

For the implementation 1 the function  $T$  is using recursion

So let,

$$T(n) = 2T(n-1) + c$$

$$T(n) = T(n-1) + T(n-2) + c$$

$$T(0) = T(1) = 1$$

$$T(n-2) \approx T(n-1)$$

$$T(n) = 2T(n-1) + c$$

$$= 4T(n-2) + 3c$$

$$= 8T(n-3) + 7c$$

$$\vdots \quad \vdots \quad \vdots$$

$$= 2^k T(n-k) + (2^k - 1)c \quad \rightarrow \textcircled{i}$$

Now

$$n-k=0$$

$$n=k$$

eqn (i) becomes

$$T(n) = 2^n T(0) + (2^n - 1)c$$

$$T(n) = 2^n (1+c) - c$$

$$T(n) \approx 2^n$$

So, for the implementation 1 the time complexity is  $O(2^n)$ .

This function is exponential function and have a bad time complexity.

The implementation 2 is using simple loop for loop & memoization method.

let's take time complexity of  $n$  to be  $T(n)$

① hence  $T(n) = T(n-1) + T(n-2)$ .

This is because  $T(n-2)$  is in the cache

When we calculate  $T(n-1)$ , so the operation of

$$T(n-1) \text{ is } 1, \text{ so } T(n) = T(n-1) + 1$$

$$= T(n-2) + 2 = T(n-n) + n.$$

And  $T(0)$  is 1, so

$$T(n) = O(n+1)$$

$$T(n) = O(n)$$

This is linear function so that why implementation 2 has better time complexity than implementation 1.