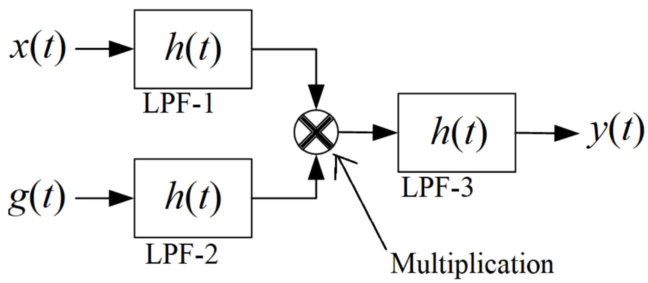
HABIB UNIVERSITY	**Final Exam**	**Timing: 4:30 PM - 7:00 PM**
	EE-252: Signals and Systems	**May 07, 2022**
	Dr. A.A. Chaudhry and Dr. S. Abrar	**Duration: 150 min**

Q1 [20 points]: Consider the system depicted in Figure 1. The impulse response of the ideal low-pass filter (LPF) is given by $h(t) = \text{sinc}(2Wt)$, where $2W = 11$. There are two input signals given by

$$x(t) = \sum_{m=1}^{\infty} \frac{1}{m^2} \cos(5\pi m t), \qquad g(t) = \sum_{m=1}^{10} \cos(8\pi m t)$$

Using properties of Fourier transform, determine $y(t)$.



Steps:
(a) Express $y(t)$ in terms of $x(t)$, $g(t)$ and $h(t)$ [**5 pt.**].
(b) Using CTFT multiplication and convolution properties, express $Y(f)$ in terms of $X(f)$, $G(f)$, and $H(f)$ [**5 pt.**].
(c) Think about what tone frequencies that will be extracted out when signals $x(t)$ and $g(t)$ are passed through the low-pass filters LPF-1 and LPF-2, respectively. Explain in words, plot spectrum, and/or express mathematically [**5 pt.**].
(d) Finally obtain $y(t)$ while discussing the role of LPF-3 [**5 pt.**].

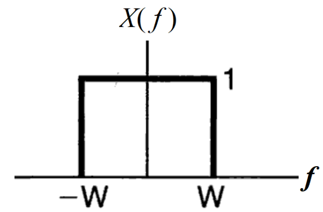
Note: the final answer will have two cosine terms.
You may need the following transform pair:

$$A\,\text{sinc}(2Bt) \longleftrightarrow \frac{A}{2B}\Pi\left(\frac{f}{2B}\right)$$

Q2 [15 points]: A continuous-time signal $x(t)$ is composed of a linear combination of sinusoidal signals (tones) of frequencies 300 Hz, 500 Hz, 1.2 kHz, 2.15 kHz, and 3.5 kHz. The signal $x(t)$ is sampled (ideally using periodic impulse train) at a 2.0 kHz rate, and the sampled sequence is passed through an ideal low-pass filter with a cut-off frequency of 900 Hz, generating a signal $y(t)$. What are the frequency components present in the reconstructed signal $y(t)$? Show your working by sketching the Fourier transforms of input $X(f)$ [**5 pt.**], sampled sequence $X_S(f)$ [**5 pt.**], filter $H(f)$, and output $Y(f)$ [**5 pt.**].

Q3 [15 points]: Assume that the Nyquist sampling frequency of a continuous-time signal $x(t)$ is f_S. That is, if W is the bandwidth of $x(t)$ in Hertz, then $f_S = 2W$.

Determine the Nyquist sampling frequencies of the following continuous-time signals derived from $x(t)$. **[3 pt. each]**



(a) $y_1(t) = x(t)x(t)$.

(b) $y_2(t) = x(t/3)$.

(c) $y_3(t) = x(3t)$.

(d) $y_4(t) = x(t) * x(t)$.

(e) $y_5(t) = \frac{d}{dt}x(t)$.

Note: Provide proper sketches of frequency domain spectrum of derived signals $y_i(t)$. Assume that the spectrum of $x(t)$ is given as:

Q4 [20 pt.]: The output of a causal LTI system is related to the input $x(t)$ and output $y(t)$ by the differential equation given by:

$$x(t) = \frac{dy(t)}{dt} + 2y(t)$$

(a) Determine the frequency response $H(f) = Y(f)/X(f)$, and compute phase and magnitude of $H(f)$. **[10 pt.]**

(b) If $x(t) = e^{-t}u(t)$, determine $Y(f)$. **[5 pt.]**

(c) Find $y(t)$ for the given input in part (b). **[5 pt.]**

Q5 [15 points]: Determine the DTFT of the following signals:

(a) $x[n] = ne^{j(\pi/8)n}\alpha^{n-3}u[n-3]$

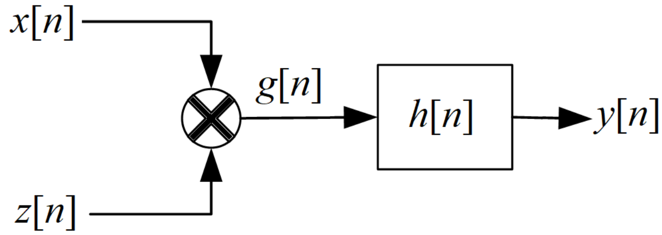
(b) $y[n] = (0.5)^n e^{j\pi n/2}\big(u[n] - u[n-5]\big)$

(c) $g[n] = (1.25)^n u[-n]$

Q6 [15 points]: Consider a discrete-time system depicted in Figure 3. This system has the following impulse response

$$h[n] = \frac{\sin\left(\frac{\pi}{2}n\right)}{\pi n}$$

Use the discrete-time Fourier transform (DTFT) to determine the output $y[n]$ for the following signals. Also sketch $G(\Omega)$, the DTFT of $g[n]$.



(a) $x[n] = \frac{\sin\left(\frac{\pi}{4}n\right)}{\pi n}$, $\quad z[n] = (-1)^n$ $\qquad$ **(b)** $x[n] = \frac{\sin\left(\frac{\pi}{2}n\right)}{\pi n}$, $\quad z[n] = \cos\left(\frac{\pi}{2}n\right)$