Every Unit Matters: Dynamic Programming for Medical Supply Chain Decisions

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Abstract: Efficient medical inventory management is a critical challenge in healthcare systems, where uncertainty in demand and the high cost of shortages can directly impact patient outcomes. This report presents a dynamic programming based decision support system for optimizing medical inventory ordering decisions over a finite planning horizon. The proposed approach explicitly balances regular ordering costs, inventory holding costs, and emergency shortage costs, while respecting warehouse capacity constraints.

The algorithm is implemented from first principles without relying on built in optimization solvers, allowing full transparency and control over the decision process. In addition, a visualization and testing framework is developed to analyze optimal policies, backtracking behavior, and cost accumulation over time. The obtained results are discussed and qualitatively compared with alternative approaches reported in the literature, and potential extensions for real hospital settings are proposed.

I. INTRODUCTION

Modern healthcare delivery relies heavily on efficient medical supply chains to ensure continuous access to essential items such as medications, consumables, and protective equipment. Inventory decisions made at hospitals directly influence patient safety, operational efficiency, and financial sustainability.

Unlike traditional commercial inventory systems, medical inventory management operates under stricter service level requirements and higher penalties for failure. A missing medical item is not merely an inconvenience, but a potential risk to patient health. For example, a shortage of a critical medication during peak demand may delay treatment, while excessive stocking may lead to expiration and waste.

These characteristics motivate the use of structured optimization methods that explicitly model time, uncertainty, and future consequences of current decisions. Dynamic programming provides a natural framework for such problems, as it decomposes complex sequential decisions into manageable stages. In this project, we employ a finite horizon dynamic programming approach to model hospital inventory replenishment decisions. The objective is to achieve an optimal balance between ordering cost, storage cost, and emergency shortage cost over time.

II. BIOMEDICAL PROBLEM DESCRIPTION

Hospitals must continuously decide how much inventory to order and when to place these orders. Ordering too little may result in stockouts that delay or compromise patient care, while ordering too much increases storage costs and the risk of expiration.

The biomedical impact of poor inventory decisions is significant. For instance, shortages of surgical supplies or emergency medications can directly affect treatment timelines and clinical outcomes. These risks are even more pronounced in regions with limited supplier flexibility, longer replenishment times, or constrained budgets.

In this work, we focus on a single hospital managing the inventory of a critical medical item. While simplified, this setting captures the core challenges faced by healthcare inventory systems and serves as a foundation that can later be extended to multiple items or more complex supply chain structures.

III. PROBLEM STATEMENT AND SYSTEM REQUIREMENTS

The objective of this project is to design and implement a decision support system for hospital inventory management of a critical medical supply.

The system is required to:

- Operate over a finite planning horizon of T periods.
- Determine optimal regular order quantities at each period.
- Respect a maximum storage capacity constraint.
- Automatically place emergency orders when demand exceeds available inventory.
- Minimize the total cost composed of ordering, holding, and emergency shortage costs.
- Output both the minimum total cost and the corresponding optimal ordering policy.

IV. RELATED WORK

Inventory optimization has been extensively studied in operations research and supply chain literature. Classical models such as the Economic Order Quantity and base stock policies provide closed form solutions under simplifying assumptions such as deterministic demand and infinite planning horizons [**?**]. While these models are computationally efficient, they are often insufficient for healthcare settings where demand varies over time and shortages carry severe consequences.

Stochastic inventory control problems are commonly formulated using Markov Decision Processes and solved via dynamic programming techniques [**?**]. These approaches explicitly model the sequential nature of inventory decisions and account for uncertainty in demand. Applications in healthcare include blood bank inventory management, pharmaceutical supply planning, and emergency preparedness systems [**?**].

Several studies have explored heuristic and approximate approaches such as rolling horizon optimization, simulation based policies, and reinforcement learning methods. While these techniques can scale to larger systems, they often sacrifice optimality guarantees and interpretability [**?**]. In contrast, dynamic programming offers a transparent and mathematically grounded framework, which is particularly valuable for safety critical medical supply systems.

This work adopts a deterministic finite horizon dynamic programming formulation to maintain full control over model behavior, cost structure, and decision logic. Unlike black box solvers, the proposed approach is implemented from first principles, enabling direct inspection of decisions, backtracking paths, and cost trade offs.

V. SOLUTION METHODOLOGY

The inventory optimization problem is solved using a backward dynamic programming approach. At each time period, the algorithm evaluates all feasible inventory states and ordering decisions, then selects the action that minimizes the total future cost.

The solution process can be understood through a simple analogy. Just as a clinician plans a treatment by considering long term patient outcomes rather than only immediate symptoms, the algorithm plans inventory decisions by accounting for future costs and constraints rather than short term savings alone. This perspective prevents decisions that appear cheap in the current period but lead to expensive emergency actions later.

The solution consists of three tightly coupled stages:

1. Construction of the dynamic programming table using backward recursion.

2. Extraction of the optimal ordering policy through systematic backtracking.

3. Visualization and performance analysis of inventory levels, costs, and emergency orders.

This structure ensures both optimality and interpretability of the resulting policy.

VI. DYNAMIC PROGRAMMING FORMULATION

The objective is to minimize the total cost incurred over the planning horizon while ensuring continuous availability of the medical supply.

A. Cost Function

At each time period, the system incurs three types of cost:

- Regular ordering cost, consisting of a fixed administrative component and a per unit purchase cost.

- Inventory holding cost for excess stock carried to the next period.

- Emergency shortage cost incurred when demand exceeds available inventory.

This cost structure reflects real hospital operations, where emergency procurement is significantly more expensive than planned ordering.

B. Bellman Equation

Let $V_t(x)$ denote the minimum cost to go from time t onward given an inventory level x. The optimality equation is:

$$V_t(x) = \min_{u \geq 0} \left[c(x, u, d_t) + V_{t+1}(x') \right]$$

where $x' = \max(0, x + u - d_t)$.

The terminal condition is defined as:

$$V_T(x) = 0$$

indicating that no future cost is incurred beyond the planning horizon.

C. Dynamic Programming Table Construction

A two dimensional dynamic programming table $V_t(x)$ is constructed, where each entry represents the minimum cost to go from time t with inventory level x. The table is filled backward starting from the final period.

For each state (t, x), all feasible order quantities are evaluated subject to storage capacity constraints. The algorithm computes the immediate cost and adds the previously computed cost to go from the next state.

D. Optimal Policy Extraction via Backtracking

After the table is fully populated, the optimal ordering policy is recovered using backtracking. Starting from the initial inventory level, the algorithm follows the sequence of decisions that produced the minimum cost.

This backtracking process generates a complete operational plan, including regular order quantities, emergency orders, inventory evolution, and cost per period. The extracted policy is guaranteed to be optimal under the defined model assumptions.

VII. CORRECTNESS ARGUMENT

The proposed algorithm satisfies the principle of optimality, which states that an optimal policy has the property that its remaining decisions constitute an optimal policy with respect to the state resulting from the first decision.

At each time period, the algorithm considers all feasible actions and selects the one that minimizes the sum of immediate cost and future optimal cost. Since the state space and decision space are exhaustively evaluated, and the recursion is applied consistently across periods, the resulting policy is globally optimal for the given model.

Therefore, the dynamic programming solution guarantees correctness and optimality within the defined system constraints.

VIII. COMPUTATIONAL COMPLEXITY

Let T denote the planning horizon, N the number of inventory states, and U the number of feasible order quantities.

The time complexity of the algorithm is:

$$O(T \cdot N \cdot U)$$

as each state requires evaluating all possible order decisions.

The space complexity is:

$$O(T \cdot N)$$

due to storage of the dynamic programming table.

This complexity is acceptable for moderate horizon lengths and inventory discretization, which aligns with typical hospital planning cycles [**?**].

IX. IMPLEMENTATION DETAILS

The dynamic programming algorithm was implemented in Python using explicit table based recursion without relying on external optimization solvers. A two dimensional dynamic programming table is constructed, where each entry stores the minimum cost to go for a given time period and inventory level.

The algorithm iterates backward in time and evaluates all feasible ordering quantities while enforcing warehouse capacity constraints. Emergency orders are triggered automatically whenever available inventory is insufficient to meet demand.

The implementation outputs the minimum total cost as well as the optimal regular ordering policy. Additional data structures are used to store inventory trajectories and cost components for later visualization and analysis.

X. EXPERIMENTAL SETUP

Simulation experiments are designed using synthetic demand sequences in order to evaluate the behavior of the proposed inventory optimization framework under controlled conditions.

Experiments are conducted over a finite planning horizon with predefined demand profiles, storage capacity limits, and cost parameters. Multiple scenarios are considered, including baseline demand, high demand variability, and stress testing conditions.

Performance is evaluated using total cost, frequency of emergency orders, inventory level evolution, and ordering patterns. These metrics are used consistently across all scenarios to enable meaningful comparison.

XI. DISCUSSION AND LIMITATIONS

This section will analyze the observed results and discuss the strengths and limitations of the proposed approach. Emphasis will be placed on cost tradeoffs, policy behavior under stress scenarios, and computational feasibility.

Model assumptions and practical limitations will also be examined, with attention to demand forecasting accuracy and scalability.

XII. CONCLUSION

This work presents a structured dynamic programming framework for medical inventory decision making. By explicitly modeling ordering, holding, and emergency costs, the proposed system provides a transparent and extensible decision support tool for healthcare supply chains.

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