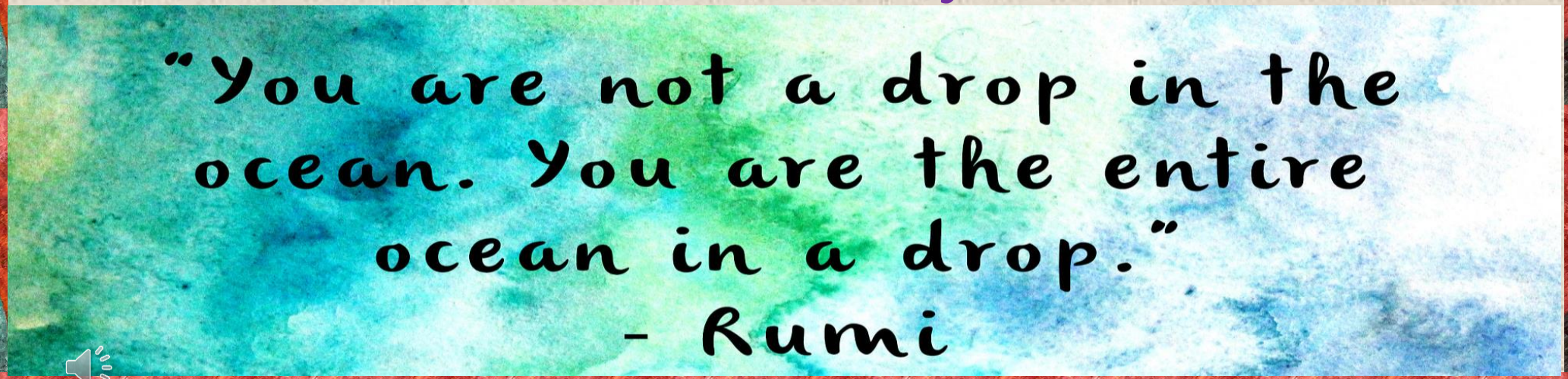
EE 354/CE 361/MATH 310 - Introduction to Probability and Statistics

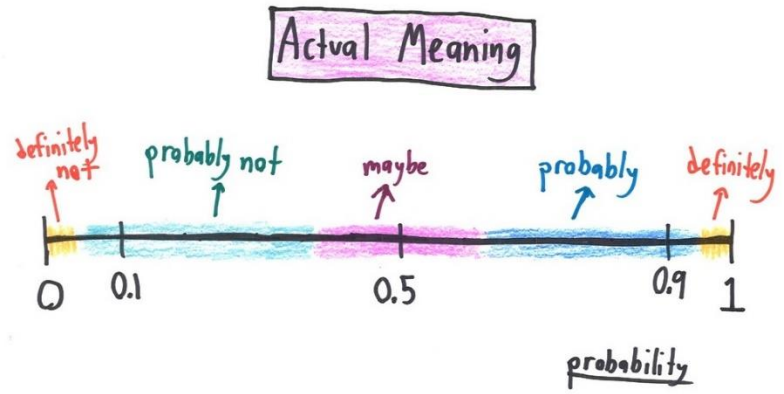


EE 354/CE 361/MATH 310 – Introduction to Probability and Statistics

Fall 2023

Aamir Hasan

Lecture No 01 – 21st August 2023



Announcements!

- Welcome to Fall 2023 semester & to EE 354/CE 361/MATH 310 class
- Fun class!

Agenda for today

- Get to know the ‘Elephant in the room’
- Introduction – personnel (You & Me) & **the course**
- Course Aims & objectives
- Course - Structure, Expectations & share previous experience

Course Character & Objectives!

Introductions

- Instructor
  - Dr Aamir Hasan, Vice President Academic Affairs & Dean of Faculty, Acting Associate Dean Undergraduate Education & Accreditation & Associate Professor in Electrical & Computer Engineering
- Students?
- What is Probability?
  - MATH 310/EE 354 – Introduction to Probability & Statistics
    - Probability? & Random Variables?
- Why study Probability?

Introductions

Why 'Study of Probability' is important now?

Science & Engineering together have developed complex systems that inherently contain uncertainties

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Role of information using data analysis has an important role in making decisions

Course Character & Objectives

- **Parallel Objectives**
  - Probabilistic concepts, and languages: menu of common models
  - Mathematical tools
  - Intuition
  - Acquire working knowledge of the subject

Not a mathematics class but to develop intuition through mathematical arguments & constructs

How do we do it?

- To acquire working knowledge, we:
  - Cover relevant material &
  - Capitalize on effective organization of the material

Course Character & Objectives

CE 361/
MATH 310/
EE 354

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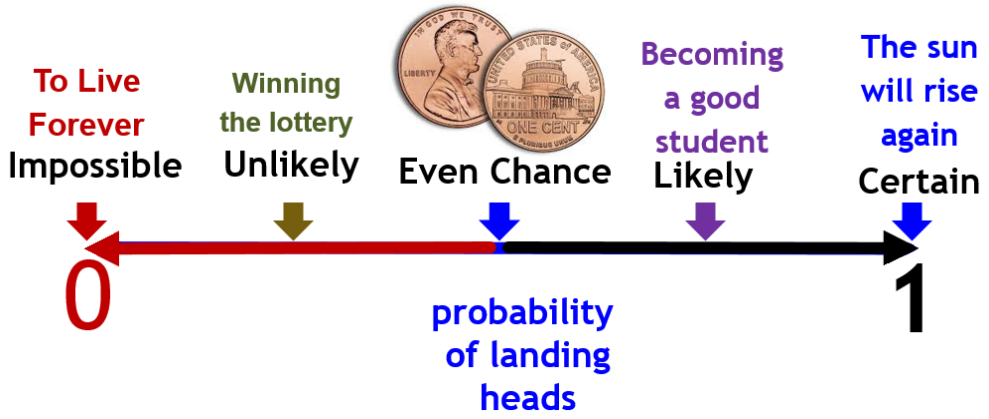
- **An introduction/overview course**
- **Rich in specific applications**
- **Rich in computational exercise**
- **Rich in demos**

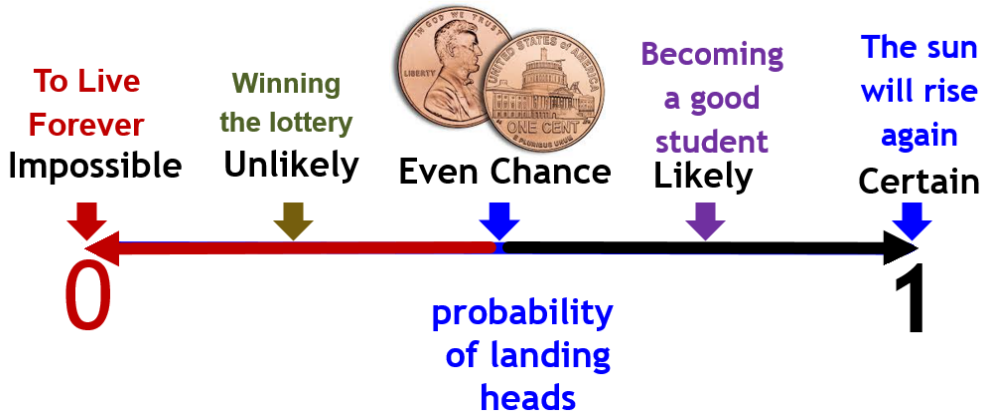
Probability

- In real life, engineers make decisions on the basis of incomplete or imperfect information
- The theory of probability helps provide a formal mechanism for understanding, quantifying, and dealing with uncertainty, which is ever present in our lives, pure science, or engineering applications
- Simply, by uncertainty we mean the condition when outcomes or future are not completely determined or can be captured by a deterministic function; they depend on a number of factors and perhaps just on pure chance
- A lot of our present day technologies will not be possible without an understanding of how to make decisions under uncertainties



Where is Probability useful?

- Everywhere
  - Physics (quantum ….)
  - Biology (evolution …..)
  - Communications & Signal processing (Noise ….)
  - Management (demand model …)
  - Finance
  - Transportation
  - Social networks (trends….)



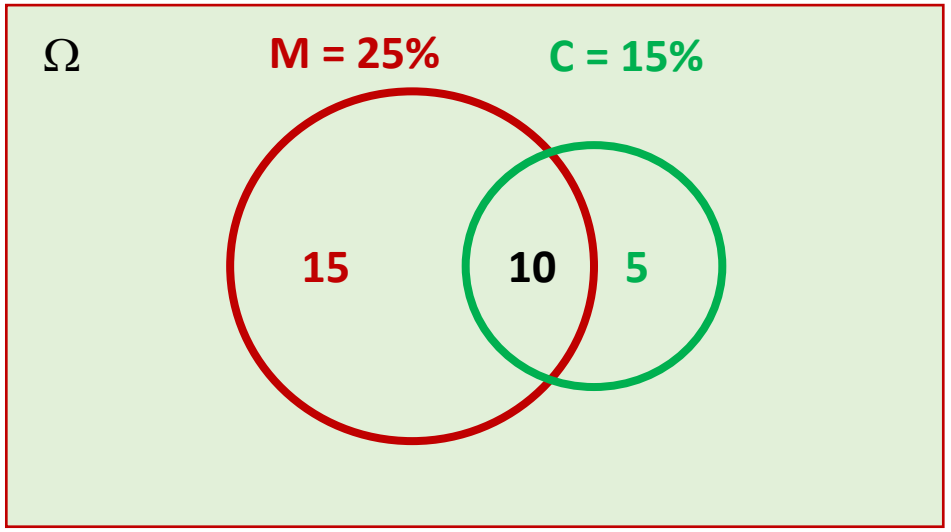
Example - Probability (I)

Question : How many (unbiased) dice must be thrown so that there is a better than 0.5 chance (probability) of obtaining at least one six?

- Question No I: What is the probability of getting a Head in a throw of an un-biased coin?
- Question No II: What is the probability of getting n-heads in n throws of a un-biased coin?
- Question No III: Assume now a biased coin with Probability of Head = 1/6 & probability of tail = 5/6. What's the probability of getting at least one head (≥ 1) in n-throws?
- Question No IV: How many times should this biased coin be thrown that there is a better than 0.5 chance (probability) of obtaining at least one head?

Example - Probability (II)

- **Question No 2**: In a certain college, 25% of the students failed mathematics, 15% of the students failed chemistry, and 10% of the students failed both mathematics and chemistry. A student is selected at random. **If he failed chemistry, what is the probability that he failed mathematics too?**



Example - Probability (III)

- **Question No 3**: The amount of rain in Karachi in the month of August is X cm.
- Question No 3b: What is X? Discuss
- Question No 3c: Predict any formula for X
- Question No 3d: Discuss the variability in X from 2000 to 2017
- Question No 3e: Draw or predict a histogram of X?

Course Character & Objectives - Summary

MATH 310/EE 354 Course

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- **An introduction/overview course**
- **Rich in specific applications**
- **Rich in computational exercise**
- **Rich in demos**

- Not a mathematics class but to develop intuition through mathematical arguments & constructs
- We need to able to understand, model, analyze & predict uncertainties
- We need to able to have a grip on the mathematical models, tools and procedures for determining the inherent randomness in complex systems

Aim of the Course

- Course will cover foundations of probability and statistics, with examples from electrical engineering, computer science, and everyday life.
- Course contents can broadly be divided:-
  1. **fundamentals of probability**
  2. **common probability models, and**
  3. **statistics**

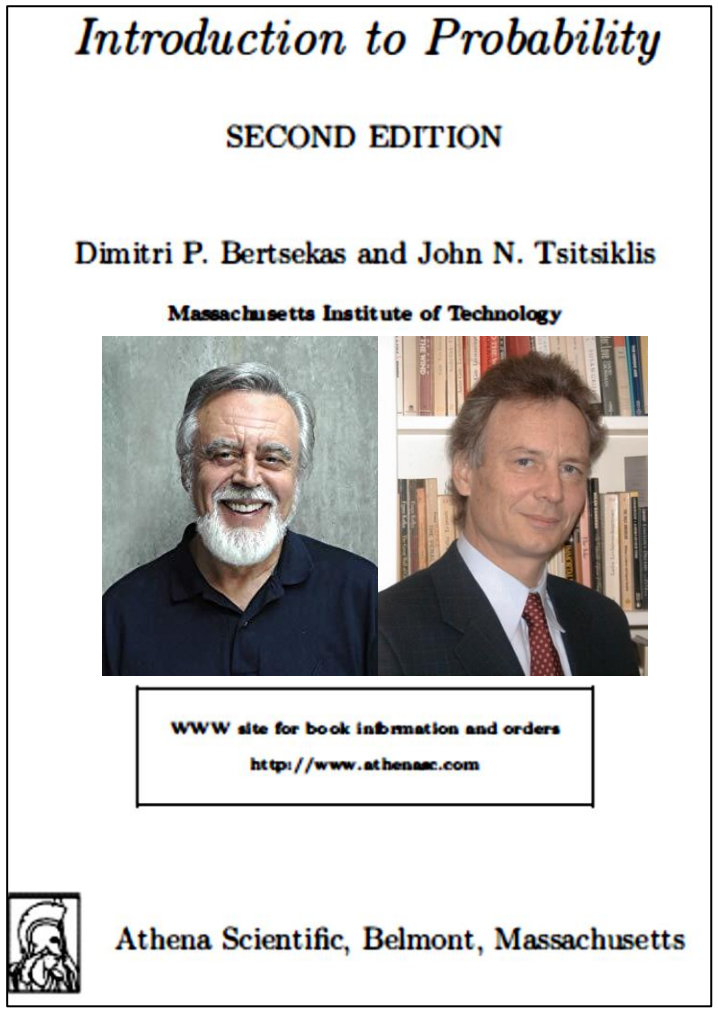
- Objectives:-
  - Gaining an appreciation for the limitation of human intuition when faced with finding optimal solution for a problem in presence of uncertainty
  - Recognizing presence of uncertainty in a scenario and modeling it using formal notions of probability
  - Identifying the right questions to ask when drawing inferences from a probability model and interpreting results correctly

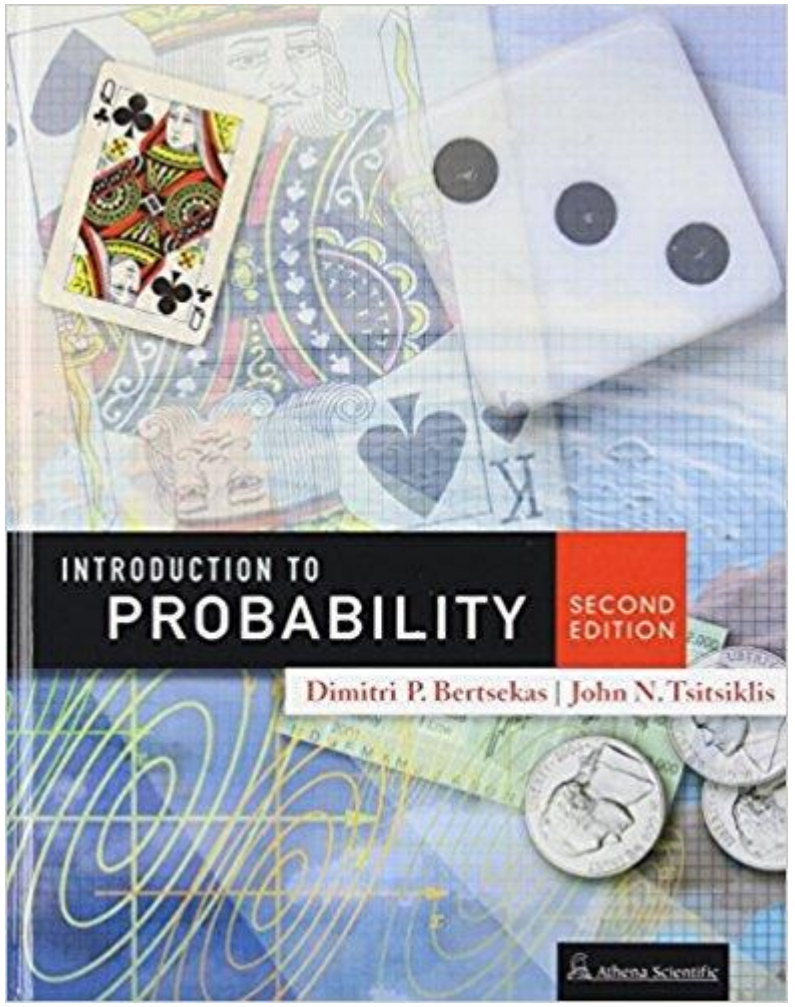
Aim of the Course

- Summary
  - To develop the art of describing uncertainty in terms of probabilistic models, as well as the skill of probabilistic reasoning
  - Therefore, the first we will describe the generic structure of such models and their basic properties. The models we consider assign probabilities to collections (sets) of possible outcomes
  - For this reason, we will do a short review of set theory

Required Course Reading

- Required text - Bertsekas, Dimitri P., and John N. Tsitsiklis. Introduction to probability. Vol. 1. Belmont, MA: Athena Scientific, 2002.





Structure of Required Course Reading

Chapter **Title**

1. **Sample Space and Probability**
   - Sets, Probabilistic Models, Conditional Probability, Total Probability, Theorem and Bayes' Rule, Independence, Counting

2. **Discrete Random Variables**
   - Basic Concepts, Probability Mass Functions, Functions of Random Variables, Expectation, Mean, and Variance, Joint PMFs of Multiple Random Variables, Conditioning, Independence

3. **General Random Variables**
   - Continuous Random Variables and PDFs, Cumulative Distribution Functions, Normal Random Variables, Joint PDFs of Multiple Random Variables, Conditioning, The Continuous Bayes' Rule

4. **Further Topics on Random Variables**
   - Derived Distributions, Covariance and Correlation, Conditional Expectation and Variance Revisited, Transforms, Sum of a Random Number of Independent Random Variables

Structure of Required Course Reading

Chapter **Title**

5. **Limit Theorems**
   - Markov and Chebyshev Inequalities, The Weak Law of Large Numbers, Convergence in Probability, The Central Limit Theorem, The Strong Law of Large Numbers
6. **The Bernoulli and Poisson Processes**
   - The Bernoulli Process, The Poisson Process
7. **Markov Chains**
   - Discrete-Time Markov Chains, Classification of States, Steady-State Behavior, Absorption Probabilities and Expected Time to Absorption, Continuous-Time Markov Chains
8. **Bayesian Statistical Inference**
   - Bayesian Inference and the Posterior Distribution, Point Estimation, Hypothesis Testing, and the MAP Rule, Bayesian Least Mean Squares Estimation
9. **Classical Statistical Inference**
   - Classical Parameter Estimation, Linear Regression, Binary Hypothesis Testing, Significance Testing

Course Plan

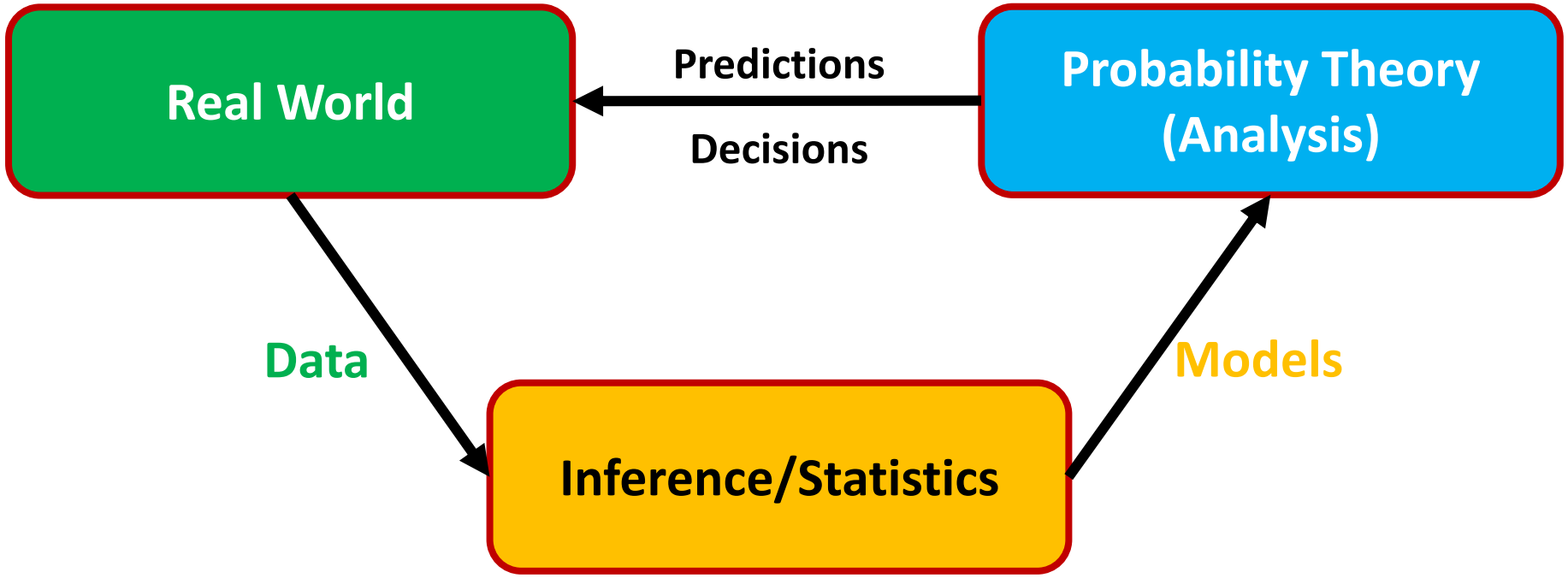
MATH 310/EE 354/CE 361 – Probability & Statistics

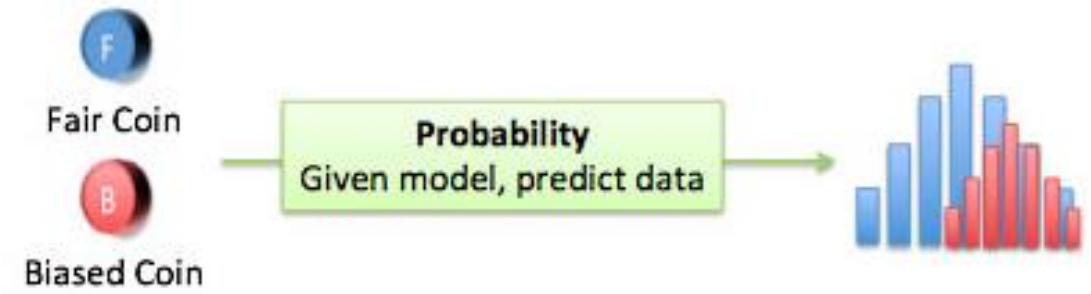
Tentative course schedule – Ambitious
Do Not Worry!

	Assessment type	Total	Weight (%)	Remarks
1	Short 15-20 mins quiz	8	40	These will be held every alternate week – these will mostly be based on the lectures and practice questions from the lectures. During the midterm and end term week the quizzes may not happen.
2	Assignments	4	16	Two assignment on module No 1 (chapter No 1), one assignment from module 2 (chapter No 2) & one assignment from module 3 (Chapter No 3)
3	Mid-term Exam	1	20	An in-person exam will be conducted that would be comprehensive with the material taught till end of week No 7/8
4	End-term Exam	1	24	An in-person exam will be conducted that would be comprehensive

GRADING SCALE		
LETTER GRADE	GPA POINTS	PERCENTAGE
A+	4.00	[95-100]
A	4.00	[90-95)
A-	3.67	[85-90)
B+	3.33	[80-85)
B	3.00	[75-80)
B-	2.67	[70-75)
C+	2.33	[67-70)
C	2.00	[63-67)
C-	1.67	[60-63)
F	0.00	[0-60)

Summary


Real World
Predictions
Decisions
Probability Theory (Analysis)
Data
Inference/Statistics
Models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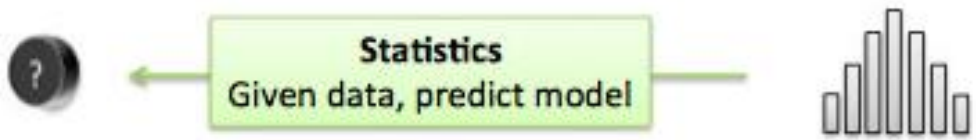
Fair Coin
B
Biased Coin
Probability
Given model, predict data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Statistics
Given data, predict model

Norm setting – for course

Inputs?