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## Assignment 2

### Question 1:

In the algorithm we will first sort red colored sticks and blue colored sticks from minimum to maximum and we will pair up the sticks according to their sorted order meaning we'll take blue stick number 1 and pair it with red stick number 1, then take blue stick number 2 and pair it up with stick number 2 and so on. This will ensure that the average length difference is minimum.

Yes it does.

Consider  $X$  as the set with pairs formed by the algorithm above and  $O$  the set of pairs formed with an optimal solution.

If  $X$  is not equal to  $O$  then there exists a pair  $(ORI, OBI)$  that doesn't belong to  $X$ . Instead two different pairs exist in  $A$ ,  $(ORI, OBJ)$  and  $(ORH, OBI)$ .

There will be two cases:

Either the difference between  $O1$  is greater than or equal to either of the pairs in  $X$ . Which technically makes  $X$  a better performing algorithm or the same as the optimal solution.

Or

The Difference of  $(ORI, OBI)$  is lesser than one of the two pairs. Then the pair of  $ORH$  or  $OBJ$  in  $O$  will have a bigger difference than the pairs of  $X$ , which will make  $X$ 's answer more optimal than  $O$  as the sum of difference will be lower. Also there will eventually come a pair  $(XRX, XBY)$  in  $A$  which will have a significantly lower difference than the corresponding sticks in  $O$ . Since if  $XRX$  increases then  $XBY$  also increases as the pairs were made by sorted lists. This shows that eventually the difference will be compensated in the pairs of  $O$ .

Hence proven.

### Question 2:

In the algorithm, the first step is filling the tank to the full. Then move to next destination and decrease fuel accordingly. In station check if tank has capacity to go to the next station if it does move to next station. If it does not fill tank until capacity to reach next station, fill tank so that it reaches hospital or if hospital distance is more than max capacity then fill it up full. Repeat until hospital is reached.

Let there be a station  $O$  where the car stops which is more optimal than the location stopped in  $A$ . The location  $O$  can be either of the two:

$O$  is reached after  $A$ . This contradicts our algorithm as in our algorithm the car only stops when the next destination is not reachable. So  $O$  would have only been made reachable if the car in Optimal solution already stopped before to fuel up. Which makes time taken to reach lesser than  $O$ , making  $A$  a more optimal location to stop at as compared to  $O$ .

or

$O$  is reached before  $A$ , which makes  $O$  less optimal than  $A$  as the car stopping at  $O$  increases time while at  $A$  you don't stop at all. Making  $A$  more optimal than  $O$ .

This proves that the algorithm is optimal.