

The resulting matrix **T0_E** is:

$$\begin{bmatrix} \cos(q_2 + q_3)\cos(q_1), & -\sin(q_2 + q_3)\cos(q_1), & \sin(q_1), & \cos(q_1)(\cos(q_2 + q_3)(l_3^2 + l_5^2)^{1/2} + \cos(q_2)(l_2^2 + l_4^2)^{1/2}) \\ \cos(q_2 + q_3)\sin(q_1), & -\sin(q_2 + q_3)\sin(q_1), & -\cos(q_1), & \sin(q_1)(\cos(q_2 + q_3)(l_3^2 + l_5^2)^{1/2} + \cos(q_2)(l_2^2 + l_4^2)^{1/2}) \\ \sin(q_2 + q_3), & \cos(q_2 + q_3), & 0, & l_1 + \sin(q_2 + q_3)(l_3^2 + l_5^2)^{1/2} + \sin(q_2)(l_2^2 + l_4^2)^{1/2} \\ 0, & 0, & 0, & 1 \end{bmatrix}$$

$$\mathbf{q1_sol} = [\text{atan2}(Py, Px), \pi + \text{atan2}(Py, Px)]$$

$$\mathbf{q2_sol} = [\text{atan2}(Pz - l_1, (Px^2 + Py^2)^{1/2}) - \text{atan2}(2*(1 - (Px^2 + Py^2 + l_2^2 - l_3^2 + l_4^2 - l_5^2 + (Pz - l_1)^2)^{1/2}/(4*(l_2^2 + l_4^2)*(Px^2 + Py^2 + (Pz - l_1)^2)))^{1/2}, (Px^2 + Py^2 + l_2^2 - l_3^2 + l_4^2 - l_5^2 + (Pz - l_1)^2)/((l_2^2 + l_4^2)^{1/2}*(Px^2 + Py^2 + (Pz - l_1)^2)^{1/2})), \text{atan2}(Pz - l_1, (Px^2 + Py^2)^{1/2}) - \text{atan2}(-2*(1 - (Px^2 + Py^2 + l_2^2 - l_3^2 + l_4^2 - l_5^2 + (Pz - l_1)^2)^{1/2}/(4*(l_2^2 + l_4^2)*(Px^2 + Py^2 + (Pz - l_1)^2)))^{1/2}, (Px^2 + Py^2 + l_2^2 - l_3^2 + l_4^2 - l_5^2 + (Pz - l_1)^2)/((l_2^2 + l_4^2)^{1/2}*(Px^2 + Py^2 + (Pz - l_1)^2)^{1/2}))]$$

$$\mathbf{q3_sol} = [\text{atan2}(2*(1 - (l_2^2 - Py^2 - Px^2 + l_3^2 + l_4^2 + l_5^2 - (Pz - l_1)^2)^{1/2}/(4*(l_2^2 + l_4^2)*(l_3^2 + l_5^2)))^{1/2}, -(l_2^2 - Py^2 - Px^2 + l_3^2 + l_4^2 + l_5^2 - (Pz - l_1)^2)/((l_2^2 + l_4^2)*(l_3^2 + l_5^2))^{1/2}), \text{atan2}(-2*(1 - (l_2^2 - Py^2 - Px^2 + l_3^2 + l_4^2 + l_5^2 - (Pz - l_1)^2)^{1/2}/(4*(l_2^2 + l_4^2)*(l_3^2 + l_5^2)))^{1/2}, -(l_2^2 - Py^2 - Px^2 + l_3^2 + l_4^2 + l_5^2 - (Pz - l_1)^2)/((l_2^2 + l_4^2)*(l_3^2 + l_5^2))^{1/2})]$$

-----Test case 1-----

Px, Py, Pz from forward kinematic

$$Px = 0.94528$$

$$Py = 0.39966$$

$$Pz = 0.59145$$

q1, q2, q3 from inverse kinematic

$$q1_sol =$$

$$0.4000 \quad 3.5416$$

$$q1_val = 0.4$$

$$q2_sol =$$

$$0.2000 \quad 0.5288$$

$$q2_val = 0.2$$

$$q3_sol =$$

$$0.3000 \quad -0.3000$$

$$q3_val = 0.3$$

Verifying Px, Py, and Pz from inverse kinematics

Solution 1: q1 = 0.4000, q2 = 0.2000, q3 = 0.3000

$$Px_sol1 = 0.94528$$

$$Py_sol1 = 0.39966$$

$$Pz_sol1 = 0.59145$$

Solution 2: q1 = 0.4000, q2 = 0.5288, q3 = -0.3000

$$Px_sol2 = 0.94528$$

$$Py_sol2 = 0.39966$$

$$Pz_sol2 = 0.59145$$

Solution 3: q1 = 3.5416, q2 = 2.9416, q3 = -0.3000

$$Px_sol3 = 0.94528$$

$$Py_sol3 = 0.39966$$

$$Pz_sol3 = 0.59145$$

Solution 4: q1 = 3.5416, q2 = 2.6128, q3 = 0.3000

$$Px_sol4 = 0.94528$$

$$Py_sol4 = 0.39966$$

$$Pz_sol4 = 0.59145$$

-----Test case 2-----

Px, Py, Pz from forward kinematic

Px = 0.51295

Py = -0.38995

Pz = 0.50355

q1, q2, q3 from inverse kinematic

q1_sol =

-0.6500 2.4916

q1_val = -0.65

q2_sol =

-0.1000 1.3876

q2_val = -0.1

q3_sol =

1.2000 -1.2000

q3_val = 1.2

Verifying Px, Py, and Pz from inverse kinematics

Solution 1: q1 = -0.6500, q2 = -0.1000, q3 = 1.2000

Px_sol1 = 0.51295

Py_sol1 = -0.38995

Pz_sol1 = 0.50355

Solution 2: q1 = -0.6500, q2 = 1.3876, q3 = -1.2000

Px_sol2 = 0.51295

Py_sol2 = -0.38995

Pz_sol2 = 0.50355

Solution 3: q1 = 2.4916, q2 = 3.2416, q3 = -1.2000

Px_sol3 = 0.51295

Py_sol3 = -0.38995

Pz_sol3 = 0.50355

Solution 4: q1 = 2.4916, q2 = 1.7540, q3 = 1.2000

Px_sol4 = 0.51295

Py_sol4 = -0.38995

Pz_sol4 = 0.50355

-----Test case 3-----

Px, Py, Pz from forward kinematic

Px = -0.31682

Py = 0.081202

Pz = 0.905

q1, q2, q3 from inverse kinematic

q1_sol =

2.8907 6.0323

q1_val = -0.2509

q2_sol =

0.5008 1.7895

q2_val = 1.3521

q3_sol =

0.8300 -0.8300

q3_val = 0.83

Verifying Px, Py, and Pz from inverse kinematics

Solution 1: q1 = 2.8907, q2 = 0.5008, q3 = 0.8300

Px_sol1 = -0.31682

Py_sol1 = 0.081202

Pz_sol1 = 0.905

Solution 2: q1 = 2.8907, q2 = 1.7895, q3 = -0.8300

Px_sol2 = -0.31682

Py_sol2 = 0.081202

Pz_sol2 = 0.905

Solution 3: q1 = 6.0323, q2 = 2.6408, q3 = -0.8300

Px_sol3 = -0.31682

Py_sol3 = 0.081202

Pz_sol3 = 0.905

Solution 4: q1 = 6.0323, q2 = 1.3521, q3 = 0.8300

Px_sol4 = -0.31682

Py_sol4 = 0.081202

Pz_sol4 = 0.905

-----Test case 4-----

Px, Py, Pz from forward kinematic

$$Px = 0.75506$$

$$Py = 0.15479$$

$$Pz = 0.55174$$

q1, q2, q3 from inverse kinematic

$$q1_sol =$$

$$0.2022 \quad 3.3438$$

$$q1_val = 0.2022$$

$$q2_sol =$$

$$0.0275 \quad 0.6242$$

$$q2_val = 0.6242$$

$$q3_sol =$$

$$0.9485 \quad -0.9485$$

$$q3_val = -0.9485$$

Verifying Px, Py, and Pz from inverse kinematics

Solution 1: q1 = 0.2022, q2 = 0.0275, q3 = 0.9485

$$Px_sol1 = 0.75506$$

$$Py_sol1 = 0.15479$$

$$Pz_sol1 = 0.55174$$

Solution 2: q1 = 0.2022, q2 = 0.6242, q3 = -0.9485

$$Px_sol2 = 0.75506$$

$$Py_sol2 = 0.15479$$

$$Pz_sol2 = 0.55174$$

Solution 3: q1 = 3.3438, q2 = 3.1141, q3 = -0.9485

$$Px_sol3 = 0.75506$$

$$Py_sol3 = 0.15479$$

$$Pz_sol3 = 0.55174$$

Solution 4: q1 = 3.3438, q2 = 2.5174, q3 = 0.9485

$$Px_sol4 = 0.75506$$

$$Py_sol4 = 0.15479$$

$$Pz_sol4 = 0.55174$$

-----Test case 5-----

Px, Py, Pz from forward kinematic

$$Px = 0.77785$$

$$Py = -0.3211$$

$$Pz = 0.18612$$

q1, q2, q3 from inverse kinematic

$$q1_sol =$$

$$-0.3915 \quad 2.7501$$

$$q1_val = -0.3915$$

$$q2_sol =$$

$$0.0743 \quad 0.1426$$

$$q2_val = 0.0743$$

$$q3_sol =$$

$$0.0799 \quad -0.0799$$

$$q3_val = 0.0799$$

Verifying Px, Py, and Pz from inverse kinematics

Solution 1: q1 = -0.3915, q2 = 0.0743, q3 = 0.0799

$$Px_sol1 = 0.77785$$

$$Py_sol1 = -0.3211$$

$$Pz_sol1 = 0.18612$$

Solution 2: q1 = -0.3915, q2 = 0.1426, q3 = -0.0799

$$Px_sol2 = 0.77785$$

$$Py_sol2 = -0.3211$$

$$Pz_sol2 = 0.18612$$

Solution 3: q1 = 2.7501, q2 = 3.0673, q3 = -0.0799

$$Px_sol3 = 0.77785$$

$$Py_sol3 = -0.3211$$

$$Pz_sol3 = 0.18612$$

Solution 4: q1 = 2.7501, q2 = 2.9990, q3 = 0.0799

$$Px_sol4 = 0.77785$$

$$Py_sol4 = -0.3211$$

$$Pz_sol4 = 0.18612$$