

Chapter summaries in MAT2400 - Real analysis

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Chapter 1

Preliminaries: Proofs, Sets, and Functions

1.1 Proofs

1.2 Sets and boolean operations

1.3 Families of sets

1.4 Functions

1.5 Relations and partitions

1.6 Countability

Chapter 2

Metric Spaces

2.1 Definitions and examples

2.2 Convergence and Continuity

2.3 Open and closed sets

2.4 Complete spaces

2.5 Compact Sets

2.6 An alternative description of compactness

2.7 The completion of a metric space

Chapter 3

Space of continuous functions

3.1 Modes of continuity

3.2 Modes of convergence

3.3 The spaces $C(X, Y)$

3.4 Application to differential equations

3.5 Compact subsets of $C(X, \mathbb{R}^m)$

3.6 Differential equations revisited

3.7 Polynomials are dense in $C([a, b], \mathbb{R})$

3.8 Baire's Category Theorem

Chapter 4

Series of functions

4.1 \limsup and \liminf

4.2 Integrating and differentiating sequences

4.3 Power series

4.4 Abel's Theorem

4.5 Normed spaces

4.6 Inner product spaces

4.7 Linear operators

Chapter 5

Measure and integration

5.1 Measure spaces

5.2 Complete measures

5.3 Measurable functions

5.4 Integration of simple functions

5.5 Integrals of nonnegative functions

5.6 Integrable functions

5.7 $L^1(X, A, \mu)$ and $L^2(X, A, \mu)$

Chapter 6

Constructing measures

6.1 Outer measure

6.2 Measurable sets

6.3 Carathéodory's Theorem

6.4 Lebesgue measure on \mathbb{R}

6.5 Approximation results

6.6 The coin tossing measure

6.7 Product measures

6.8 Fubini's Theorem