<u>Assignment 03</u>

<u>MAT120</u>

MATHEMATICS II: INTEGRAL CALCULUS & DIFFERENTIAL EQUATION

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Section = 04

Set = 04

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1. Evaluate $\displaystyle\int_0^2\int_{-1}^{\sqrt{4-x^2}}\int_{-5+x^2+y^2}^{3-x^2-y^2} x\,dz\,dy\,dx$

Solution:-

$$= \int_0^2\int_{-1}^{\sqrt{4-x^2}}\int_{-5+x^2+y^2}^{3-x^2-y^2} x\,dz\,dy\,dx$$

$$= \int_0^2\int_{-1}^{\sqrt{4-x^2}} \Big[xz\Big]_{-5+x^2+y^2}^{3-x^2-y^2} dy\,dx$$

$$= \int_0^2\int_{-1}^{\sqrt{4-x^2}} \Big[3x - x^3 - xy^2 + 5x - x^3 - xy^2\Big]\,dy\,dx$$

$$= \int_0^2\int_{-1}^{\sqrt{4-x^2}} \left(-2x^3 - 2xy^2 + 8x\right) dy\,dx$$

$$= \int_0^2 \left[-2x^3y - \frac{2xy^3}{3} + 8xy\right]_{-1}^{4-x^2} dx$$

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$$= \int_0^2 \left[\left(-2x^3\sqrt{4-x^2} - \frac{2x(4-x^2)^{3/2}}{3} + 8x\sqrt{4-x^2} \right) \right.$$

$$\left. - \left(+2x^3 - \frac{2x}{3} - 8x \right) \right]$$

$$= \int_0^2 \left(-2x^3\sqrt{4-x^2} - 2x^3 + 8x\sqrt{4-x^2} \right.$$

$$\left. + 8x - \frac{2x\left((4-x^2)^{3/2}+1\right)}{3} \right) dx$$

$$= \left[\int_0^2 -2x^3\sqrt{4-x^2}\, dx - \int_0^2 -2x^3 + \int_0^2 8x\sqrt{4-x^2} \right.$$

$$\left. + \int_0^2 8x - \int_0^2 \frac{2x\left((4-x^2)\right)^{3/2}+1}{3}\, dx \right]$$

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$$= -\frac{128}{15} + \frac{64}{3} - 8$$

$$+ 16 - \frac{28}{5}$$

$$= \frac{76}{5} = 15.2$$

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2. Use the transformation $u = x - 2y$, $v = 2x + y$ to find $\iint_R \frac{x-2y}{2x+y}\, dA$ where R is the rectangular region enclosed by the linear $x-2y=1$, $x-2y=4$, $2x+y=1$, $2x+2y=3$.

Solution:

R is the rectangular region enclosed by the linear

$x-2y=1$, $x-2y=4$, $2x+y=1$, $2x+2y=3$

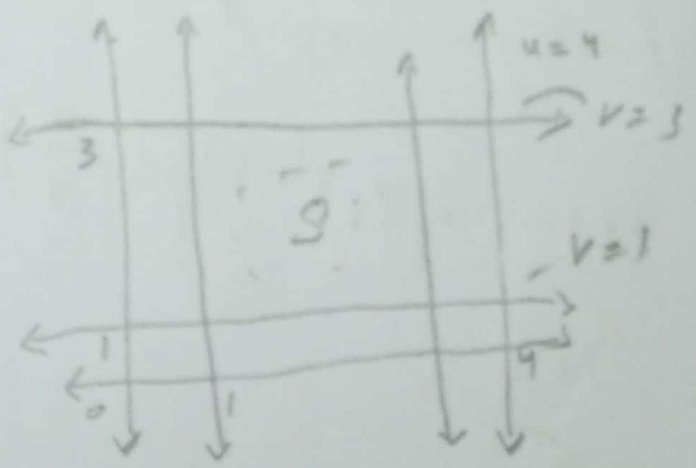
The interpretation given transformation

$U = x - 2y$ —— (i)

$V = 2x + y$ ——> (ii)

$x - 2y = 1$ $\quad$ $x-2y=4$ $\quad$ $2x+y=1$ $\quad$ $2x+y=3$

$\Rightarrow u = 1$ $\quad$ $\Rightarrow u = 4$ $\quad$ $\Rightarrow V = 1$ $\quad$ $\Rightarrow V = 3$



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$S = \{(u,v) : 1 \le u \le 4,\ 1 \le v \le 3\}$ --- (iii)

Now Jacobian

$$\frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix}$$

$$= \begin{vmatrix} \frac{\partial}{\partial x}(x-2y) & \frac{\partial}{\partial y}(x-2y) \\ \frac{\partial}{\partial x}(2x+y) & \frac{\partial}{\partial y}(2x+y) \end{vmatrix}$$

$$= \begin{vmatrix} 1-0 & 0-2 \\ 2+0 & 0+1 \end{vmatrix}$$

$$= 1 \times 1 - (-2)(2)$$

$$= 1+4$$

$$= 5$$

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Jacobian of the given transformation

$$\frac{\partial(x,y)}{\partial(u,v)} = \left|\left(\frac{\partial(u,v)}{\partial(x,y)}\right)^{-1}\right| = \left|5^{-1}\right|$$

$$= \left|\frac{1}{5}\right|$$

$$= 1/5$$

Hence Differential is

$$dA = \frac{1}{5}\,du\,dv \quad \text{—— (iv)}$$

$$\Rightarrow \iint_R \frac{x-2y}{2x+y}\,dA$$

$$= \iint_S \frac{u}{v}\cdot\frac{1}{5}\,du\,dv$$

$$= \int_1^3\int_1^4 \frac{1}{5}\left(\frac{u}{v}\right)du\,dv$$

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$$= \frac{1}{5} \int_1^3 \int_1^4 \frac{u}{v} \, du dv$$

$$= \frac{1}{5} \int_1^3 \frac{1}{v} dv \int_1^4 u \, du$$

$$= \frac{1}{5} \ln|v| \Big|_1^3 \frac{u^2}{2} \Big]_1^4$$

$$= \frac{1}{5} \left[\ln(3) - \ln(1) \right] \left[\frac{16}{2} - \frac{1}{2} \right]$$

$$= \frac{1}{5} \left(\ln(3) - 0 \right) \left(8 - \tfrac{1}{2} \right)$$

$$= \tfrac{1}{5} \left(\ln(3) \right) \left(\tfrac{15}{2} \right)$$

$$= \tfrac{3}{2} \ln(3)$$

$$\iint_R \frac{x - 2y}{2x + y} \, dA = \frac{3}{2} \ln(3)$$

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3. Verify that the Piece-Wise defined function

$$y = \begin{cases} x^2 & x < 0 \\ x-2 & x \geq 0 \end{cases}$$

is a Solution of differential equation $xy' - 2y = 0$ on $(-\infty, \infty)$

Solution :-

if $x < 0$

$$y' = 2x$$

Put in equ $xy' - 2y = 0$

$$x(2x) - 2(x^2) = 0$$

$$2x^2 - 2x^2 = 0$$

$$0 = 0$$

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If $x \geq 0$

$y = x - 2$

$y' = 1$

$xy' - 2y = 0$

$x(1) - 2(x-2) = 0$

$x - 2x + 4 = 0$

$-x + 4 = 0$

Not Solution

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Q4

Solution :-

$$\sqrt{1-y^2}\,dx - \sqrt{1-x^2}\,dy = 0$$

$$-\sqrt{1-x^2}\,dy = -\sqrt{1-y^2}\,dx$$

$$\sqrt{1-x^2}\,dy = \sqrt{1-y^2}\,dx$$

$$\frac{dy}{\sqrt{1-y^2}} = \frac{dx}{\sqrt{1-x^2}}$$

Integrating b/s

$$\int \frac{dy}{\sqrt{1-y^2}} = \int \frac{dx}{\sqrt{1-x^2}}$$

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$$\int \frac{1}{\sqrt{1-y^2}}\,dy = \int \frac{1}{\sqrt{1-x^2}}\,dx$$

$$\sin^{-1}(y) = \sin^{-1}(x) + C$$

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5. Find the Jacobian $J(u,v)$ while $x(u,v) = u^2 - v^2$
$y(u,v) = 2uv$.

Solution:

$$x(u,v) = u^2 - v^2$$

$$y(u,v) = 2uv$$

Jacobian of x,y w.r.t to $J(u,v)$

$$\frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial x}{\partial v} \end{vmatrix}$$

$$= \begin{vmatrix} 2u & -2v \\ 2v & 2u \end{vmatrix}$$

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$$= (2v)(2u) - (-2v)(2v)$$

$$= 4u^2 - (-4v^2)$$

$$= 4u^2 + 4v^2$$

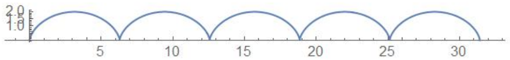
In[53]:=

```
Qoustion(6);

Ans(a);


x = θ - Sin[θ];
y = 1 - Cos[θ];
ParametricPlot[{x, y}, {θ, 0, 10 π}]
```

Out[57]=

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Ans (b);


k = (Abs[D[x, θ] * D[y, {θ, 2}] - D[y, θ] * D[x, {θ, 2}]] / (D[x, θ]^2 + D[y, θ]^2)^(3/2));
k /. θ → π
```

Out[46]= $\frac{1}{4}$

In[47]:=

```
Ans (c);


Clear[x, y]
x = r Cos[θ];
y = r Sin[θ];
k = (Abs[D[x, θ] * D[y, {θ, 2}] - D[y, θ] * D[x, {θ, 2}]] / (D[x, θ]^2 + D[y, θ]^2)^(3/2));
k /. θ → 0 // N
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Out[52]= $\frac{\text{Abs}[r]^2}{(r^2)^{3/2}}$