DGIM algorithm

Datar-Gionis-Indynk-Motwani Algorithm Counting 1's in a stream

Amazon example:

- For every product X we keep 0/1 stream of whether that product was sold in the n-th transaction
- We want answer queries, how many times have we

Sliding Window: 1 Stream

Sliding window on a single stream:

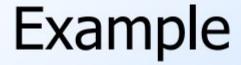
N = 6

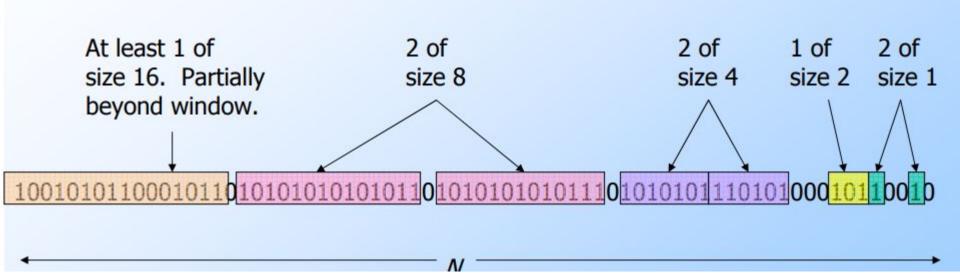
DGIM Algorithm

Representing a Stream by Buckets

- Either one or two buckets with the same power-of-2 number of 1's.
- Buckets do not overlap in timestamps.
- Buckets are sorted by size (# of 1's).
 - Earlier buckets are not smaller than later buckets.
- Buckets disappear when their end-time
 is > N time units in the past.

There can be 2 buckets of size 2 or 1 buckets of size 2, as in this case.





Example 2

. . 1 0 1 1 0 1 1 0 0 0 1 0 1 1 1 0 1 1 0 0 1 0 1 1 0

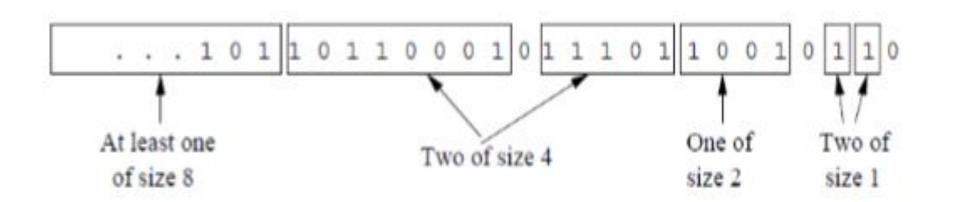


Figure: A bit-stream divided into buckets following the DGIM rules

Updating Buckets --- (1)

- When a new bit comes in, drop the last (oldest) bucket if its end-time is prior to N time units before the current time.
- If the current bit is 0, no other changes are needed.

Updating Buckets when 0 comes in

Example

Updating Buckets when 0 comes in

Example

Updating Buckets when 1 comes in

Current state of the stream:

Bit of value 1 arrives

001010110001011 010101010101011 010101010111 0101010111010101000 101100101

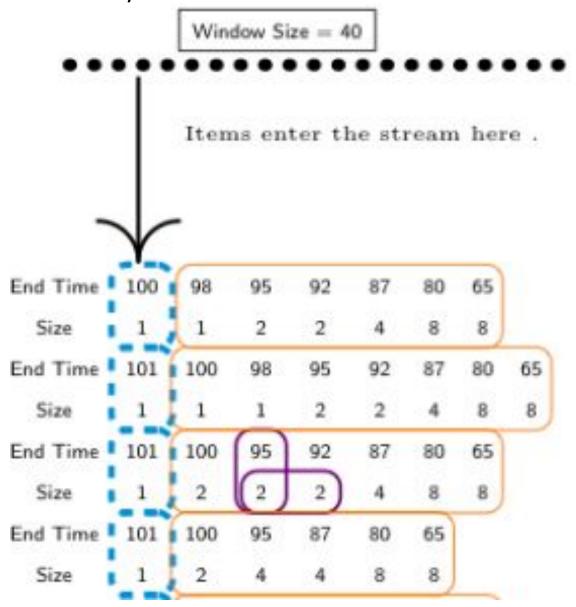
Two orange buckets get merged into a yellow bucket

Next bit 1 arrives, new orange bucket is created, then 0 comes, then 1:

Buckets get merged...

State of the buckets after merging

Sliding window of length 40. The current timestamp is 100. Suppose that at times 101 through 105, 1's appear in the stream. Compute the set of buckets that would exist in the system at time 105.



End Time	102	101	100	95	87	80	65		
Size	1	1	2	4	4	8	8		
End Time	103	102	101	100	95	87	80	65	
Size	1	1	1	2	4	4	8	8	
End Time	103	102	100	95	87	80	65		
Size	1	2	2	4	4	8	8		The difference between the latest timestamp(105) and the oldest(65) equals the windows size(40). So the oldest bucket is dropped.
End Time	104	103	102	100	95	87	80	65	/
Size	1	1	2	2	4	4	8	8	∠
End Time	105	104	103	102	100	95	87	80	65
Size	1	1	1	2	2	4	4	8	8
End Time	105	104	102	100	95	87	80		
Size	1	2	(2	2	4	4	8		
End Time	105	104	102	95	87	80			
Size	1	2	(4)	4)	4	8			
End Time	105	104	102	95	80				
Size	1	2	4	8	8				