ARITHMETIC SEQUENCES

What is an Arithmetic Sequence?

A sequence formed by adding a fixed number repeatedly.

2 → 4 → 6 (+2, +2)

Common Difference (+2)

You start with a first term and add the same number over and over to get the next terms.

5 → 7 ... (+2)

The "Common Difference" is the key.

The common difference is the constant value found by subtracting any term from the term that immediately follows it. It can be positive, negative or zero.

How to check if a sequence is arithmetic

Subtract each term from the one after it. If the result is always the same, it's an arithmetic sequence.

E.g., 1, 4, 7... (4-1=3, 7-4=3), common difference is 3.

Position and Term Relationships

Increasing: 2, 4, 6, ... (+2)

Decreasing: 90, 85, 80, ... (-5)

Constant: 360, 360, 360, ... (0)

Term Difference = Position Difference × Common Difference

$$(X_m - X_n) = (m - n) \times d$$

X_m, X_n: Terms at positions m, n

d : Common Difference

Change in Term is proportional to Change in Position.

Problem: Find the sequence if the the 3rd term is 37 and the 7th term is 73.

1. Position Difference: 7 - 3 = 4
2. Term Difference: 73 - 37 = 36
3. Find Common Difference (d): 4 x d = 36 → d = 9
4. Find First Term: 1st term = 3rd term - 2d → 37 - (2 x 9) = 19

The Sequence: **19, 28, 37, 46, ...**

Properties of Terms and Sums

The Sum of Equidistant Term Pairs is Constant.

(1st term + 6th term) | (2nd term + 5th term)

(1st term + 6th term) = (2nd term + 5th term)

=

If the 2nd term + 5th term = 35...

...then the 3rd term + 4th term = 35, and the 1st term + 6th term = 35 (since 2+5 = 3+4 = 1+6).

The Middle Term is the Average.

Term 1 – Term 2 – Term 3

Sum of first & last = 2 x Middle Term

Sum of an Odd Number of Consecutive Terms.

Middle Term x Number of Terms = Sum

The sum is equal to the middle term multiplied by the number of terms.

E.g., sum of first five terms is 5 times the 3rd term.

Problem: The sum of the first 5 terms is 250, and the first term is 10. Find the sequence.

1. Find the Middle Term: 3rd term = $\frac{\text{Total Sum}}{\text{Number}} = \frac{250}{5} = 50$.
2. Find Common Differernce: 1st term = 10, 3rd term = 50. Difference (40) = 2d → d = 20.
3. The Sequence: 10, 30, 50, 70, 90.

Is This Number in the Sequence?

Step 1: Find the Term Difference. Subtract the first term of the sequence from the number you are checking.

Step 2: Check for Divisibility. Divide the result ffrom Step 1 by the common difference. If it divides perfectly with no reminder, the number is a term.

Is 1000 a term in the sequence 19, 28, 37, ...?

1. Common Difference (d) = 28 - 19 =9.
2. Term Difference = 1000 - 19 = 981.
3. Check Divisibility: 981 ÷ 9 = 109.

Since it divides perfectly, 1000 is a term in the sequence.

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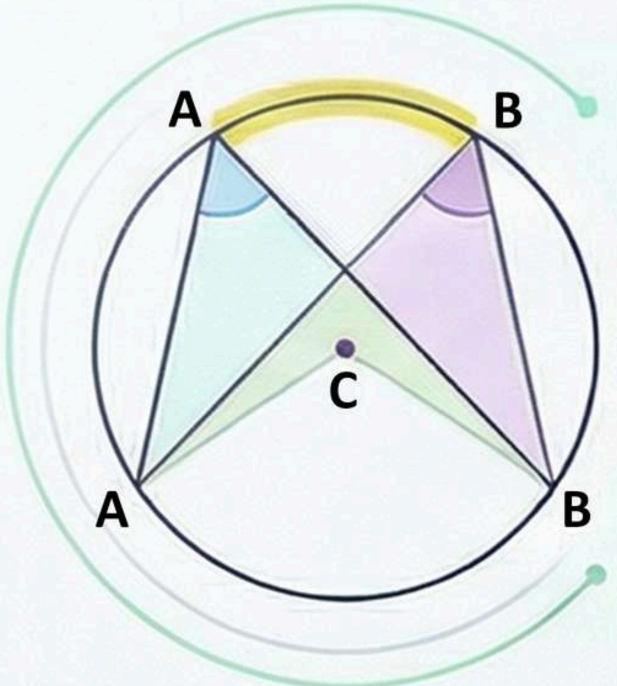
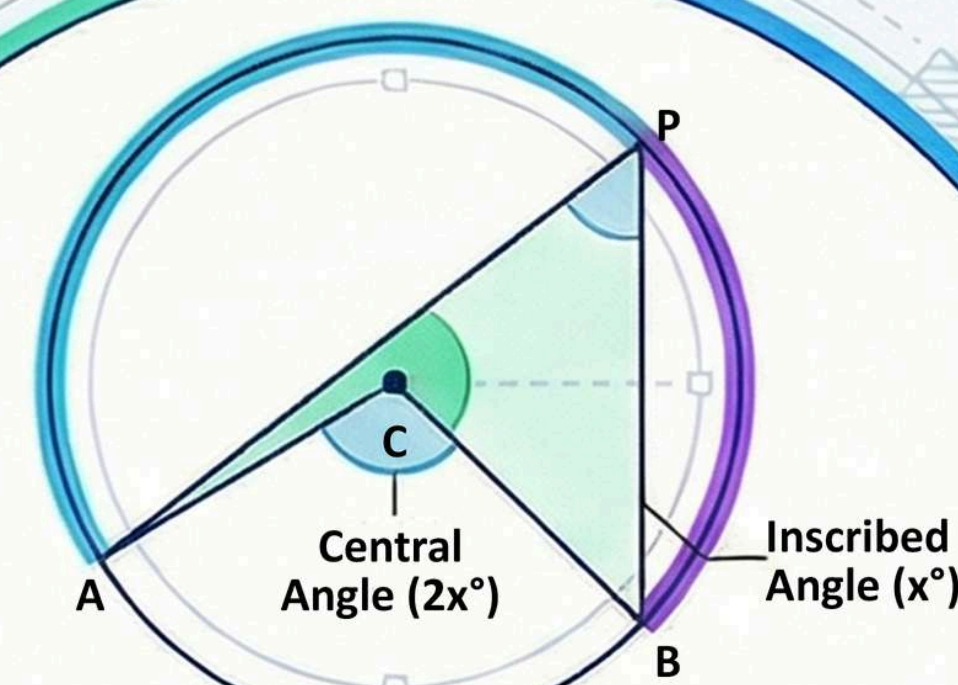
CIRCLES AND ANGLES



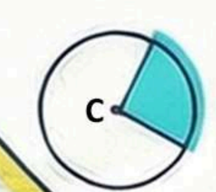
Angles in the Same Segment are Equal

Since all inscribed angles subtended by the same arc are half of the same central angle, they must all be equal.

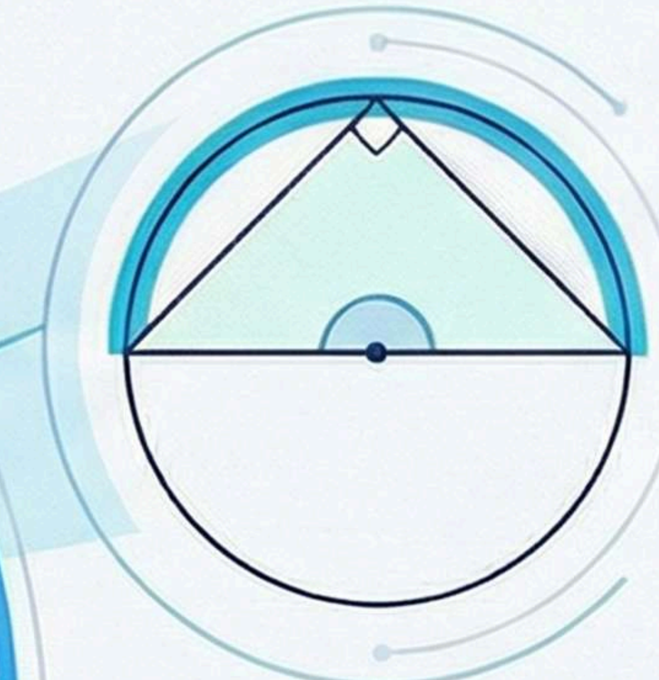


The Central Angle Theorem

An arc creates a Central Angle at the center and an inscribed. Angle on the circumference. The relationship is foundational: The central angle is double the inscribed angle subtended by the same arc.



EXAMPLE:
If are is 1/3 of circle, Central Angle 120°, Inscribed Angle = 1/2 of 120° = 60°.



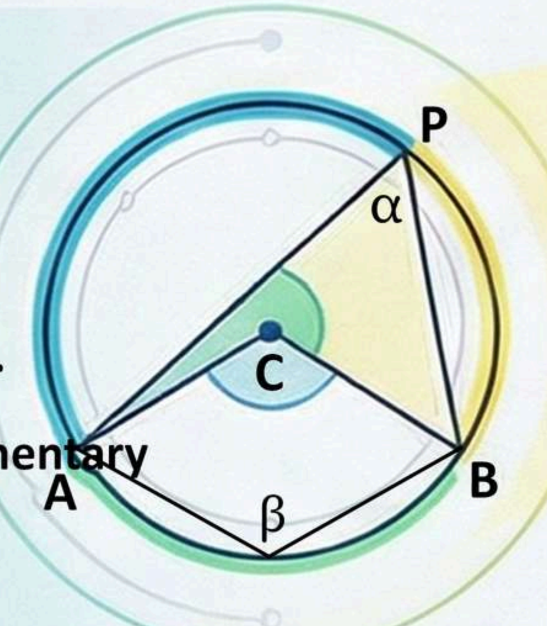
The Angle in a Semicircle is a Right Angle (90°)

A semicircle has a central angle of 180°. The inscribed angle is half of 180°, which is always 90°.

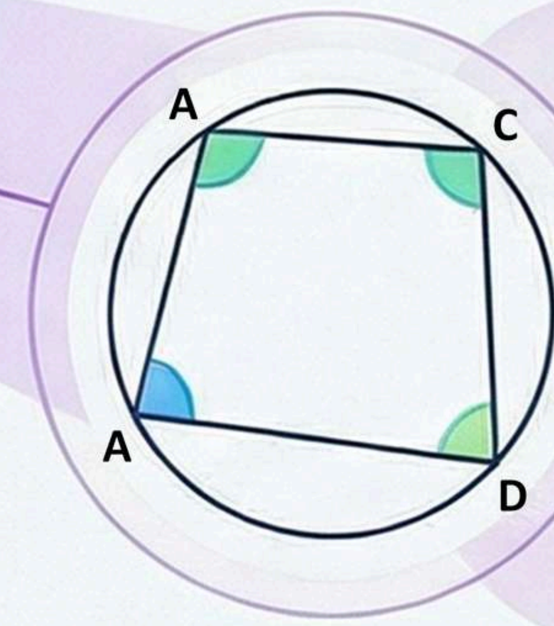
Alternate Arcs and Supplimentary Angles

A pair of arcs that make a full circle are called Alternate Arcs. the angles subtended in these two alternate arcs are supplementary (they add up to 180°).

$\alpha + \beta = 180°$



Cyclic Quadrilaterals



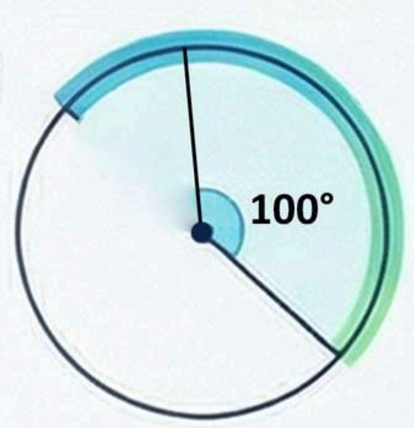
A cyclic quadrilateral's four vertices all lie on a single circle. **Opposite angles are supplementary** (add up to 180°).

$\angle A + \angle C = 180°$ $\angle B + \angle D = 180°$

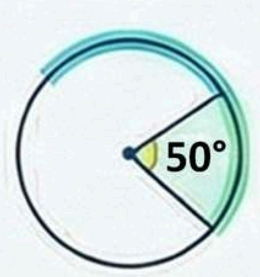
- Converse: If opposite angles of a quadrilatoral add to 180°, it is cyclic.
- Example: leosceles trapesoids are always cyclic.

Problem-Solving Examples

1

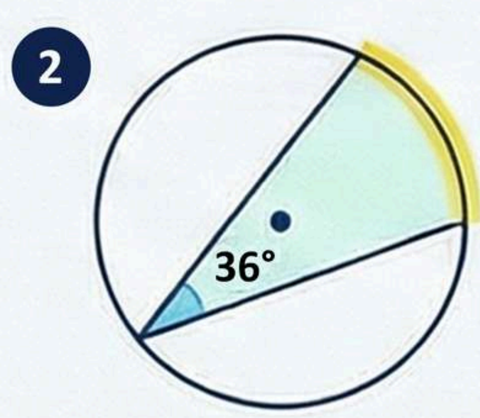


An arc has an central angle of 100°. What is the angle on the alternate arc?

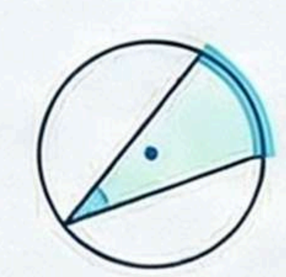


Solution:
The inscribed angle is half the central angle, so it is 50°.

2

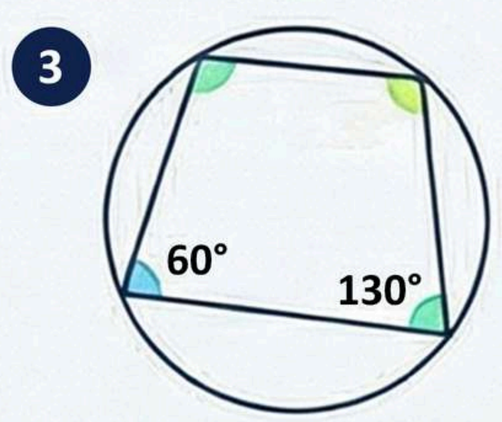


An arc has an inscribed angle of 36°. What is its central angle?

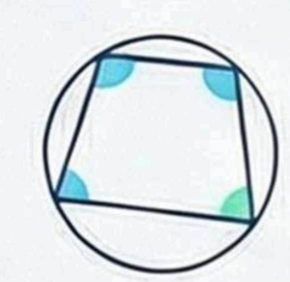


Solution:
The central angle is double the inscribed angle, so it is 72°.

3



In the cyclic quadrilateral, find the remaining angles.

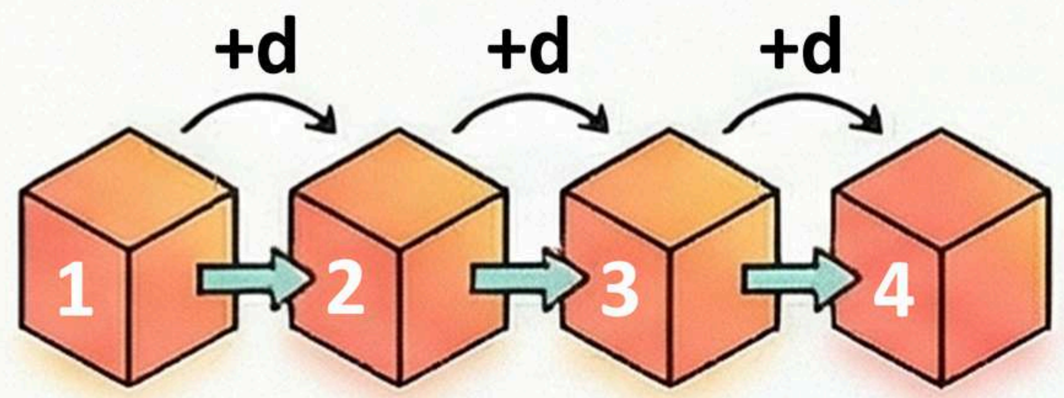


Solution:
Other angles must be 180° – 60° = 120° and 180° – 130° = 50°. Opposite angles are supplimentary.

SEQUENCE AND ALGEBRA

A concise summary of key formulas for finding terms and sums, serving as aquick study guide for dequences and series.

FINDING ANY TERM (THE ALGEBRAIC FORM)



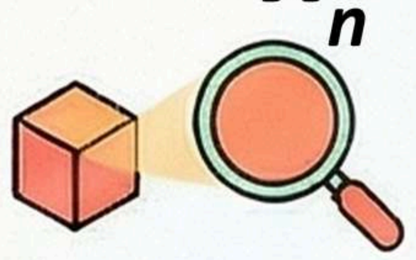
What is an Arithmetic Sequence ?

A sequence where each term is found by adding a constant value to the previous one.

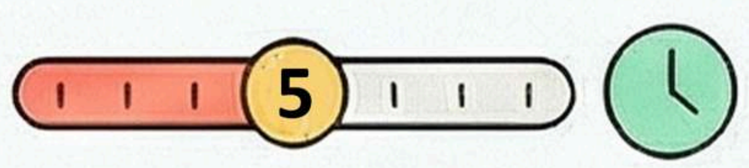
$(x_1 = f)$

$$x_n = x_1 + (n - 1)d$$

$$x_n = dn + (f-d)$$

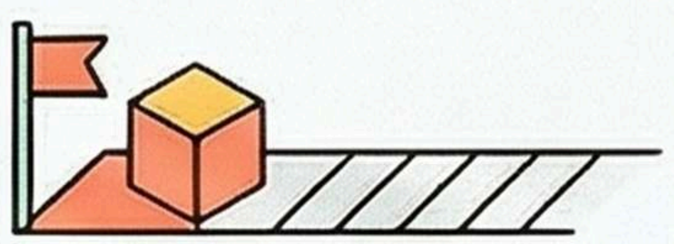


x_n: The term you want to find (the n-th term)

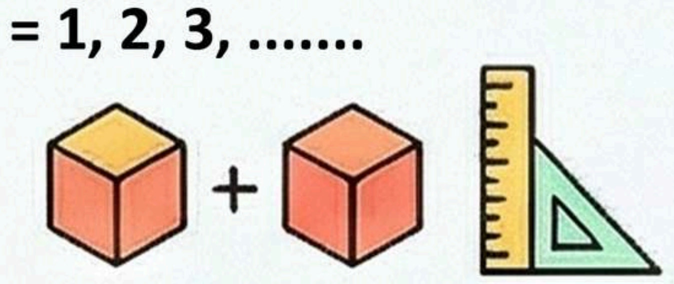


n: The position of the term in the sequence (e.g., 5th, 20th)

$n = 1, 2, 3, ...$



x_1: The very first term of the sequence *(f)*

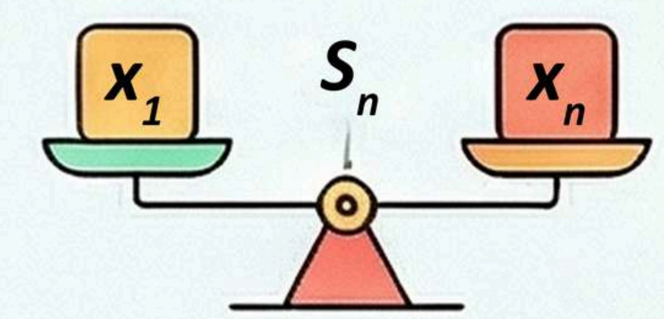


d: The common difference (the constant value added between terms)

CALCULATING THE SUM OF TERMS $\sum$

How to Find the Sum of the First 'n' Terms (S_n)

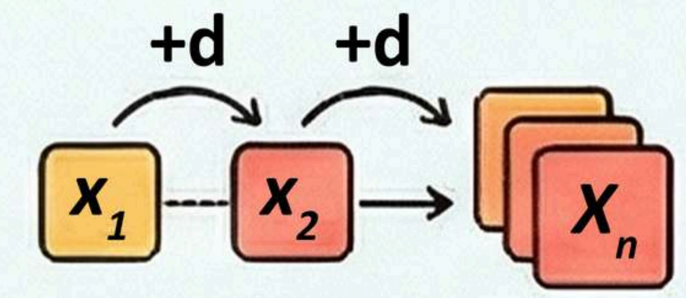
$$S_n = n/2\ (x_1 + x_n)$$



Use this when you know the first and the last term you're adding.

Alternative Sum Formula

$$S_n = n/2\ [(2x_1 + n -1)d]$$

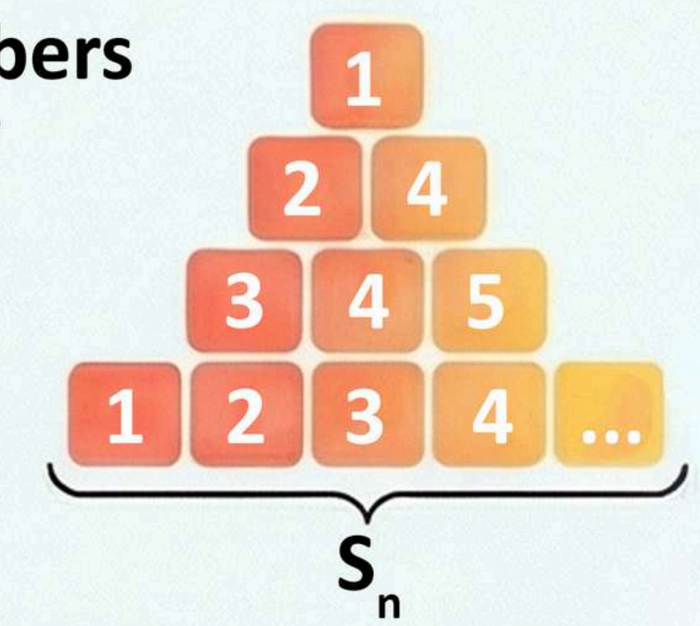


Use this when you only know the first term and the common difference.

Sum of First 'n' Natural Numbers

A foundational formula for the sum of 1+2+3+ ... +n.

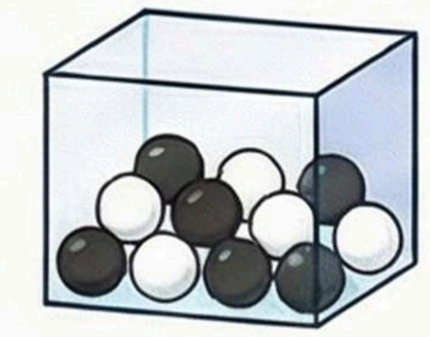
$$S_n = \frac{n\ (n + 1)}{2}$$



MATHEMATICS OF CHANCE

Probability measures the chance of an event happening. It is calculated by diving the number of ways a specific event can occur by the total number of possible outcomes.

$$\text{Probability (Event)} = \frac{\text{Favorable Outcomes}}{\text{Total Outcomes}}$$



Example: Box with 6 black, 4 white balls.
P(black ball) = 6/10 = 3/5.
P(white ball) = 4/10 = 2/5.

1. Numerical Probability Problems

1 2 3 4 5 6 7 9 10 11 12 13 14 15 16 17 19 20 21 23 25
(2, 6, 12, 13, 25 marked)

Finding an even number from 1 to 25.
Out of 25 numbers, there are 12 even numbers.
P(even number) = 12/25.

1000
2^3 1000
2^3 5^3
2^3 5^3

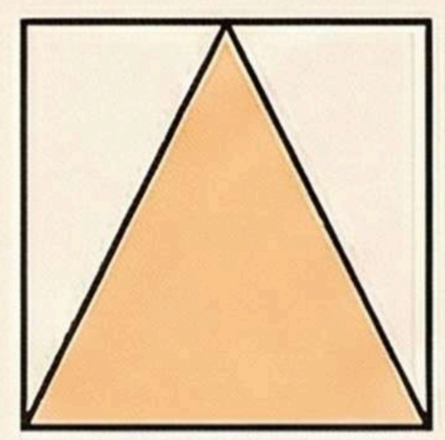
Picking a factor of 1000 from 1 to 1000.
1000 = $2^3 \times 5^3$.
Total factors = (3+1) x (3+1) = 16
P(facotor of 1000) = 16/1000.

Probability can be shown in multiple formats:
16/1000 = 2/125 = 0.016 = 1.6%

2. Geometric Probability Problems

Geometric probability is based on are. Its' the chance of a random point falling within a specific shape inside a larger area.

$$\text{Probability} = \frac{\text{Favorable Area}}{\text{Total Area}}$$

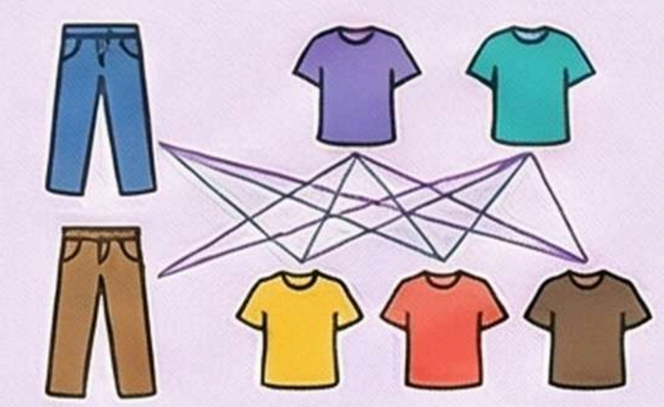


Point randomly inside a square containing a triangle
(Area of triangle = 1/2 Area of square).
P(point in triangle) = 1/2.

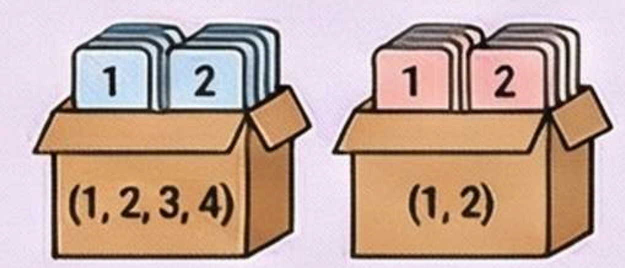
Spinner with 8 equal sectors
(3 are yellow).
P(landing on yellow) = 3/8.

3. Probability of Pairs and Combinations

Calculating Total Possible Pairs: If there are 'm' choices for the first item and 'n' choices for the second, total possible pairs is m x n.



Outfit Combinations:
2 pairs of pants x 3 shirts
= 6 possible unique outfits.



Two boxes of numbered cards.
Total pairs = 4 x 2 = 8
P(picking two odd numbers, e.g., (1,1) or (3,1)) = 2/8 = 1/4.

Two baskets of mangoes.
P(picking a ripe mango from both)
= (30x25)/(50x40) = 750/2000 = 3/8.

4. Theoretical vs Experimental Probability

A fair coin has a theoretical probability of 1/2 for heads. In theory, each outcome is equally likely.

Experimental Probability gets closer to theoretical probability with more trials.

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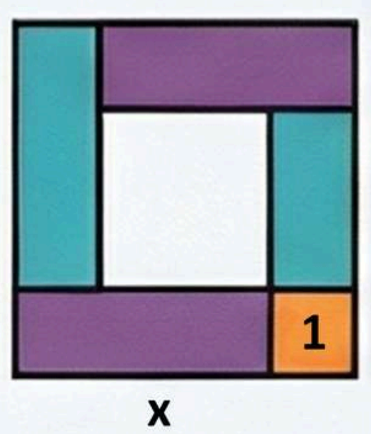
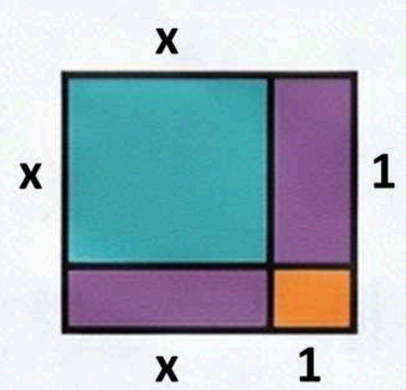
SECOND DEGREE EQUATIONS

1. From Word Problem to Equation

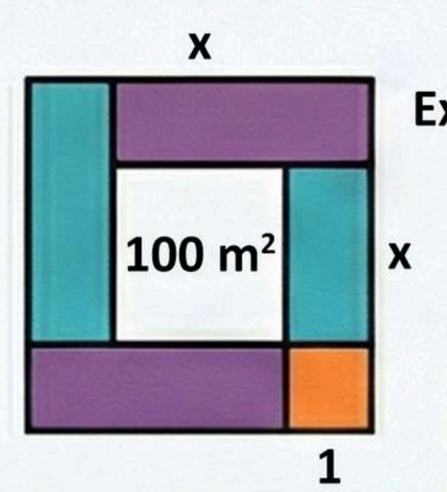


What is a Second-Degree Equation?:
Highest power of variable is 2.
General form: $ax^2 + bx + c = 0$

Step 1: Represent the Unknown
Identify the quantity, label it with a variable, e.g., let ‘x’ be the side of a square.



Step 2: Build the Equation
Translate relationships into a mathematical equation.



Example: The Geometric Puzzle

$x^2 + 2x + 1 = 100$

2. The Power of Perfect Squares

The Perfect Square Identity

$$(x + a)^2 = x^2 + 2ax + a^2$$

Step 3: Simplify the Equation.
$x^2 + 2x + 1$ matches identity.
Rewrite as $(x + 1)^2 = 100$.

Step 4: Solve by Taking the Square Root
Square root of both sides:

$x + 1 = 10$

Find the Final Answer
Solve simple equation:
$x = 9$.
Side of square is 9 meters.

3. Technique: Completing the Square

What if it's not a perfect square?
Equation $x^2 + 20x = 224$

The Golden Rule

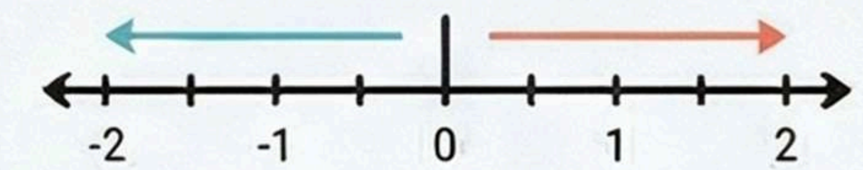
For $x^2 + bx$, add $(b/2)^2$

Step 1: Find the Missing Place
b=20.
Half of b is 10
$(10)^2$ is 100. Add 100.

Step 2: Add to Both Sides
$x^2 + 20x + 100 = 224 + 100$

Step 3: Factor and Solve
The equation becomes
$(x + 10)^2 = 324$.
Taking the square root gives
$x + 10 = 18$, which means $x = 8$.

4. Two Answers: Choosing the Right Answer



A Number Has Two Square Roots
Square root can be positive or negative.
$x^2 = 1$ means $x = 1$ OR $x = -1$.

Finding Both Algebraic Solutions
From $(x - 2)^2 = 1$, we get two possibilities: one is $x - 2 = 1$ (giving $x = 3$), and the other is $x - 2 = -1$ (giving $x = 1$).

Context is Everything
Check if both solutions are valid in the real world.

Discarding Invalid Answers
Length cannot be negative.
If solution is $x = -16$, ignore it.

5. Exam Practice Problems

Rectangle Problem
Perimeter 100 m, Area 525 m²
Algebraic Setup: $x (50 - x) = 525$
simplifies to $(x - 25)^2 = 100$.
Solution: Sides are 15m and 35m.

Arithmetic Progression
99, 97, ...
95, 96, ...

Sum of first ‘n’ terms of 99, 97, 95, ... is 900.
Algebraic Setup: $100n - n^2 = 900$
simplifies to $(n -50)^2 = 1600$.
Solution: n = 10 or n = 90. Both are valid.

Right Triangle
Perpendicular side 5cm longer than the other.
Area 12 cm²
Algebraic Setup: $(1/2) + x + (x +5) = 12$
simplifies to $(x + 5/2)^2 = 121/4$.
Solution: Sides are 3cm and 8cm.

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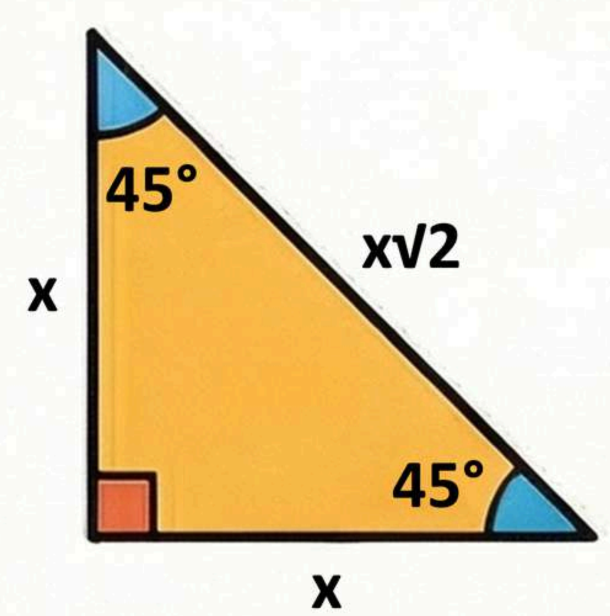
TRIGONOMETRY

SPECIAL RIGHT-ANGLED TRIANGLES

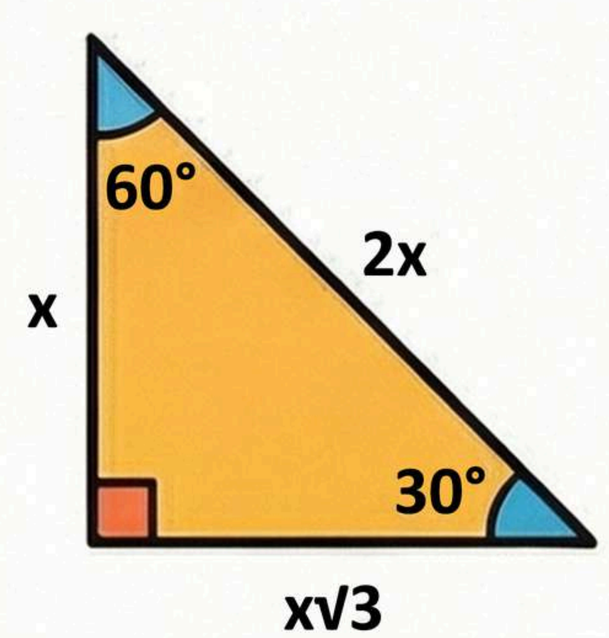


The 45° - 45° - 90° Triangle (Isosceles Right Triangle)

The two shorter sides (legs) are equal in length. The hypotenuse is √2 times the length of a leg.

Side Ratio: 1 : 1 : √2

If the legs have length 'x', the sides are in the proportion x : x : x√2.



The 30° - 60° - 90° Triangle

This triangle can be formed by splitting an equilateral triangle in half. The side lengths are always in a fixed proportion.

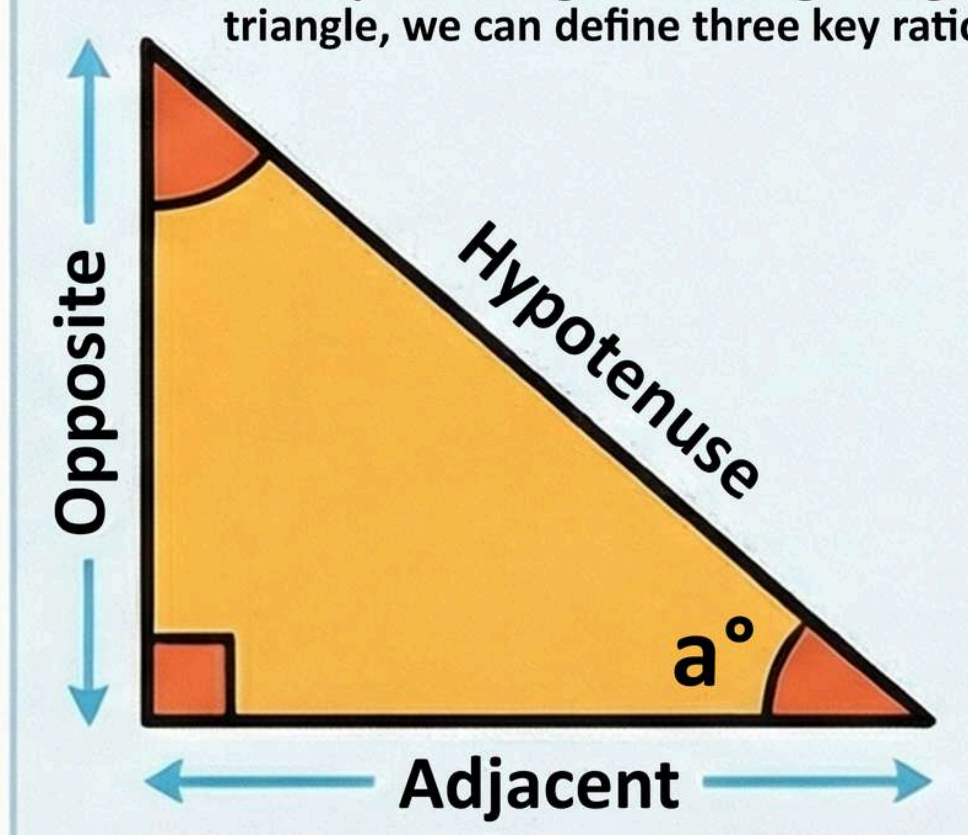
Side Ratio: 1 : √3 : 2

The sides opposite the 30°, 60°, and 90° angles are in the proportion x : √3x : 2x.

THE TRIGONOMETRIC RATIOS

Defining the Ratios

For any acute angle 'a' in a right-angled triangle, we can define three key ratios.



$$\text{Sine } (\sin a°) = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

The ratio of the length of the side opposite the angle to the length of the hypotenuse.

$$\text{Cosine } (\cos a°) = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

The ratio of the length of the side adjacent to the angle to the length of the hypotenuse.

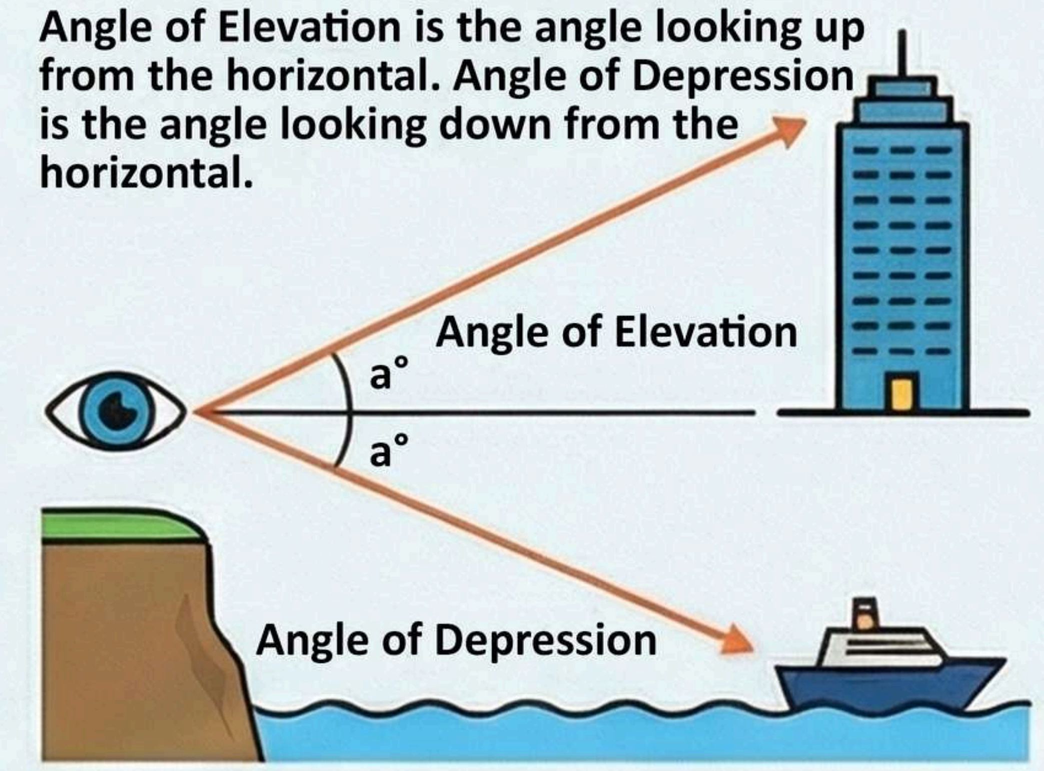
$$\text{Tangent } (\tan a°) = \frac{\text{Opposite}}{\text{Adjacent}}$$

The ratio of the length of the side opposite the angle to the length of the side adjacent.

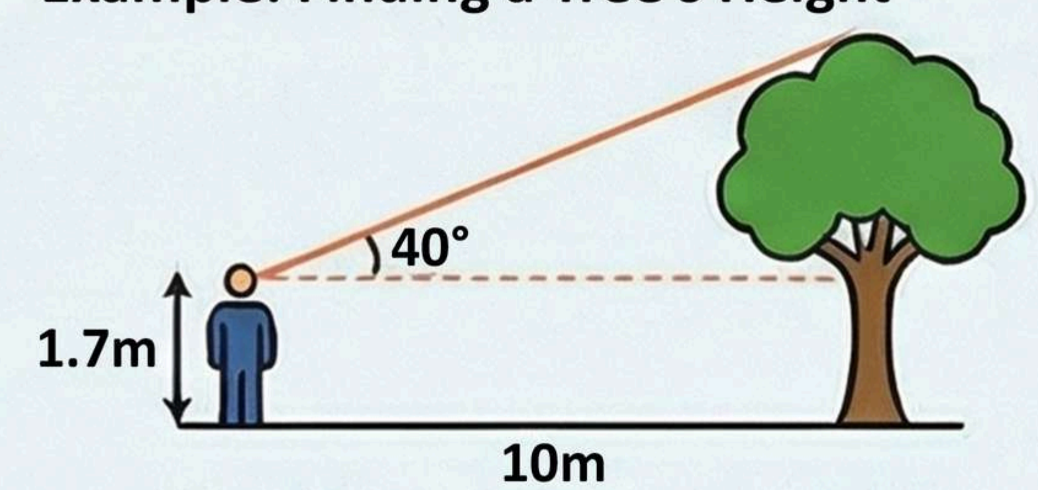
Angle (a)	30°	45°	60°
sin a°	1/2	1/√2	√3/2
cos a°	√3/2	1/√2	1/2
tan a°	1/√3	1	√3

Angle of Elevation & Depression

Angle of Elevation is the angle looking up from the horizontal. Angle of Depression is the angle looking down from the horizontal.

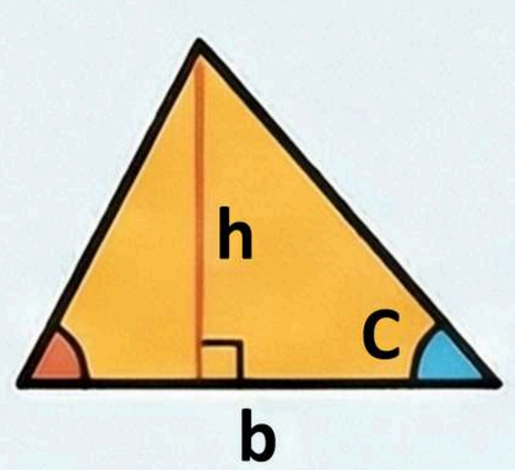


Example: Finding a Tree's Height



Tree's total height = 1.7m + (10 x tan 40°), which is approximately 1.7m + 8.39m = 10.09m.

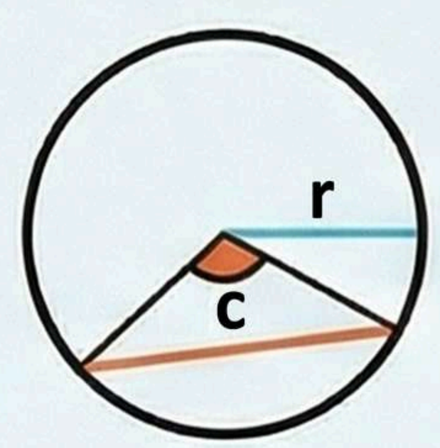
APPLICATIONS AND PROBLEM SOLVING



Calculating the Area of a Triangle

To find the area, calculate the height (h) using the formula h=b sin(C), where b is a known side and C is the angle adjacent to the base.

Area = 1/2 x base x h

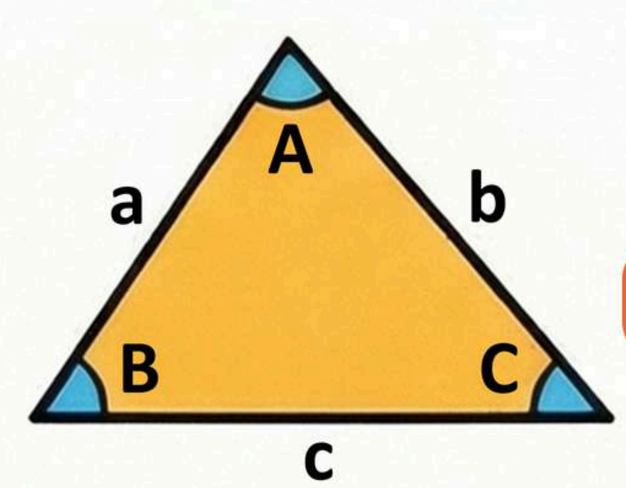


Finding the Length of a Chord in a Circle

The length of a chord can be calculated using the circle's radius (r) and the central angle (c) subtended by the chord.

Chord Length = 2r sin (c/2)

THE SINE RULE



Angles Determine Side Ratios

In any triangle, the ratio of the length of the sides is the same as the ratio of the sines of their opposite angles.

Ratio of Sides = a : b : c = sin(A) : sin(B) : sin(C)

The rule connects the sides of any triangle (a, b, c) to the sines of their opposite angles (A, B, C).

CO-ORDINATES

The Coordinate System

The Coordinate Plane is defined by two perpendicular lines.

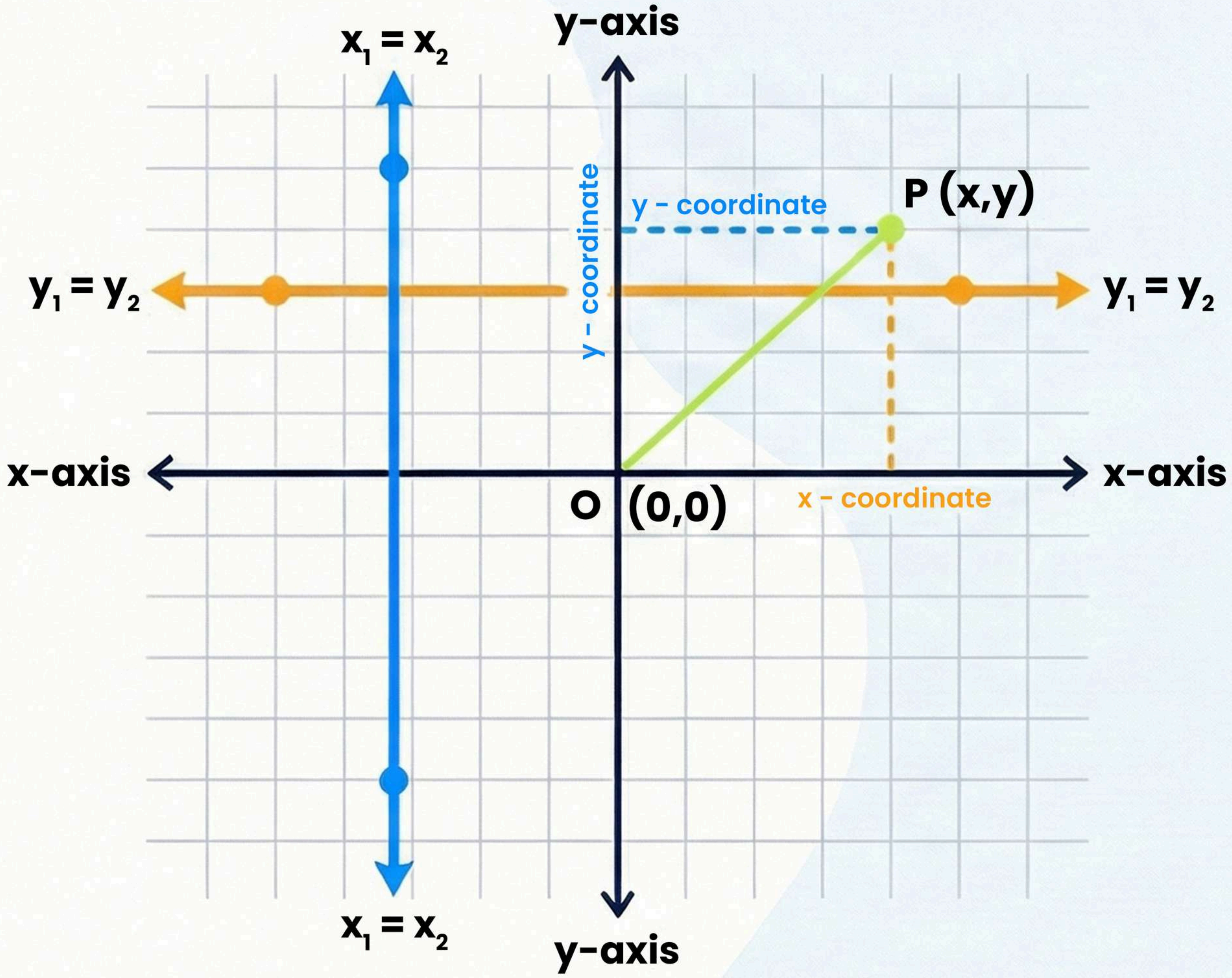
The horizontal line is the x-axis, and the vertical line is the y-axis.

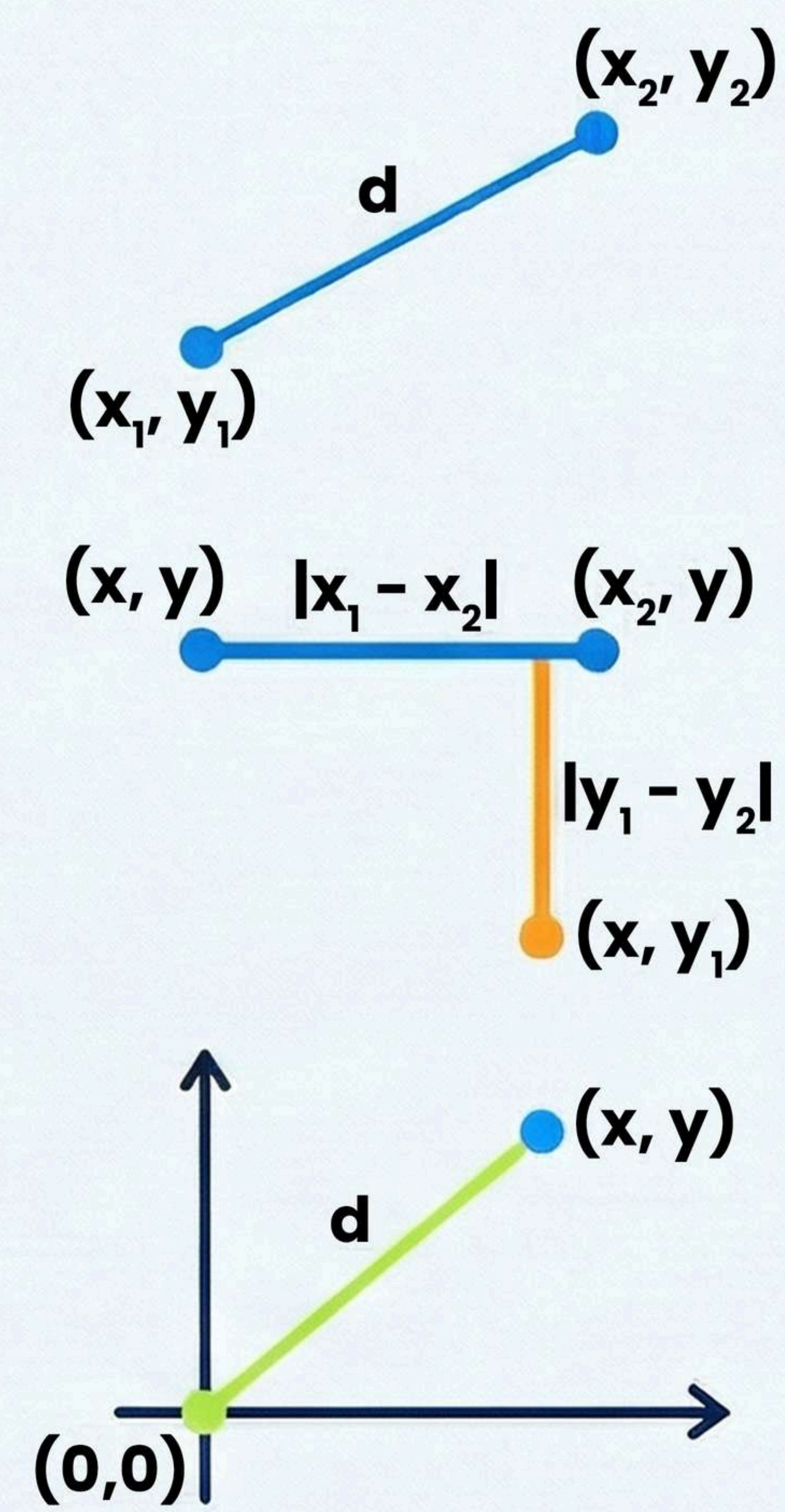
A point's position is given by coordinates (x, y).

The x-coordinate is the horizontal position; the y-coordinate is the vertical position.

Lines Parallel to the Axes Have a Key Property.

If x-coordinates are equal, the line is parallel to the y-axis.
If y-coordinates are equal, it's parallel to the x-axis.





Calculating Distances

Distance Between Two Points:

For points (x_1, y_1) and (x_2, y_2), the distance is

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Special Case: Distance for Lines Parallel to an Axis

For horizontal lines, distance is $|x_1 - x_2|$.
For vertical lines, distance is $|y_1 - y_2|$.

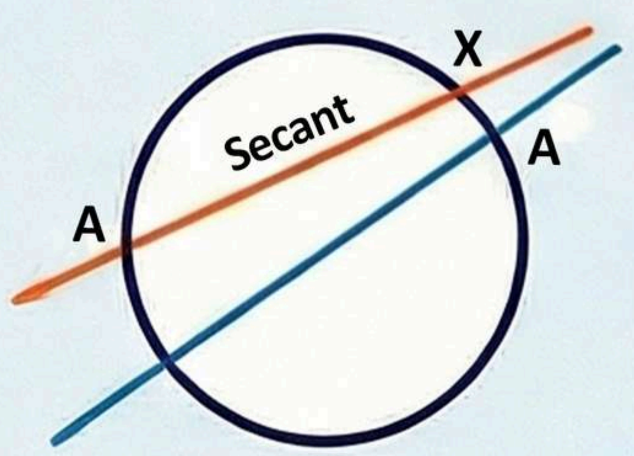
Shortcut: Distance of a Point from the Origin (0,0)

$$d = \sqrt{x^2 + y^2}$$

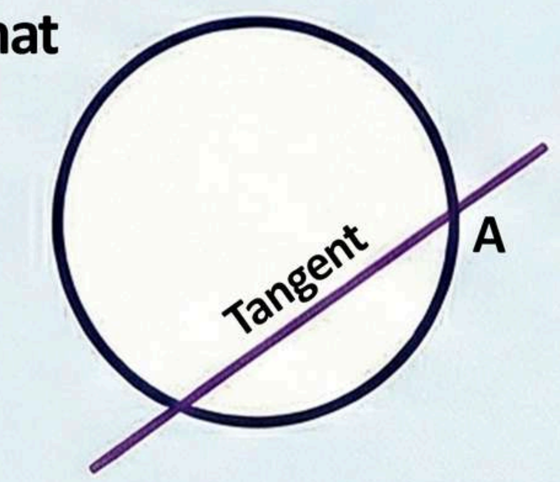
The distance between (x, y) and the origin is simply the square root of the sum of x squared and y squared.

TANGENTS

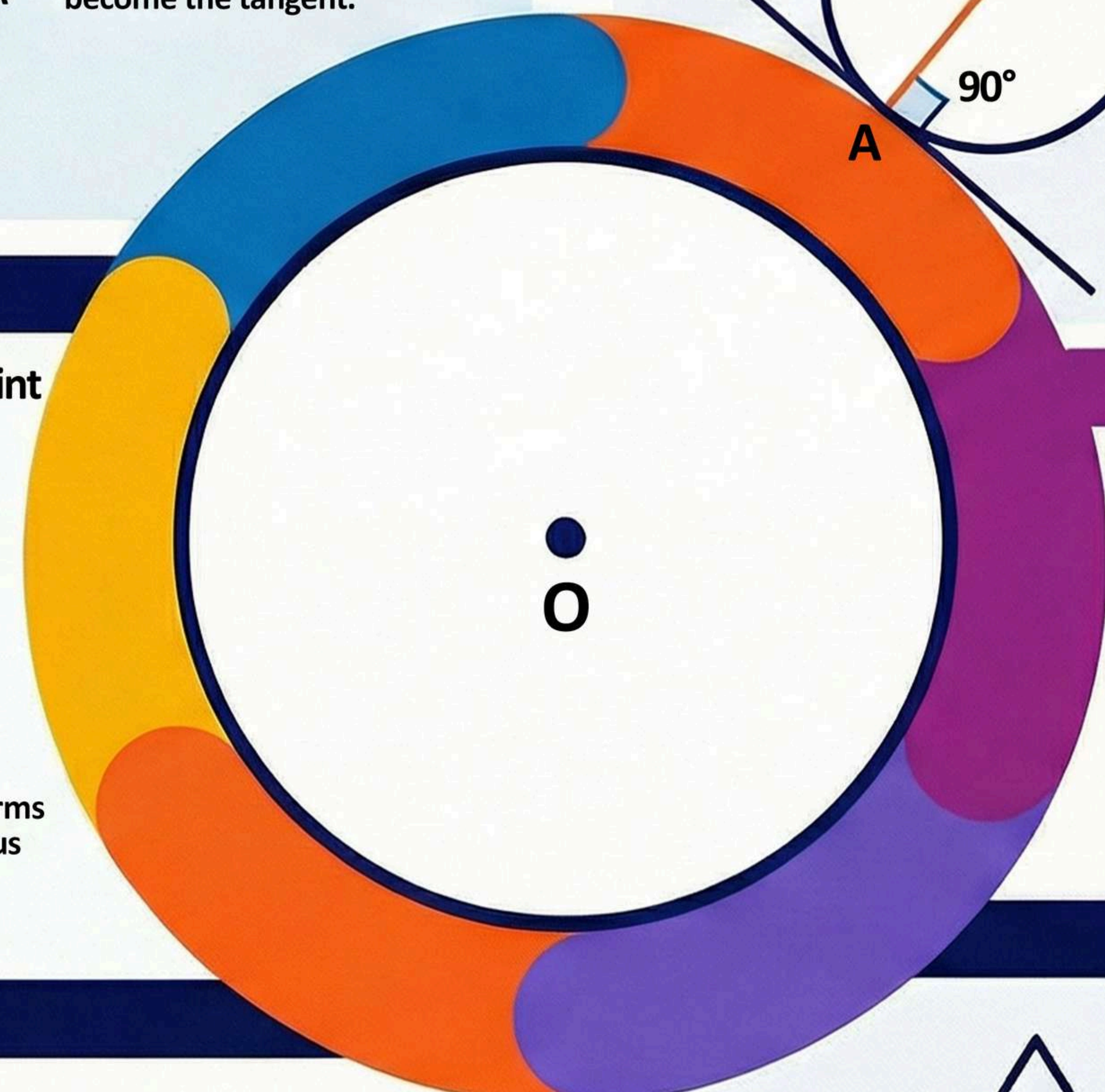
1. What is a Tangent?



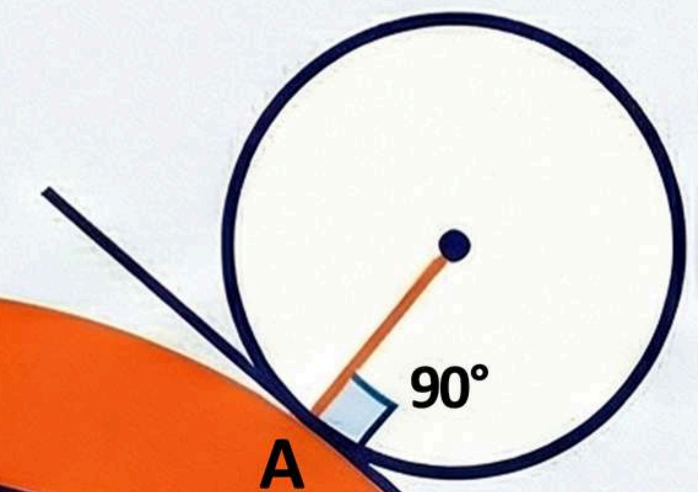
A tangent is a line that touches a circle at exactly one point.
The single point is the point of tangency.



A tangent is the limit of a secant.
As X gets infinitely close to A, the secant line pivots to become the tangent.

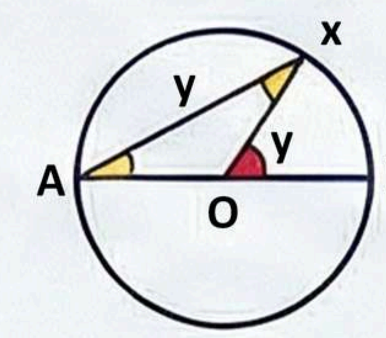


2. The Fundamental Theorem: Tangent & Radius



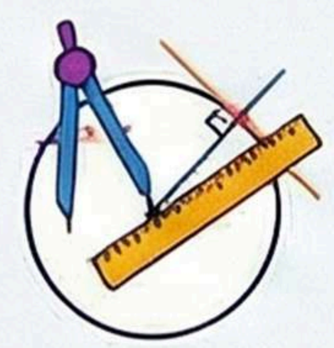
A tangent is perpendicular to the radius at the point of contact.
The angle between the tangent and the radius at the point of tangency is always 90°.

Proof Insight: The angle approaches 90°



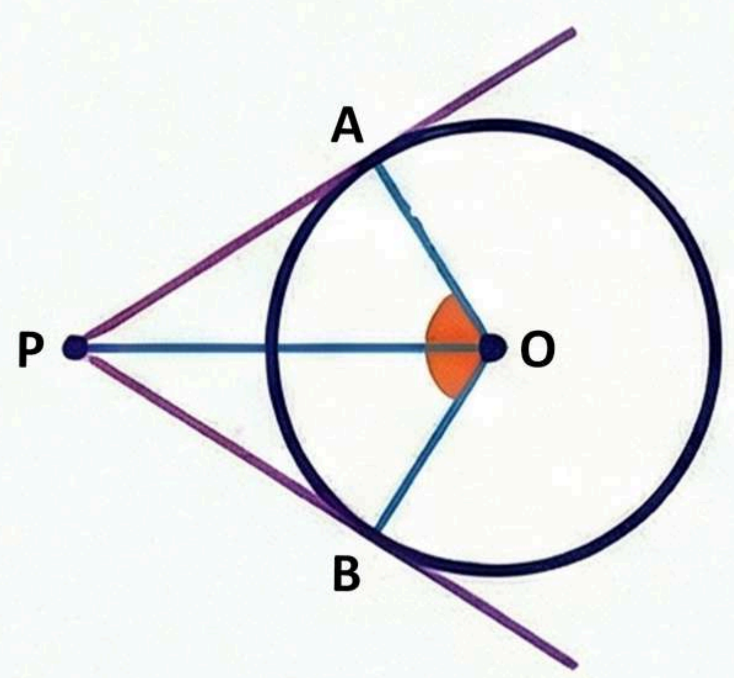
$$x = 90^\circ - \frac{y^\circ}{2}$$

As y approaches 0°, x becomes exactly 90°.



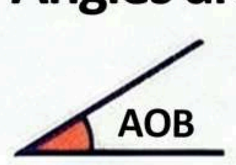
Application: How to Construct a Tangent
Draw radius to point, construct perpendicular line at that point.

3. Tangents From an External Point



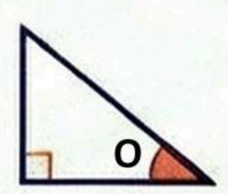
Two tangents from an external point have equal length.
If two tangents from P touch the circle at A and B, then PA = PB.

Angles are Supplementary



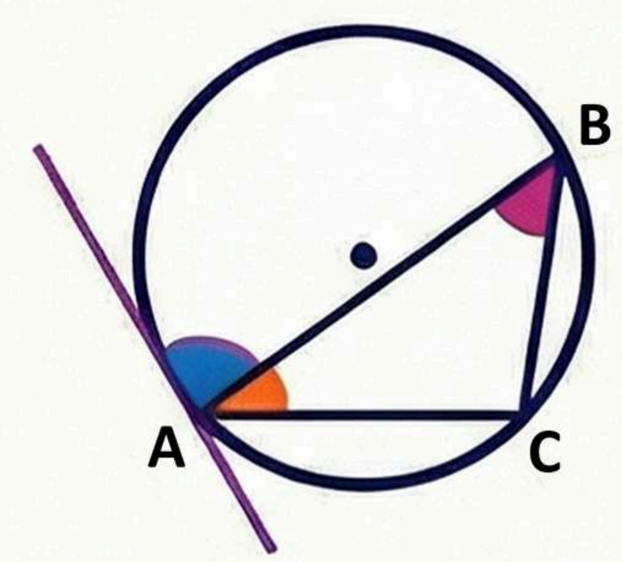
Angle between tangents and central angle of radii add to 180°.

Calculating Tangent Length



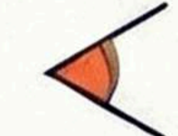
Use Pythagorean theorem: Forms right-angled triangle with radius and line connecting center to external point.

4. Tangents and Chords (Alternate Segment Theorem)



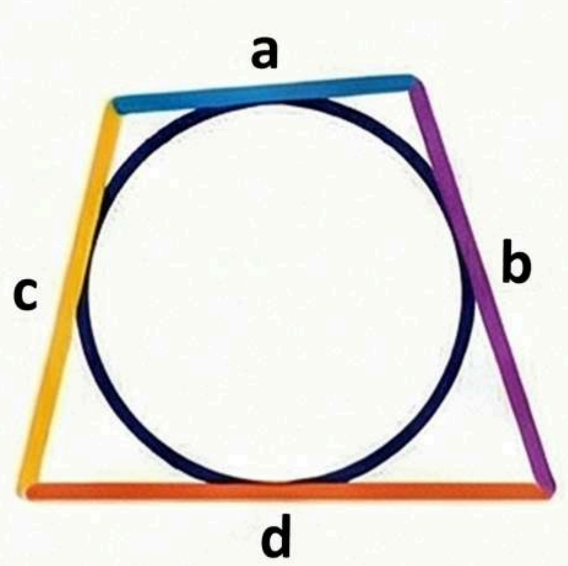
The angle between a tangent and a chord equals the angle in the alternate segment.
Angle between chord and tangent at one endpoint equals the angle subtended by the chord in the alternate segment.

Angle Relationship



Angle between tangent and chord is half the measure of the central angle subtended by the chord.

5. Polygons and Tangents

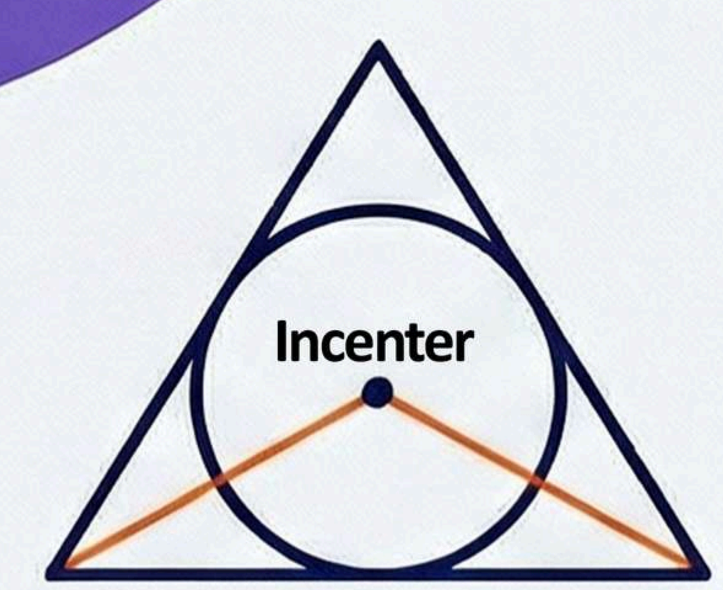


Tangential Quadrilateral Property
For a quadrilateral, whose sides are tangent to a circle, sum of opposite sides are equal.

$$a + c = b + d$$

Sum of first and third sides equals the sum of the second and fourth sides.

6. The Incircle of a Triangle

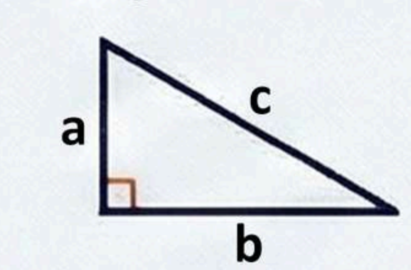


The incircle is a circle inside a triangle that is tangent to all three sides.
Its center, the incenter, is the point where the angle bisectors intersect

Radius of the Incircle

$$r = \frac{A}{S}$$

r = Radius
A = Area of Triangle
S = Semi-perimeter ((a+b+c)/2)
This formula connects the incircle's radius directly to the triangle's overall dimensions.



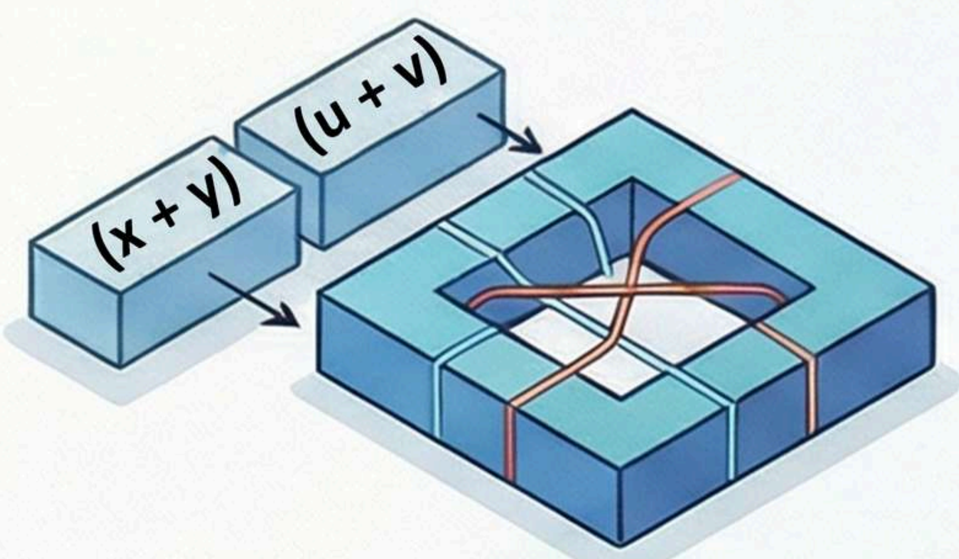
Inradius of a Right-Angled Triangle

$$r = \frac{a+b+c}{2}$$

POLYNOMIALS AND QUADRATIC EQUATIONS

A cohesive visual guide on multiplying polynomials, factoring quadratics, and solving quadratic equations, culminating in the general formula

1 The Foundation: Multiplying Two Sums



$(x + y)(u + v) = xu + xv + yu + yv$

The general algebraic rule for expanding the product of two binomials.

↓

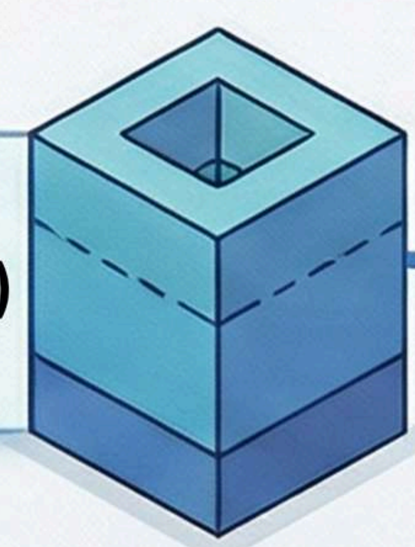
$(x + a)(x + b) = x^2 + (a + b)x + ab$

↓

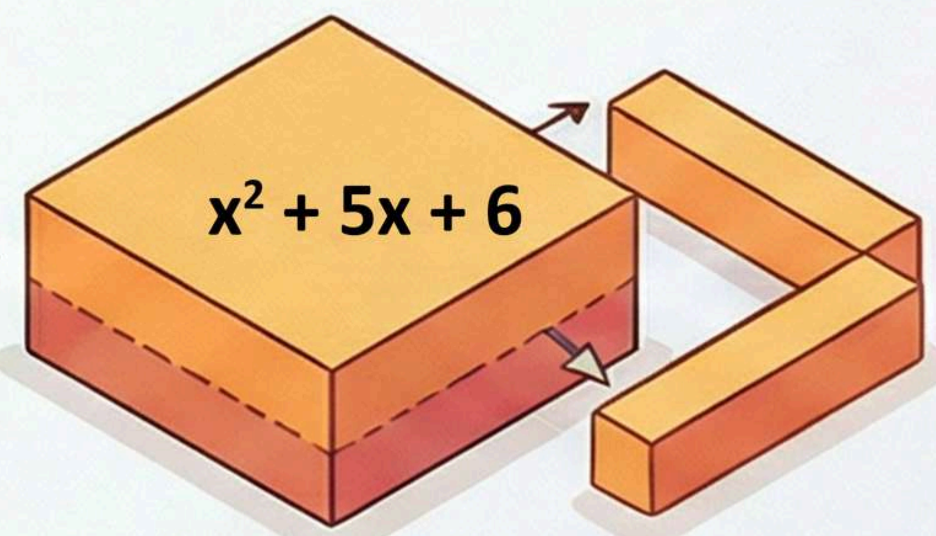
$(x + a)(x + b) = x^2 + (a + b)x + ab$

The coefficient of X is the sum of a and b, and the constant term is their product.

$(x + 5)(x + 3)$
$\rightarrow x^2 + (5 + 3)x + (5*3)$
$\rightarrow x^2 + 8x + 15$



2 Factoring Second-Degree Polynomials (The Reverse Process)



The Goal: Find the Original Factors

Step 1: Identify the Sum and Product

To factor a quadratic $x^2 + (\text{sum})x + (\text{product})$, we need to find two numbers that add up to the 'sum' and multiply to the 'product'.

↓

Step 2: Find the Two Numbers

2 3 Sum of 5 and product of 6. Numbers are 2 and 3.

↓

Step 3: Write the Factored Form

3 → $(x + 2)(x + 3)$

Example with Negatives:
$x^2 - 5x + 6 \rightarrow$ Sum to -5 and multiply to 6
→ Numbers are -2 and -3
→ Factors are $(x - 2)(x - 3)$

3 Solving Equations by Factoring

The Zero Product Property

$(A)(B) = 0$ → $A = 0$ or $B = 0$

If the product of two numbers is zero, at least one must be zero.

Step 1: Set the Equation to Zero

$x^2 - 4x + 3 = 0$

Step 2: Factor the Polynomial

$(x - 1)(x - 3) = 0$

Step 3: Solve for Each Factor

$x - 1 = 0$ $\quad$ $x - 3 = 0$

$x = 1$ $\quad$ $x = 3$

4 General Solution: Quadratic Formula

When Factoring is Hard

For any quadratic equation $ax^2 + bx + c = 0$, there is a universal formula.

$$X = \frac{-b \pm \sqrt{b^2 4ac}}{2a}$$

↓

E.g.: $2x^2 + 3x - 2 = 0$

Step 1: Identify a, b, and c

$a = 2,$ $b = 3,$ $c = -2,$

↓

Step 2: Substitute and Calculate

$$X = \frac{-3 \pm \sqrt{3^2 - 4(2)(-2)}}{2(2)}$$

↓

Step 3: Simplify to Find Solutions

$$X = \frac{-3 \pm \sqrt{9 + 16}}{4} = \frac{-3 \pm \sqrt{25}}{4} = \frac{-3 \pm 5}{4}$$

↓

Two Solutions are

$X = \frac{-3 + 5}{4} = \frac{1}{2}$ $\quad$ $X = \frac{-3 - 5}{4} = -2$

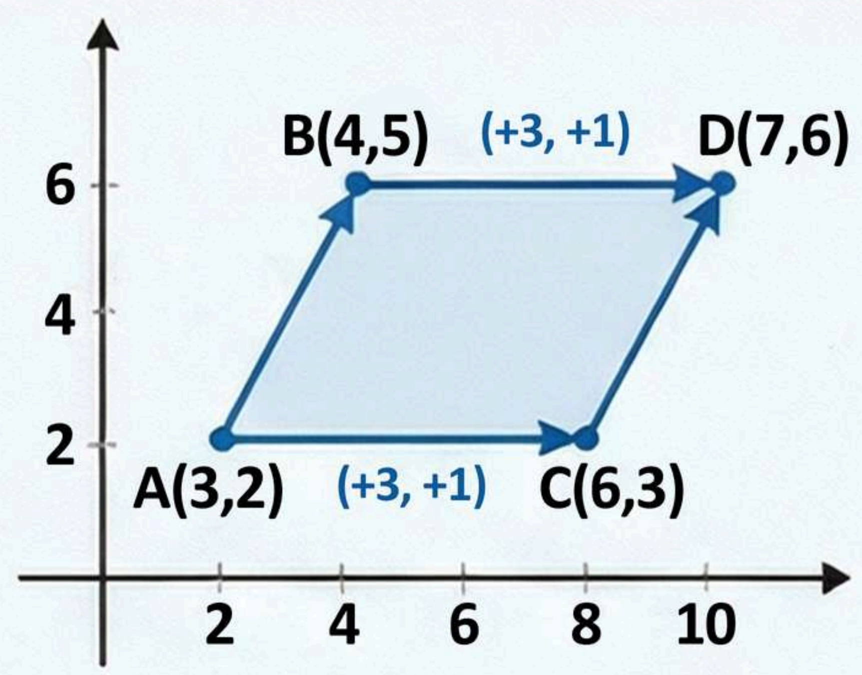
Excellency
The Experience

Subject Expo

CIRCLES AND LINES

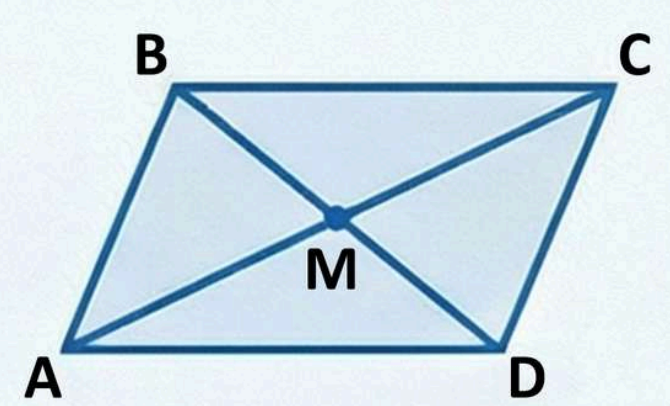
CORDINATE GEOMETRY AND ALGEBRA

Parallelograms on the Coordinate Plane



Parallel sides have equal coordinate shifts.

Example: Finding the 4th Vertex.
Given vertices A(3, 2) B(4, 5) and C(6,3), find D.
Shift from A(3, 2) to C(6,3) is (x+3, y+1).
Apply to B(4, 5) to find D: (4+3, 5+1) = (7,6).

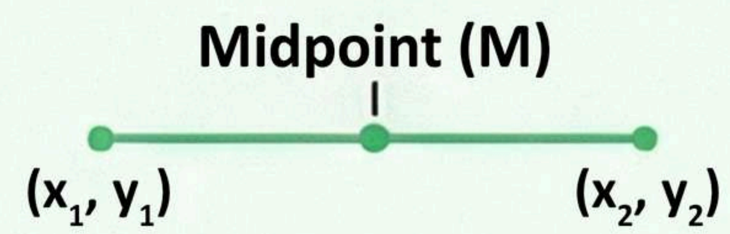


Diagonal Midpoint Property
The diagonals of a parallelogram bisect each other.

Midpoints:

$$\left(\frac{x_1+x_3}{2}, \frac{y_1+y_3}{2}\right) = \left(\frac{x_2+x_4}{2}, \frac{y_2+y_4}{2}\right)$$

The Midpoint Formula

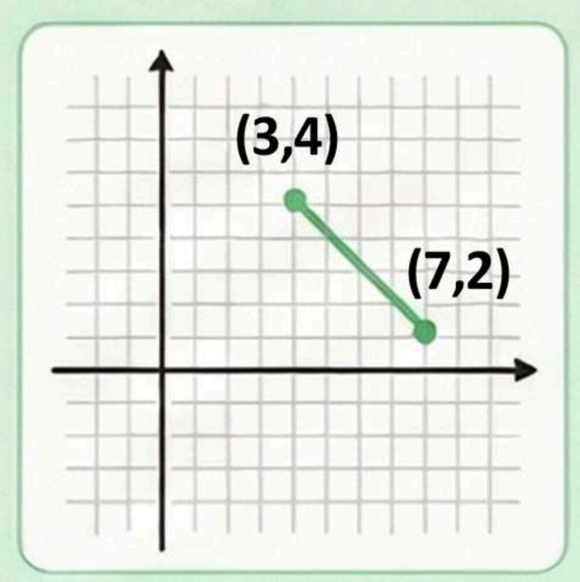


What is the Midpoint?
The point exactly halfway between two endpoints.

Midpoint Formula:

$$M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

Example: Find the midpoint of (3,4) to (7,1).

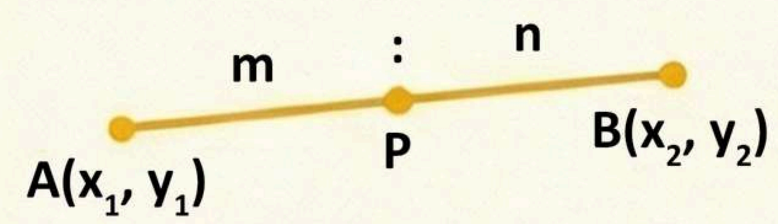


Midpoint

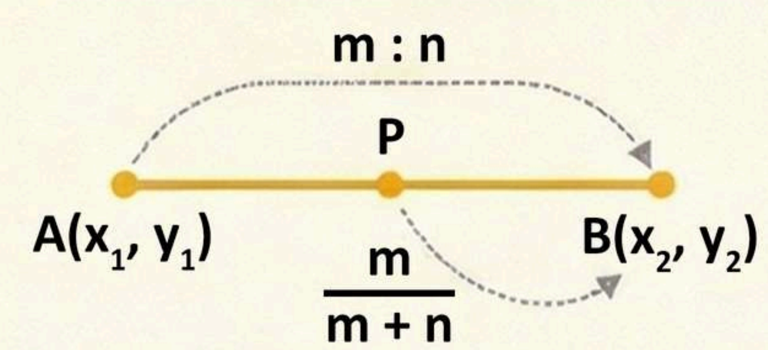
$$= \left(\frac{3+7}{2}, \frac{4+2}{2}\right)$$

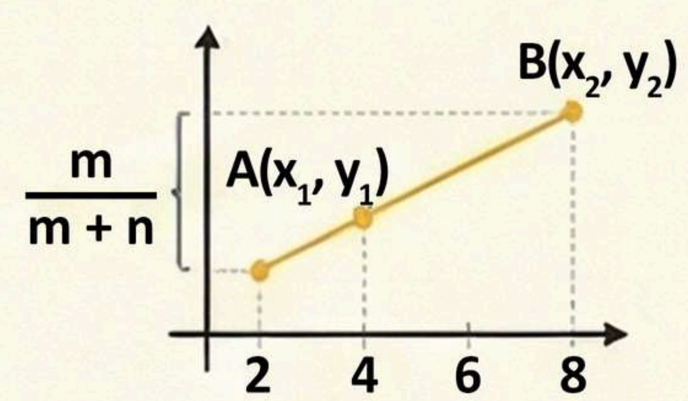
$= (5,3)$.

Dividing a Line Segment in a Ratio



Section Formula:
Finds the point P dividing segment AB in a ratio m:n.



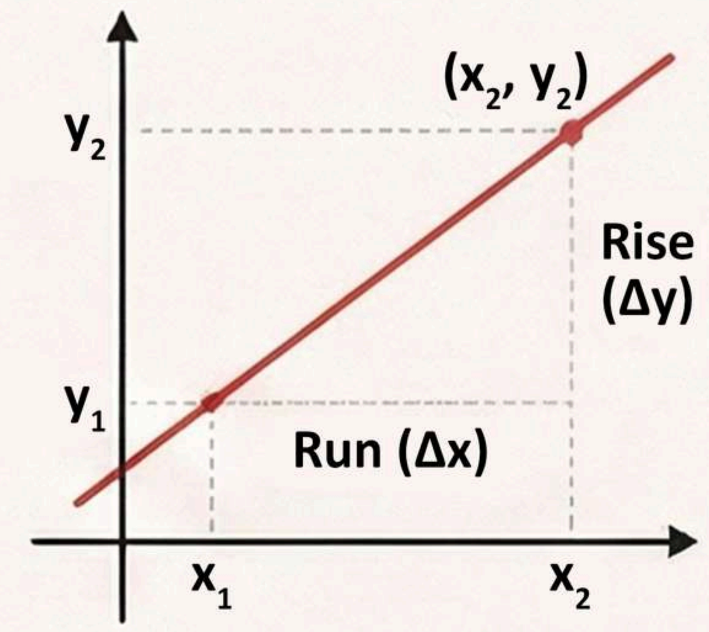


Example: Divide line from (2, 4) to (8, 7) in ratio 1:2.

Point is $\frac{1}{1+2} = \frac{1}{3}$ of way.

Total change in x is 6; in y is 3.
New coords:
x = 2 + (1/3)*6 = 4;
y = 4 + (1/3)*3 = 5.
The point is (4,5).

Slope of a Line

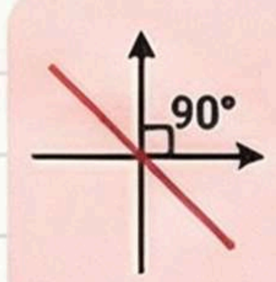


What is Slope?
Measures steepness.
Ratio of vertical change (rise) to horizontal change (run).

Slope (m) Formula:

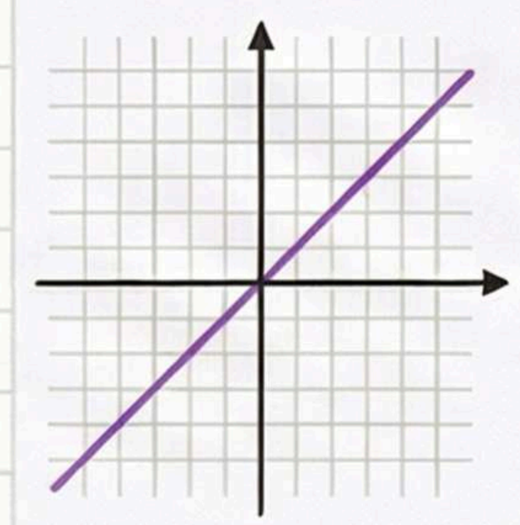
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Headline: How it Works.
Supporting Detail: The change in coordinates from the starting point to the dividing point is a fraction (m/(m + n)) of the total change coordinates between the endpoints.



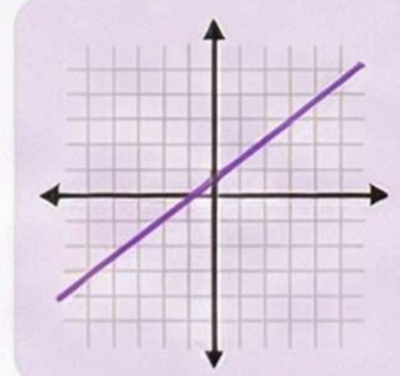
Perpendicular Lines:
Product of slopes is -1
$(m_1 * m_2 = -1)$

Equation of a Line



What is the Equation of a Line?
An algebraic equation true for all (x, y) points on the line.

Point-Slope Form
Formula: $y - y_1 = m(x - x_1)$
(Given (x_1, y_1) and slope m)

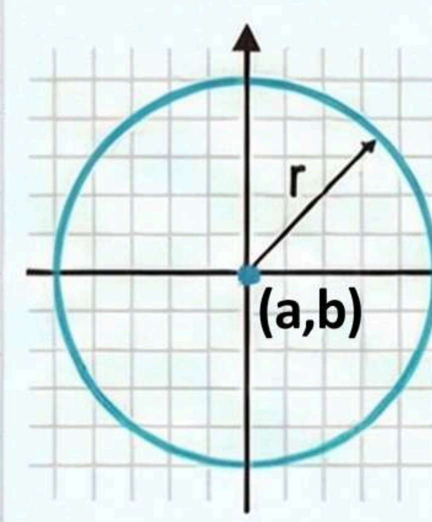


Example: Line through (2, 4) with slope 2/3.

Equation: y - 4 = 2/3 (x - 2)

General form: 2x - 3y + 8= 0

Equation of a Circle



What is a Circle?
All points on a plane at a fixed distance (radius r) from a fixed center (a, b).

Standard Equation of a Circle:

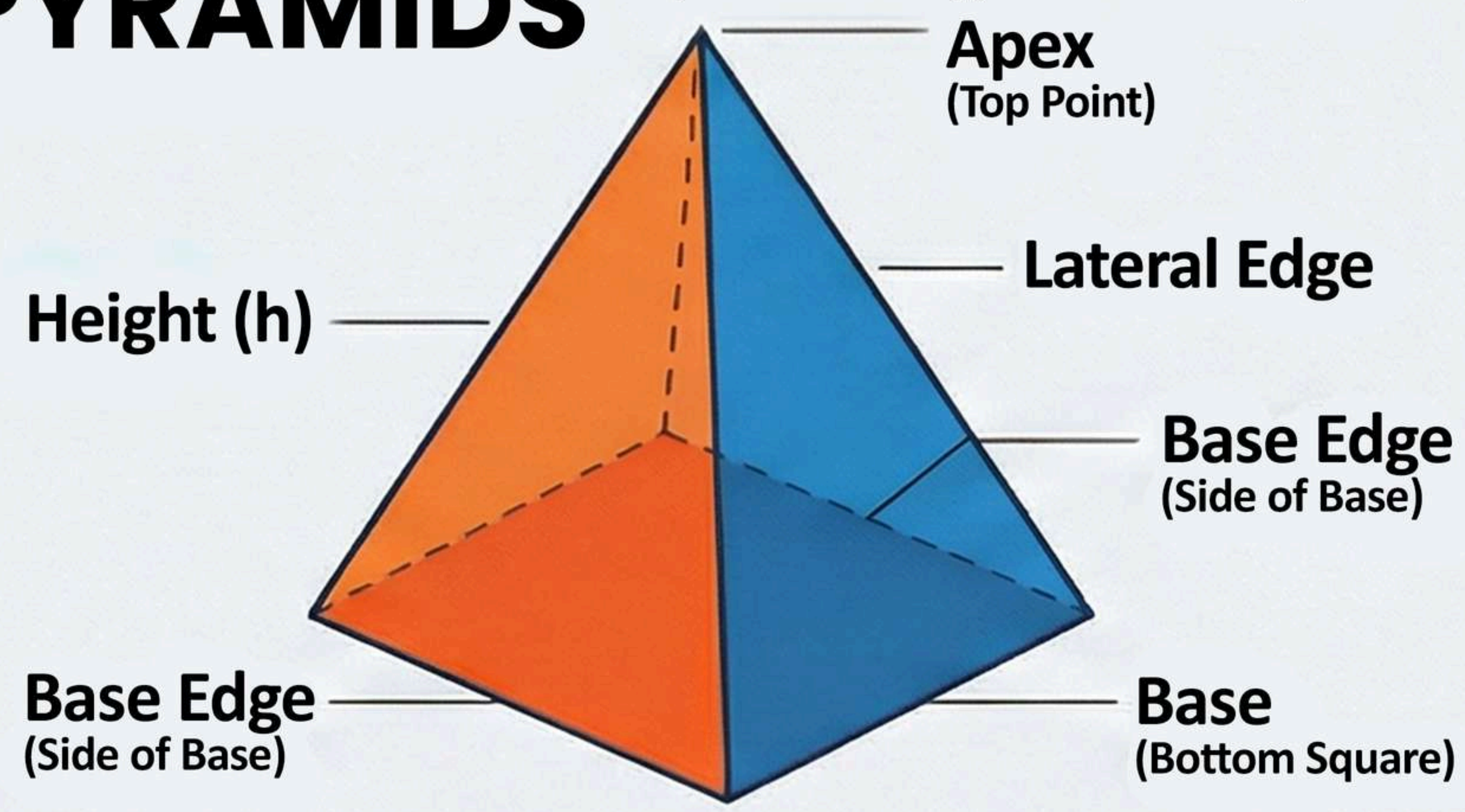
$$(x - a)^2 + (y - b)^2 = r^2$$

Example: Circle with center (1, 4) and radius 2.
Equation: $(x - 1)^2 + (y - 4)^2 = 2^2$ which is $(x - 1)^2 + (y - 4)^2 = 4$.

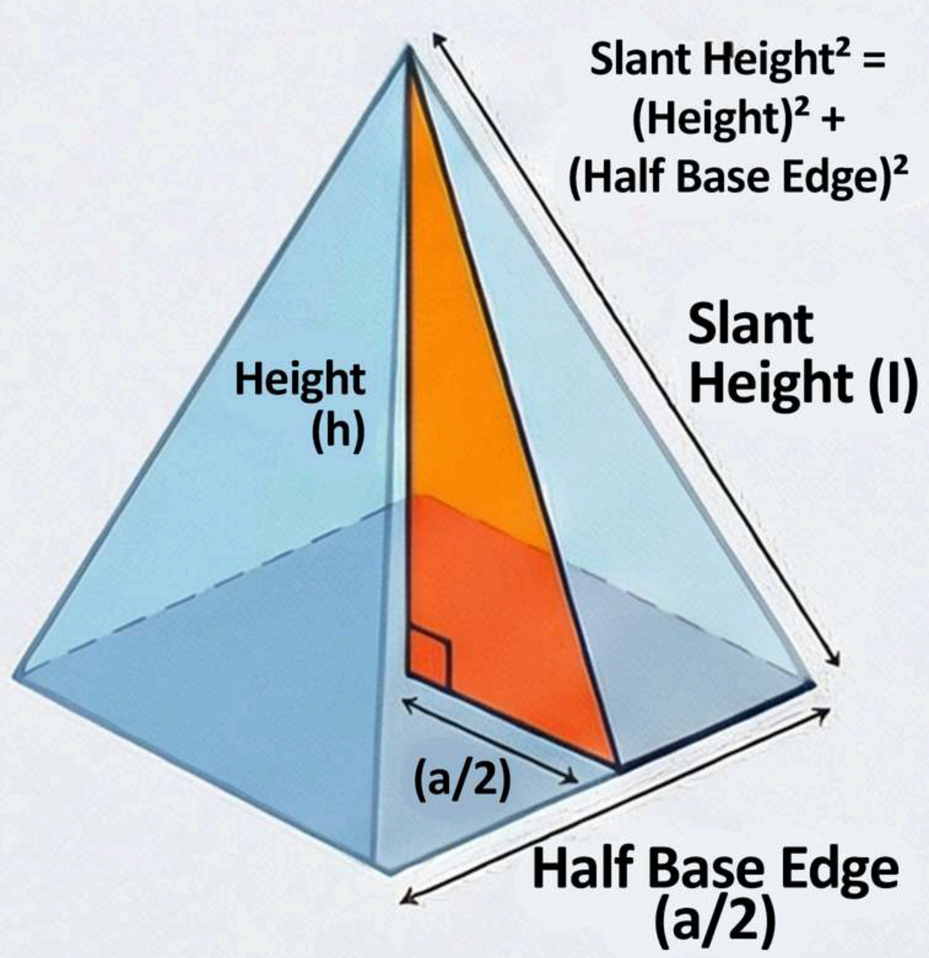
SOLIDS

UNDERSTANDING PYRAMIDS

A solid shape with a polygon base, triangular faces that meet at a single point at the top, known as the apex.

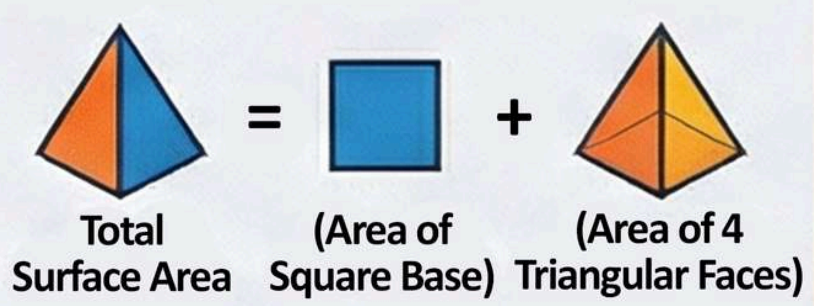


The Hidden Right-Angled Triangle



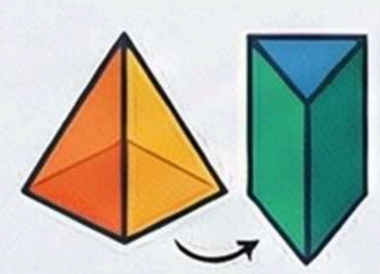
Area & Volume

Surface Area of a Square Pyramid



$A = a^2 + 2al$

Volumeof a Square Pyramid

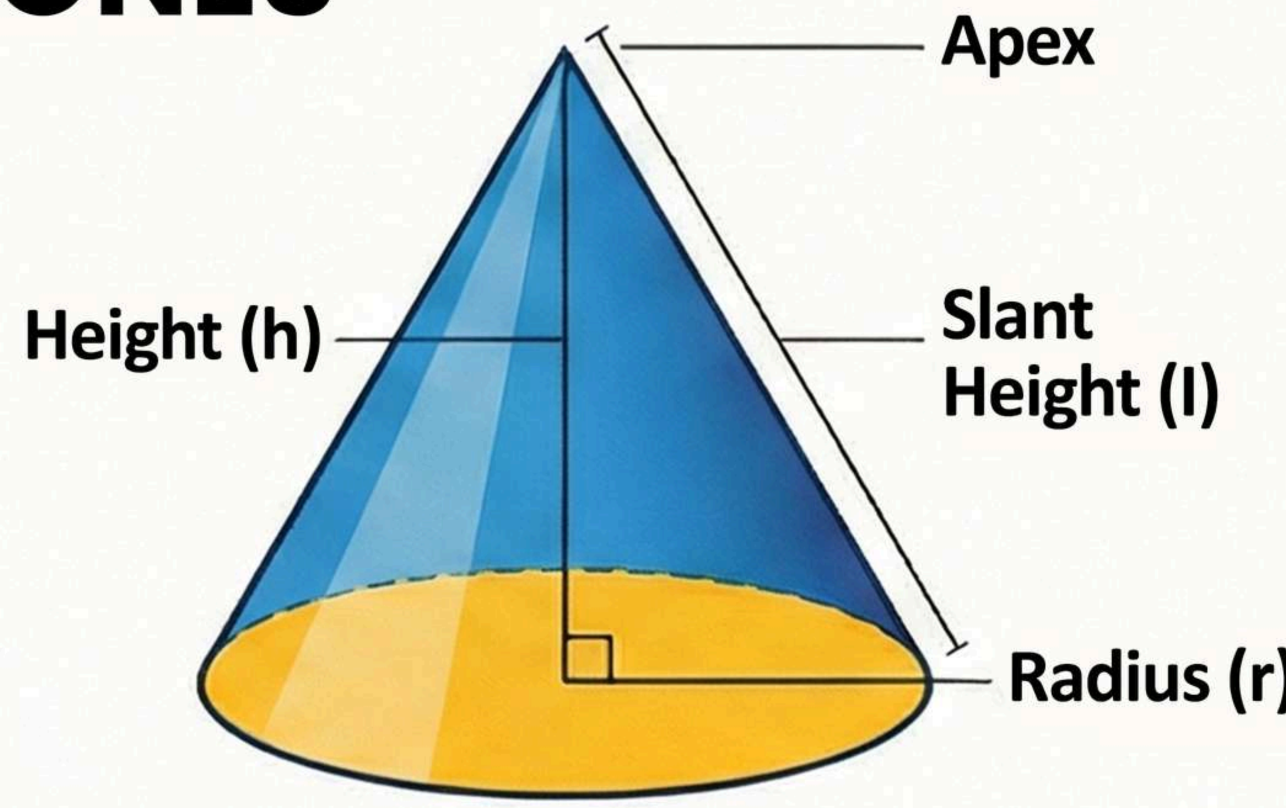


Volume is exactly 1/3 of a prism with the same base and height

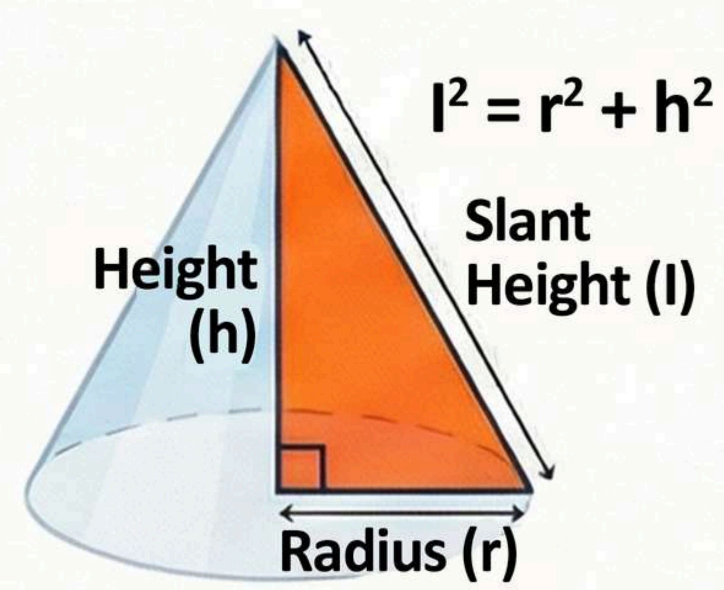
$V = 1/3a^2h$

UNDERSTANDING CONES

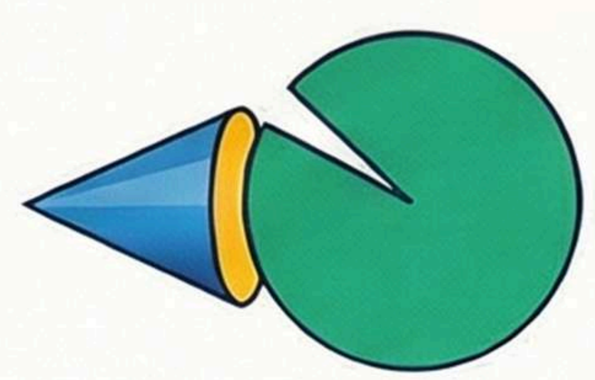
A solid shape with a circular base that tapers to a single point (apex).



The Cone's Right-Angled Triangle



$l^2 = r^2 + h^2$

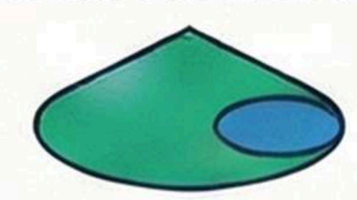


Area & Volume

Curved Surface Area

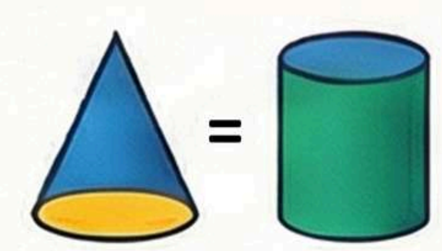
$A = \pi rl$

Total Surface Area



$A = \pi rl + \pi r^2$

Volume of a Cone

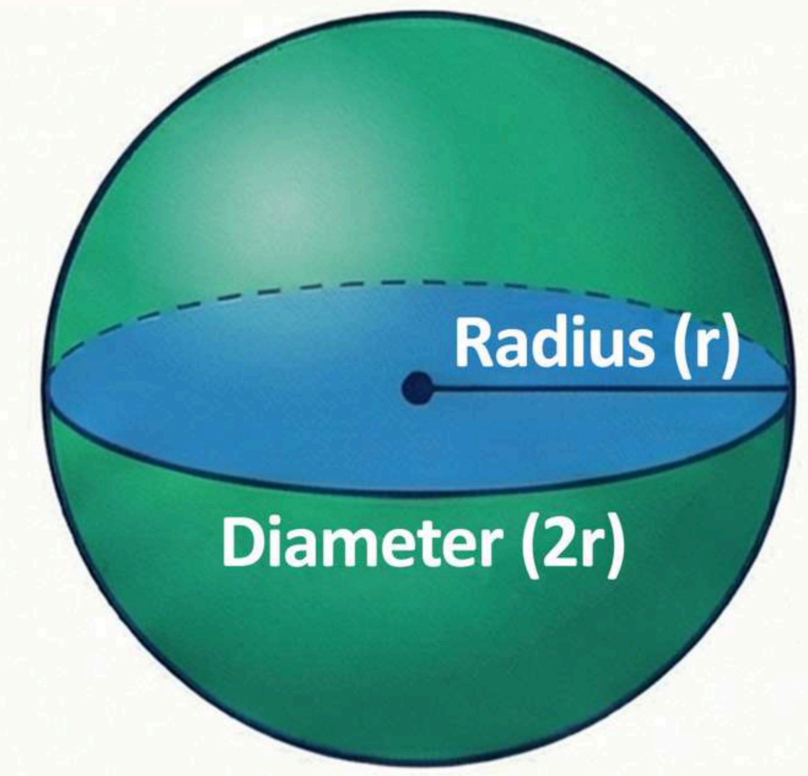


Volume is exactly 1/3 of a prism with the same base and height.

$V = 1/3\ \pi r^2h$

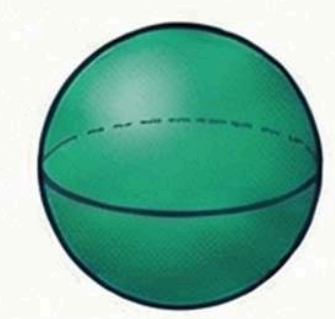
UNDERSTANDING SPHERES

A perfectly round 3D object where every point on its surface is equidistant from its center.



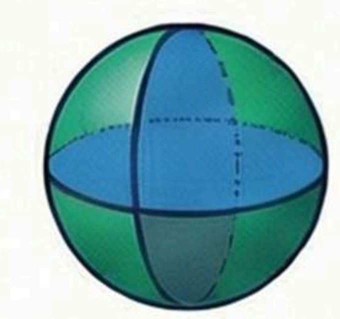
A hemisphere is a sphere cut exactly in half. It consists of a curved surface and a flat circular base.

Surface Area of a Sphere



$A = 4\pi r^2$

Volume of a Sphere



$V = 4/3\pi r^3$

Surface Area of a Hemisphere

= (Curved Area) + (Flat base Area)

$2\pi r^2 + \pi r^2 = 3\pi r^2$

STATISTICS

THE MEAN

THE MEAN: A QUICK REVIEW

What is the Mean?

The mean (or average) is calculated by adding up all the numbers in a dataset and then dividing by the number of values.

Consider 10 families with these monthly incomes (in Rupees):

16500, 17000, 17500, 18000, 18600, 19500, 21000, 21050, 21700, 22000

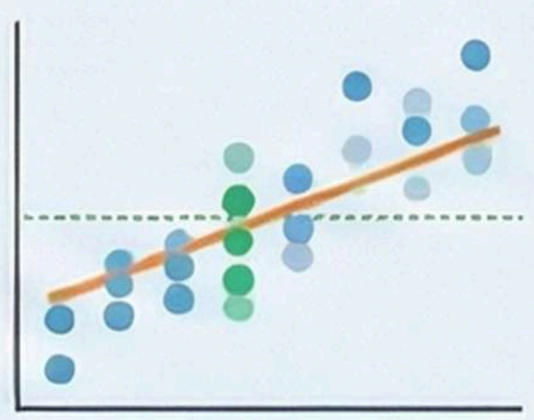
$$\underbrace{\text{16500, 17000, 17500, 18000, 18600, 19500, 21000, 21050, 21700, 22000}}$$

Sum of Incomes ÷ 10 Families

THE MEAN INCOME IS

₹19,285

This value seems to be a fair representation of the group, as most incomes are clustered around this figure.



THE PROBLEM: HOW OUTLIERS SKEW THE MEAN

Now, a new family moves in with a very high monthly income.

A new family with an income of 175,000 (an outlier) joins the original 10 families.

16500, 17000, 17500, 18000, 18600, 19500, 21000, 21050, 21700, 22000, 175,000

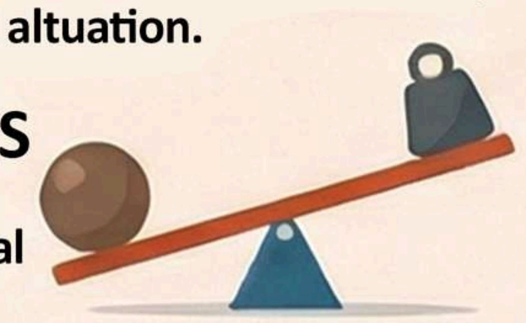
Sum of Incomes ÷ 11 Families

THE NEW MEAN INCOME SKYROCKETS TO ₹33,441!

This new mean is much higher than the income of 10 out of the 11 families, giving a misleading picture of their overall financial altuation.

THE MEAN IS SENSITIVE TO OUTLIERS

A single extreme value can drastically change the mean, making it a poor representation of the typical value in the dataset.



THE MEDIAN

THE SOLUTION: THE ROBUST MEDIAN

What is the Median?

The median is the middle value of a dataset when the numbers are arranged in ascending or descending order.

Let's find the median for the 11 families.

First, arrange all 11 incomes in order:

16500, 17000, 17500, 18000, 18600, 19500, 21000, 21050, 21700, 22000, 175000

THE MEDIAN INCOME IS

₹19,500

This value is a much better representation of the group's typical income, as 5 families earn less and 5 families earn more. The extreme outlier of ₹175,000 did not significantly effect it.

HOW TO CALCULATE THE MEDIAN: AN EXAM CUIDE

CALCULATING MEDIAN FROM A FREQUENCY TABLE

This table shows the hemoglobin levels of 25 children.
We need to find the median level.

Hemoglobin (g/dL)	Frequency	Cumulative Freq.
12.0	2	2
12.4	3	5
12.7	5	10
13.1	6	16
13.3	4	20
13.6	3	23
14.0	2	25

Find the Middle Position

With 25 children, the middle position is the 12th child.

Locate the Position in the Table

The 1st-10th children have levels up to 12.7 g/dL. The 11th through 16th children are in the next group.

THE MEDIAN IS 13.1 g/dL

Since the 18th child falls into the group with a hemoglobin level of 13.1 g/dL, this is the median value.

Step 1: Always Arrange Data in Order

Before finding the median, you must sort the data from smallest to largest.

If you have an ODD number of values...
The median is the single middle number.

Value 1 | Value 2 | MEDIAN | Value 4 | Value 5

MEDIAN

(Example: For 11 values, the median is the 6th value).

If you have an EVEN number of values...
The median is the mean of the two middle numbers.

Value 1 | Value 2 | Value 3 | Value 4

(Value 2 + Value 3)/2

(Example: For the original 10 families, the middle values are 18800 and 19500. The median is (18800 + 19500)/2 = 19060).

CALCULATING MEDIAN FROM A GROUPED FREQUENCY TABLE

This table shows the daily wages of 41
We need to find the median wage.

Daily Wage (Rupees)	Frequency	Cumulative Freq.
400 - 500	6	6
500 - 600	7	13
600 - 700	10	23
700 - 800	9	32
800 - 900	5	37
900 - 1000	4	41

1. Find the Middle Position

For 41 workers, the middle person is the 21st worker.

2. Identify the Median Class

Using the cumulative frequency, we use the 14th to 25th workers are in the 800-700 wage class. Therefore, the 21st worker is in this class.

3. Calculate the Exact Median

Accoming workers is the median class are evenly apraad, we find the 21st marker's estimated wage. The 14th worker is at the start (₹600) and the 25th is at the end (₹700). The 21st worker is the 8th person in this group of 10. A simplified calculation estimates the median wage to be ₹675.