Chapter 1

Overview and Descriptive Statistics

1.1 Populations and Samples

1.2 Pictorial and Tabular Methods in Descriptive Statistics

Procedure 1 (Steps for constructing a stem-and-leaf display)**.**

1. Select one or more leading digits for the stem values. The trailing digits becomes the leaves.
2. List possible stem values in a vertical column.
3. Record the leaf for every observation beside the corresponding stem value.
4. Order the leaves from smallest to largest on each line.
5. Indicate the units for stems and ;eaves someplace in the display.

Procedure 2 (A histogram for counting data)**.** First, determine the frequency and relative frequency of each x value. Then mark possible x values on a horizontal scale. Above each calue, draw a rectangle whose height is the relative frequency (or alternatively, the frequency of that value).

Procedure 3 (A histogram for measurement data: equal class widths)**.** Determine the frequency and relative frequency for each class. Mark the class boundaries on a horizontal measurement axis. Above each class interval, draw a rectangle whose height is the corresponding relative frequence (or frequency).

Procedure 4 (A histogram for measurement data: unequal class widths)**.** After determining frequencies and relative frequencies, calculate the height of each rectangle using the formula

$$\text{rectangle height} = \frac{\text{relative frequency of the class}}{\text{class width}}$$

The resulting rectangle heights are usually called *densities*, and the vertical scale is the **density scale**. This prescription will also work when class widths are equal.

1.3 Measures of Location

Definition 1. The **sample mean** $\bar{x}$ of observations $x_1, x_2, \ldots, x_n$ is given by

$$\bar{x} = \frac{x_1 + x_2 + \ldots + x_n}{n} = \frac{\sum_{i=1}^{n} x_i}{n}. \tag{1.1}$$

The numerator of $\bar{x}$ can be written more informally as $\sum x_i$ where the summation is over all sample observations.

Definition 2. The **sample mean** is obtained by first ordering the n observations from smallest to largest (with any repeated values included so that every sample observation appears in the ordered list). Then,

$$\bar{x} = \begin{cases} \text{The single middle value if } n \text{ is odd} \\ \text{The average of the two middle values if } n \text{ is even} \end{cases}$$

1.4 Measures of Variability

Definition 3. The **sample variance**, denoted by s^2, is given by

$$s^2 = \frac{\left(\sum(x_i - \bar{x})^2\right.}{n-1} = \frac{S_{xx}}{n_1}$$

The **sample standard deviation**, denoted by s, is the (positive) square root of the variance:

$$s = \sqrt{s^2}$$

Proposition 1. *Let $x_1, x_2, \ldots, x_n$ be a sample and c be a constant.*

1. *If $y_1 = x_1 + c, y_2 = x_2 + c, \ldots, y_n = x_n + c$, then $s_y^2 = s_x^2$, and*
2. *If $y_1 = cx_1, \ldots, y_n = cx_n$, then $s_y^2 = c^2 s_x^2, sy = |c| s_x$,*

where s_x^2 is the sample variance of the x's and s_y^2 is the sample variance of the y's.

Definition 4. Order the n observations from smallest to largest and separate the samllest half from the largest half; the median $\bar{x}$ is included in both halves if n is odd. Then the **lower fourth** is the median of the smallest half and the **upper fourth** is the median of the largest half. A measure of spread that is resistant to outliers is the **fourth spread** f_s, given by

$$f_s = \text{upper fourth - lower fourth}$$

Definition 5. Any observation farthen than $1.5f_s$ from the closts fourth is an **outlier**. An outlier is **extreme** if it is more than $3f_s$ from the nearest fourth, and it is **mild** otherwise.

Chapter 2

Probability

2.1 Sample Spaces and Events

Definition 6. The **sample space** of an experiment, denoted by $\mathcal{S}$, is the set of all possible outcomes of that experiment.

Definition 7. An **event** is any collection (subset) of outcomes contained in the sample space $\mathcal{S}$. An event is said to be **simple** if it consists of exactly one outcome and **compound** if it consists of more than one outcome.

Definition 8. Some relations from set theory:

1. The **union** of two events A and B, denoted by $A \cup B$ and read "A or B" is the event consisting of all outcomes that are *either in A or in B or in both events* (so that the union includes outcomes for which both A and B occur as well as the outcomes for which exactly one occurs) - that is, all outcomes in at least one of the events.

2. The **intersection** of two events A and B, denoted by $A \cap B$ and read " *A and B* " is the event consisting of all outcomes that are in *both A and B*.

3. The **complement** of an event A, denoted by A', is the set of all outcomes in $\mathcal{S}$ that are not contained in A.

Definition 9. When A and B have no outcomes in common, they are said to be **disjoint** or **mutually exclusive** events. Mathematicians write this compactly as $A \cup B = \emptyset$ where $\emptyset$ denotes the event consisting of no outcomes whatsoever (the "null" or "empty" event).

2.2 Axioms, Interpretations, and Properties of Probability

Definition 10. Basic properties of probability:

1. For any event $A, P(A) \geq 0$.

2. $P(S) = 1$.

3. If $A_1, A_2, A_3, \cdots$ is an infinite collection of disjoint events, then

$$P(A_1 \cup A_2 \cup \cdots) = \sum_{i=1}^{\infty} P(A_i).$$

Proposition 2. *$P(\emptyset) = 0$ where $\emptyset$ is the null event. This in turn implies that the property contained in Axiom 3 is valid for a finite collection of events.*[1]

Proposition 3. *For any event $A, P(A) = 1 - P(A')$.*[2]

Proposition 4. *For any event $A, P(A) \leq 1$.*

Proposition 5. *For any events A and B,*

$$\begin{aligned}
P(A \cup B) &= P(A) + P(B) - P(A \cap B).^3 \\
P(A \cup B \cup C) &= P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) \\
&\quad - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)
\end{aligned}$$

2.3 Counting Techniques

Proposition 6. *If the first element or object of an ordered pair can be selected in n_1 ways, and for each of these n_1 ways the second element of the pair can be selected in n_2 ways, then the number of pairs is $n_1 n_2$.*

Proposition 7 (Product rule for K-tuples)**.** *Suppose a set consists of ordered collections of k elements (k-tuples) and that there are n_1 possible choices for the first element; for each choice of the first element, there are n_2 possible choices of the second element;...; for each possible of the first $k-1$ elements, there are n_k choices for kth element. Then there are $n_1 n_2 \cdot \cdots \cdot n_k$ possible k-tuples.*

Definition 11. Any ordered sequence k objects taken from a set of n distinct objects is called a **permutation** of size k of the objects. The numbers of permutations of size k that can be constructed from the n objects is denoted $P_{k,n}$.

Definition 12. For any positive integer m, $m!$ is read "m factorial" and is defined by $m! = m(m-1)(m-2)\dots(2)(1)$. Also, $0! = 1$.

Definition 13. Given a set of n distinct objects, any unordered subset of size k of the objects is colled a **combination**. The number of combinations of size k that can be formed from n will be denoted by $\binom{n}{k}$. This notation is more common in probability than $C_{k,n}$, which would be analogous to notation for permutations.

2.4 Conditional Probability

Definition 14. For any two events A and B with $P(B) > 0$, the **conditional probability of A given B that has occured** is defined by

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}.$$

[1] Proof page 57.
[2] Proof page 60.

Proposition 8 (The multiplication rule). $P(A \cap B) = P(A \mid B) \cdot P(B)$

Proposition 9 (The law of total probability). *Let $A_1, \ldots, A_k$ be mutually exclusive and exhaustive events. Then for any other event B.*

$$\begin{aligned} P(B) &= P(B \mid A_1) \cdot P(A_1) + \cdots + P(B \mid A_k) \cdot P(A_k) \\ &= \sum_{i=1}^{k} P(B \mid A_i) P(A_i) \end{aligned}$$

Proposition 10 (Bayes' Theorem). *Let $A_1, \ldots, A_k$ be a collection of mutually exclusive and exhaustive events with $P(A_i) > 0$ for $i = 0, \ldots, k$. Then for any other event B, for which $P(B) > 0$*

$$P(A_j \mid B) = \frac{P(A_j \cap B)}{P(B)} = \frac{P(B \mid A_j) P(A_j)}{\sum\limits_{i=1}^{k} P(B \mid A_i) P(A_i)} \qquad j = 0, \ldots, k$$

2.5 Independence

Definition 15. Two events A and B are **independent** if $P(A \mid B) = P(A)$ and are **dependend** otherwise.

Proposition 11. *A and B are independent if and only if*

$$P(A \cap B) = P(A) \cdot P(B)$$