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2.16 Req VC dim of H is exactly $(D+1)$

a) Does H shatters $(D+1)$ Points
Choose $D+1$ points in R: $x_0, x_1 - x_D$

$$X = \begin{bmatrix} 1 & x_0^1 & x_0^D \\ 1 & x_1^1 & x_1^D \\ \vdots & \vdots & \vdots \\ 1 & x_D^1 & x_D^D \end{bmatrix} \qquad |X| \neq 0$$

& let y (any dichotomy) $= (y_0, .. y_D) \in \{-1, 1\}^{D+1}$

then solving $C = (C_0, -- C_D) = X^{-1} y$

$$XC = y$$

$$\text{So } h_c(x_k) \text{ Sign}\left(\sum_{i=0}^{D} x_k^i c_i\right) = \text{Sig}(y_k)$$

$$= y_k \qquad \forall k = 0, -- D$$

So H Shatters $x_0, - x_D$ ~~$\not\in$~~

$m_H(D+1) = 2^{D+1}$, $d_{vc}(H) \geq D+1$

(1^{st} Req)

b) let (D+2) Points in R: $x_0, - x_D, x_{D+1}$

Then The D+2 vectors $(x_k^0 \dots x_k^D)$ are in D+1 dimension

So They are linearly depedent

So There excist vector that is Linear Combination of the other D+1 vectors

$$(x_l^0 \dots x_l^D) = \sum_{k \neq l} a_k (x_k^0, - x_k^D) \rightarrow ①$$

$a_k \neq 0$

let Pick a dichotomy $\boxed{y_l = -1}$ & $y_k = \text{Sign}(a_k)$ $\underline{\underline{l \neq k}}$

From ①

$$(x_l^0 \dots x_l^D) = \sum_{k \neq l} \sum_{i=0}^{D} c_i a_k x_k^i$$

$$y_l = h_c(x_l) = \text{sign}\left(\sum_{i=0}^{D} c_i x_l^i\right)$$

$$y_k = \text{sign}(a_k) = \text{Sign}\left(\sum_{i=0}^{D} c_i x_k^i\right) \Rightarrow \sum_{i=0}^{D} c_i a_k x_k^i > 0$$

So for any k, $a_k \neq 0$

$$\sum_{k \neq l} \sum_{i=0}^{D} c_i a_k x_k^i = (x_l^0, x_l^1 \dots x_l^D) \begin{pmatrix} c_0 \\ \vdots \\ c_D \end{pmatrix}$$

$$= \sum_{i=0}^{D} c_i x_l^i > 0$$

$$\therefore y_l = \text{Sign}\left(\sum_{i=0}^{D} c_i x_l^i\right) + 1$$

So There excist a dichotomy in R^{D+2} That can't be generated by H

Scanned with CamScanner

) a) $\bar{g}(x) \rightarrow$ For st line hypoth ($y = mx + C$)

$\therefore m_H(D+2) < 2^{D+2}$ & $d_{VC}(H) \leq D+1$

(No $(D+2)$ Points shattered by H)

2^{nd} Req

2.24) a) $\bar{g}(x) \rightarrow$ For st line hypoth $(y = mx + C)$

$\bar{g}(x) = E_D(g^{(D)}(x))$

$= E_D(mx + C)$

$m = \dfrac{y_1 - y_2}{x_1 - x_2}$

$= E_D\left[\dfrac{y_1 - y_2}{x_1 - x_2}x + \dfrac{x_1 y_2 - x_2 y_1}{x_1 - x_2}\right]$

$C = y_1 - mx_1$

$= y_1 - \left(\dfrac{y_1 - y_2}{x_1 - x_2}\right)x_1$

$= \dfrac{y_1(x_1 - x_2)}{(x_1 - x_2)} - \dfrac{(y_1 - y_2)x_1}{(x_1 - x_2)}$

$= \dfrac{\cancel{y_1 x_1} - x_2 y_1 - \cancel{y_1 x_1} + y_2 x_1}{x_1 - x_2}$

$= \dfrac{x_1 y_2 - x_2 y_1}{x_1 - x_2}$

$\therefore$ Targ func $\rightarrow x^2$

$\therefore y_1 = x_1^2$

$y_2 = x_2^2$

$\bar{g}(x) = E_D\left[\dfrac{x_1^2 - x_2^2}{x_1 - x_2}x + \dfrac{x_1 x_2^2 - x_2 x_1^2}{x_1 - x_2}\right]$

$= x E_D\left[\dfrac{x_1^2 - x_1^2}{x_1 - x_2}\right] + E_D\left[\dfrac{x_1 x_2^2 - x_2 x_1^2}{x_1 - x_2}\right]$

Continous $\Rightarrow$ i.e $\int$ i)

$X \rightarrow$ uniform.

(2 variables $\therefore$ double $\int$)

$= \dfrac{1}{4}x \displaystyle\int_{-1}^{1}\int_{-1}^{1} \dfrac{\overbrace{x_1^2 - x_2^2}^{(x_1 + x_2)(x_1 - x_2)}}{\cancel{x_1 - x_2}}\, dx_1\, dx_2 + \dfrac{1}{4}\int_{-1}^{1}\int_{-1}^{1} \dfrac{x_1 x_2^2 - x_2 x_1^2}{x_1 - x_2}\, dx_1\, dx_2$

$= \dfrac{1}{4}x \displaystyle\int_{-1}^{1}\int_{-1}^{1} (x_1 + x_2)\, dx_1\, dx_2 + \dfrac{1}{4}\int_{-1}^{1}\int_{-1}^{1} \dfrac{-x_1 x_2 \cancel{(x_2 - x_1)}}{\cancel{x_1 - x_2}}\, dx_1\, dx_2$

$= \dfrac{1}{4}x \displaystyle\int_{-1}^{1}\int_{-1}^{1} (x_1 + x_2)\, dx_1\, dx_2 - \dfrac{1}{4}\int_{-1}^{1}\int_{-1}^{1} x_1 x_2\, dx_1\, dx_2$

$$= \frac{1}{4} x \int_{-1}^{1} \left[\frac{1}{2} x_1^2 + x_1 x_2\right]_{x_1=-1}^{1} dx_2 - \frac{1}{4} \int_{-1}^{1} \left[\frac{x_1^2 x_2}{2}\right]_{x_1=-1}^{x_1=1} dx_2$$

$$\frac{1}{4} x \int_{-1}^{1} \cancel{\frac{1}{2}(1)} + x_2 - \left[\cancel{\frac{1}{2}} - x_2\right) dx_2 - \frac{1}{4} \int_{-1}^{1} \cancel{\left[\frac{1}{2} x_2 - \frac{1}{2} x_2\right]}$$

$$0$$

$$= \frac{1}{4} x \int_{-1}^{1} 2 x_2 \, dx_2 + -0$$

$$= \frac{1}{4} x \left[\frac{\cancel{2} x_2^2}{\cancel{2}}\right]_{-1}^{1} = 0$$

[Consistent with Coding Results]

0

c) $Var(x) = E_D[(g(x) - \bar{g}(x))^2]$ From ⓐ $\bar{g}(x) = 0$

$$= E_D[(g(x))^2] = E_D[(ax+b)^2]$$

$$= E_D(a^2 x^2 + 2axb + b^2]$$, $E_x(ax+b) = aE(x)+b$

$$x^2 \cdot E_D(a^2) + 2x E_D[ab] + E_D[b^2]$$

a depends on D

Not x

St line hy , The parabolic func.

$$E_{out} = E_x[(g(x) - f(x))^2] = E_x[(\underbrace{ax+b}_{\text{St line hy}} - \underbrace{x^2}_{\text{The parabolic func.}})^2]$$

$$= E_x[(ax+b)^2 + x^4 + - 2x^2(ax+b)]$$

$$= E_x[a^2x^2 + 2axb + b^2 + x^4 - 2x^2(ax+b)]$$

$$-2ax^3 - 2bx^2$$

$$= E_x[x^4] - 2a E_x[x^3] + (a^2 - 2b) E_x[x^2] + 2ab E_x[x] + E_x[b^2]$$

$$= \frac{1}{2}\int_{-1}^{1} x^4 dx - 2a\frac{1}{2}\int_{-1}^{1} x^3 dx + \frac{1}{2}(a^2 - 2b)\int_{-1}^{1} x^2 dx + 2ab\int_{-1}^{1} x dx + b^2$$

$$= \frac{1}{\cancel{2}}\frac{1}{5}[\cancel{2}] - 0 + \frac{1}{\cancel{2}}(a^2 - 2b)\frac{(\cancel{2})}{3} + 0 + b^2$$

$$= \frac{1}{5} + \frac{1}{3}(a^2 - 2b) + b^2$$

$a = x_1 + x_2$

$b = -x_1 x_2$

$$E_D(E_{out}) = E_D\left(\frac{1}{5} + \frac{1}{3}[(x_1 + x_2)^2 + 2(x_1 x_2)] + E_D[x_1^2 x_2^2]\right)$$

$$= \frac{1}{5} + \frac{1}{3} E_D[(x_1 + x_2)^2 + 2x_1 x_2] + E_D(x_1^2 x_2^2)$$

$$= \frac{1}{5} + \frac{1}{3}\frac{1}{2}\frac{1}{2}\int_{-1}^{1}\int_{-1}^{1}(x_1^2 + x_2^2 + 4x_1x_2)dx_1 dx_2 + \frac{1}{4}\int_{-1}^{1}\int_{-1}^{1} x_1^2 x_2^2 dx_1 dx_2$$

$$= x^2 E_D[(x_1+x_2)^2] - 2x E_D[(x_1+x_2)(x_1 x_2)]$$
$$+ E_D[x_1^2 x_2^2]$$

$$= x^2 E_D[x_1^2 + 2x_1 x_2 + x_2^2] - 2x E_D[x_1^2 x_2 + x_1 x_2^2]$$
$$+ E_D[x_1^2 x_2^2]$$

$$= \frac{1}{4} x^2 \int_{-1}^{1}\int_{-1}^{1} (x_1^2 + 2x_1 x_2 + x_2^2)\, dx_1 dx_2$$
$$- \frac{2}{4} x \int_{-1}^{1}\int_{-1}^{1} (x_1^2 x_2 + x_1 x_2^2)\, dx_1 dx_2$$
$$+ \frac{1}{4} \int_{-1}^{1}\int_{-1}^{1} x_1^2 x_2^2\, dx_1 dx_2$$

$$= \frac{1}{4} x^2 \left(\frac{4}{3} + 4/3\right) - 0 + \frac{1}{4}\left(\frac{4}{9}\right) = \frac{2}{3} x^2 + \frac{1}{9}$$

$$\text{Var}_2 \cdot E_x(\text{var}(x)) = E_x\left[\frac{2}{3} x^2 + \frac{1}{9}\right] = \frac{2}{3}\frac{1}{2}\int_{-1}^{1} x^2 dx + 1/9$$
$$= \frac{1}{3} = 0.333$$

[Consistent with Coding Variance]
0.338

$$+ \frac{1}{5} + \frac{1}{12} \cdot \frac{8}{3} + \frac{1}{4}\left(\frac{4}{9}\right) = \frac{8}{15} = 0.533$$

Consistat with
the Coding
Results (0.536)