Measure of Central Tendency

(Measure of Location)

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Content

- Measure of central tendency
  - Mean
  - Median
  - Mode
- Calculation of the measures using raw (ungrouped) and grouped data
- Median like measures- Quartiles, Deciles, Percentiles

- In previous chapter, we have displayed the techniques for **organizing** and **displaying** the data. In this chapter, we introduce the concept of **summarization** of the data by means of **descriptive measures**. A descriptive measure computed from the values of a sample is called a "**statistic**". A descriptive measure computed from the values of a population is called a "**parameter**".

- As we have mentioned earlier, for the variable of interest there are:
- (i) "**N**" population values, and
- (ii) "**n**" sample of values.

- Let be the population values (in general, they are unknown) of the variable of interest where the population size = N and be the sample values (these values are known) where the sample size = n.
- A parameter is a measure (or number) obtained from the population values , and values of the parameters are unknown in general. We are interested to know true values of the parameters.
- A statistic is a measure (or number) obtained from the sample values, and values of statistics can be computed from the samples as functions of observations. Since parameters are unknown, statistics are used to approximate (estimate) parameters.

Measures of central tendency

- In every data set, the data have the tendency to occur mostly in a central location.
- The measures used to find and describe those locations, are collectively known as measures of central tendency
- The measures of central tendency are sometimes referred as the measures of location too.

Necessity of measuring the central tendency

- They give us an idea about the concentration of the values in the central part of the distribution.
- It is the value of the variable which is typical of the whole set.
- It represents all relevant information contained in the data as few numbers as possible.
- They give precise information, not information of a vague general type.
- It avoids confusion between the various uses of words.

Criteria of a good measure of central tendency

- It should be-
- easy to understand
- easy to compute
- based upon all observations
- rigidly defined
- Excessively effected by extreme values
- Suitable for further algebraic treatment
- Less affected by sampling fluctuation

Measures of central tendency

Daily income of 30 respondents-

50, 100, 500, 1000, 400, 100, 200, 500, 200, 500, 800, 900, 700, 500, 600, 450, 600, 500,450, 400, 350, 650, 300, 200, 800, 700, 50, 550, 600, 500

Measures of central tendency

- Measure of central tendency provide a very convenient way of describing a set of scores with a **single number** that describe the **performance of the group**

- It is also defined as a single value that is used to describe the **center of the data**.

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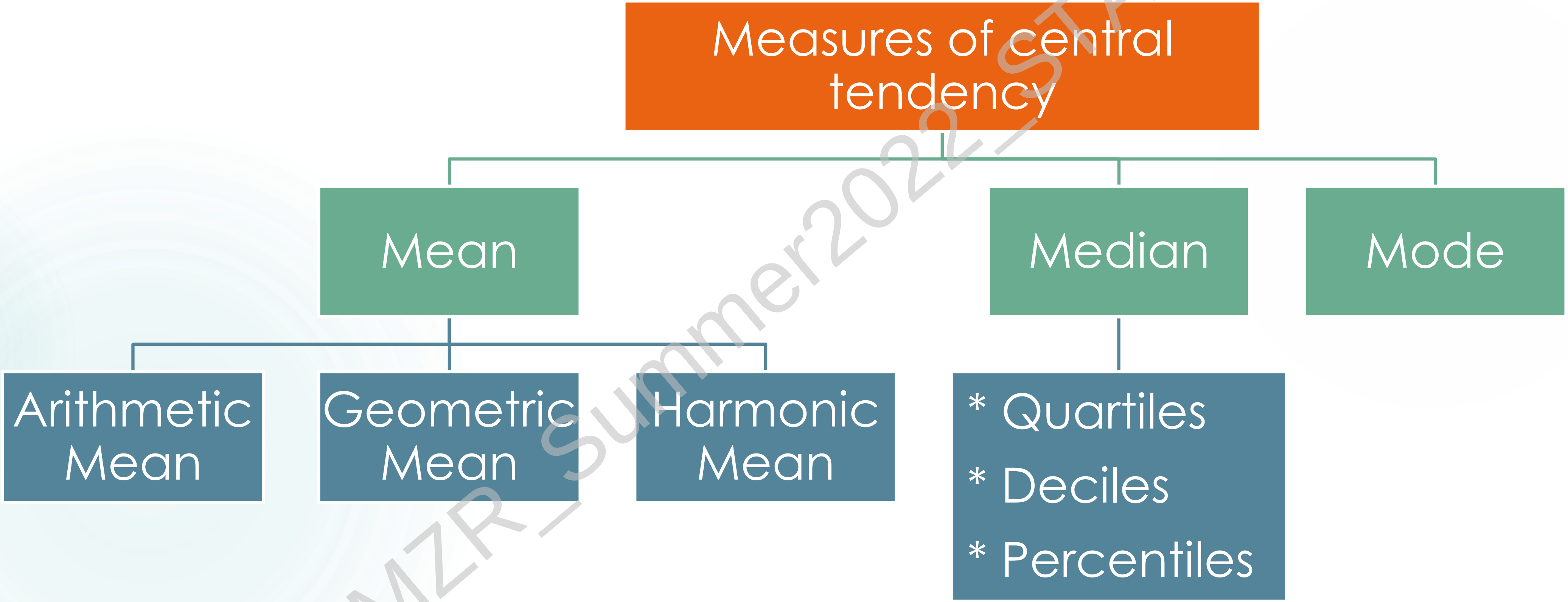
Measures of central tendency



Mean (Arithmetic Mean)

It is simply the sum of the **numbers divided by the number of numbers** in a set of data. This is also known as average.

For example, consider the values-

5, 3, 9, 2, 7, 5, 8

Mean (Arithmetic Mean)

It is simply the sum of the **numbers divided by the number of numbers** in a set of data. This is also known as average.

For example, consider the values-

5, 3, 9, 2, 7, 5, 8

$$Mean = \frac{5+3+9+2+7+5+8}{7}$$

$$= \frac{39}{7} = 5.57\ unit$$

Mean (Arithmetic Mean)

Formulas:

For raw or ungrouped data-

For Population: let, $X_1, X_2, \ldots, X_N$ are values of a variable from a population of size N. Then,

$$Population\ mean, \mu = $$

$$= \frac{X_1 + X_2 + \ \ldots + X_N}{N} = \frac{\sum_{i=1}^{N} X_i}{N}$$

(Parameter)

For Sample: let, $x_1, x_2, \ldots, x_n$ are values of a variable from a sample of size n. Then,

$$Sample\ mean, \bar{x} = $$

$$= \frac{x_1 + x_2 + \ \ldots + x_n}{n} = \frac{\sum_{i=1}^{n} x_i}{n}$$

(Statistic)

Mean (Arithmetic Mean)

Uses of Arithmetic mean:

- It has great importance in the study of social, economical, geographical, engineering & commercial problems such as production, income, price, exports etc.
- We often use the AM in different cases like average income, average price, average consumption etc.

Note:

- Mean cannot be calculated for Nominal & Ordinal level of data
- Mean is easily affected by extreme values
- Difficult to calculate in case of open end classes

Median

Median is the number present in the **middle** when the numbers in a set of data are **arranged in ascending or descending order**. If the number of numbers in a data set is **even**, then the median is the **mean of the two middle numbers**.

For example, consider the values-

5, 3, 9, 2, 7, 5, 8

Median

Organizing in **ascending** order,

2, 3, 5, 5, 7, 8, 9

Median

Organizing in **ascending** order,

$$\underbrace{2,\ 3,\ 5,}_{\text{50\% observations}}\ 5,\ \underbrace{7,\ 8,\ 9}_{\text{50\% observations}}$$

Here, n= 7 (an odd number)

Median = 5

Median

Consider – 3, 5, 5 , 7, 8, 9

Median

Consider – 3, 5, 5 , 7, 8, 9

Organizing in **ascending** order,

3, 5, 5, | 7, 8, 9

50% observations

50% observations

Median

Consider – 3, 5, 5 , 7, 8, 9

Organizing in **ascending** order,

$$\underbrace{3,\ 5,\ 5,}_{\text{50\% observations}}\ |\ \underbrace{7,\ 8,\ 9}_{\text{50\% observations}}$$

Here, n= 6 (an even number)

$$Median = \frac{3rd\ value + 4th\ value}{2}$$

$$= \frac{5+7}{2} = \frac{12}{2} = 6$$

Median

Formulas:

For raw or ungrouped data- (Sample)

For Odd n

$$Median = \left(\frac{n+1}{2}\right)^{th} value$$

For Even n

$$Median = \frac{1}{2}\left[\left(\frac{n}{2}\right)^{th} value + \left(\frac{n}{2}+1\right)^{th} value\right]$$

Median

Note:

- To find median, data has to be at least in ordinal level of measurement

- Median is not affected by extreme values

Mode

Mode is the value that occurs most frequently in a set of data

Notes: Mode can be computed for all levels of data.

For example, consider the values-

5, 3, 9, 2, 7, 5, 8

Mode

consider the values-

5, 3, 9, 2, 7, 5, 8

Mode

consider the values-

5, 3, 9, 2, 7, 5, 8

Value **5** occurred maximum 2 times

Mode = 5

Mode

Formulas:

For raw or ungrouped data-

Find the value occurred most of the times in the data

Example (raw data)

Weekly income of 6 respondents (in taka)-

2500, 3900, 3500, 5000, 4000, 3500

Find Mean, Median and Mode. Interpret the results.

Class task (raw data)

Weekly income of 6 respondents (in taka)-

2500, 3900, 3500, 5000, 4000, 3500

$$\boldsymbol{Mean}, \overline{\boldsymbol{x}} = \frac{2500 + 3900 + 3500 + 5000 + 4000 + 3500}{6} = \frac{22400}{6}$$

$$= 3733.33 \approx 3734 \, taka$$

Interpretation: Average weekly income of the respondents is 3734 taka

Example (raw data)

Weekly income of 6 respondents (in taka)-

2500, 3900, 3500, 5000, 4000, 3500

Organizing the values in ascending order-

2500, 3500, 3500, 3900, 4000, 5000

Example (raw data)

$$\boldsymbol{Median} = \frac{1}{2}\left[\left(\frac{n}{2}\right)^{th} value + \left(\frac{n}{2}+1\right)^{th} value\right]$$

$$= \frac{1}{2}[3^{rd} value + 4^{th} value] = \frac{1}{2}[3500 + 3900] = 3700\ taka$$

Interpretation: 50% of the respondents have weekly income less than or equal to 3700 taka, and 50% of the respondents have weekly income higher than or equal to 3700 taka.

Example (raw data)

Weekly income of 6 respondents (in taka)-

2500, 3900, 3500, 5000, 4000, 3500

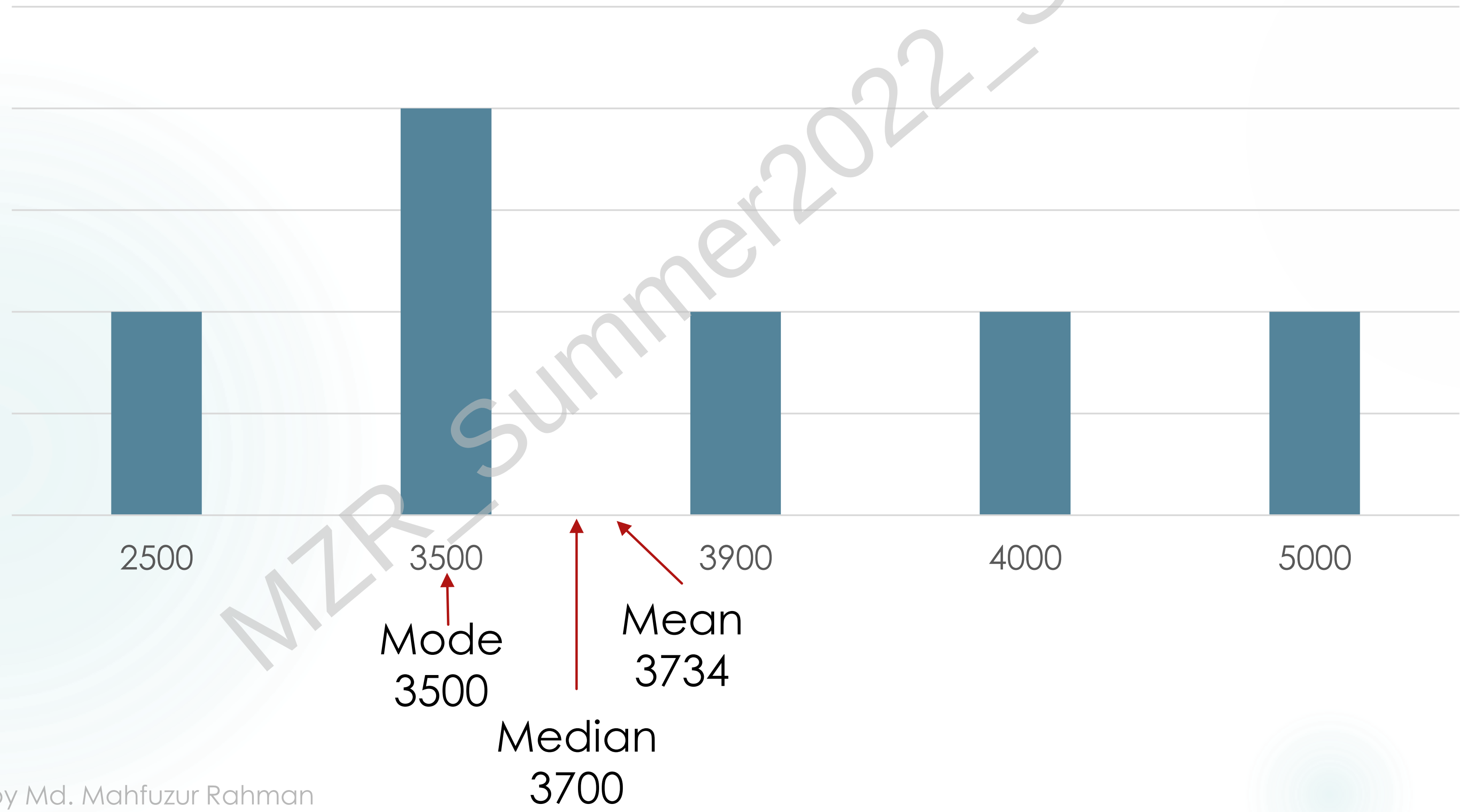
Value 3500 occurs highest 2 times in the data.

So, **Mode** = 3500

Interpretation: In the data, weekly income 3500 taka is occurred highest number of times.

Example (raw data)

In graphs-



Mean (Arithmetic Mean)

Formulas:

For grouped data- (Population)

X_i	f_i
X_1	f_1
X_2	f_2
…	…
X_K	f_K

let, $\mathbf{X_1, X_2, \ldots, X_K}$ are values of a variable from a population of size N and they occurred $\mathbf{f_1, f_2, \ldots, f_K}$ times respectively. Then,

$$Population\ mean, \mu = \frac{\sum_{i=1}^{K} f_i X_i}{N}$$

Mean (Arithmetic Mean)

Formulas:

<u>For grouped data- </u>(Sample)

x_i	f_i
x_1	f_1
x_2	f_2
…	…
x_k	f_k

let, $\mathbf{x_1, x_2, \ldots, x_k}$ are values of a variable from a sample of size n and they occurred $\mathbf{f_1, f_2, \ldots, f_k}$ times respectively. Then,

$$Sample\ mean, \bar{x} = \frac{\sum_{i=1}^{k} f_i x_i}{n}$$

Median

Formulas:

<u>For grouped data- </u>(Sample)

$$Median = L_m + \frac{\frac{n}{2} - F}{f_m} * c$$

Where,

L_m= Lower class limit of the median class

n= Total frequency

F= Cumulative frequency of the pre-median class

f_m= frequency of the median class

c= class interval

Mode

Formulas:

<u>For grouped data-</u>

$$Mode = L_o + \frac{\Delta_1}{\Delta_1 + \Delta_2} * c$$

Mode

Where,

L_o = Lower class limit of the modal class

Δ_1 = Excess of modal frequency over frequency of the next lower class (Pre-modal class) = difference between the frequencies of the modal class and pre-modal class.

Δ_2 = Excess of modal frequency over frequency of the next higher class (Post-modal class) = difference between the frequencies of the modal class and post-modal class.

c= class interval

Mode

- We can also compute mode by the following formula which is based upon an empirical relationship between mean, median and mode :

- **Mode = (3* Median – 2 * Mean)**

Example (grouped data)

Monthly income ('000 tk)	No. of respondents
5-30	7
30-55	10
55-80	6
80-105	4
105-130	3

Find Mean, Median and Mode. Interpret the results.

Example (grouped data)

Monthly income ('000 tk)	No. of respondents (f_i)	Class Midpoint (x_i)	$f_i x_i$
5-30	7	17.5	122.5
30-55	10	42.5	425
55-80	6	67.5	405
80-105	4	92.5	370
105-130	3	117.5	352.5
Total	**30**		**1675**

$$Mean, \bar{x} = \frac{\sum_{i=1}^{k} f_i x_i}{n} = \frac{1675}{30} = 55.83\ (thousand\ taka)$$

Example (grouped data)

Monthly income ('000 tk)	No. of respondents (f_i)	Cumulative frequency
5-30	7	7
30-55	10	17
55-80	6	23
80-105	4	27
105-130	3	30
Total	**30**	

50% of the respondents have monthly family income less than or equal to 50,000 taka and 50% of the respondents have monthly family income higher than or equal to 50,000 taka

$$Median = L_m + \frac{\frac{n}{2} - F}{f_m} * c = 30 + \frac{15 - 7}{10} * 25 = 50\ thousand\ taka$$

Example (grouped data)

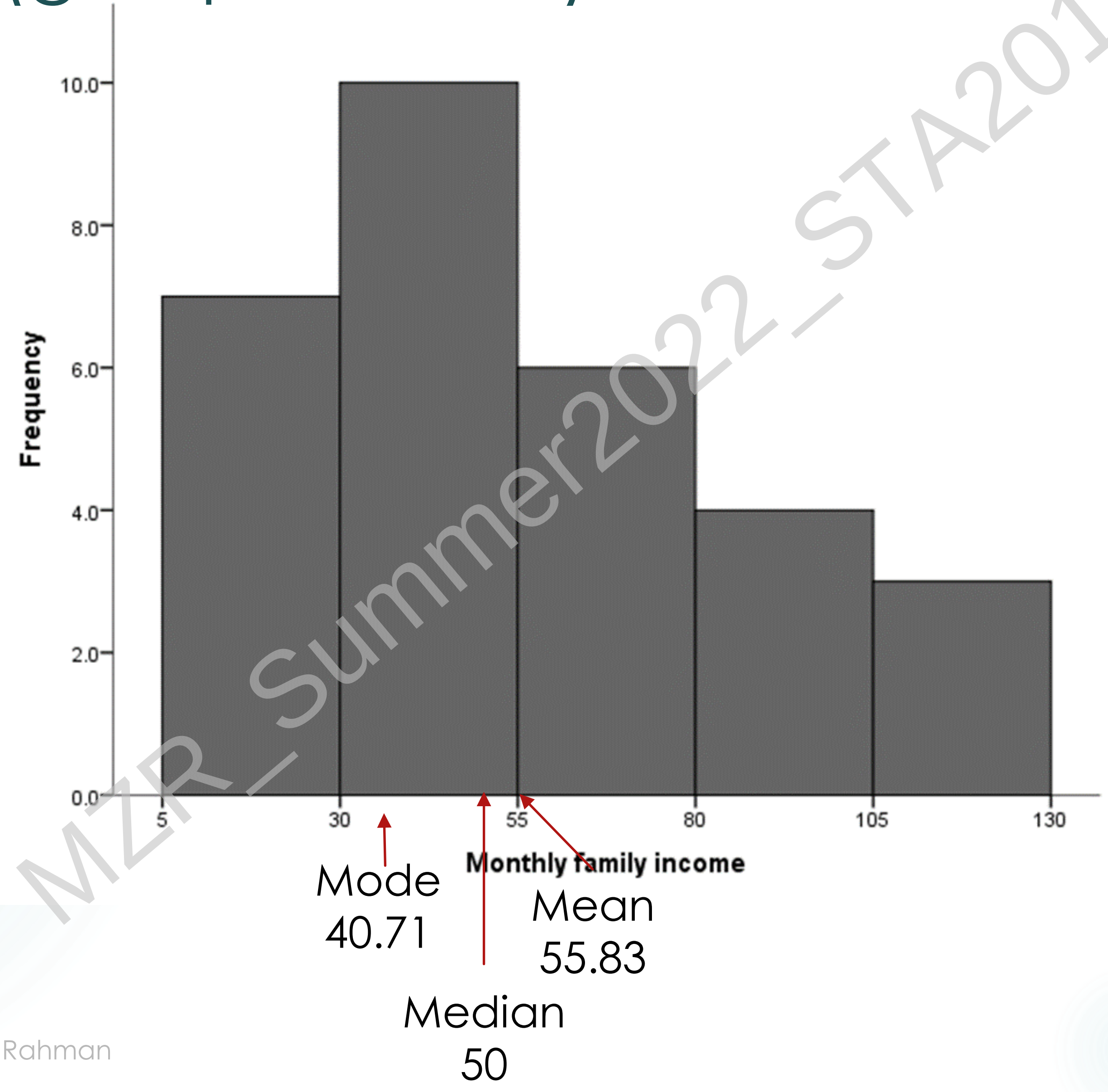
Monthly income ('000 tk)	No. of respondents (f_i)
5-30	7
30-55	10
55-80	6
80-105	4
105-130	3
Total	**30**

Comparatively a higher number of the respondents have monthly family income around 40,000 taka

$$Mode = L_o + \frac{\Delta_1}{\Delta_1 + \Delta_2} * c = 30 + \frac{3}{3+4} * 25 = 40.71\ thousand\ taka$$

Example (grouped data)

In graphs-



Class Task

Question: Below given the frequency distribution of house rents of 70 houses in a city-

Monthly rent (taka)	No. of houses
6000-8000	11 +last 1 digit of ID
8000-10000	14 +last 1 digit of ID
10000-12000	20 +last 1 digit of ID
12000-14000	15 +last 1 digit of ID
14000-16000	10 +last 1 digit of ID

Find mean, median and modal house rent (in taka) in that city. Interpret your result.

Geometric Mean

The geometric mean G of n positive rates x1, x2,…, xn is defined as the nth positive root of the product of the rates. Symbolically,

$$G = (x_1 x_2 x_3 \ldots x_n)^{\frac{1}{n}} - 1$$

Where x_i's are rate of changes.

If x_i's are the value for a given time t, then,

$$G = \sqrt[n]{\frac{x_n}{x_0}} - 1$$

G provides average rates of change.

Geometric Mean

Uses:

- Geometric mean is useful when the data is in geometric progression
- If the observations are ratios, the geometric mean is the appropriate average

Example: At year 2000, the profit was 10000.

Year	Profit
2001	12000
2002	15000
2003	20000
2004	18000
2005	22000
2006	27000

Geometric Mean

Example: At year 2000, the profit was 10000.

Year	Profit	Increase rate	Increased rate (x)
2001	12000	0.20	1.20
2002	15000	0.25	1.25
2003	20000	0.33	1.33
2004	18000	-0.10	0.90
2005	22000	0.22	1.22
2006	27000	0.23	1.23

$$G = (x_1 x_2 x_3 \dots x_n)^{\frac{1}{n}} - 1 = (1.20 * 1.25 * 1.33 \ * 0.90 * 1.22 \ * 1.23)^{\frac{1}{6}} - 1$$
$$= 1.18 - 1 = \ 0.18 = 18\% \ increase\ per\ year.$$

Or, $G = \sqrt[6]{\frac{27000}{10000}} - 1 = 1.18 - 1 = \ 0.18 = 18\% \ increase\ per\ year$

Harmonic Mean

Harmonic mean is defined as the reciprocal of the arithmetic mean of the reciprocals of the individual values.

$$H = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \ldots + \frac{1}{x_n}}$$

For grouped data,

$$H = \frac{\sum f_i}{\frac{f_1}{x_1} + \frac{f_2}{x_2} + \ldots + \frac{f_k}{x_k}} = \frac{n}{\frac{f_1}{x_1} + \frac{f_2}{x_2} + \ldots + \frac{f_k}{x_k}}$$

Harmonic Mean

Uses:

When dealing with rates, harmonic mean is more appropriate.

Example:

Suppose you are travelling to Narayanganj to Dhaka by car. The distance between Dhaka and Naraynganj is 18km. You drive your car for the first 6km at a speed of 30km per hour and the second 5km at a rate of 40km per hour and the remaining 7 kilometers at a speed of 20km per hour. What is the average speed with which you traveled from Dhaka to Narayanganj ?

The others (special) mean are-

- Weighted mean
- Quadratic mean
- Combined mean
- Trimmed mean
- Trimean

The Weighted Mean

- The weighted mean is a special case of the arithmetic mean. It occurs when there are several observations of the same value.
- **To explain:** Suppose the **Shumi's Hot Cake** offers three different kinds of burger packages small, medium and large for Tk. 100, Tk. 125 and Tk. 150. Of the last 10 burgers sold 3 were small, 4 were medium and 3 were large. To find the mean price of the last 10 burger packages sold we can calculate using the usual formula of the arithmetic mean as follows –
- $$\bar{X} = \frac{Tk.(100+100+100+125+125+125+125+150+150+150)}{10} = \frac{Tk.1250}{10} = Tk.125$$

The mean selling price of the last 10 burger packages sold is Tk. 125.

The Weighted Mean

An easier ways to find the mean selling price is to determine the weighted mean. In this method we multiply each observation by the number of times it happens as described below –

$$\bar{X}_w = \frac{(3*100) + (4*125) + (3*150)}{3+4+3} = \frac{1250}{10} = 125$$

In this case the weights are frequency counts. However, any measure of importance could be used as a weight. In general the weighted mean of a set of numbers $X_1, X_2, ..., X_n$ designated with the corresponding weights $W_1, W_2, ..., W_n$ is computed by:

$$\bar{X}_w = \frac{\Sigma(WX)}{\Sigma W} = \frac{W_1X_1 + W_2X_2 + ... + W_nX_n}{W_1 + W_2 + ... + W_n}$$

The Weighted Mean

- **Problem:**

  Madina Construction Company pays its part time employees hourly basis. For different level of employee the hourly rate are Tk. 50, Tk. 75 and Tk. 90. There are 260 hourly employees, 140 of which are paid at Tk. 50 rate, 100 at Tk. 75 and 20 at the Tk. 90 rate. What is the mean hourly rate paid to the employees?

The Weighted Mean

- **Solution:**
- To find the mean hourly rate, we multiply each of the hourly rates by the number of employees earning that rate as follows –

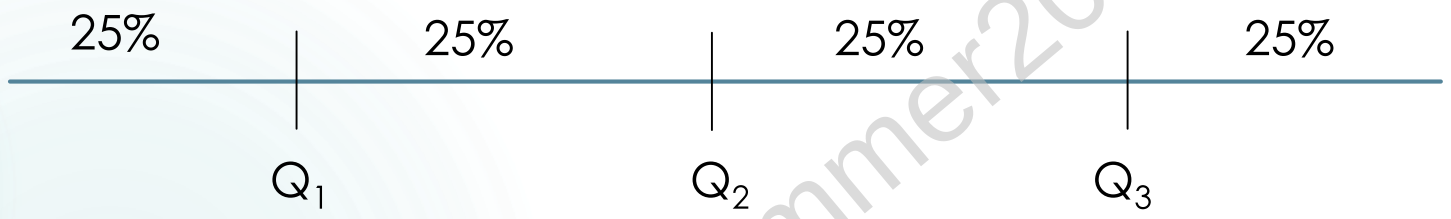
$$\bar{X}_w = \frac{\sum(WX)}{\sum W} = \frac{140*50+100*75+20*90}{140+100+20} = \frac{16300}{260} = Tk.\ 62.69$$

- The weighted mean hourly wage is Tk. 62.69 or Tk. 63.00 (approximately).

Median like measures

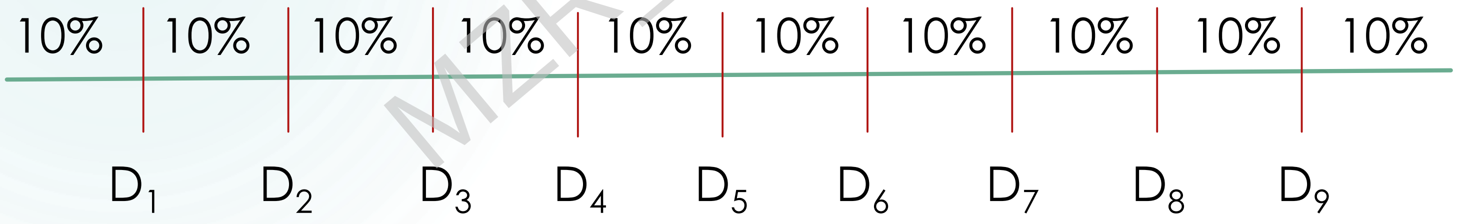
Quartiles:

Quartiles Divides the whole distribution into 4 equal parts



Deciles:

Deciles divide the whole distribution into 10 equal parts



Median like measures

Percentiles:

Percentiles divide the whole distribution into 100 equal parts.

A percentile, ***p***, is a measure used to indicate the value below which a given percentage of observations fall.

For example, **the 90th** percentile is the value below which **90 percent of the observations** may be found.

And it is also the value above which **10 percent** of the observations may be found.

Median like measures

Formulas for finding percentiles:

For raw data,

- If $\frac{in}{100}$ is an integer- (for i= 1, 2, 3, …, 99)

$$P_i = \frac{1}{2}\left[\left(\frac{in}{100}\right)^{th} value + \left(\frac{in}{100} + 1\right)^{th} value\right]$$

- If $\frac{in}{100}$ is not an integer-

$$P_i = next\ integer^{th}\ value\ of\ \frac{in}{100}$$

Median like measures

Example:

Weekly income of 6 respondents (in taka)-

2500, 3900, 3500, 5000, 4000, 3500

Find Q1, Q2, Q3.

Median like measures

Example:

Organizing in ascending order- 2500, 3500, 3500, 3900, 4000, 5000

So, the quartiles-

$$Q_1 = P_{25} = \ next\ integer^{th} value\ of \frac{25n}{100} = next\ integer^{th} value\ of\ \frac{150}{100}$$

$$= next\ integer^{th} value\ of 1.5 = 2^{nd}\ value = 3500$$

Median like measures

Example:

$$Q_2 = P_{50} = Median = \frac{1}{2}\left[\left(\frac{n}{2}\right)^{th} value + \left(\frac{n}{2}+1\right)^{th} value\right] = 3700$$

$$Q_3 = P_{75} = next\ integer^{th} value\ of\ \frac{75n}{100} = next\ integer^{th} value\ of\ \frac{450}{100}$$
$$= next\ integer^{th} value\ of\ 4.5 = 5^{th}\ value = 4000$$

Median like measures

Formulas for finding percentiles:

For grouped data-

$$P_i = L_i + \frac{\frac{in}{100} - F_i}{f_i} * c \qquad i = 1, 2, 3, \ldots, 99$$

L_i= Lower class limit of the i^{th} percentile class

n= Total frequency

F_i= Cumulative frequency of the i^{th} pre-percentile class

f_i= frequency of the i^{th} percentile class

c= class interval

Class Task

Q1. Find the following for given ungrouped data

14.6, 33.3, 24.3, 27.4, 28.2, 31.6, 30.7, 33.6, 34.3, 36.9, 24.9, 27.2, 44.0, 28.8, 31.5, 32.3, 32.8, 38.3, 27.0, 29.9

1. 1st Quartiles
2. 2nd Quartiles
3. 3rd Quartiles
4. 5th and 9th Deciles
5. 67th percentiles

Class Task

Q2. Find 30th percentile for the grouped data below given:

Class	Frequency
150 up to155	8
155 up to160	16
160 up to165	23
165 up to170	7
170 up to175	19
Total	

Relationship

Deciles	Percentiles
D_1	P_{10}
D_2	P_{20}
D_3	P_{30}
D_4	P_{40}
D_5	P_{50}
D_6	P_{60}
D_7	P_{70}
D_8	P_{80}
D_9	P_{90}

Quartiles	Percentiles
Q_1	P_{25}
Q_2	P_{50}
Q_3	P_{75}