Solution # 01:

$$\begin{aligned} y(t) &= [(x(t) * h(t))(g(t) * h(t))] * h(t) \\ &= [x_h(t) g_h(t)] * h(t) \\ &= m(t) * h(t) \end{aligned}$$

$$h(t) = \frac{\sin(11\pi t)}{\pi t} \quad \overset{FT}{\longleftrightarrow} \quad H(j\omega) = \begin{cases} 1 & |\omega| \leq 11\pi \\ 0 & \text{otherwise} \end{cases}$$

$$x(t) = \sum_{k=1}^{\infty} \frac{1}{k^2} \cos(k5\pi t) \quad \overset{FT}{\longleftrightarrow} \quad X(j\omega) = \pi \sum_{k=1}^{\infty} \frac{1}{k^2} [\delta(\omega - 5k\pi) + \delta(\omega + 5k\pi)]$$

$$g(t) = \sum_{k=1}^{10} \cos(k8\pi t) \quad = \quad \pi \sum_{k=1}^{10} [\delta(\omega - 8k\pi) + \delta(\omega + 8k\pi)]$$

$$\begin{aligned} X_h(j\omega) &= X(j\omega) H(j\omega) \\ &= \pi \sum_{k=1}^{2} \frac{1}{k^2} [\delta(\omega - 5k\pi) + \delta(\omega + 5k\pi)] \end{aligned}$$

$$\begin{aligned} G_h(j\omega) &= G(j\omega) H(j\omega) \\ &= \pi\delta(\omega - 8\pi) + \pi\delta(\omega - 8\pi) \\ M(j\omega) &= \frac{1}{2\pi} X_h(j\omega) * G_h(j\omega) \\ &= \frac{1}{2} [X_h(j(\omega - 8\pi)) + X_h(j(\omega + 8\pi))] \\ &= \pi \sum_{k=1}^{2} \frac{1}{k^2} [(\delta(\omega - 8\pi - 5k\pi) + \delta(\omega - 8\pi + 5k\pi)) + (\delta(\omega + 8\pi - 5k\pi) + \delta(\omega + 8\pi + 5k\pi))] \\ Y(j\omega) &= M(j\omega) H(j\omega) \\ &= \frac{\pi}{2} [\delta(\omega + 3\pi) + \delta(\omega - 3\pi)] + \frac{\pi}{8} [\delta(\omega - 2\pi) + \delta(\omega + 2\pi)] \\ y(t) &= \frac{1}{2} \cos(3\pi t) + \frac{1}{8} \cos(2\pi t) \end{aligned}$$

Solution # 02:

Since the continuous-time signal $x_a(t)$ is being sampled at 2 kHz rate, the sampled version of its i-th sinusoidal component with a frequency F_i will generate discrete-time sinusoidal signals with frequencies $F_i \pm 2000n$, $-\infty < n < \infty$. Hence, the frequencies F_{im} generated in the sampled version associated with the sinusoidal components present in are as follows:

$$F_1 = 300 \text{ Hz} \Rightarrow F_{1m} = 300,\ 1700,\ 2300, \ldots \text{ Hz}$$
$$F_2 = 500 \text{ Hz} \Rightarrow F_{2m} = 500,\ 1500,\ 2500, \ldots \text{ Hz}$$
$$F_3 = 1200 \text{ Hz} \Rightarrow F_{3m} = 1200,\ 800,\ 3200, \ldots \text{ Hz}$$
$$F_4 = 2150 \text{ Hz} \Rightarrow F_{4m} = 2150,\ 150,\ 4150, \ldots \text{ Hz}$$
$$F_5 = 3500 \text{ Hz} \Rightarrow F_{5m} = 3500,\ 1500,\ 5500, 500,\ 7500, \ldots \text{ Hz}$$

After filtering by a lowpass filter with a cutoff at 900 Hz, the frequencies of the sinusoidal components in $y_a(t)$ are 150, 300, 500, 800 Hz.

Solution # 03:

(a) Now, the CTFT of $y_1(t)$ is given by $Y_1(j\Omega) = \frac{1}{2\pi} G_a(j\Omega) \circledast G_a(j\Omega)$ where $G_a(j\Omega)$ denotes the CTFT of $g_a(t)$ and $\circledast$ denotes the frequency-domain convolution. The highest frequency present in $y_1(t)$ is therefore twice that of $g_a(t)$ and hence, the Nyquist frequency of $y_1(t)$ is $2\Omega_m$.

(b) The CTFT of $y_2(t)$ is given by $Y_2(j\Omega) = \int_{-\infty}^{\infty} g_a(\frac{t}{3}) e^{-j\Omega t} dt$

$= 3 \int_{-\infty}^{\infty} g_a(\tau) e^{-j3\Omega\tau} d\tau = 3G_a(j3\Omega)$. The highest frequency present in $y_2(t)$ is therefore one-third of that of $g_a(t)$ and hence, the Nyquist frequency of $y_2(t)$ is $\Omega_m / 3$.

(c) The CTFT of $y_3(t)$ is given by $Y_3(j\Omega) = \int_{-\infty}^{\infty} g_a(3t) e^{-j\Omega t} dt$

$= \frac{1}{3} \int_{-\infty}^{\infty} g_a(\tau) e^{-j\Omega\tau/3} d\tau = \frac{1}{3} G_a(j\frac{\Omega}{3})$. The highest frequency present in $y_3(t)$ is therefore three times of that of $g_a(t)$ and hence, the Nyquist frequency of $y_3(t)$ is $3\Omega_m$.

(d) The CTFT of $y_4(t)$ is given by

$$Y_4(j\Omega) = \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} g_a(t-\tau) g_a(\tau) d\tau \right] e^{-j\Omega t} dt = \int_{-\infty}^{\infty} g_a(\tau) \left[\int_{-\infty}^{\infty} g_a(t-\tau) e^{-j\Omega t} dt \right] d\tau$$

$= \int_{-\infty}^{\infty} g_a(\tau) e^{-j\Omega\tau} G_a(j\Omega) d\tau = G_a(j\Omega) \int_{-\infty}^{\infty} g_a(\tau) e^{-j\Omega\tau} d\tau = G_a(j\Omega) G_a(j\Omega)$. The highest frequency present in $y_4(t)$ is therefore the same as that of $g_a(t)$ and hence the Nyquist frequency of $y_4(t)$ is Ω_m.

(e) Now $g_a(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G_a(j\Omega) e^{j\Omega t} d\Omega$. Differentiating both sides of this equation we get $\frac{dg_a(t)}{dt} = \frac{1}{2\pi} \int_{-\infty}^{\infty} j\Omega G_a(j\Omega) e^{j\Omega t} d\Omega$. Hence, it follows that the CTFT of $y_5(t) = \frac{dg_a(t)}{dt}$ is simply $j\Omega G_a(j\Omega)$. The highest frequency present in $y_5(t)$ is therefore the same as that of $g_a(t)$ and hence, the Nyquist frequency of $y_5(t)$ is Ω_m.

Solution # 04:

(a) We are given the differential equation

$$\frac{dy(t)}{dt} + 2y(t) = x(t)$$

Taking the Fourier transform of eq. (S9.7-1), we have

$$j\omega Y(\omega) + 2Y(\omega) = X(\omega)$$

Hence,

$$Y(\omega)[2 + j\omega] = X(\omega)$$

and

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{1}{2 + j\omega},$$

$$H(\omega) = \frac{1}{2 + j\omega} = \frac{1}{2 + j\omega}\left(\frac{2 - j\omega}{2 - j\omega}\right) = \frac{2 - j\omega}{4 + \omega^2}$$

$$= \frac{2}{4 + \omega^2} - j\frac{\omega}{4 + \omega^2},$$

$$|H(\omega)|^2 = \frac{4}{(4 + \omega^2)^2} + \frac{\omega^2}{(4 + \omega^2)^2} = \frac{4 + \omega^2}{(4 + \omega^2)^2},$$

$$|H(\omega)| = \frac{1}{\sqrt{4 + \omega^2}}$$

(b) We are given $x(t) = e^{-t}u(t)$. Taking the Fourier transform, we obtain

$$X(\omega) = \frac{1}{1 + j\omega}, \qquad H(\omega) = \frac{1}{2 + j\omega}$$

Hence,

$$Y(\omega) = \frac{1}{(1 + j\omega)(2 + j\omega)} = \frac{1}{1 + j\omega} - \frac{1}{2 + j\omega}$$

(c) Taking the inverse transform of $Y(\omega)$, we get

$$y(t) = e^{-t}u(t) - e^{-2t}u(t)$$

Solution # 05 (a):

$$X(e^{jw}) = j\frac{d}{dw}\left\{\frac{e^{-j3(w-\pi/8)}}{1-\alpha e^{-j(w-\pi/8)}}\right\}$$

Solution # 05 (b):

$$Y(e^{jw}) = \frac{1+j\frac{1}{32}e^{-j5w}}{1-0.5e^{-jw}}$$

Solution # 05 (c):

$$G(e^{jw}) = \frac{1}{1-0.8e^{jw}}$$

Solution # 06 (a):

$$
\begin{aligned}
y[n] &= (x[n]w[n]) * h[n] \\
&= g[n] * h[n]
\end{aligned}
$$

$$
h[n] = \frac{\sin(\frac{\pi}{2}n)}{\pi n} \quad \overset{FT}{\longleftrightarrow} \quad H(e^{j\Omega}) = \begin{cases} 1 & |\Omega| \le \frac{\pi}{2} \\ 0 & \frac{\pi}{2} \le |\Omega| < \pi \end{cases}
$$

$H(e^{j\Omega})$ is 2π periodic.

(a) $x[n] = \frac{\sin(\frac{\pi}{4}n)}{\pi n}, \quad w[n] = (-1)^n$

$$
x[n] = \frac{\sin(\frac{\pi}{4}n)}{\pi n} \quad \overset{DTFT}{\longleftrightarrow} \quad X(e^{j\Omega}) = \begin{cases} 1 & |\Omega| \le \frac{\pi}{4} \\ 0 & \frac{\pi}{4} \le |\Omega| < \pi \end{cases}
$$

$$
w[n] = e^{j\pi n} \quad \overset{DTFT}{\longleftrightarrow} \quad W(e^{j\Omega}) = 2\pi\delta(\Omega - \pi)
$$

$$
\begin{aligned}
G(e^{j\Omega}) &= \frac{1}{2\pi} X(e^{j\Omega}) * W(e^{j\Omega}) \\
&= \begin{cases} 1 & |\Omega - \pi| \le \frac{\pi}{4} \\ 0 & \frac{\pi}{4} \le |\Omega - \pi| < \pi \end{cases} \\
g[n] &= e^{j\pi n} \frac{\sin(\frac{\pi}{4}n)}{\pi n} \\
Y(e^{j\Omega}) &= G(e^{j\Omega}) H(e^{j\Omega}) \\
&= 0 \\
y[n] &= 0
\end{aligned}
$$

Solution # 06 (b):

$$x[n] = \frac{\sin(\frac{\pi}{2}n)}{\pi n}, \quad w[n] = \cos(\frac{\pi}{2}n)$$

$$\begin{aligned}
W(e^{j\Omega}) &= \pi\left[\delta(\Omega - \frac{\pi}{2}) + \delta(\Omega + \frac{\pi}{2})\right], \; 2\pi \text{ periodic} \\
G(e^{j\Omega}) &= \frac{1}{2\pi}X(e^{j\Omega}) * W(e^{j\Omega}) \\
&= \begin{cases} \frac{1}{2} & |\Omega - \frac{\pi}{2}| \leq \frac{\pi}{2} \\ 0 & \frac{\pi}{2} \leq |\Omega - \frac{\pi}{2}| < \pi \end{cases} + \begin{cases} \frac{1}{2} & |\Omega + \frac{\pi}{2}| \leq \frac{\pi}{2} \\ 0 & \frac{\pi}{2} \leq |\Omega + \frac{\pi}{2}| < \pi \end{cases} \\
g[n] &= \frac{1}{2}\frac{\sin(\frac{\pi}{2}n)}{\pi n}\left(e^{j\frac{\pi}{2}n} + e^{-j\frac{\pi}{2}n}\right) \\
&= \frac{\sin(\frac{\pi}{2}n)}{\pi n}\cos(\frac{\pi}{2}n) \\
&= \frac{\sin(\pi n)}{2\pi n} \\
&= \frac{1}{2}\delta(n) \\
y[n] &= g[n] * h[n] \\
&= \frac{1}{2}h[n] \\
&= \frac{\sin(\pi n)}{2\pi n}
\end{aligned}$$