

**Theorem 1.** *Any normal modal logic containing  $\Diamond\Box\perp$  is the inconsistent logic.*

*Proof.* Formal proof deriving  $\perp$ :

1.  $\perp \rightarrow p$  (taut.)
2.  $\perp \rightarrow \neg\Box\perp$  (unif, 1.)
3.  $\Box(\perp \rightarrow \neg\Box\perp)$  (nec, 2.)
4.  $\Box(p \rightarrow q) \rightarrow (\Box p \rightarrow \Box q)$  (Ax. K.)
5.  $\Box(\perp \rightarrow \neg\Box\perp) \rightarrow (\Box\perp \rightarrow \Box\neg\Box\perp)$  (unif, 4)
6.  $\Box\perp \rightarrow \Box\neg\Box\perp$  (MP, 3, 5.)
7.  $(p \rightarrow q) \rightarrow (\neg q \rightarrow \neg p)$  (taut.)
8.  $(\Box\perp \rightarrow \Box\neg\Box\perp) \rightarrow (\neg\Box\neg\Box\perp \rightarrow \neg\Box\perp)$  (unif, 7.)
9.  $\neg\Box\neg\Box\perp \rightarrow \neg\Box\perp$  (MP, 6, 8.)
10.  $\Diamond p \leftrightarrow \neg\Box\neg p$  (Dual Ax.)
11.  $\Diamond\Box\perp \leftrightarrow \neg\Box\neg\Box\perp$  (unif, 10.)
12.  $(p \leftrightarrow q) \rightarrow ((q \rightarrow r) \rightarrow (p \rightarrow r))$  (taut.)
13.  $(\Diamond\Box\perp \leftrightarrow \neg\Box\neg\Box\perp) \rightarrow ((\neg\Box\neg\Box\perp \rightarrow \neg\Box\perp) \rightarrow (\Diamond\Box\perp \rightarrow \neg\Box\perp))$  (unif, 12.)
14.  $(\neg\Box\neg\Box\perp \rightarrow \neg\Box\perp) \rightarrow (\Diamond\Box\perp \rightarrow \neg\Box\perp)$  (MP, 11, 13.)
15.  $\Diamond\Box\perp \rightarrow \neg\Box\perp$  (MP, 9, 14)
16.  $\Diamond\Box\perp$  (Ax.)
17.  $\neg\Box\perp$  (MP, 15, 16.)
18.  $\Box\neg\Box\perp$  (nec, 17.)
19.  $q \rightarrow ((p \leftrightarrow \neg q) \rightarrow (p \rightarrow \perp))$  (taut.)
20.  $\Box\neg\Box\perp \rightarrow ((\Diamond\Box\perp \leftrightarrow \neg\Box\neg\Box\perp) \rightarrow (\Diamond\Box\perp \rightarrow \perp))$  (unif, 19.)
21.  $(\Diamond\Box\perp \leftrightarrow \neg\Box\neg\Box\perp) \rightarrow (\Diamond\Box\perp \rightarrow \perp)$  (MP, 18, 20.)
22.  $\Diamond\Box\perp \rightarrow \perp$  (MP, 11, 21.)
23.  $\perp$  (MP, 16, 22.)

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