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Chapter summaries in MAT2400 - Real analysis

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Chapter 1

Preliminaries: Proofs, Sets, and Functions

1.1 Proofs

Implication

Many mathematical statements are on the form *If A, then B*. These are denoted $A \Rightarrow B$. If $A \Rightarrow B$ then it is not neccessarily so that $B \Rightarrow A$. That is, these two mean different things. Note that $A \Rightarrow B$ is logically equivalent to $\sim B \Rightarrow \sim A$.

Equivalence

If it is true that $A \Rightarrow B$ and $B \Rightarrow A$. Then we call these statements equivalent. This is denoted $A \iff B$. When proving that two statements are equivalent, it is often easier to prove that they both imply eachother. That is, by proving $A \Rightarrow B$ and $B \Rightarrow A$, you have proven that $A \iff B$.

Methods of proof

There are several ways of proving mathematical hypotheses.

1. Instead of proving $A \Rightarrow B$, prove $\sim B \Rightarrow \sim A$. This is called a *contrapositive proof.*
2. Another common method of proof is *proof by contradiction.* Assume the opposite of what you want to prove, and by showing that this leads to a contradiction, our assumption must be false, and hence the original hypothesis is true.
3. When dealing with natural numbers, the go-to method of proof is *proof by induction.* Show that a statement holds for a given number, and then show that if it holds for an arbitrary number n, it must also hold for the successor $n+1$.

This list is of course non-exhaustive, but these are some of the more common methods one should consider when faced with a problem to solve.

1.2 Sets and boolean operations

A set is a collection of mathematical objects. It may be finite, it may be infinite. A is a subset of B if all the elements in A are also in B. This is denoted $A \subseteq B$. Two sets are equal if they contain exactly the same elements. This is denoted $A = B$, where A and B are sets. If A is a subset of B and B is a subset of A, the sets are equal.
The set that contains no elements is called the empty set and is denoted $\emptyset$.

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