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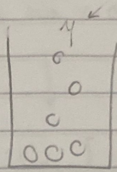
Assig (1)

problem 1.6

$\mu = 0.05 - 0.5 - 0.8$



what $P[\nu = 0] = ??$

$N = 10$

For $\mu = 0.05 \leftarrow$ probability of drawing red Marble from the bin

$\hookrightarrow \therefore \nu = 0 \rightarrow$ only 1

$\hookrightarrow$ Prob of success :) in each trial

a)

10 indepdent trial

$\hookrightarrow$ This is binomial distribution

So μ is prob of success in a single trial

$\therefore$ we have 10 trials Then P of failure in the 10 trials

$= \prod_{i=1}^{10} (1-\mu) = (1-\mu)^{10}$ (indepedent trial)

a) $P(\nu = 0) = (1-\mu)^{10}$

For $\mu = 0.05$ $P(\nu = 0) = (1 - 0.05)^{10} = 0.599$

For $\mu = 0.5$ $P(\nu = 0) = (1 - 0.5)^{10} = 9.77 \times 10^{-4}$

For $\mu = 0.8$ $P(\nu = 0) = (1 - 0.8)^{10} = 1.024 \times 10^{-7}$

b) 1,000 indepdent Samples P atleast one of Samples has $\nu = 0$

$\hookrightarrow$ To get P That @ least one of The samples has $\nu = 0$

$\equiv$ 1 - P that None of the samples has $\nu = 0$

$(1-(1-\mu))^{10}$

①

Since we have 1,000 indep Samples o/o P that None of the 1000 Sample has $r=0$ is $[1-(1-\mu)^{10}]^{1000}$

$\downarrow$

$$1-[1-(1-\mu)^{10}]^{1000}$$ [atleast one Sample has $r=0$!!] outof 1k drawn Samples.

b)

For $\mu=0.05$ $p(\ ) = 1-[1-(1-0.05)^{10}]^{1000}$ $=1$

For $\mu=0.5$ $= 1-[1-(1-0.5)^{10}]^{1000}$ $=0.6236$

$\mu=0.8$ $= 1-[1-(1-0.8)^{10}]^{1000}$ $=1.024\times10^{-4}$

Comment $\mu=0.05$ $\rightarrow$ 5% chance of drawing a red marble in a single trial drawing 1k indep Sample we get $P=1$ we are almost certain That @ least one Sample outof The 1000 will have No red Marble

Similarly $\mu=0.5$ $\rightarrow$ 50% Certain
$\mu=0.8$ $\rightarrow$ 80% Certain

c) 1,000,000 indep sample
$\hookrightarrow 1-[1-(1-\mu)^{10}]^{10^6}$

For $\mu=0.05$ $1-[1-(1-0.05)^{10}]^{10^6}$ $=1$

For $\mu=0.5$ $1-[1-(1-0.5)^{10}]^{10^6}$ $=1$

For $\mu=0.8$ $1-[1-(1-0.8)^{10}]^{10^6}$ $=0.0973$

problem 2.5 $\sum_{i=0}^{D} \binom{N}{i} \leq N^D + 1$ Then $m_H(N) \leq N^{d_{vc}} + 1$

base Case $D=0$

$$\sum_{i=0}^{0} \binom{N}{i} = {}^N C_0 = 1 \leq N^0 + 1$$

Let The base result above be Correct for D ($D \geq 1$) & Try to prove it For $D+1$

$$\sum_{i=0}^{D+1} \binom{N}{i} = {}^N C_{D+1} + \sum_{i=1}^{D} {}^N C_i$$

$$= \binom{N}{D+1} + \sum_{i=1}^{D} \binom{N}{i}$$

$$\sum_{i=1}^{D} \binom{N}{i} \leq N^D + 1$$ [Assumption Above]

$$\sum_{i=1}^{D+1} \binom{N}{i} \leq \binom{N}{D+1} + (N^D + 1)$$

Rem $\binom{N}{r} = \frac{N!}{r! \times (N-r)!}$

$$\leq \frac{N!}{(D+1)!\,(N-D-1)!} + N^D + 1 \rightarrow (1)$$

To Continue we need to prove $\frac{N!}{(N-D-1)!} \leq N^{D+1}$ (Our Aim)

$$\frac{N!}{(N-D-1)!} = \frac{N*(N-)\ldots(N-D)\cancel{(N-D-1)!}}{\cancel{(N-D-1)!}} = N(N-1)\ldots(N-D)$$

$$= \prod_{i=0}^{i=D} (N-i)$$

$\therefore \dfrac{N!}{(N-D-1)!} = \prod_{i=0}^{D} (N-i)$

$\therefore \dfrac{1}{N^{D+1}} \prod_{i=0}^{D} (N-i) \leq 1 \Rightarrow \boxed{\dfrac{N!}{(N-D-1)!} * \dfrac{1}{N^{D+1}} \leq 1}$

$\hookrightarrow \dfrac{N!}{(N-D-1)!} \leq N^{D+1}$

sub in ①

$$\sum_{i=1}^{D+1} \binom{N}{i} \leq \frac{N!}{(D+1)!\,(N-D-1)!} + N^D + 1$$

$$\leq \frac{N^{D+1}}{(D+1)!} + N^D + 1 \rightarrow ②$$

$\hookrightarrow \because$ we are in case $D \geq 1 \Rightarrow (D+1)! \geq 2$

$\hookrightarrow \dfrac{1}{(D+1)!} \leq 1/2$ sub in ②

$$\sum_{i=1}^{D+1} \binom{N}{i} \leq \frac{1}{2} N^{D+1} + N^D + 1 \rightarrow ③$$

Assumption $N \geq D+1$ & $\rightarrow D \geq 1$ $\therefore N \geq 2$

why?

bec we compute

$\sum_{i=0}^{D} \binom{N}{i} \rightarrow$ There exist Term ${}^N C_D \Rightarrow$ For This to be right $\Rightarrow N \geq D+1$:)

$\therefore N \geq 2 \Rightarrow \dfrac{1}{N} < \cancel{2}\ \tfrac{1}{2} \quad * \dfrac{N^D}{N^D}$

$\dfrac{N^D}{N^{D+1}} < 1/2 \rightarrow N^D < \dfrac{1}{2} N^{D+1}$

Term appears in eq. ③ :)

From eq 3

$$\sum_{i=1}^{D+1} \binom{N}{i} \leq \frac{1}{2} N^{D+1} + N^{D} + 1$$

$$\leq \frac{1}{2} N^{D+1} + \frac{1}{2} N^{D+1} + 1$$

$$\boxed{\sum_{i=1}^{D+1} \binom{N}{i} \leq N^{D+1} + 1}$$

Proved for D+1

$$\text{as} \quad m_H(N) \leq \sum_{i=0}^{d_{vc}(H)} \binom{N}{i} \quad \text{--- } 4$$

using prove above

$$\sum_{i=0}^{d_{vc}(H)} \binom{N}{i} \leq N^{d_{vc}(H)} + 1 \quad 5$$

From 4 & 5

$$m_H(N) \leq N^{d_{vc}(H)} + 1$$

Finally, [The required]

From eq 3

$$\sum_{i=1}^{D+1} \binom{N}{i} \le \frac{1}{2} N^{D+1} + N^{D} + 1$$

$$\le \frac{1}{2} N^{D+1} + \frac{1}{2} N^{D+1} + 1$$

$$\boxed{\sum_{i=1}^{D+1} \binom{N}{i} \le N^{D+1} + 1}$$

Proved for D+1

$$\therefore \; m_H(N) \le \sum_{i=0}^{d_{vc}(H)} \binom{N}{i} \quad (4)$$

using prove above

$$\sum_{i=0}^{d_{vc}(H)} \binom{N}{i} \le N^{d_{vc}(H)} + 1 \quad (5)$$

From (4) & (5)

$$m_H(N) \le N^{d_{vc}(H)} + 1$$

Finally, [The required]