Real Analysis — Exercises

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Week 1

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Problem 1.1.1

Assume that the product of two integers x and y is even. Show that at least one of the numbers is even.

Solution. We want to prove the implication xy even $\implies x$ even or y even. We prove this by contraposition. Assume that x and y are both odd. Then for some $n, m \in \mathbb{N}$ we have $x = 2n + 1$ and $y = 2m + 1$. It then follows that $xy = (2n + 1)(2m + 1) = 4nm + 2(n + m) + 1 = 2(2nm + (n + m)) + 1$. Hence xy is odd. By the contrapositive proof our original implication holds.

Problem 1.1.2

Assume that the sum of two integers x and y is even. Show that x and y are either both even or both odd.

Solution. Again, we proceed by contrapositive. Assume that x is even and y is odd (the other case follows by symmetry). We then have that $x + y = 2n + 2m + 1$ for some $n, m \in \mathbb{N}$. Hence $x + y$ is odd. We have therefore proved the contrapositive statement, so the original implication holds.

Problem 1.1.3

Show that if n is a natural number such that n^2 is divisible by 3, then n is divisible by 3. Use this to show that $\sqrt{3}$ is irrational.

Solution. Assume that n is not divisible by 3. This means that $n = 3m + r$ for some integer $0 < r < 3$. Then $n^2 = (3m+r)^2 = 9m+6mr+r^2 = 3m(3+2r)+r^2$. This is only divisible by 3 is r^2 is divisible by 3, but $r = 1$ or $r = 2$ are the only two cases we have, hence $r^2 = 1$ or $r^2 = 4$, with neither being divisible by 3. In other words n^2 is not divisible by 3. This concludes the proof.

We now want to show that $\sqrt{3}$ is irrational. We assume for contradiction that it is rational. Let $\sqrt{3} = m/n$. Also assume that m, n have no common factors. Then $3 = m^2/n^2 = q$. Since q is divisible by 3 we have that $\sqrt{3}$ is divisible by 3. So $\sqrt{3} = 3p$ for some $p \in \mathbb{N}$.