Practice Problem Set 1

(Sample Space, Sets and Outcomes, Simple Probability Model)

Questions

Probability

1. A retail establishment accepts either the American Express or the VISA credit card. A total of 29 percent of its customers carry an American Express card, 72 percent carry a VISA card, and 14 percent carry both cards. What percentage of its customers carry a credit card that the establishment will **not** accept?
2. A total of 28 percent of American males smoke cigarettes, 7 percent smoke cigars, and 5 percent smoke both cigars and cigarettes.
    a. What percentage of males smokes neither cigars nor cigarettes?
    b. What percentage smokes cigars but not cigarettes?
3. A fair six-sided dice is thrown twice and their result recorded. What is the probability of:
    a) Getting a 3 or a 4 in at least one of throw.
    b) Getting an even number in both throws.
    c) The sum of both the throws equals 12.
4. A bag stores 3 red pens, 6 blue pens, and 1 white pens. The pens are selected at random, one at a time and not replaced. Find the compound probability in each case.
    a) A blue, and red pen.
    b) A red, blue and, white pen.
    c) Three red pens.
    d) Two white and 1 red pen.

Sample Space and outcomes

1. How many 5-digit zip codes are possible if digits can be repeated? If there cannot be repetitions?
2. Two fair spinners are spun.

    Spinner 1 has four equal sections labelled 1, 3, 4, and 5.
    Spinner 2 has three equal sections labelled 5, 6 and 7.

    Each spinner is spun once.
    The numbers are added together.
    a) Make a table showing all the possible scores.
    b) Find the probability of scoring an 8.
    c) Find the probability of scoring an odd number.
3. Sara has 9 pairs of Jeans, 2 pairs of shoes and 6 shirts. How many outfits can she make?

Sets

1. Let A = {1, 2, 3, 5, 6}, B = {3, 4, 6, 8} be two subsets of the universal set ξ = {1, 2, 3, 4, 5, 6, 7, 8}

State the elements contained in each set and draw Venn diagrams to represent the following sets:

a) A'
b) B'
c) $A \cup B$
d) $A \cap B$
e) $(A \cup B)'$
f) $(A \cap B)'$

2. If A and B are two sets such that $A \cup B$ has 60 elements. A has 32 elements and B has 40 elements. How many elements does $A \cap B$ have?

3. If X and Y are two sets such that X has 30 elements and $X \cup Y$ has 50 elements and $X \cap Y$ has 8 elements, how many elements does Y have?

Solutions on the next page

Answers

Probability

1) 0.13

$A = American\ Express, B = Visa P(A) = 0.29, P(B) = 0.72, P(A \cap B) = 0.14$

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.29 + 0.72 - 0.14 = 0.87$

$P(\neg(A \cup B)) = 1 - 0.87 = 0.13$

2)

$A = Cigrettes, B = Cigars P(A) = 0.28, P(B) = 0.07, P(A \cap B) = 0.05$

a) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.28 + 0.07 - 0.05 = 0.30$

$P(No\ smokers) = P(\acute{A} \cap B) = 1 - P(A \cup B) = 1 - 0.30 = 0.7$

b) We need to find people who only smoke cigars.

$P(\acute{A} \cap B) = P(A \cup B) - P(A) = 0.30 - 0.28 = 0.2$

3)

a) $Total\ outcomes = 6\ x\ 6 = 36$

$P(\{(3,4),(4,3),(3,3),(4,4)\}) = \frac{4}{36} = \frac{1}{9}$

b) $P(\{(2,2),(4,4),(6,6),(2,4),(4,2),(2,6),(6,2),(4,6),(6,4)\}) = \frac{9}{36} = \frac{1}{4}$

c) $P(\{(6,6)\}) = \frac{1}{36}$

4)

a) $P(A) = \frac{1}{5}$

b) $P = \frac{3}{10} \times \frac{6}{9} \times \frac{1}{8} = \frac{1}{40}$

c) $P = \frac{3}{10} \times \frac{2}{9} \times \frac{1}{8} = \frac{1}{120}$

d) $P = 0$

Sample Space and outcomes

1)

If they can be repeated then we will have 10^5 possible outcomes.

It digits cannot be repeated then we will have $10 * 9 * 8 * 7 * 6$ possible outcomes.

2)

a)

	Spinner 1				
Spinner 2		**1**	**3**	**4**	**5**
	5	6	8	9	10
	6	7	9	10	11
	7	8	10	11	12

b) $P(x=8)=\frac{2}{12}=\frac{1}{6}$

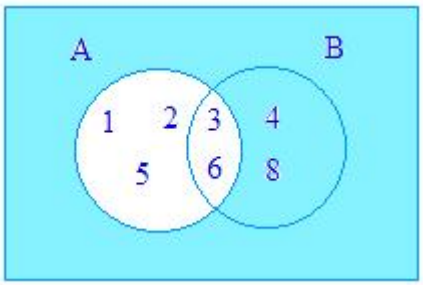
c) $P(x=odd)=\frac{5}{12}$

3) 9 jeans x 2 shoes x 6 shirts = 108 different outfits

Sets

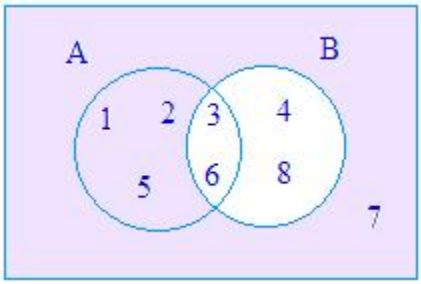
1)

(a)



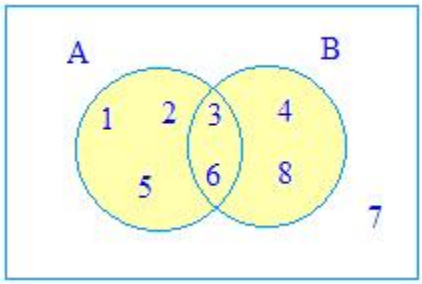
A' = {4, 7, 8}

(b)



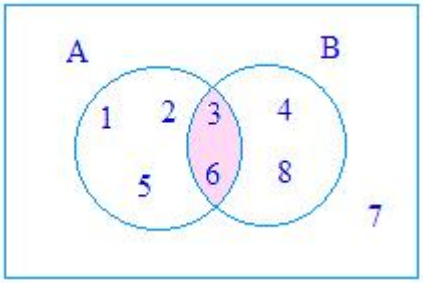
B’ = {1, 2, 5, 7}

(c)



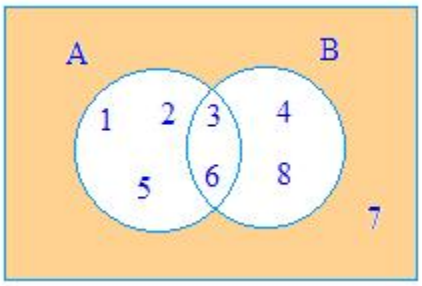
A ∪ B = {1, 2, 3, 4, 5, 6, 8}

(d)



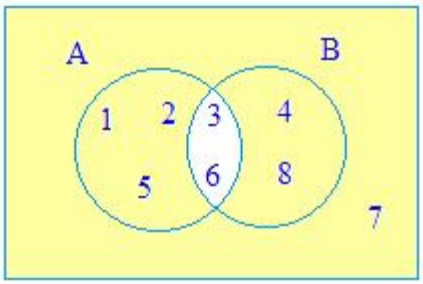
A ∩ B = {3, 6}

(e)



(A ∪ B)’ = {7}

(f)



(A ∩ B)’ = {1, 2, 4, 5, 7, 8}

2)

$$n(A \cup B) = n(A) + n(B) - n(A \cap B) n(A \cap B) = n(A) + n(B) - n(A \cup B)$$

$$n(A \cap B) = 40 + 32 - 60 n(A \cap B) = 12$$

3)

$$n(X \cup Y) = n(X) + n(Y) - n(A \cap Y) 50 = 30 + n(Y) - 850 + 8 - 30 = n(Y) n(Y) = 28$$