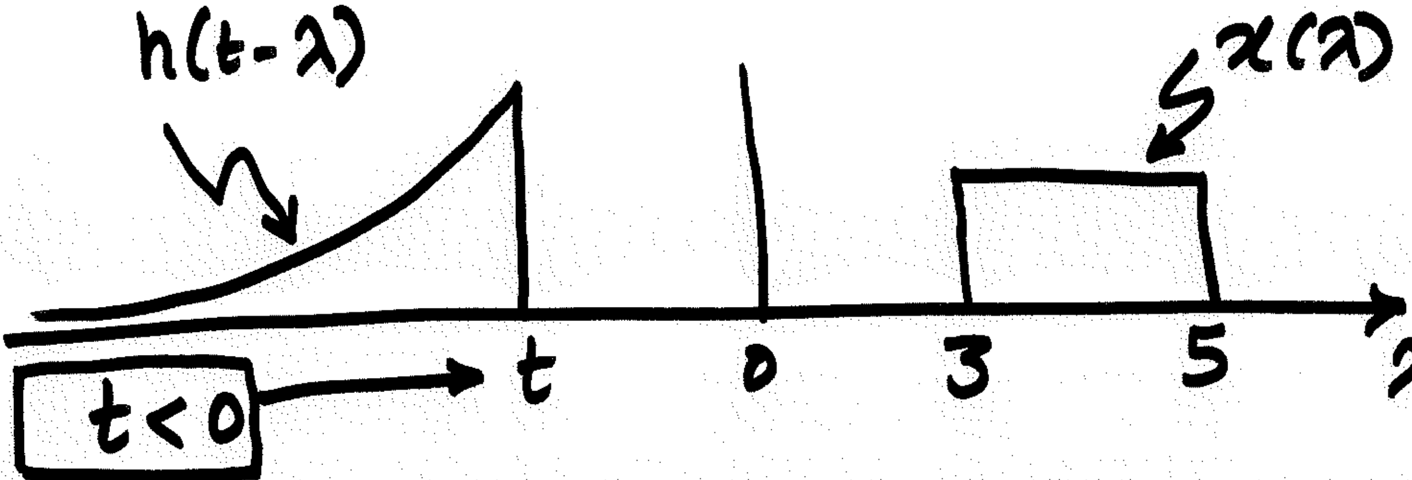
$$y_2[n] = \text{sys}\{g_2[n]\} = x^2[n-n_0-2]$$

Since $y_1[n] = y_2[n]$

System is time invariant.

Q4.

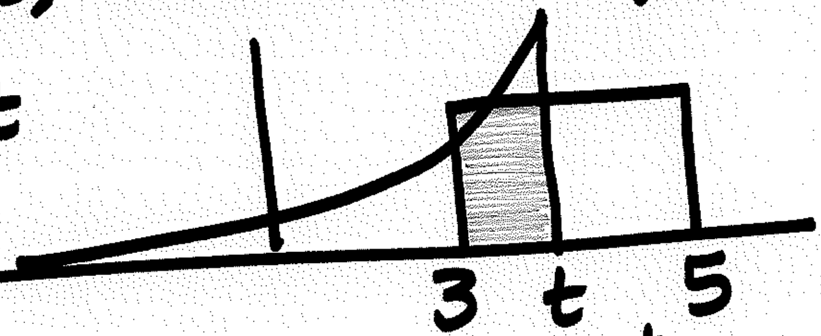




For $t < 3$, no overlap, $y(t) = 0$

For $3 < t < 5$, There is overlap.

$3 < \lambda < t$



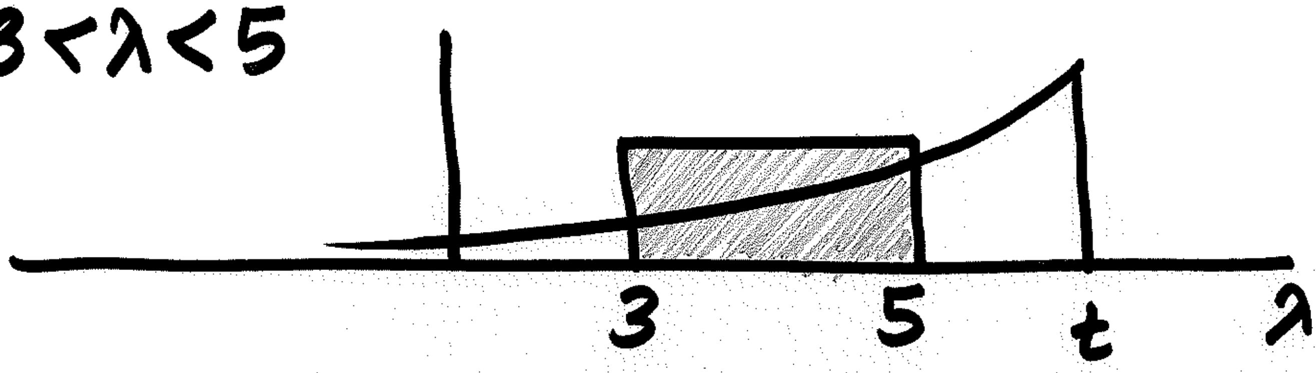
$$y(t) = \int_3^t e^{-3(t-\lambda)} d\lambda = e^{-3t} \int_3^t e^{3\lambda} d\lambda$$

$$= e^{-3t} \left. \frac{e^{3\lambda}}{3} \right|_3^t = \frac{1}{3} e^{-3t} \left(e^{3t} - e^{9}\right)$$

$$= \frac{1}{3}\left(1 - e^{-3t+9}\right) \text{ for } 3<t<5.$$

For $5 < t < \infty$, there is always an overlap.

$3 < \lambda < 5$



$$y(t) = e^{-3t} \left. \frac{e^{3\lambda}}{3} \right|_3^5$$

$$= \frac{1}{3} e^{-3t} \left(e^{15} - e^{9} \right)$$

$$= \frac{(e^6 - 1)}{3} e^{-3t+9}$$

$$5 \leq t < \infty$$