"Introduction to Artificial Intelligence: Homework #2"

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Contents

1 Problem 1

1.1 1-A

1. 3: Indeed, C has to be true (else $x \wedge C$ would be false). Then, since we want $A \vee B$ to be true, we know there is only 3 ways to make it true ((True, True), (False, True), (True, False)).
   In that case, there is only 3 models for that sentence (Or $3 * 2 = 6$ if we consider that D is still part of the problem)

2. 1: Since we have $\neg A \wedge B \wedge C \wedge D$, we know that (A, B, C, D) has to be (False, True, True, True).
   Then, it only remain $(A \Rightarrow B)$. Since we have already constrained our 4 letters, we can just check if this sentence is correct. Hopefully, it is. (If it wasn't, there were only 0 models for that sentence, so the problem woudl be unsatisfiable)

1.2 1-B

1. Neither: The sentence cannot be:

   - Unsatisfiable since (Smoke=True, Fire=True) is True
   - Valid since (Smoke=True, Fire=False) is False

2. Unsatisfiable: Since we have $Fire \wedge Fire$, we cannot find a value for $Fire$ such that the sentece is correct. So we cannot find any model in which this sentence is true, so it is unsatisfiable.

3. Valid: To do that, let us manipulate the sentence:

$$\begin{aligned}((Smoke \wedge Heat) \Rightarrow Fire) &\Leftrightarrow \neg(Smoke \wedge Heat) \vee Fire \\ &\Leftrightarrow \neg Smoke \vee \neg Heat \vee Fire \\ &\Leftrightarrow \neg Smoke \vee \neg Heat \vee Fire \vee Fire \\ &\Leftrightarrow (\neg Smoke \vee Fire) \vee (\neg Heat \vee Fire) \\ &\Leftrightarrow (Smoke \Rightarrow Fire) \vee (Heat \Rightarrow Fire)\end{aligned}$$

1.3 1-C

Let's add first the negation of our goal sentence in our set of "known sentence":

- S0: $A \vee B$
- S1: $A \Leftrightarrow (B \vee E)$
- S2: $E \Rightarrow D$
- S3: $C \wedge F \Rightarrow \neg B$
- S4: $E \Rightarrow B$

- S5: $B \Rightarrow F$
- S6: $F \Rightarrow C$

Then, let us develop those sentences:

1. $A \vee B$
2. (a) $\neg A \vee B \vee E$
   (b) $\neg B \vee A$
   (c) $\neg E \vee A$
3. $\neg E \vee D$
4. $\neg C \vee \neg F \vee \neg B$
5. $\neg E \vee B$
6. $\neg B \vee F$
7. $\neg F \vee C$

One we have this, we can solve the problem according to the following diagram:

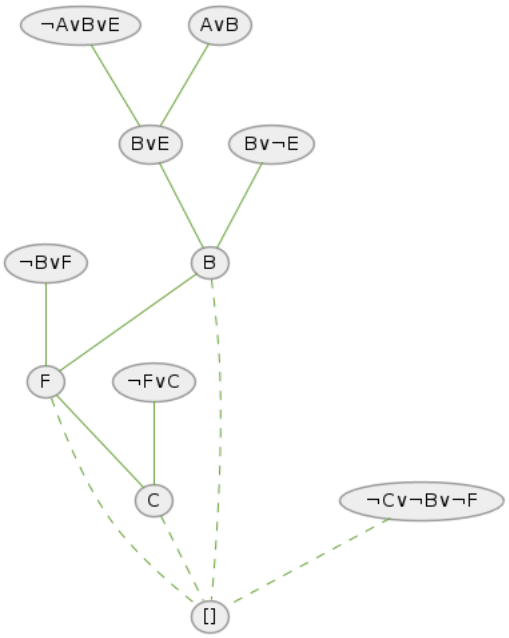

Figure 1: Solution

We can then assume that $\neg A \wedge \neg B$

1.4 1-D

1. $\exists x \, \forall y \, Dog(x) \wedge Human(y) \wedge Loves(x, y)$
2. $\exists x \, \forall y \, Barber(x) \wedge ((Shaves(x, y) \vee Shaves(y, y)))$
3. $\forall x, \; y \, Smart(x) \wedge StudiesHard(x) \wedge (\neg Smart(y) \vee \neg StudiesHard(y)) \Rightarrow gt(Grade(x), Grade(y))$

2 Problem 2

2.1 2-A

$Wrote(Gershwin, TheManILove)$

2.2 2-B

$\neg Wrote(Gershwin, EleanorRigby)$

2.3 2-C

$Wrote(Gershwin, TheManILove) \vee Wrote(McCartney, TheManILove)$

2.4 2-D

$\exists song \, Wrote(Joe, song)$

2.5 2-E

$\exists disk \, Owns(Joe, disk) \wedge CopyOf(disk, Revolver)$

2.6 2-F

$\forall song \, Sings(McCartney, song, Revolver) \Rightarrow Wrote(McCartney, song)$

2.7 2-G

$\forall song \, \exists singer \, Sings(singer, song, Revolver) \Rightarrow \neg Wrote(Gershwin, song)$

2.8 2-H

$\forall song \, Wrote(Gershwin, song) \Rightarrow \exists album, singer \, Sings(singer, song, album)$

2.9 2-I

$\exists album \, \forall song \, Wrote(Joe, song) \Rightarrow \exists singer \, Sings(singer, song, album)$

2.10 2-J

$\exists disk(Owns(Joe, disk) \wedge \exists album(CopyOf(disk, album) \wedge Sings(BillyHoliday, TheManILove, album))$

2.11 2-K

$\forall album(\exists songSings(McCartney, song, album)) \Rightarrow (\exists diskCopyOf(disk, album) \wedge$
$Owns(Joe, disk))$

2.12 2-L

$\forall song \exists album, singer(Sings(singer, osng, album) \wedge (singer! = BillyHoliday) \Leftrightarrow$
$(\exists disk\, CopyOf(disk, album) \wedge Owns(Joe, disk))$

3 Problem 3

Let us recall the property given in the exercice:

1. $0 \leq 4$
2. $5 \leq 9$
3. $\forall x,\ x \leq x$
4. $\forall x,\ x \leq x + 0$
5. $\forall x,\ x + 0 \leq x$
6. $\forall x, y,\ x + y \leq y + x$
7. $\forall w, x, y, z,\ x \leq y \wedge x \leq z \Rightarrow w + x \leq y + z$
8. $\forall x, y, z,\ x \leq y \wedge y \leq z \Rightarrow x \leq z$

3.1 3-A

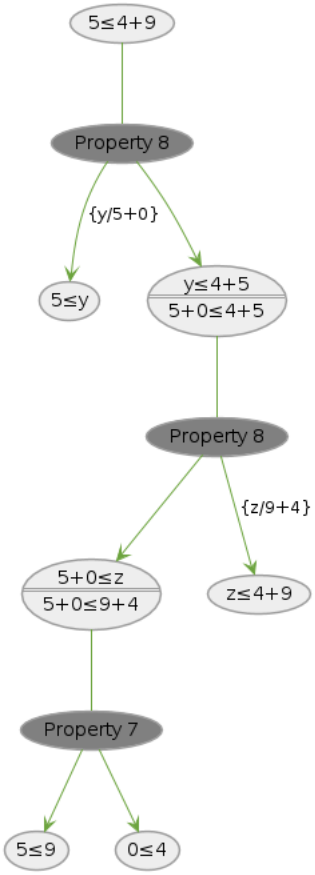

Figure 2: Backward chaining

3.2 3-B

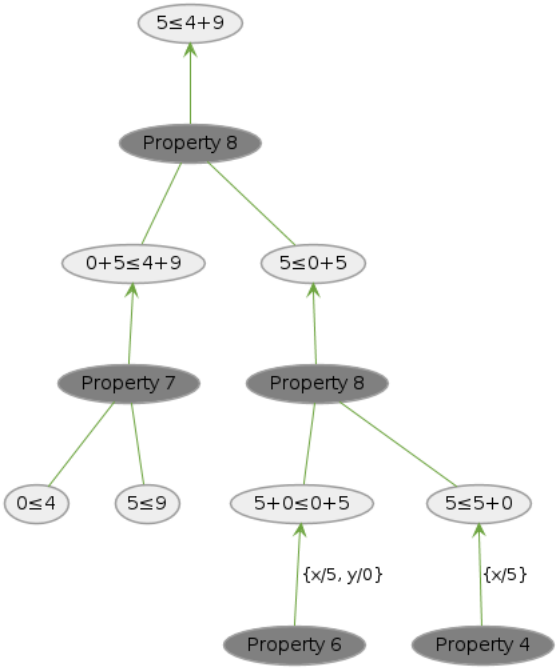

Figure 3: Forward Chaining