HABIB UNIVERSITY

Final Exam

Last Name:__________ ID:__________

Course: Signals and Systems **Date:** June 30, 2020

Instructor: Dr. Shafayat Abrar **Marks:** 200 points

Duration: 12 Hours **Term:** Spring 2020

YOUR ILLUSTRATIONS SHOULD BE NEATLY DRAWN, PROPERLY LABELLED, AND NEAR TO-THE-SCALE.

YOUR WORK SHOULD PROVIDE ALL REQUIRED STEPS IN A NEAT AND TIDY MANNER.

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1. **[40 points]** In zero-order hold (**ZOH**) sampling, a given sample of analog signal is hold for a duration of dT_s as shown in `Figure 6.22` in the book. When $dT_s = T_s/2$ (where $T_s = 1/f_s$ is sampling time), this is called return-to-zero (**RZ**) flat-top sampling; and when $dT_s = T_s$, non-return-to-zero (**NRZ**) flat-top sampling. The Fourier transform of ZOH sampled signal $x_{\text{ZOH}}(t)$ is expressed below

$$X_{\text{ZOH}}(f) = \frac{dT_s}{T_s}\,\text{sinc}(dT_s f)\,e^{-j2\pi f dT_s/2}\sum_{k=-\infty}^{\infty} X_a(f - kf_s)$$

where $X_a(f)$ is the Fourier transform of the analog sampling signal $x_a(t)$.

Consider an analog sampling signal $\boxed{x_a(t) = \text{sinc}(2t)}$ and assume that $f_s = 1/T_s = 3$ Hertz.

Task 1.1 Draw the sampling signal $[x_a(t)]$ together with ZOH sampled signal $[x_{\text{ZOH}}(t)]$ when **RZ** scheme is employed; show for the time interval $-2 \le t \le 2$. **[5 points]**

Task 1.2 Draw the sampling signal $[x_a(t)]$ together with ZOH sampled signal $[x_{\text{ZOH}}(t)]$ when **NRZ** scheme is employed; show for the time interval $-2 \le t \le 2$. **[5 points]**

Task 1.3 For **RZ** scheme, draw the amplitude spectrum $|X_{\text{ZOH}}(f)|$; show spectrum for the range $-6 \le f \le 6$. **[5 points]**

Task 1.4 With the aid of diagram, explain if it is possible to reconstruct the original signal $x_a(t)$ using an ideal lowpass filter (LPF), if yes, find the bandwidth of the filter. **[5 points]**

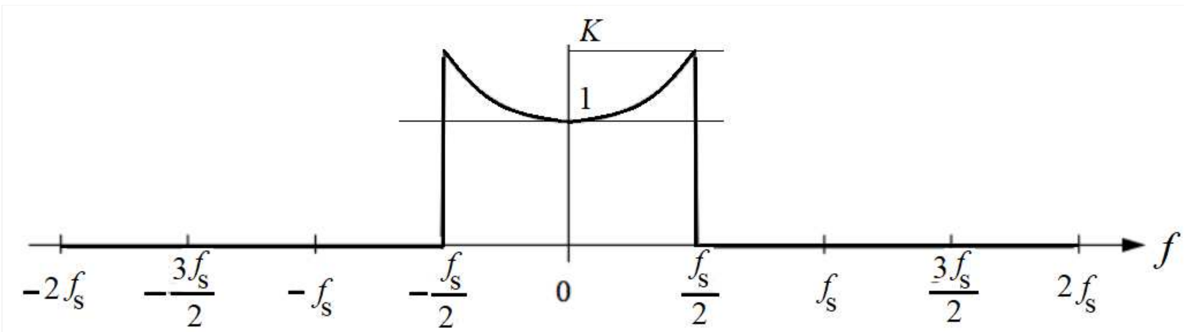
Task 1.5 Based on **Task 1.4**, Obtain and draw the amplitude spectrum of the reconstructed signal in the interval $-2 \le f \le 2$. Discuss if there is any distortion in the reconstructed spectrum.
. **[5 points]**

Task 1.6 For **NRZ** scheme, draw the amplitude spectrum $|X_{\text{ZOH}}(f)|$; show spectrum for the range $-6 \le f \le 6$. **[5 points]**

Task 1.7 With the aid of diagram, explain if it is possible to reconstruct the original signal $x_a(t)$ using an ideal lowpass filter (LPF), if yes, find the bandwidth of the filter. **[5 points]**

Task 1.8 Based on **Task 1.7**, obtain and draw the amplitude spectrum of the reconstructed signal in the interval $-2 \le f \le 2$. Discuss if there is any distortion in the reconstructed spectrum.
. **[5 points]**

2. **[20 points]** Fatima is a newly appointed research engineer. She was given an assignment to design a reconstruction filter for ZOH sampled signals (the NRZ one, where $dT_s = T_s$). She realized that due to the **sinc** function in its Fourier transform, all sampled signals suffer some sort of distortion as the frequency spectrum of **sinc** function is not flat and it tends to **bow down** for large frequencies. She came up with an idea to design a reconstruction lowpass filter such that the spectrum amplitude **rises** with frequency in a proportionate manner (so that to compensate the *aperture* distortion caused by the **sinc** function). She suggested the following transfer function $H_r(f)$ for the reconstruction filter:



Assuming that the Nyquist criterion $f_s = 2W$ is fulfilled, where W is the bandwidth of the modulating signal.

Task 2.1 Express the transfer function $H_r(f)$ mathematically.

Task 2.2 Find the value of K in the amplitude spectrum of $H_r(f)$.

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3. **[30 points]** Mubeen is a newly appointed research engineer. He was given an assignment to design an amplifier for a bandlimited signal (with bandwidth W) which has significant low-frequency contents. However, the available amplifier in his lab-inventory performs poorly for low-frequency contents, and it is described by the following frequency response:

$$H_a(f) = \begin{cases} 0 & |f| < f_L - \epsilon \\ K/2 & f_L \leq |f| \leq f_L + W/2 \\ K & f_L + W/2 \leq |f| \leq f_L + B \\ 0 & |f| > f_L + B \end{cases}$$

where $|f|$ is used to express both sides of the spectrum. Note that $f_L \gg W$ and $B \gg f_L$.
Mubeen came-up with an idea to use a bipolar chopper to first sample the amplifying signal such that the sampled signal gets modulated (or translated in frequency domain) to the range of frequencies where amplifier has flat spectrum, and then use another bipolar chopper to demodulate the amplified signal back to its original baseband position.

Task 3.1 Plot the amplitude spectrum $H_a(f)$ for arbitrary values of W, f_L and B. Keeping in mind that $f_L \gg W$ and $B \gg f_L$. **[5 points]**

Task 3.2 Bipolar chopper is a sampling device where the sampling signal is multiplied with a periodic signal $\widetilde{p}(t)$ containing alternate polarities ($+1$ and -1); each polarity exists for T_s seconds, where T_s is the sampling time. **(a)** What is the time period of signal $\widetilde{p}(t)$? **(b)** With the aid of Fourier series, prove that the even-symmetric signal $\widetilde{p}(t)$ may be synthesized as follows:

$$\widetilde{p}(t) = \sum_{k=1,3,5,\cdots}^{\infty} a_k \cos\left(\pi k \frac{t}{T_s}\right) \quad \text{where} \quad a_k = \begin{cases} 0 & k = 2, 4, 6, \cdots \\ \dfrac{4}{k\pi} & k = 1, 5, 9, \cdots \\ -\dfrac{4}{k\pi} & k = 3, 7, 11, \cdots \end{cases}$$

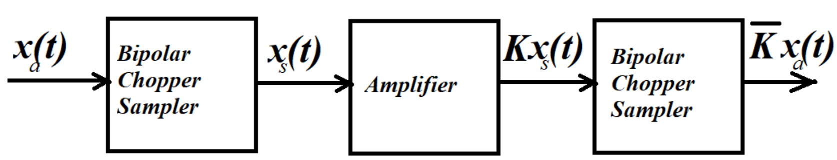
[10 points]

Task 3.4 Assuming the transform pair $X_a(f) \leftrightarrow x_a(t)$, obtain the Fourier transform $X_s(f)$ of the sampled signal: **[5 points]**

$$x_s(t) = \widetilde{p}(t) x_a(t)$$

Task 3.5 Assuming an arbitrary but symmetric shape of spectrum $X_a(f)$, plot the amplitude spectrum of $X_s(f)$ in the frequency range $-10f_s \leq f \leq 10f_s$, where $f_s := 1/T_s$. **[5 points]**

Task 3.6 With the aid of $X_s(f)$ and $H_a(f)$, obtain an appropriate value of f_s such that a distortion free amplification is achieved. **[5 points]**



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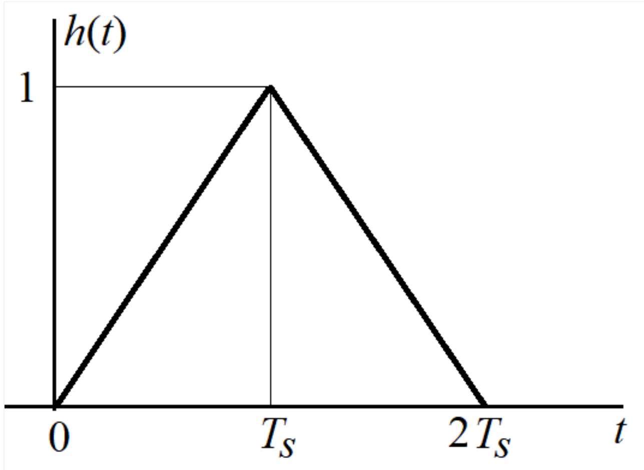
4. **[30 points]** The impulse response of a certain reconstruction filter is given as



where T_s is the sampling time. In the sequel, assume $T_s = 1$.

The impulse-sampled signal is expressed as

$$x_S(t) = a_1\delta(t) + a_2\delta(t - T_s) + a_3\delta(t - 2T_s)$$

where the constants a_1 to a_3 are unique integer values (no two a_is are same). You have to select these three values from the following set in a random fashion:

$$\{-7, -5, -3, -1, 1, 3, 5, 7, 9\}$$

You have to select only one set of coefficients a_is.

Task 4.1 Express $h(t)$ in a mathematical form. **[0 points]**

Task 4.2 Find the output of the reconstruction filter, $x_R(t)$, which is given as the convolution of $h(t)$ and $x_S(t)$, that is: **[20 points]**

$$x_R(t) = x_S(t) \star h(t)$$

Task 4.3 Provide the plots of $x_S(t)$ and $x_R(t)$. **[10 points]**

Note: The chances that two students will select the same sequence of a_i is very slim. So better select your own values and do not try to copy the work of others.

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5. **[30 points]** Consider the two periodic signals as shown below:

$$\widetilde{x}[n] = \{\cdots, 1, 2, 2, 1, \cdots\}$$

and

$$\widetilde{h}[n] = \{\cdots, 0, 1, 1, 1, \cdots\}$$

Task 5.1 Compute the periodic convolution by using its definition

$$\widetilde{y}[n] = \widetilde{x}[n] \bigotimes \widetilde{h}[n]$$

of the two signals. **[10 points]**

Task 5.2 Determine the DTFS coefficients of signals $\widetilde{x}[n]$, $\widetilde{h}[n]$ and $\widetilde{y}[n]$. Verify that the periodic convolution property holds. You may use MATLAB to validate your answers but you have to show all steps. **[20 points]**

6. **[20 points]** Showing all steps, obtain the DTFT of the signals

Task 6.1 $x[n] = \frac{1}{2}\left(\frac{1}{2^n} + \frac{1}{4^n}\right)u[n]$

Task 6.2 $x[n] = (-0.8)^n(u[n-3] - u[n+5])$

7. **[20 points]** Showing all steps, obtain the inverse DTFT of

Task 7.1 $X(\Omega) = \begin{cases} e^{\Omega} & \text{for } -\pi \leq \Omega < 0 \\ e^{-\Omega} & \text{for } 0 \leq \Omega < \pi \end{cases}$

Task 7.2 $X(\Omega) = \dfrac{\alpha e^{-j\Omega}}{(1 - \alpha e^{-j\Omega})^2}$

8. **[10 points]** Consider the finite-length signal

$$x[n] = \left\{ \overset{n=0}{1}, 2, 3, 0, 0 \right\}$$

Compute the 5-point DFT $X[k]$ for $k = 0, \cdots, 4$.