Planning and Scheduling: Propositional Satisfiability Techniques

Prof. Dr.-Ing. Gerhard K. Kraetzschmar

Hochschule Bonn-Rhein-Sieg

b-it Bonn-Aachen International Center for Information Technology

Acknowledgements

- These slides are based on those slides by
  Dana Nau, Stuart Russell, Sheila McIlrath and Alen Fern

Motivation

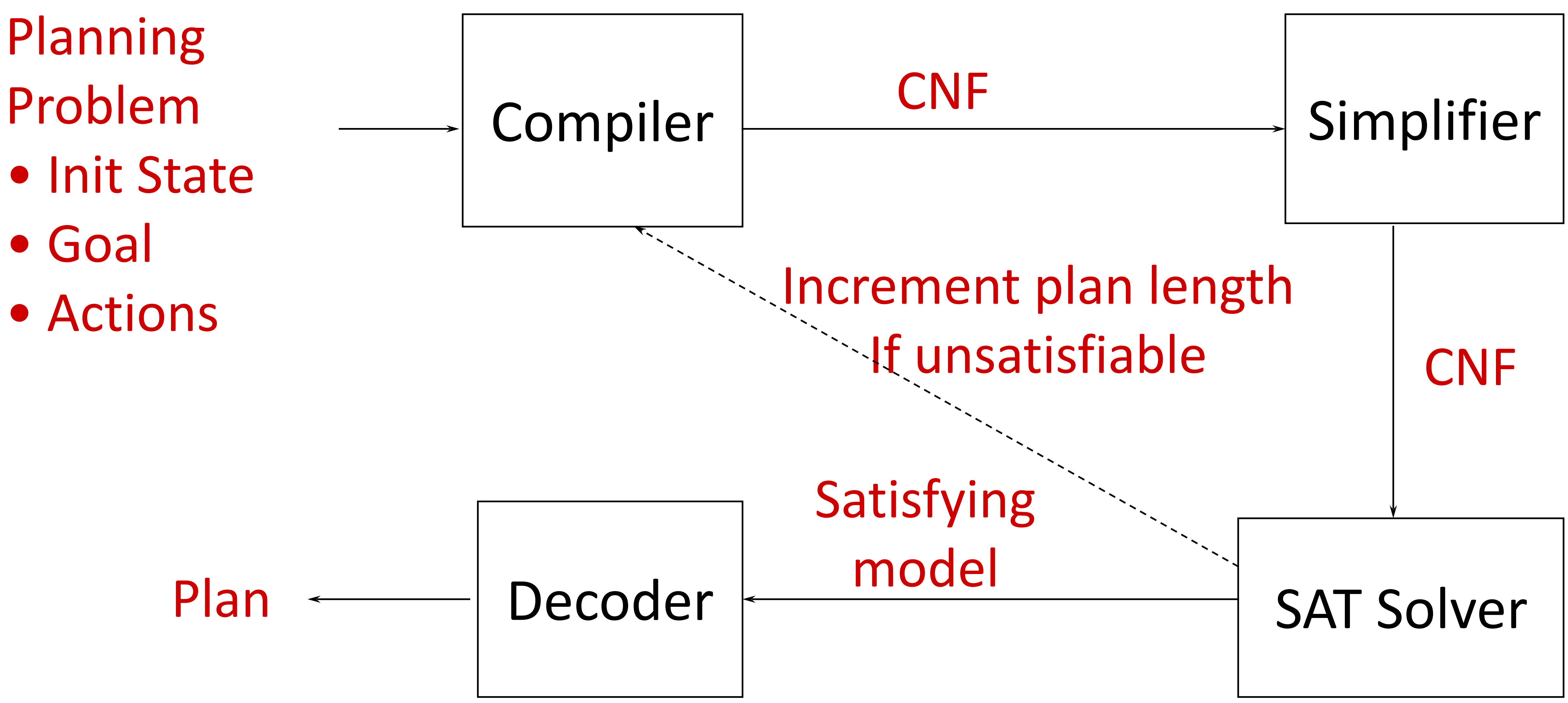
- Propositional satisfiability:
  - Given a Boolean formula, e.g. $(P \vee Q) \wedge (\neg Q \vee R \vee S) \wedge (\neg R \vee \neg P)$ does there exist a model?
    - Model: an assignment of truth values to the propositional variables that makes the formula true?
- This was the very first problem shown to be NP-complete
- Lots of research on algorithms for solving it
  - Algorithms are known for solving all but a small subset in average-case polynomial time
- Therefore:
  - Try translating classical planning problems into satisfiability problems, and solving them that way

Outline

- Brief review of propositional logic & satisfiability
- Planning as propositional satisfiability:
  - Encoding planning problems as satisfiability problems
  - Checking for satisfiability with a SAT solver
  - Extracting plans from truth values
- Satisfiability techniques
  - Davis-Putnam (actually DPLL, sometimes referred to as DP)
  - Local search
  - GSAT and WalkSAT
- Combining satisfiability techniques with planning graphs
  - BlackBox & SATPlan

* Terminology: "SATPLAN approach" (circa 1992) vs. the SATPLAN planner of 2004, 2006 etc., the successor of Blackbox.

Architecture of a SAT-based Planner



History of SAT-based approach

- 1969 Plan synthesis as theorem proving (Green IJCAI-69)
- 1971 STRIPS (Fikes & Nilsson AIJ-71)
- …
- 1992 Satplan Approach (Kautz & Selman ECAI-92)
- 1996 (Kautz & Selman AAAI-96) (Kautz, McAllester & Selman KR-96)
- 1997 MEDIC (Ernst et al. IJCAI-97)
- 1998 Blackbox (Kautz & Selman AIPS98 workshop)
- 1998 IPC-1 Blackbox performance comparable to the best
- 2000 IPC-2 Blackbox performs terribly (Graphplan-style planners dominated)
- 2002 IPC-3 No SAT-based planners entered
- 2004 IPC-4 Satplan04 was clear winner of Optimal propositional planners
- 2006 IPC-5 Satplan06 & Maxplan* (Chen Xing & Zhang IJCAI-07) dominated

BG - Propositional Logic: Syntax

- We are given a set of primitive propositions $\{P_1, \ldots, P_n\}$
  - These are the basic statements we can make about the "world"

- From basic propositions we can construct compound sentences (also called formulas)
  - If S is a sentence, $\neg S$ is a sentence (negation)
  - If S1 and S2 are sentences, $S1 \wedge S2$ is a sentence (conjunction)
  - If S1 and S2 are sentences, $S1 \vee S2$ is a sentence (disjunction)
  - If S1 and S2 are sentences, $S1 \Rightarrow S2$ is a sentence (implication)
  - If S1 and S2 are sentences, $S1 \Leftrightarrow S2$ is a sentence (biconditional)

BG - Propositional Logic: CNF

- A literal is either a proposition or the negation of a proposition
- A clause is a disjunction of literals
- A formula is in conjunctive normal form (CNF) if it is the conjunction of clauses:
  - E.g.: $(\neg R \vee P \vee Q) \wedge (\neg P \vee Q) \wedge (\neg P \vee R)$
- Any formula can be represented in conjunctive normal form (CNF)

BG - Propositional Logic: Semantics

- A truth assignment assigns a truth value to each propositions:
  - E.g. P1 = false, P2 = true, P3 = false
- A formula is either true or false with respect to a truth assignment
- The truth of a formula is evaluated recursively as given below: (let P and Q be formulas)

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
$false$	$false$	$true$	$false$	$false$	$true$	$true$
$false$	$true$	$true$	$false$	$true$	$true$	$false$
$true$	$false$	$false$	$false$	$true$	$false$	$false$
$true$	$true$	$false$	$true$	$true$	$true$	$true$

- A model is a truth assignment that makes the formula true

BG - Propositional Satisfiability

- A formula is satisfiable iff there exists some model
  - e.g. $A \vee B$, C
- A formula is unsatisfiable iff there does not exist any model
  - e.g. $A \wedge \neg A$

Encoding Planning as Satisfiability: Basic Idea

- A bounded planning problem is a pair (P,n):
  - P is a planning problem; n is a positive integer
  - Find a solution for P of length n
- Create a propositional formula that represents:
  - Initial state
  - Goal
  - Action dynamics
- for n time steps

- We will define the formula for (P,n) such that:
  1) any model of the formula represents a solution to (P,n)
  2) if (P,n) has a solution then the formula is satisfiable

Example of Complete Formula for (P,1)

at(r1,l1,0) ∧ ¬at(r1,l2,0) ∧

at(r1,l2,1) ∧

move(r1,l1,l2,0) ⇒ at(r1,l1,0) ∧ at(r1,l2,1) ∧ ¬at(r1,l1,1) ∧

move(r1,l2,l1,0) ⇒ at(r1,l2,0)] ∧ at(r1,l1,1) ∧ ¬at(r1,l2,1) ∧

¬move(r1,l1,l2,0) ∨ ¬move(r1,l2,l1,0) ∧

¬at(r1,l1,0) ∧ at(r1,l1,1) ⇒ move(r1,l2,l1,0) ∧

¬at(r1,l2,0) ∧ at(r1,l2,1) ⇒ move(r1,l1,l2,0) ∧

at(r1,l1,0) ∧ ¬at(r1,l1,1) ⇒ move(r1,l1,l2,0) ∧

at(r1,l2,0) ∧ ¬at(r1,l2,1) ⇒ move(r1,l2,l1,0)

Overall Approach

- Do iterative deepening (like we did with GraphPlan):
  - for n = 0, 1, 2, …,
    - encode (P,n) as a satisfiability problem Φ
    - if Φ is satisfiable, then
      - From the set of truth values that satisfies Φ, a solution plan can be constructed, so return it and exit
- With a complete satisfiability tester,
  this approach will produce optimal layered plans for solvable problems
- We can use a GraphPlan analysis to determine an upper bound on n, providing us with a way to detect unsolvability

Notation

- For satisfiability problems we need to use propositional logic
- Need to encode ground atoms into propositions
  - For set-theoretic planning we encoded atoms into propositions by rewriting them as shown here:
    - Atom: at(r1,loc1)
    - Proposition: at-r1-loc1
- For planning as satisfiability we'll do the same thing
  - But we won't bother to do a syntactic rewrite
  - Just use at(r1,loc1) itself as the proposition
- Also, we'll write plans starting at a_0 rather than a_1
  - $\pi = \langle a_0, a_1, \ldots, a_{n-1} \rangle$

Fluents

- If $\pi = \langle a_0, a_1, \ldots, a_{n-1} \rangle$ is a solution for (P,n), it generates these states:

  $$s_0, s_1 = \gamma(s_0, a_0), s_2 = \gamma(s_1, a_1), \ldots, s_n = \gamma(s_{n-1}, a_{n-1}$$

- A fluent is a proposition used to describe what's true in each s_i
  - $at(r1, loc1, i)$ is a fluent that's true iff $at(r1, loc1)$ is true in s_i
  - We'll use l_i to denote the fluent for a literal l in state s_i
    - e.g., if $l = at(r1, loc1)$ then $l_i = at(r1, loc1, i)$
  - a_i is a fluent saying that a is the i'th step of π
    - e.g., if $a = move(r1, loc2, loc1)$ then $a_i = move(r1, loc2, loc1, i)$

Encoding Planning Problems

- Encode (P,n) as a formula Φ such that
  - $\pi = \langle a_0, a_1, \ldots, a_{n-1} \rangle$ is a solution for (P,n) iff
  - Φ can be satisfied in a way that makes the fluents $a_0, a_1, \ldots, a_{n-1}$ true
- Let
  - A = {all actions in the planning domain}
  - S = {all states in the planning domain}
  - L = {all literals in the language}
- Φ is the conjunction of many other formulas …

Formulas in Φ

- Formula describing the initial state:

$$\bigwedge\{l_0 \mid l \in s_0\} \wedge \bigwedge\{\neg l_0 \mid l \in \mathcal{L} \setminus s_0\}$$

Describes the complete initial state (both positive and negative facts)
E.g. on(A,B,0) ∧ ¬on(B,A,0)

- Formula describing the goal:

$$\bigwedge\{l_n \mid l \in g^+\} \wedge \bigwedge\{\neg l_n \mid l \in g^-\}$$

Says that goal facts must be true in the final state (i.e. at time-step n)
E.g. on(B,A,n)

- Is this enough?

Formulas in Φ

- For every action a and timestep i, formula describing what fluents must be true if a were in the i'th step of the plan:

$$a_i \Rightarrow \bigwedge\{p_i \mid p \in Precond(a)\} \wedge \bigwedge\{e_{i+1} \mid e \in Effects(a)\}$$

- Complete exclusion axiom:
  - For all actions a and b and time-steps i, formulas saying a and b can't occur at the same time, e.g.

$$\neg a_i \vee \neg b_i$$

- Is this enough?
  - The formulas say nothing about what happens to facts if they are not effected by an action.

Frame Axioms

- Frame axioms:
  - Formulas describing what doesn't change between steps i and i+1
- Several ways to write these…

- One way: explanatory frame axioms
  - One axiom for every possible literal l at every timestep i
  - Says that if l changes between s_i and s_{i+1},
  - then the action at step i must be responsible:

$$(\neg l_i \wedge l_{i+1} \Rightarrow \bigvee \{a_i \mid l \in effects^+(a)\})$$
$$\wedge (l_i \wedge \neg l_{i+1} \Rightarrow \bigvee \{a_i \mid l \in effects^-(a)\})$$

Example

- Planning domain:
  - one robot r1
  - two adjacent locations l1, l2
  - one operator (move the robot)

- Encode (P,n) where n = 1
  - Initial state: {at(r1,l1)}
    - Encoding: $at(r1,l1,0) \wedge \neg at(r1,l2,0)$
  - Goal: {at(r1,l2)}
    - Encoding: $at(r1,l2,1) \wedge \neg at(r1,l1,1)$
  - Operator: …

Example (continued)

- Operator: move(r,l,l')
  - precond: at(r,l)
    effects: at(r,l'), ¬at(r,l)
- Encoding:
  - $\text{move}(r1,l1,l2,0) \Rightarrow \text{at}(r1,l1,0) \wedge \text{at}(r1,l2,1) \wedge \neg\text{at}(r1,l1,1)$

    $\text{move}(r1,l2,l1,0) \Rightarrow \text{at}(r1,l2,0) \wedge \text{at}(r1,l1,1) \wedge \neg\text{at}(r1,l2,1)$

    $\text{move}(r1,l1,l1,0) \Rightarrow \text{at}(r1,l1,0) \wedge \text{at}(r1,l1,1) \wedge \neg\text{at}(r1,l1,1)$

    $\text{move}(r1,l2,l2,0) \Rightarrow \text{at}(r1,l2,0) \wedge \text{at}(r1,l2,1) \wedge \neg\text{at}(r1,l2,1)$

    contradictions (easy to detect)

    $\text{move}(l1,r1,l2,0) \Rightarrow \ldots$

    $\text{move}(l2,l1,r1,0) \Rightarrow \ldots$

    $\text{move}(l1,l2,r1,0) \Rightarrow \ldots$

    $\text{move}(l2,l1,r1,0) \Rightarrow \ldots$

    nonsensical
- How to avoid generating the last four actions?
  - Assign data types to the constant symbols like we did for state-variable representation

Example (continued)

- Locations: l1, l2
- Robots: r1

- Operator: move(r : robot, l : location, l' : location)
  - precond: at(r,l)
  - effects: at(r,l'), ¬at(r,l)

- Encoding:
  - $move(r1,l1,l2,0) \Rightarrow at(r1,l1,0) \wedge at(r1,l2,1) \wedge \neg at(r1,l1,1)$
  - $move(r1,l2,l1,0) \Rightarrow at(r1,l2,0) \wedge at(r1,l1,1) \wedge \neg at(r1,l2,1)$

Example (continued)

- Complete-exclusion axiom:
  - $\neg move(r1,l1,l2,0) \vee \neg move(r1,l2,l1,0)$
- Explanatory frame axioms:
  - $\neg at(r1,l1,0) \wedge at(r1,l1,1) \Rightarrow move(r1,l2,l1,0)$
  - $\neg at(r1,l2,0) \wedge at(r1,l2,1) \Rightarrow move(r1,l1,l2,0)$
  - $at(r1,l1,0) \wedge \neg at(r1,l1,1) \Rightarrow move(r1,l1,l2,0)$
  - $at(r1,l2,0) \wedge \neg at(r1,l2,1) \Rightarrow move(r1,l2,l1,0)$

Example of Complete Formula for (P,1)

Initial State

$at(r1,l1,0) \wedge \neg at(r1,l2,0) \wedge$

Goal

$at(r1,l2,1) \wedge$

Operator

$move(r1,l1,l2,0) \Rightarrow at(r1,l1,0) \wedge at(r1,l2,1) \wedge \neg at(r1,l1,1) \wedge$

$move(r1,l2,l1,0) \Rightarrow at(r1,l2,0)\ ] \wedge at(r1,l1,1) \wedge \neg at(r1,l2,1) \wedge$

Complete exclusion axiom

$\neg move(r1,l1,l2,0) \vee \neg move(r1,l2,l1,0) \wedge$

Explanatory frame axioms

$\neg at(r1,l1,0) \wedge at(r1,l1,1) \Rightarrow move(r1,l2,l1,0) \wedge$

$\neg at(r1,l2,0) \wedge at(r1,l2,1) \Rightarrow move(r1,l1,l2,0) \wedge$

$at(r1,l1,0) \wedge \neg at(r1,l1,1) \Rightarrow move(r1,l1,l2,0) \wedge$

$at(r1,l2,0) \wedge \neg at(r1,l2,1) \Rightarrow move(r1,l2,l1,0)$

Convert to CNF and give it to a SAT solver

Extracting a Plan

- Suppose we find an assignment of truth values that satisfies Φ.
  - This means P has a solution of length n

- For i=0,…,n-1, there will be exactly one action a such that a_i = true
  - This is the i'th action of the plan.

- Example (from the previous slides):
  - Φ can be satisfied with move(r1,l1,l2,0) = true
  - Thus, 〈move(r1,l1,l2,0)〉 is a solution for (P,0)
  - It's the only solution — no other way to satisfy Φ

Supporting Layered Plans

- Complete exclusion axiom:
  - For all actions a and b and time steps i include the formula

    $$\neg a_i \lor \neg b_i$$
  - This guaranteed that there could be only one action at a time
- Partial exclusion axiom:
  - For any pair of incompatible actions a and b and each time step i include the formula $\neg a_i \lor \neg b_i$
  - This encoding allows more than one action to be taken at a time step — resulting in layered plans

SAT Algorithms

- Systematic Search
  - DPLL (Davis Putnam Logemann Loveland)
    - backtrack search and unit propagation
    - Extend partial assignment into complete assignment
    - Sound and complete
- Local Search
  - GSAT
  - Walksat
    - Greedy local search and noise to escape minima
    - Modify randomly chosen total assignment
    - Sound but not complete (but very fast!)

Planning

- How to find an assignment of truth values that satisfies Φ?
  - Use a satisfiability algorithm

- DPLL is a complete SAT-solver:
  - First need to put Φ into conjunctive normal form
    - e.g., $\Phi = D \wedge (\neg D \vee A \vee \neg B) \wedge (\neg D \vee \neg A \vee \neg B) \wedge (\neg D \vee \neg A \vee B) \wedge A$
  - Write Φ as a set of clauses (disjunctions of literals)
    - $\Phi = \{\{D\},\ \{\neg D, A, \neg B\},\ \{\neg D, \neg A, \neg B\},\ \{\neg D, \neg A, B\},\ \{A\}\}$
  - Two special cases:
    - If $\Phi = \varnothing$ (I.e. no clauses) is a formula that is always true
    - If $\Phi = \{\ldots, \varnothing, \ldots\}$ (i.e. an empty clause) is a formula that's always false and hence unsatisfiable
  - DPLL simply searches the space of truth assignments, assigning one proposition a value at each step of the search tree

The DPLL Algorithm

- Early termination
  - A clause is true if any literal is true
  - A sentence is false if any clause is false. E.g. $(\neg A \vee \neg B) \wedge (A \vee C)$
- Pure symbol heuristic
  - Pure symbol: always appears with the same "sign" in all clauses
  - e.g., In the three clauses $(A \vee \neg B)$, $(\neg B \vee \neg C)$, $(C \vee A)$
    A and B are pure, C is impure
  - Make a pure symbol literal true
- Unit clause heuristic
  - Unit clause: only one literal in the clause
  - The only literal in a unit clause must be true

Basic Observations

- If literal L1 is true,
  then clause (L1∨L2∨…) is true
- If clause C1 is true,
  then C1∧C2 ∧C3… has the same value as C2∧C3
  - Therefore: Its ok to delete clauses containing true literals!
- If literal L1 is false,
  then clause (L1∨L2∨L3∨…) has the same value as (L2∨L3∨…)
  - Therefore: It is ok to shorten clauses containing false literals!
- If literal L1 is false,
  then clause (L1) is false
  - Therefore: the empty clause means false!

The Davis-Putnam Procedure

μ = **{literals to which we have assigned the value *true*}**; initially empty

DPLL:

if Φ contains an empty clause
  then return false; // backtrack
if Φ is a consistent set of literals
  then return true; // solution
Unit-Propagate // see below
Choose a literal P from Φ;
DPLL($\Phi \wedge P, \mu$);
DPLL($\Phi \wedge \neg P, \mu$);

Unit-propagate: // simplify - B.C. P.
For every unit clause l in Φ
  Add l to the set of true literals
  Delete all clauses containing l
  Delete all occurrences of ¬l

```
Davis-Putnam(Φ,μ)
    if Ø ∈ Φ then return
    if Φ = Ø then exit with μ
    Unit-Propagate(Φ,μ)
    select a variable P such that P or ¬P occurs in φ
    Davis-Putnam(Φ ∪ {P},μ)
    Davis-Putnam(Φ ∪ {¬P},μ)
end

Unit-Propagate(Φ,μ)
    while there is a unit clause {l} in Φ do
        μ ← μ ∪ {l}
        for every clause C ∈ Φ
            if l ∈ C then Φ ← Φ − {C}
            else if ¬l ∈ C then Φ ← Φ − {C} ∪ {C − {¬l}}
end
```

Backtracking search through alternative assignments of truth values to literals

DPLL Example

- Given: formula Φ, with list of symbols {A,B,C,D,E}
  - DPLL({(¬D∨¬B∨C), (B∨¬A∨¬C), (¬C∨¬B∨E), (¬E∨D∨B), (B∨E∨¬C)}, {A, B, C, D, E}, [])
- Step 1: not every clause true, none false, pure symbol ¬A
  - DPLL({(¬D∨¬B∨C), (¬C∨¬B∨E), (¬E∨D∨B), (B∨E∨¬C)}, {B,C,D,E}, [A=false])
- Step 2: not every clause true, none false, no pure symbols, no unit clause
  - DPLL({(¬E∨D), (E∨¬C)}, {C,D,E}, [A=false, B=false])
  - DPLL({(¬D∨C), (¬C∨E)}, {C,D,E}, [A=false, B=true])
- Step 3b: not every clause true, none false, pure D, ¬C, no unit clause
  - DPLL({}, {E}, [A=false, B=false, C=false, D=true]) DONE!
- Step 3a: not every clause true, none false, pure ¬D, E, no unit clause
  - DPLL({}, {C}, [A=false, B=true, D=false, E=true]) DONE!
- Done! (We found even two models.)

DPLL: Example 1

$$(\neg a \lor b) \land (a \lor \neg b) \land (\neg a \lor \neg b)$$

a=T

a=F

$$(\neg a \lor b) \land (a \lor \neg b) \land (\neg a \lor \neg b) \land a$$

$$(\neg a \lor b) \land (a \lor \neg b) \land (\neg a \lor \neg b) \land \neg a$$

Simplify

Simplify

$$b \land \neg b$$

...Satisfiable!

$$\neg b$$

Simplify

{} ...Unsatisfiable!

Assignment: $a \rightarrow 0$, $b \rightarrow 0$

Remove: all clauses which become true

all literals that become false from the remaining clauses

DPLL: Example 2

$$(P \lor R) \land (\neg P \lor \neg Q \lor R) \land (P \lor \neg R)$$

P=T

P=F

$$(P \lor R) \land (\neg P \lor \neg Q \lor R) \land (P \lor \neg R) \land P$$

$$(P \lor R) \land (\neg P \lor \neg Q \lor R) \land (P \lor \neg R) \land \neg P$$

Simplify

Simplify

$\neg Q \lor R$ ...Satisfiable!

$R \land \neg R$

Simplify

$\{\}$

Assignment: $P \rightarrow 1,\ Q \rightarrow 0,\ R \rightarrow 1$

...Unsatisfiable!

Remove: all clauses which become true

all literals that become false from the remaining clauses

Local Search - Basic Idea

- Keep only a single (complete) state in memory
- Generate only the neighbours of that state
- Keep one of the neighbours and discard others

- Key features:
  - no search paths
  - neither systematic nor incremental

- Key advantages:
  - use very little memory (constant amount)
  - find solutions in search spaces too large for systematic algorithms

Local Search

- Local search over space of complete truth assignments
- Let u be an assignment of truth values to all of the variables
  - cost(u,Φ) = number of clauses in Φ that aren't satisfied by u
  - flip(P,u) = u except that P's truth value is reversed
- Local search:
  - Select a random assignment u
  - while cost(u,Φ) ≠ 0
    - if there is a P such that cost(flip(P,u),Φ) < cost(u,Φ) then
      - randomly choose any such P
      - u ← flip(P,u)
    - else return failure
- Local search is sound
- If it finds a solution it will find it very quickly
- Local search is not complete: can get trapped in local minima

GSAT

- Basic-GSAT:
  - Select a random assignment u
  - while cost(u,Φ) ≠ 0
    - choose the P that minimizes cost(flip(P,u),Φ), and flip it
- Not guaranteed to terminate
- GSAT:
  - restart after a max number of flips
  - return failure after a max number of restarts
- WalkSAT is like GSAT
  but differs in the method used to pick which variable to flip
  - Both algorithms may restart with a new random assignment
    if trapped in local minima
  - Many versions of GSAT/WalkSAT. WalkSAT superior for planning

WalkSat

- With probability P:
  - flip any variable in any unsatisfied clause
- With probability (1-P):
  - flip best variable in any unsatisfied clause
  - The best variable is the one that
    when flipped causes the most clauses to be satisfied
- P controls the randomness of search
- Randomness can help avoid local minima
- Best DPLL-based solvers (e.g. Siege) are currently best!

What SAT-based planning shows

- General propositional reasoning can compete with state of the art specialized planning systems
  - Radically new stochastic approaches to SAT can provide very low exponential scaling
  - Best solvers for SAT-based planning are currently DPLL-based solvers such as Satzilla, PrecoSAT (and previously RelSAT and before that Siege and before that ZChaff) that have the option of using random restarts and some other local-search tricks.
- Why does it work?
  - More flexible than forward or backward chaining
  - Randomized algorithms less likely to get trapped along bad paths

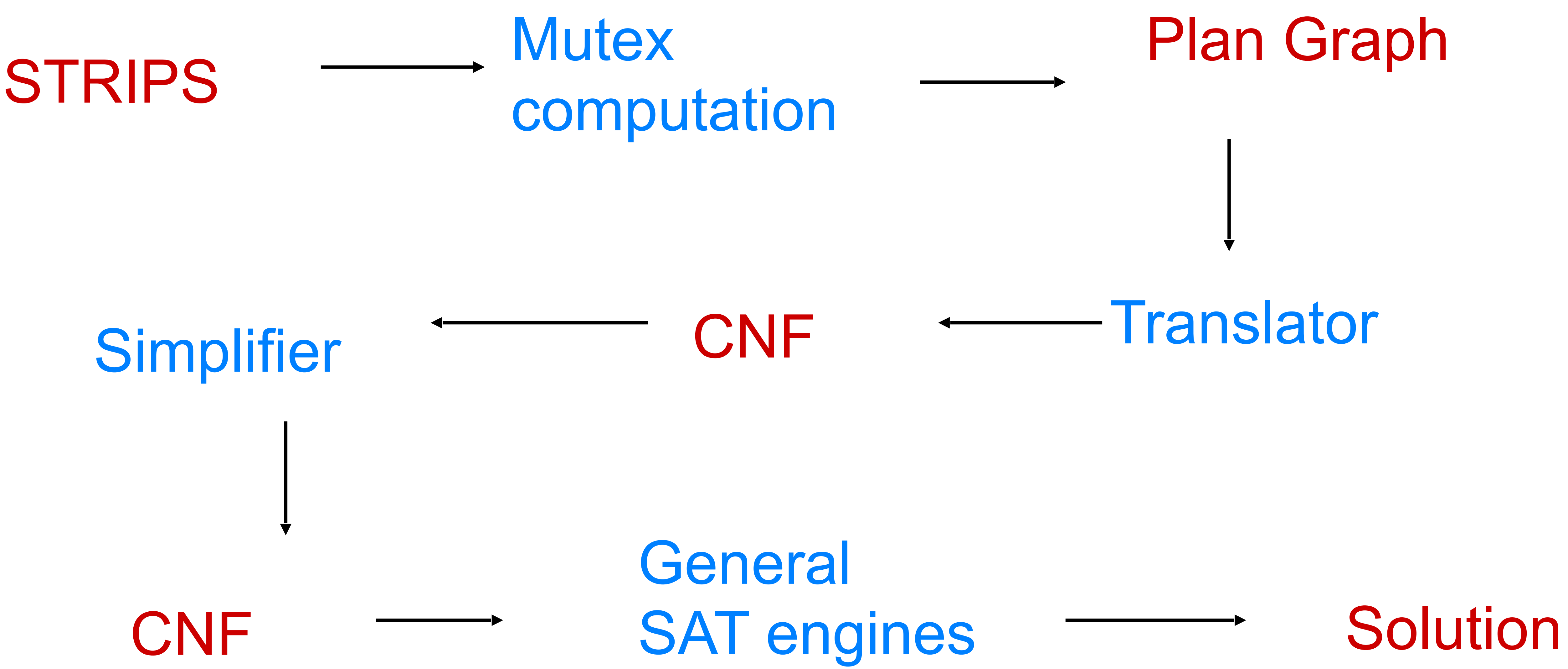
Discussion:

- Recall the overall approach:
  - for n = 0, 1, 2, …,
    - encode (P,n) as a satisfiability problem Φ
    - if Φ is satisfiable, then
      - From the set of truth values that satisfies Φ, extract a solution plan and return it
- How well does this work?

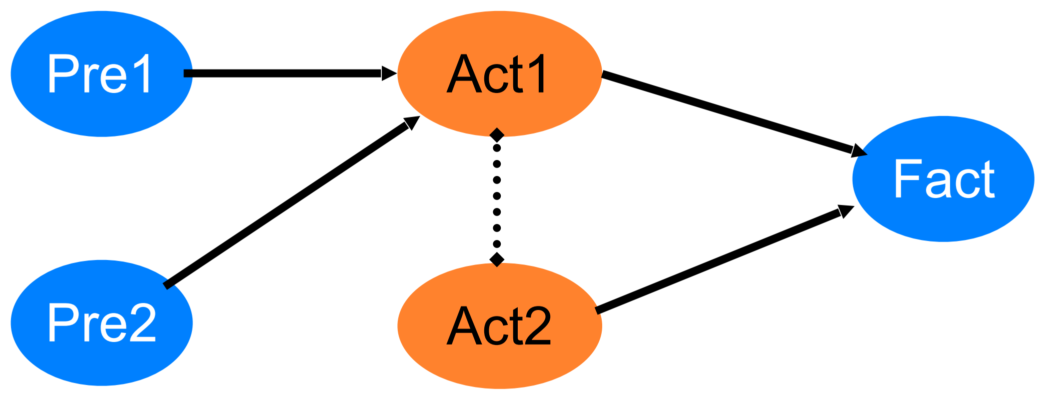
Discussion:

- Recall the overall approach:
  - for n = 0, 1, 2, …,
    - encode (P,n) as a satisfiability problem Φ
    - if Φ is satisfiable, then
      - From the set of truth values that satisfies Φ, extract a solution plan and return it
- How well does this work?
  - By itself, not very practical (takes too much memory and time)
  - But it can be combined with other techniques
    - e.g., planning graphs

BlackBox


STRIPS
Mutex computation
Plan Graph
Translator
CNF
Simplifier
CNF
General SAT engines
Solution

Translation of Plan Graph



- Fact ⇒ Act1 ∨ Act2
- Act1 ⇒ Pre1 ∧ Pre2
- ¬Act1 ∨ ¬Act2

Can create such constraints for every node in the planning graph

BlackBox

- The BlackBox procedure combines planning-graph expansion and satisfiability checking
- Roughly as follows:
  - for n = 0, 1, 2, …
    - Graph expansion: create a "planning graph" that contains n "levels"
    - Check whether the planning graph satisfies a necessary (but insufficient) condition for plan existence
    - If it does, then
      - Encode (P,n) as a satisfiability problem Φ but include only the actions in the planning graph
      - If Φ is satisfiable then return the solution

BlackBox

- Memory requirement still is combinatorially large, but less than satisfiability alone
- It was one of the two fastest planners in the 1998 Planning Competition

- When is BlackBox not a good idea?
  - When the domain is too large for a propositional planning approach
  - When long sequential plans are needed
  - When the solution time is dominated by graph-expansion and not plan extraction.

SatPlan: BlackBox's successor

- SatPlan combines planning-graph expansion and satisfiability checking, roughly as follows:
  - for n = 0, 1, 2, …
    - Create a planning graph that contains n levels
    - Encode the planning graph as a satisfiability problem
    - Try to solve it using a SAT solver
      - If the SAT solver finds a solution within some time limit,
        - Remove some unnecessary actions
        - Return the solution
- Memory requirement still is combinatorially large
  - but less than what's needed by a direct translation into satisfiability
- SatPlan was one of the best planners in the 2004 and 2006 planning competitions

* Terminology: "SATPLAN approach" (circa 1992) vs. the SATPLAN planner of 2004, 2006 etc., the successor of Blackbox.