EE 354/CE 361/MATH 310 - Introduction to Probability and Statistics

EE 354/CE 361/MATH 310 L1 section (Fall 2023) –

Introduction to Probability and Statistics

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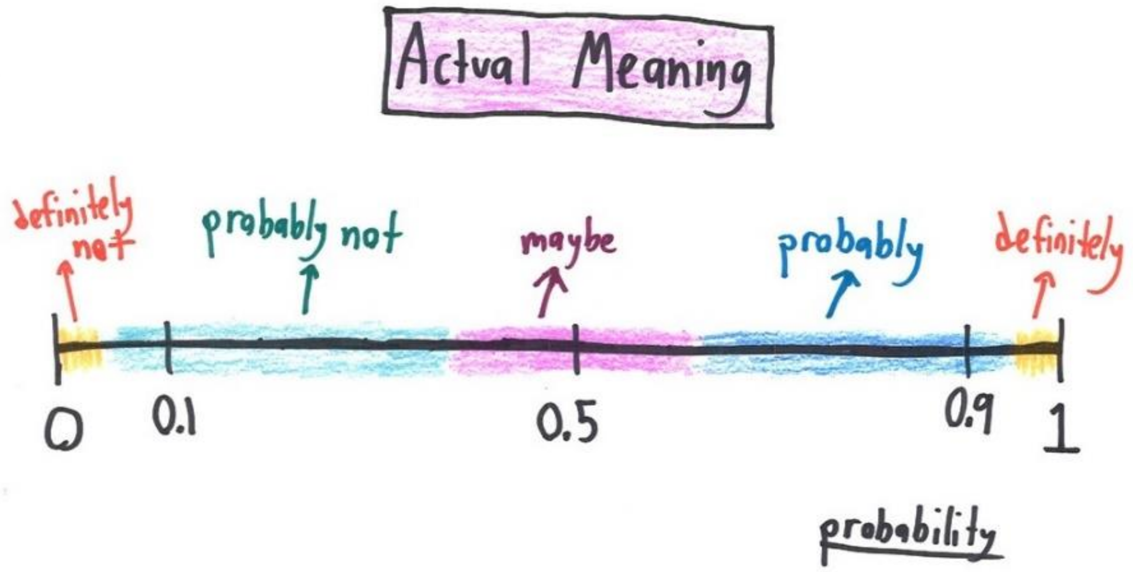
Lecture No 04 – 28th August 2023



Announcements!

- Slide deck & & attendance up to lecture No 03 posted
- **Quiz No 01 – Wednesday August 30, 2023 – Good Luck**
- **Makeup sessions**
  1. Wednesday August 30, 2023 - 5:40 – 6:30 PM at GF-C109 Arif Habib classroom (no class on Sep 6, 2023, Wednesday)
  2. Monday September 04, 2023 - 5:40 – 6:30 PM at GF-C109 Arif Habib classroom (no class on Sep 8, 2023, Friday)

Agenda for today

- *Unit 1: Probability models and axioms Lecture outline (continue)*

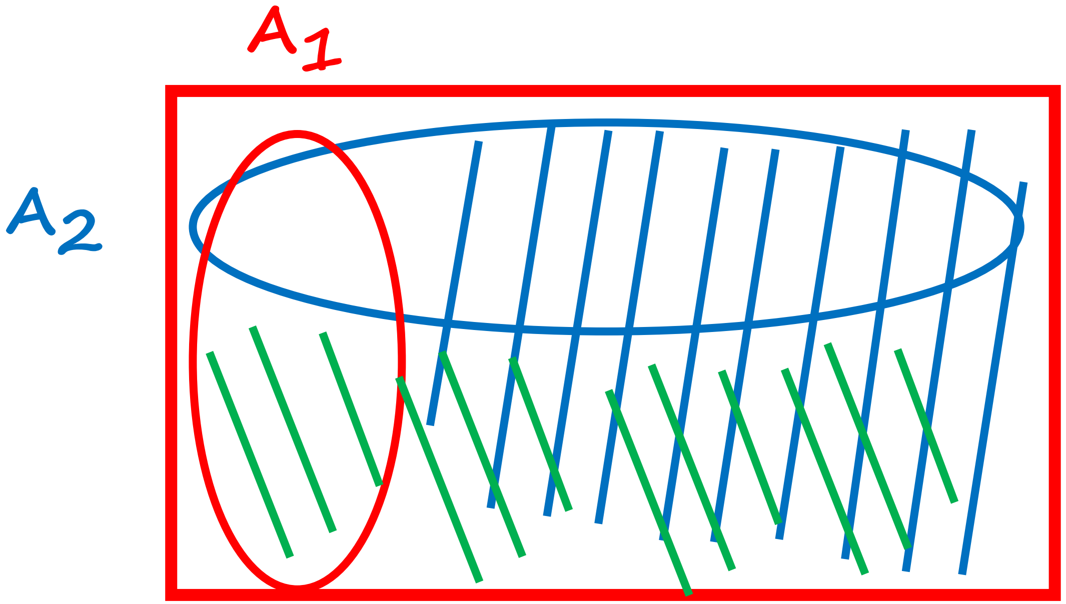
Unit 1: Probability models and axioms Lecture outline

- Sample space
- Probability laws
  - Axioms
  - Some properties
- Examples
  - Discrete
  - Continuous
- Interpretations of probabilities

Interpreting the union bound and the Bonferroni inequality

$$P\,(A_1 \cup A_2) \le P(A_1) + P(A_2)$$

- Suppose that:
  - Very few students are smart A_1
  - Very few students are beautiful A_2
- Then: very few students are smart or beautiful



- Suppose that:
  - Most of the students are smart
  - Most students are beautiful
- Then: most students are smart and beautiful

$$P\,(A_1 \cap A_2) \ge \underline{P(A_1)} + \underline{P(A_2)} - 1$$

The Bonferroni inequality

$$\boldsymbol{P\,(A_1 \cap A_2) \geq P(A_1) + P(A_2) - 1}$$

$$\boldsymbol{P\,(A_1 \cup A_2) \leq P(A_1) + P(A_2)}$$

$$P\,(A_1 \cap A_2)^c = P\left(A_1^c \cup A_2^c\right) \leq P\left(A_1^c\right) + P\left(A_2^c\right)$$

$$\cancel{1} - P\,(A_1 \cap A_2) \qquad\qquad \leq \cancel{1} - P(A_1) + 1 - P(A_2)$$

$$\boldsymbol{\underline{P\,(A_1 \cap \ldots \cap A_n)} \geq \underline{P(A_1) + \cdots + P(A_n)} - \underline{\underline{(n-1)}}}$$

$$P\,(A_1 \cap \ldots \cap A_n)^c = P\left(A_1^c \cup \ldots \cup A_n^c\right) \leq P\left(A_1^c\right) + \ldots + P(A_n^c)$$

$$1 - P\,(A_1 \cap \ldots \cap A_n) \qquad \leq \qquad (1 - P(A_1)) + \ldots + (1 - P(A_n))$$

Interpretations of probability theory

- A narrow view: a branch of math
  - Axioms → theorems

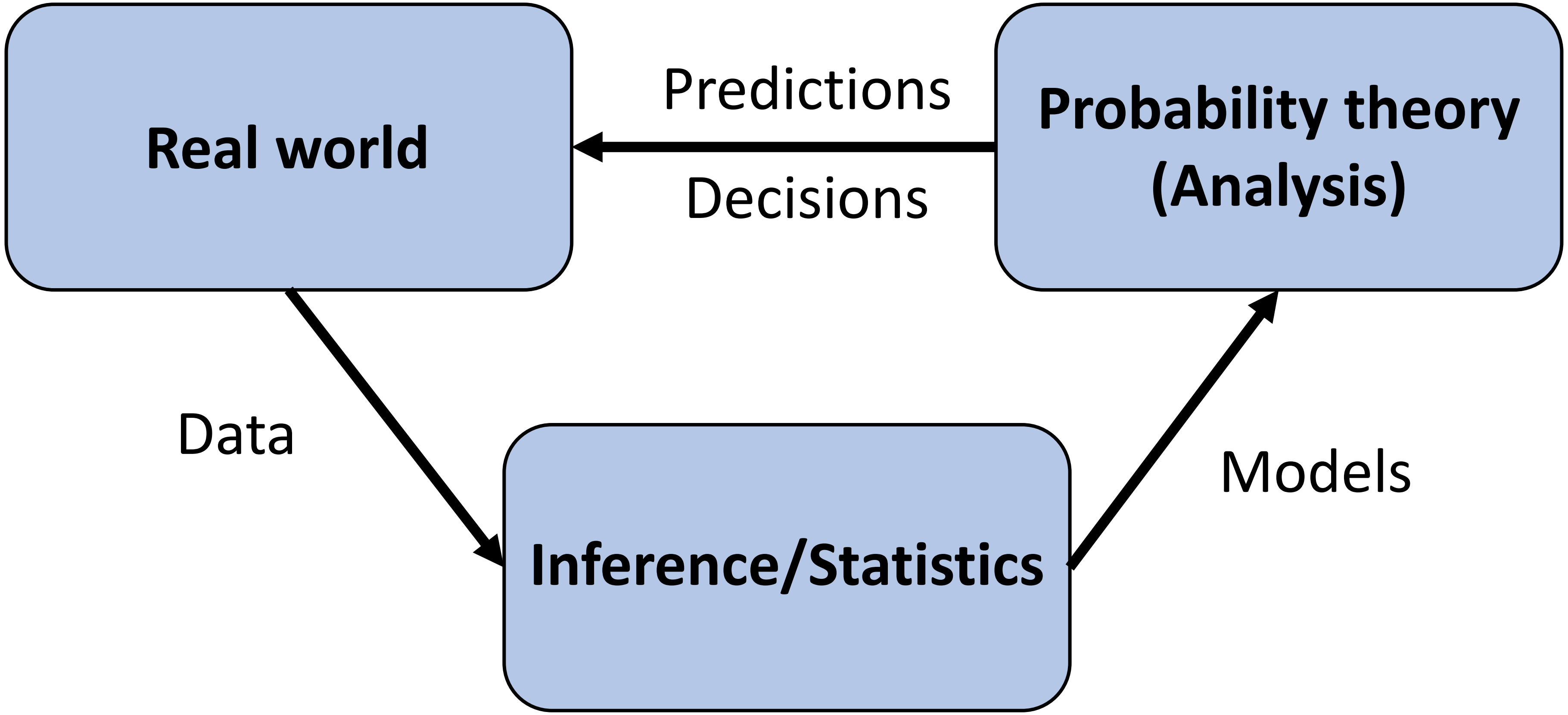
"Thm:" "Frequency" of event A "is" P (A)

- Are the probabilities frequencies?
  - P (coin toss yields heads) = ½
  - P (the prime Minister, Imran Khan will be reelected) = 0.7

- Probabilities are often interpreted as:
  - Description of beliefs
  - Betting preferences

Interpretations of probability theory

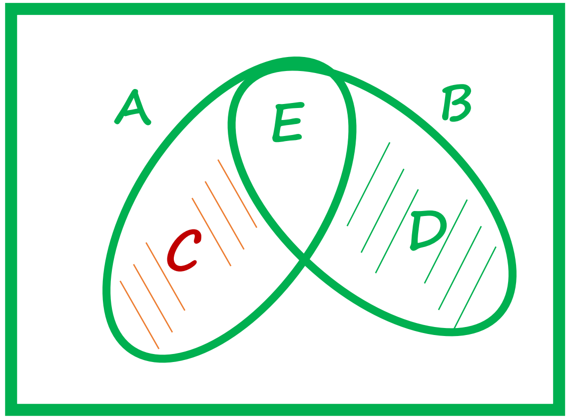
- A framework for analyzing phenomena with uncertain outcomes
  - Rules for consistent reasoning
  - Used predictions and decisions



4 Practise Problems

Practice Problem 1

Show $P(C \cup D) = P(A) + P(B) - 2P(E)$



$$P(C \cup D) = P(C) + P(D)$$

$$= P(C) + P(E) + P(D) + P(E) - 2P(E)$$

$$= P(A) + P(B) - 2P(E)$$

Practice Problem 2

Tim has a peculiar four-sided dice. When he rolls the dice, the probability of getting any particular outcome is proportional to result of the dice.

(a) What is the probability of the number being even?

Sample Space

Probability Assignment

Practice Problem 2

Tim has a peculiar four-sided dice. When he rolls the dice, the probability of getting any particular outcome is proportional to result of the dice.

(a) What is the probability of the number being even?

Sample Space | *Probability Assignment*

No	Probability
1	p
2	$2p$
3	$3p$
4	$4p$

$10p = 1$

$p = 1/10$

Sample Space | Probability Assignment

No	Probability
1	*1/10*
2	*2/10*
3	*3/10*
4	*4/10*

P(even No) = 6/10

Practice Problem 3 – *do it in pairs*

Hasan has a peculiar pair of four-sided dice. When he rolls the dice, the probability of getting any particular outcome is proportional to the sum of the results of the two individual dice.

(a) What is the probability of the sum being even?

(b) What is the probability of Hasan rolling a 2 and a 3, in any order?

Practice Problem 3 - Solution

Hasan has a peculiar pair of four-sided dice. When he rolls the dice, the probability of getting any particular outcome is proportional to the sum of the results of the two individual dice.

(a) What is the probability of the sum being even?

(b) What is the probability of Hasan rolling a 2 and a 3, in any order?

Die 1	Die 2	Sum	$\mathbf{P}$(Sum)
1	1	2	2p
1	2	3	3p
1	3	4	4p
1	4	5	5p
2	1	3	3p
2	2	4	4p
2	3	5	5p
2	4	6	6p
3	1	4	4p
3	2	5	5p
3	3	6	6p
3	4	7	7p
4	1	5	5p
4	2	6	6p
4	3	7	7p
4	4	8	8p
		Total	80p

$$\mathbf{P}(\text{All outcomes}) = 80p \text{ (Total from the table)}$$

and therefore

$$p = \frac{1}{80}$$

(a)

$$\mathbf{P}(\text{Even sum}) = 2p + 4p + 4p + 6p + 4p + 6p + 6p + 8p = 40p = \boxed{1/2}$$

(b)

$$\mathbf{P}(\text{Rolling a 2 and a 3}) = \mathbf{P}(2,3) + \mathbf{P}(3,2) = 5p + 5p = 10p = \boxed{1/8}$$

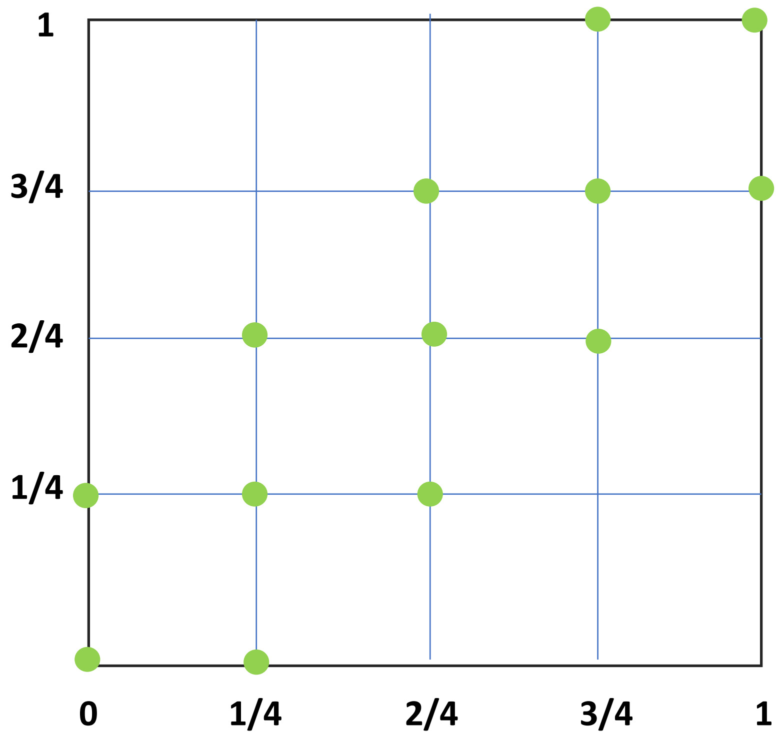
Practice Problem 4

"S" and "A" have a meeting at a given time and each will arrive at the meeting place with a delay between 0 and 1 hour – **all pairs of delay being equally likely**. The first to arrive will wait for 15 minutes and will leave if the other has not yet arrived. **What is the probability that they will meet?**

Two Cases of equally likely times –

A) Arrival times are Discrete for both 'S' & 'A' - in steps of 15 mins (t = 0, 1/4 , 2/4, 3/4 , 1)

B) Arrival times are Continuous



P(Meeting) = 13/25

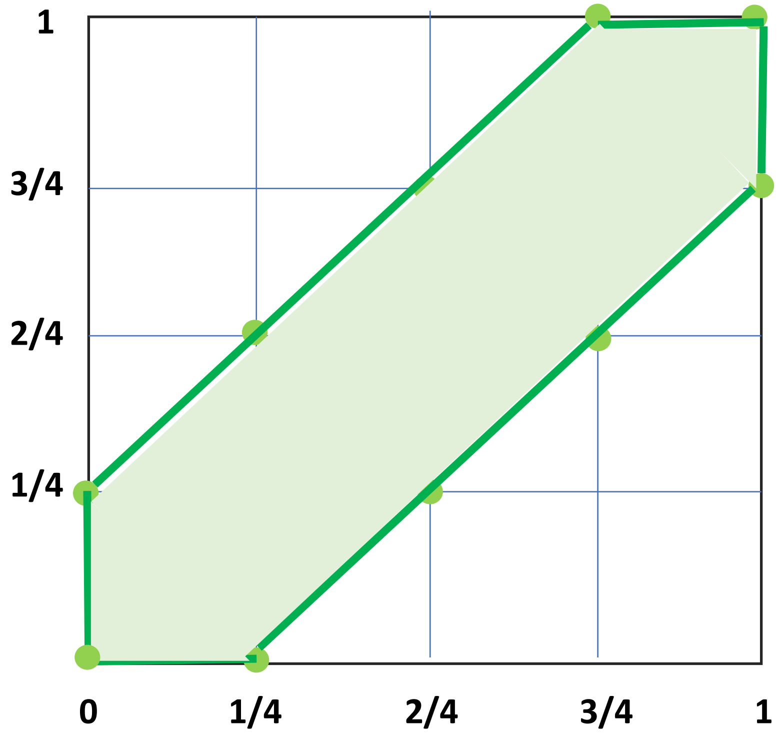
Practice Problem 4

"S" and "A" have a meeting at a given time and each will arrive at the meeting place with a delay between 0 and 1 hour – **all pairs of delay being equally likely**. The first to arrive will wait for 15 minutes and will leave if the other has not yet arrived. **What is the probability that they will meet?**

Two Cases of equally likely times –

A) Arrival times are Discrete for both Romeo & Juliet - in steps of 15 mins (t = 0, 1/4 , 2/4, 3/4 , 1)

B) Arrival times are Continuous



Conditioning & Bayes' rule

- Let's say that you roll two dice and their sum comes out to be S.

    ❖ If S = 9, what can we say about the first and second roll?

    ✓ What is the probability that first roll is 1 or 2?

New knowledge should be used to modify our original beliefs.

↓

Revised probabilities are called the conditional probabilities.