Lemma 1. *Linear approximation of spiking nodes*

$$X_j(t) = E[Y_j(t)]$$

where j is the j^{th} neuron

Proof by induction. Let $t, k \in \mathbb{Z}$. Given $X_j(1) = 1$ or 0.

Show for $k = 1$.

If $X_j(1) = 1$, then $Y_j(1) = 1$ and $E[Y_j(1)] = 1 = X_j(1)$.

If $X_j(1) = 0$, then $Y_j(1) = 0$ and $E[Y_j(1)] = 0 = X_j(1)$.

Show that $k \to k + 1$. Assume $X_j(k) = E[Y_j(k)]$

$$\begin{aligned}
X_j(k+1) &= E[Y_j(k+1)] \\
&= E[\sum_{i \to j} \sum_{Y_i(k)} R(A_{ij})] && \text{By definition,} \\
&= \sum_{i \to j} E[\sum_{Y_i(k)} R(A_{ij})] \\
&= \sum_{i \to j} E[Y_i(k)] A_{ij} && \text{Bernoulli process, R} \\
&= \sum_{i \to j} X_j(k) A_{ij} && \text{Assumption} \\
&= X_j(k+1) && \square
\end{aligned}$$