Habib University
Course Code: EE 354/CE 361
Course Title: Intro to Probability & Statistics
Instructor's name: Dr. M. Shahid Shaikh
Examination: Assignment #2 Given: 1 October 2023
Total Marks: 40 Due: 15 October 2023

Problem 1: **[5]**

You and your friends are playing a game of *Dungeons and Dragons* with a fair, 20-sided die. One of your friends is the Dungeon Master and rolls the die behind a curtain where only they can see the result of a roll. Your strategy depends on whether you land an 18 or more when the die is rolled.

(a) What are the odds that the die will land on an 18 or more when rolled? [2]

To make matters interesting, your friend offers you a deal:

> *I'll let you know if the die lands on an even number or not. You can then decide how you want to play the game...*

(b) Does having this information improve your odds? Using what you know about conditional probabilities, reason why or why not. [3]

Problem 2: **[7]**

Two out of the three prisoners are shortlisted for parole. One of the prisoners asks a guard to tell him the identity of a prisoner other than himself that will be released. The guard refuses with the following rationale:

> *At your present state of knowledge, your probability of being released is 2/3, but after you know my answer, your probability of being released will become 1/2, since there will be two prisoners (including yourself) whose fate is unknown and exactly one of the two will be released.*

Using *Bayes' Theorem*, argue why the guard is incorrect?

Problem 3: **[6]**

A particular class has had a history of low attendance. The annoyed professor decides that she will not lecture unless at least k of the n students enrolled in the class are present. Each student will independently show up with probability p_g if the weather is good, and with probability p_b if the weather is bad. Given the probability of bad weather on a given day, obtain an expression for the probability that the professor will teach her class on that day.

Problem 4: **[8]**

An academic department offers 8 lower level courses: $\{L_1, L_2, \ldots, L_8\}$ and 10 higher level courses: $\{H_1, H_2, \ldots, H_{10}\}$. A valid curriculum consists of 4 lower level courses and 3 higher level courses.

(a) How many different curricula are possible? [3]

(b) Suppose that $\{H_1, H_2, \ldots, H_5\}$ have L_1 as a prerequisite, and $\{H_6, H_7, \ldots, H_{10}\}$ have L_2 and L_3 as prerequisites, i.e. any curricula which involve, say, one of $\{H_1, H_2, \ldots, H_5\}$ must also include L_1. How many different curricula are there? [5]

Problem 5: **[7]**

Two, fair, 6-sided dice are rolled. Let X be the difference in the number of dots facing up.

(a) Find and plot the pmf of X. [3]

(b) Find the probability that $|X| \geq k$ for all k. [2]

(c) Find the expected value of the random variable X. [2]

Problem 6: **[7]**

A stock trader buys 100 shares of stock A and 200 shares of stock B. Let X and Y be the price changes of A and B, respectively, over a certain time period. Assuming that the joint PMF of X and Y is uniform over the set of integers x and y satisfying:

$$-2 \leq x \leq 4, \quad -1 \leq y - x \leq 1$$

(a) Find the marginal PMFs of X and Y. [3]

(b) Find the mean values of X and Y. [2]

(c) Find the mean of the trader's profit. [2]