Midterm II Exam	Signals and Systems	Spring 2021	Points 120
Dr Shafayat Abrar	Electrical Engineering	Habib University, KHI	

INSTRUCTIONS:

1. YOU ARE NOT ALLOWED TO DISCUSS WITH EACH OTHER.
2. DO NOT SHARE YOUR ROUGH/FAIR WORK WITH OR COPY/CHEAT FROM OTHERS; YOUR WORK MUST BE YOUR "INDEPENDENT" WORK.
3. NO PENCIL WORK, USE PROPER BLACK MARKER OR POINTER.
4. PROVIDE A NEAT WORK WHILE AVOIDING EXCESSIVE NUMBER OF PAGES.
5. ENCLOSE YOUR FINAL RESULTS IN RECTANGULAR BOXES.
6. SINGLE COLUMN ONLY IN PORTRAIT FORMAT. LEAVE SOME SPACES AS LEFT/RIGHT MARGINS.
7. IF REQUIRED, ADJUST CONTRAST LEVEL IN SCANS TO REMOVE DARK BACK-GROUND.
8. IF REQUIRED, CROP YOUR IMAGES TO REMOVE UNNECESSARY SURROUND-INGS.

Question 1 [5]:

Use the following transformation pair

$$v(t) * w(t) \leftrightarrow V(f)W(f)$$

to find the expression of $y(t) = v(t) * w(t)$, where
$v(t) = \delta(t - t_1)$ and $w(t) = \delta(t - t_2)$.

Note: You do not have to perform convolution explicity; use Fourier transform of impulse function directly and the given Fourier transform property to evaluate the value of $y(t)$. You may also use any other property Fourier transform directly if needed.

Question 2 [20]:

In a linear time-invariant system, the output $y(t)$ and the input $x(t)$ are related as follows:

$$y(t) = \int z(t)dt$$
$$z(t) = x(t) - w(t)$$
$$w(t) = x(t - T)$$

where $z(t)$ and $w(t)$ are intermediate variables; and T denotes delay.

(a) Obtain the block diagram representation of the system. **[5]**
(b) Find $Y(f)/Z(f)$; note $Y(f) \leftrightarrow y(t)$ and $Z(f) \leftrightarrow z(t)$. **[5]**
(c) Find $W(f)/X(f)$; note $W(f) \leftrightarrow w(t)$ and $X(f) \leftrightarrow x(t)$. **[5]**
(d) Find $Y(f)/X(f)$. **[5]**

Question 3 [20]:

In a linear time-invariant system, the output $y(t)$ and the input $x(t)$ are related as follows:

$$y(t) = \int x(\lambda)h(t - \lambda)d\lambda$$

where $h(t) \leftrightarrow H(f)$. Let $H(f) = T\,\mathrm{sinc}(fT)$. Let $x(t) = A\Pi(t/\tau)$ be the input signal. Use frequency-domain analysis to find $y(t)$ when $\tau \ll T, \tau = T$, and $\tau \gg T$.

Question 4 [30]:

(a) Find the Fourier series coefficients $\tilde{c}_k$ of the periodic signal $\tilde{x}(t) = \sin(2\pi t)$, for $-\frac{1}{4} \leq t \leq \frac{1}{4}$. Note: the period is 0.5 seconds.
(b) Write a MATLAB code to synthesize the signal $\tilde{x}(t)$ using $\tilde{c}_k$. Provide neat and tidy figures properly labelled. Also provide your code.

Question 5 [25]:

Consider that the input is given by

$$x(t) = \cos(\omega_0 t) - \frac{1}{3}\cos(3\omega_0 t)$$

where $\omega_0 = 2\pi$. Consider that the input signal passes through a system whose transfer function is given as

$$H(f) = e^{-j\omega/3}$$

(a) Using Fourier transform properties, find the output response $y(t)$ of this system. **[5]**
(b) Compare the signal $x(t)$ and response $y(t)$ using MATLAB for $0 \leq t \leq 3$ sec. Provide a clear figure with proper line-type, line-width and legend; also provide your code in your answer sheet **[5]**
(c) Provide a few lines of commentary on your findings in part (b). **[2.5]**

Now consider a new scenario; the input signal $x(t)$ (as mentioned above) passes through another system where the output response is obtained as

$$y(t) = \cos\left(\omega_0 t - \frac{2\pi}{3}\right) - \frac{1}{3}\cos\left(3\omega_0 t - \frac{2\pi}{3}\right)$$

(d) Find out the transfer function of this system or discuss if it is not possible, otherwise **[5]**
(e) Compare the signal $x(t)$ and response $y(t)$ using MATLAB for $0 \leq t \leq 3$ seconds. Provide a clear figure with proper line-type, line-width and legend; also provide your code in your answer sheet **[5]**
(f) Provide a few lines of commentary on your findings in part (b). **[2.5]**

Question 6 [20]:

(a) Consider a nonlinear system where the input $x(t)$ and the output $y(t)$ are related as follows:

$$y(t) = x(t) + \frac{1}{2}x^2(t) + \frac{1}{3}x^3(t)$$

Let $Y(f) \leftrightarrow y(t)$ and $X(f) \leftrightarrow x(t)$. Express $Y(f)$ as a function of $X(f)$. You may use Fourier transform properties directly. **[5]**

(b) Let $x(t) = \cos(\omega_0 t)$. Find $y(t)$. **[5]**
(c) Find $Y(f)$ and plot its amplitude spectrum. **[10]**