Chapter 1

Probability

1.1 Sample Spaces and Events

The Sample Space of an Experiment: The **sample space** of an experiment, denoted by S, is the set of all possible outcomes of that experiment.

Events: An **event** is any collection (subset) of outcomes contained in the sample space S. An event is said to be **simple** if it consists of exactly one outcome and **compound** if it consists of more than one outcome.

Some Relations from Set Theory:

1. The **union** of two events A and B, denoted by $A \cup B$ and read "A or B" is the event consisting of all outcomes that are *either in A or in B or in both events* (so that the union includes outcomes for which both A and B occur as well as the outcomes for which exactly one occurs) - that is, all outcomes in at least one of the events.

2. The **intersection** of two events A and B, denoted by $A \cap B$ and read " A *and* B " is the event consisting of all outcomes that are in *both A and B*.

3. The **complement** of an event A, denoted by A', is the set of all outcomes in S that are not contained in A.

Mutually exlusive events: When A and B have no outcomes in common, they are said to be **disjoint** or **mutually exclusive** events. Mathematicians write this compactly as $A \cup B = \emptyset$ where $\emptyset$ denotes the event consisting of no outcomes whatsoever (the "null" or "empty" event).

1.2 Axioms, Interpretations, and Properties of Probability

Basic properties of probability:

1. For any event $A, P(A) \geq 0$.

2. $P(S) = 1$.

3. If $A_1, A_2, A_3, \cdots$ is an infinite collection of disjoint events, then

$$P(A_1 \cup A_2 \cup \cdots) = \sum_{i=1}^{\infty} P(A_i).$$

1.2.1 Proposition

$P(\emptyset) = 0$ where $\emptyset$ is the null event. This in turn implies that the property contained in Axiom 3 is valid for a *finite* collection of events.[1]

1.2.2 Proposition

For any event $A, P(A) = 1 - P(A')$.[2]

1.2.3 Proposition

For any event $A, P(A) \leq 1$.

1.2.4 Proposition

For any events A and B,

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B).^{3} \\ P(A \cup B \cup C) &= P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) \\ &\quad - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C) \end{aligned}$$

1.3 Counting Techniques

The Product Rule for Ordered Pairs

1.3.1 Proposition

If the first element or object of an ordered pair can be selected in n_1 ways, and for each of these n_1 ways the second element of the pair can be selected in n_2 ways, then the number of pairs is $n_1 n_2$.

Product Rule for K-Tuples:

Suppose a set consists of ordered collections of k elements (k-tuples) and that there are n_1 possible choices for the first element; for each choice of the first element, there are n_2 possible choices of the second element;...; for each possible of the first $k-1$ elements, there are n_k choices for kth element. Then there are $n_1 n_2 \cdot \cdots \cdot n_k$ possible k-tuples.

[1] Proof page 57.
[2] Proof page 60.

Permutations:

Any ordered sequence k objects taken from a set of n distinct objects is called a **permutation** of size k of the objects. The numbers of permutations of size k that can be constructed from the n objects is denoted $P_{k,n}$.