Homework 3 – Logical Reasoning

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Date assigned: Wed, 7 October 2015
Date due: Mon, 26 October 2015

Submit one pdf file containing your report via KLMS (naming format of the pdf file: <student#>_<name>.pdf, for example "20150724_john.pdf"). Late submissions will not be accepted. (Write your name and student ID in your report.) This homework should be done individually and written in English.

1. Propositional logic and first-order logic (30 points)

A. Consider a vocabulary with only four propositions, A, B, C, and D. How many models are there for the following sentences? Write your answer and justify it. (4 points)
   - $(A \vee B) \wedge C$ (2 points)
   - $(A \Rightarrow B) \wedge \neg A \wedge B \wedge C \wedge D$ (2 points)

B. Decide whether each of the following sentences is valid, unsatisfiable, or neither. Justify your answer. (6 points)
   - $Smoke \Rightarrow Fire$ (2 points)
   - $Smoke \wedge Fire \wedge \neg Fire$ (2 points)
   - $((Smoke \wedge Heat) \Rightarrow Fire) \Leftrightarrow ((Smoke \Rightarrow Fire) \vee (Heat \Rightarrow Fire))$ (2 points)

C. Use resolution to prove the sentence $(\neg A \wedge \neg B)$ from the following set of sentences: (5 points)
   - S1: $A \Leftrightarrow (B \vee E)$
   - S2: $E \Rightarrow D$
   - S3: $C \wedge F \Rightarrow \neg B$
   - S4: $E \Rightarrow B$
   - S5: $B \Rightarrow F$
   - S6: $B \Rightarrow C$

D. Represent the following sentences in first-order logic, using a consistent vocabulary which you must define in your answer. (15 points)
   - There is a dog that loves all humans. (3 points)
   - There is a barber who shaves all men in town who do not shave themselves. (5 points)
   - Every person who is smart and studies hard gets a higher score than every person who is not smart or does not study hard. (7 points)

2. First-order logic (30 points)

Consider a first-order logical knowledge base that describes worlds containing people, songs, albums (e.g., "Meet the Beatles") and disks (i.e., particular physical instances of CDs). The vocabulary contains the following symbols:

CopyOf(d, a): Predicate. Disk *d* is a copy of album *a*.
Owns(p, d): Predicate. Person *p* owns disk *d*.
Sings(p, s, a): Album *a* includes a recording of song *s* sung by person *p*.
Wrote(p, s): Person *p* wrote song *s*.
McCartney, *Gershwin*, *BHoliday*, *Joe*, *EleanorRigby*, *TheManILove*, *Revolver*: Constants with the obvious meanings.

Express the following statements in first-order logic:

A. Gershwin wrote "The Man I Love". (1 points)
B. Gershwin did not write "Eleanor Rigby". (1 points)
C. Either Gershwin or McCartney wrote "The Man I Love". (2 points)
D. Joe has written at least one song. (2 points)
E. Joe owns a copy of Revolver. (2 points)
F. Every song that McCartney sings on Revolver was written by McCartney. (3 points)
G. Gershwin did not write any of the songs on Revolver. (3 points)
H. Every song that Gershwin wrote has been recorded on some album. (Possibly different songs are recorded on different albums.) (3 points)
I. There is a single album that contains every song that Joe has written. (3 points)
J. Joe owns a copy of an album that has Billie Holiday singing "The Man I Love". (3 points)
K. Joe owns a copy of every album that has a song sung by McCartney. (Of course, each different album is instantiated in a different physical CD.) (3 points)
L. Joe owns a copy of every album on which all the songs are sung by Billie Holiday. (4 points)

3. Inference in first-order logic (40 points)

Suppose you are given the following axioms:

1. $0 \leq 4$.
2. $5 \leq 9$.
3. $\forall x \quad x \leq x$.
4. $\forall x \quad x \leq x + 0$.
5. $\forall x \quad x + 0 \leq x$.
6. $\forall x, y \quad x + y \leq y + x$.
7. $\forall w, x, y, z \quad w \leq y \ \wedge \ x \leq z \Rightarrow w + x \leq y + z$.
8. $\forall x, y, z \quad x \leq y \ \wedge \ y \leq z \ \Rightarrow \ x \leq z$.

A. Give a backward-chaining proof of the sentence $5 \leq 4 + 9$. (Be sure, of course, to use only the axioms given here, not anything else you may know about arithmetic.) Show only the steps that leads to success, not the irrelevant steps. (20 points)

B. Give a forward-chaining proof of the sentence $5 \leq 4 + 9$. Again, show only the steps that lead to success. (20 points)