①) $P(X=1) = 0.2$
②) $P(X=2) = 0.3$

$\sum 1 \cdot P(X=1)$
$1 \cdot 0.2$ + ~~0.2 mean~~

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PRACTICE expectation.

→ mean is also expectation.

- The expected value or mean value of a discrete random variable is defined as.

$$E[X] = \sum_{p(u)>0} u \cdot P(X=x) = \sum_{p(u)>0} u(pu)$$

means that if we have random variable 1 and 2 and their respective probs is 0.8 and 0.2 then the mean would be

$$(1)(P X=1) + (2)\ P(X=2)$$

mean $= (1)(0.8) + (2)(0.2)$

$0.8 + 0.4$

mean. 1.2

-) $E[X]$ is a weighed average of the possible values u that X can take on, each value being weighted by the probability $p(u)$.

Q). X is the outcome when we roll a 4 sided fair die, find $E[X]$ $X =$ is the random variable

step 1) X is the outcome [illegible]

X is $[1, 2, 3, 4]$.

step 2)

now mean $= P(X=x) \cdot (u)$

$P(X=1) \cdot (1) + P(X=2) \cdot 2 + P(X=3) \cdot 3 + P(X=4) \cdot 4$.

SOLO

x , y $\quad \Sigma\, g(x) \cdot p(x)$

$(-1)^2 \cdot \frac{1}{4} + 0^2 \cdot \frac{1}{2} + 1^2 \cdot \frac{1}{4}$ $\quad Y = x^2 \to g(x)$ $\quad \Sigma (x^2) \cdot p(x)$

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$1 \cdot \frac{1}{4} + 2 \cdot \frac{1}{4} + 3 \times \frac{1}{4} + 4 \times \frac{1}{4}$ $\quad (1)^2 \cdot (p = x = 1)$ $\quad 1 \cdot \frac{1}{4}$

$\frac{1}{4} + \frac{1}{2} + \frac{3}{4} + 1$

$\frac{1+2+3+4}{4}$ $\quad \frac{\cancel{10}\,5}{\cancel{4}\,2}$ $\quad \frac{5}{2} = 2.5$

mean = 2.5.

If I throw this dice for a large number of times then I will get the expected value of a mean value of 2.5

- Expectation of a function of a random variable
  - If X is a discrete random variable and g is a real valued function then the expectation (or expected value) of $\underbrace{y = g(x)}_{}$ is → new random variable

$$E[g(X)] = \sum_{u : p_X(u) > 0} g(u) \cdot p_X(u)$$

X is a discrete random variable with pmf.

$u =$	-1	0	1
$p_X(u) =$	$1/4$	$1/2$	$1/4$

$Y = X^2$ = compute $E[Y]$.

$g(u) = y.$
$x^2 = y$

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$E(Y) = 1\cdot(1/4) + 1(1/4)$

$\frac{1}{4} + \frac{1}{4}$

$\frac{2}{4}$

$E(y) = \frac{1}{2}$

This is useful because by this method we can find out about the expectation of function of X as well.
From previous example we could only find out about the expectency of X.

Also,

$\sum_{p_X(u)>0} g(u)\cdot P_X(u)$

$\sum x^2 \cdot P_X(u)$

why is this?

well we were supp to find $E[Y]$ and as we saw in the generic form that
$E[Y] = g(u)\cdot P_X(u)$
is some function of Y.
so in this example we have
$E[Y] = x^2.$
so $x^2 \cdot P_X(u)$
where $g(x)$ is x^2.

Properties of expected values:

→ If a and b are constants then,

$$E[aX+b] = aE[X]+b$$

holds for all random variables X,

→ This property is saying that we can take constants outside and then do the calc with them.

•) Proof for discrete random variable: let $g(X) = aX+b$

$$E[g(X)] = \sum_{u:p(u)>0} g(u)\cdot p(u)$$

$$E[aX+b] =$$

$$\sum (aX+b)\cdot p(u)$$

$E[X] = u \cdot pu$ $\quad z = x + y.$

$E[z] = p(g(u)) \cdot f(u).$

Date $\quad g(u) = ax + b$

$\sum (ax+b) \cdot p(x).$ $\quad \boxed{\sum p(u) = 1}$

$\sum \underbrace{ax \cdot p(x)}_{E[x]} + \sum b(pu)$

$a \sum \underbrace{u \cdot p(u)}_{\hookrightarrow E[x].} + b \sum p(u)$

$a\, E[x] + b$

$g(x) = aX + b.$

$E[aX+b]$ = proofed above.

•) Constant factor.

$$E[aX] = a E[x]$$

•) Constant

→ expectation of a constant is itself.

$E[b] = b.$

For two random variable's X and Y.

$$E[X+Y] = E[X] + E[Y].$$

$y = 3x + 2$ $\quad p(k) = 1$

$\sum (3k+2) \cdot p(k)$

$\sum (3k) \cdot p(k) + \sum 2 \cdot p(k)$

$3 \underbrace{\sum (k) \cdot p(k)} + 2$

$3\, E[X] + 2.$

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PRACTICE.

Variance and Standard deviation:

•) Variance:

•) Expected value / mean represented by μ = $E[X]$ yields the weighted average of the possible values of X

μ = is the weighted average of all the possible values of x.

Variance measures the variation (or spread) of these values

$$\sigma^2 = Var(X) = E[(X - E(X))^2] = E[(X - \mu)^2]$$

This holds for all random variable X.

Simplification: $\mu = E[X]$

$$Var(X) = E(X^2) - \mu^2 = E[X^2] - (E[X])^2$$

Standard deviation:

$$\sigma = SD(X) = \sqrt{Var(X)}$$

Q) compute the S.D of fair 4 sided dice.

$Var = E(X^2) - \mu^2$

$\mu = (1) \cdot \frac{1}{4} + 2 \cdot \frac{1}{4} + 3 \cdot \frac{1}{4} + 4 \cdot \frac{1}{4}$

$\mu = \frac{1}{4} + \frac{1}{2} + \frac{3}{4} + 1 = 2.5$.

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$\mu^2 = (2.5)^2$

$\mu^2 = 6.25$

$E[X^2] = \sum_4 x^2 \cdot p(x)$

$E[X^2] = (1)^2 \cdot \frac{1}{4} + (2)^2 \cdot \frac{1}{4} + (3)^2 \cdot \frac{1}{4} + (4)^2 \cdot \frac{1}{4}$

$\frac{1}{4} + 1 + \frac{9}{4} + 4 = \cancel{6.75}\ 7.5$

$var(X) = \overset{7.5}{\cancel{6.75}} - 6.25$

$var(X) = \cancel{0.5}\ 1.25$

$S.D = \sqrt{var(X)}$

$S.D = \sqrt{\cancel{0.5}\ 1.25}$

$S.D = \cancel{0.707\ ans.}\ 1.12\ ans$

→ ~~Higher the variance the further the random variable is set apart, lies on a greater region.~~

→ larger the S.D or variance the random variable is spread over a larger area, the higher the spread.