## Chapter summaries in MAT2400 - Real analysis

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#### Chapter 1

# **Preliminaries: Proofs, Sets, and Functions**

- 1.1 Proofs
- 1.2 Sets and boolean operations
- 1.3 Families of sets
- 1.4 Functions
- 1.5 Relations and partitions
- 1.6 Countability

#### **Chapter 2**

## **Metric Spaces**

- 2.1 Definitions and examples
- 2.2 Convergence and Continuity
- 2.3 Open and closed sets
- 2.4 Complete spaces
- 2.5 Compact Sets
- 2.6 An alternative description of compactness
- 2.7 The completion of a metric space

# Chapter 3

# **Space of continuous functions**

- 3.1 Modes of continuity
- 3.2 Modes of convergence
- **3.3** The spaces C(X, Y)
- 3.4 Application to differential equations
- **3.5** Compact subsets of  $C(X, \mathbb{R}^m)$
- 3.6 Differential equations revisited
- **3.7** Polynomials are dense in  $C([a,b],\mathbb{R})$
- 3.8 Baire's Category Theorem