Temporal and Spatial Reasoning

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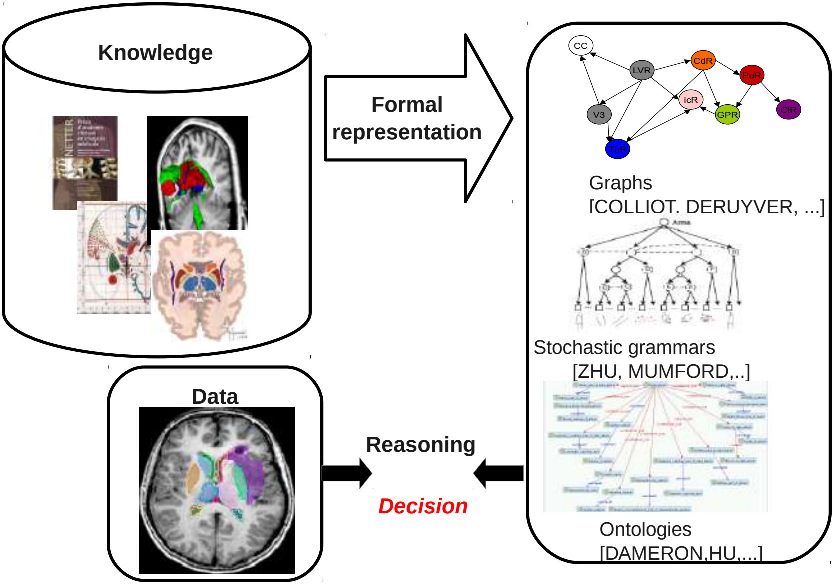
Spatial Reasoning

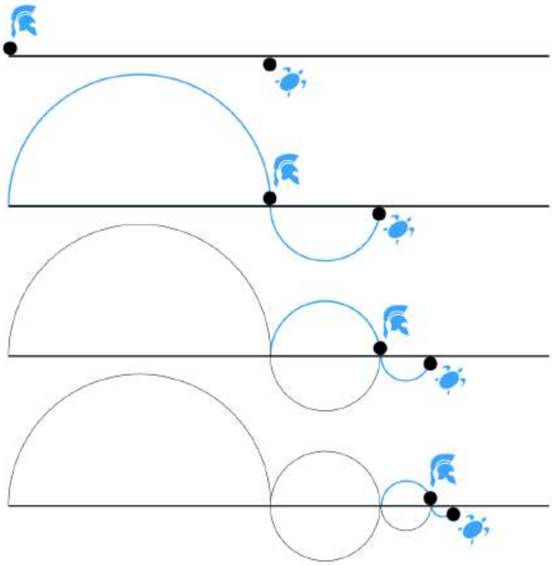
Knowledge representation and reasoning on spatial entities and spatial relationships

- largely developed in the artificial intelligence community
  - mainly topological relations
  - formal logics (ex: mereotopology)
  - inference
- less developed in image interpretation
  - need for imprecise knowledge representation
  - (semi-)quantitative framework ($\Rightarrow$ numerical evaluation)
  - examples: structural recognition in images under imprecision
- main ingredients:
  - knowledge representation (including spatial relations)
  - imprecision representation and management
  - fusion of heterogeneous information
  - reasoning and decision making


Knowledge
Formal representation
CC
LVR
CdR
PuR
icR
V3
GPR
ClR
ThR
Graphs
[COLLIOT, DERUYVER, ...]
Stochastic grammars
[ZHU, MUMFORD,..]
Ontologies
[DAMERON,HU,...]
Data
Reasoning
Decision

Philosophy

- From Pythagoras (c. 570-495 BC) to Zeno (c. 490-430 BC): concept of space linked to the first developments in arithmetics and Pythagorian geometry - Problem of infinitely subdivision possibility.
- Descartes (1596-1650): spatial extension is specific to material entities, governed by the only laws of mechanics.
- Newton (1643-1727): notion of absolute space.
- Hume (1711-1776): space reduced to a pure psychological function.
- Leibniz (1646-1716): space cannot be an absolute reality, motion and position are real and detectable only in relation to other objects, not in relation to space itself.
- Kant (1724-1804): objectivity of space.



- Poincaré (1854-1912): empiricist point of view where spatial knowledge is mainly derived from motor experience. Relativity of space.
- Bergson (1859-1941): a position in the space can be considered as an instantaneous cut of the movement, but the movement is more that a sum of positions in the space.
- Einstein (1879-1955): geometry is linked to the sensible and perceptible space. The geometrical configuration of the world itself becomes relative.
- Purely philosophical views of space developed by the phenomenologists and the existentialists.
- Reichenbach (1891-1953): geometry as a theory of relations.

Linguistics

- Rich variety of lexical terms for describing spatial location of entities.
- Concern all lexical categories (nouns, verbs, adjectives, adverbs, prepositions).
  - French, and other Romance languages, shows a typological preference for the lexicalization of the path in the main verb.
  - In Germanic and Slavic languages, the path is rather encoded in satellites associated to the verb (particle or prefix).
- Source of inspiration of many works on qualitative spatial information representation and qualitative spatial reasoning.
- Asymmetry, importance of reference, of context, of functional properties of the considered physical entities
- Imprecision (too precise statements can even become inefficient if they make the message too complex).

Human perception: example of distance

- Purely spatial measures, in a geometric sense, give rise to "metric distances", and are related to intrinsic properties of the objects.
- Temporal measures lead to distances expressed as travel time, and can be considered of extrinsic type, as opposed to the previous class.
- Economic measures, in terms of costs to be invested, are also of extrinsic type.
- Perceptual measures lead to distances of deictic type; they are related to an external point of view, which can be concrete or just a mental representation, which can be influenced by environmental features, by subjective considerations, leading to distances that are not necessarily symmetrical.
- Influence of other objects.

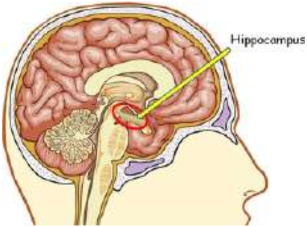
Cognition

Cognitive understanding of a spatial environment is issued from two types of processes:

- route knowledge acquisition (first acquired during child development), which consists in learning from sensori-motor experience (i.e. actual navigation) and implies an order information between visited landmarks,
- survey knowledge acquisition, from symbolic sources such as maps, leading to a global view ("from above") including global features and relationships, which is independent of the order of landmarks.

Neuro-imaging:

- a right hippocampal activation can be observed for both mental navigation and mental map tasks,
- a parahippocampal gyrus activation is additionally observed only for mental navigation, when route information and object landmarks have to be incorporated.



Internal representation of space in the brain:

- egocentric representations,
- allocentric representations ("map in the head").

Intensively used in several works in the modeling and conception of geographic information systems, and in mobile robotics.

Spatial reasoning formalisms

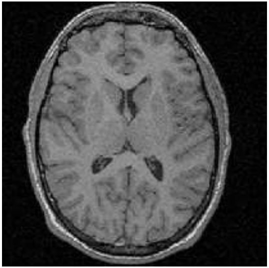
- Quantitative
- Qualitative (QSR)
- Fuzzy representations and reasoning: semi-quantitative / semi-qualitative approches

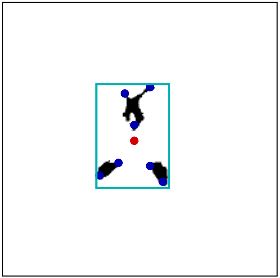
- Spatial entities
- Spatial relations
- Real world problems: dealing with imprecision and uncertainty.

Common to several representation and reasoning frameworks, used in the next parts of the course.

Spatial entities

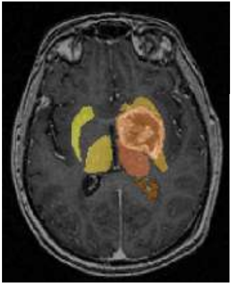
- Regions, fuzzy regions.
- Key points.
- Simplified regions (centroid, bounding box...).
- Abstract representations (e.g. in mereotopology, without referring to points, formulas in some logics...).





Spatial relations

- Useful... (see e.g. Freeman 1975, Kuipers 1978...).
- Structural stability (more than shape, size, absolute position).
- Different types (binary / n-ary, simple / complex, well-defined / vague).



Quantitative representations

- Precisely defined objects.
- Computation of well defined relations.
- Many limitations:
  - on the objects,
  - on the relations,
  - on the type of representations,
  - for reasoning.

But does not always match the usual way of reasoning (e.g. to the north, closer...).

Qualitative / symbolic representations

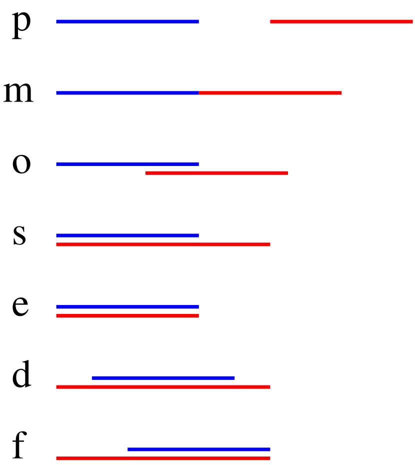
- Allen's intervals (temporal reasoning): 13 relations.
- Rectangle calculus (Allen on each axis): 169 relations.
- Cube calculus...
- Cardinal directions: 9 positions.
- Region Connection Calculus (RCC), mereotopology (based on connection and parthood predicates).
- Extensions to objects with broad or imprecise boundaries.
- Spatial logics.

Main features:

- Formal logics (propositional, first order, modal...).
- Compromise between expressiveness, completeness with respect to a class of situations, and complexity.
- Reasoning: inference, satisfiability, composition tables, CSP...

Allen's intervals: temporal reasoning

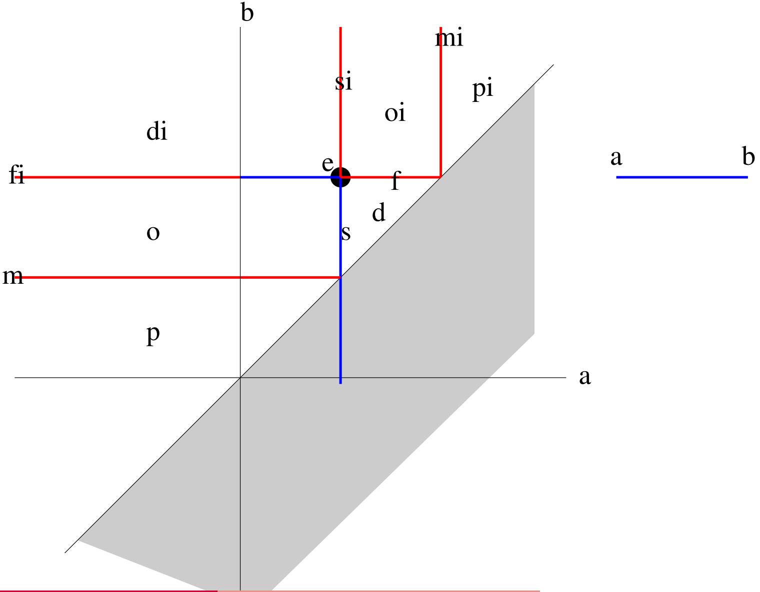
13 basic relations:



Reasoning: based on geometrical or latticial representations.

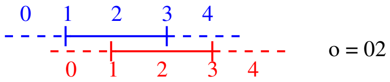
Allen's intervals: temporal reasoning

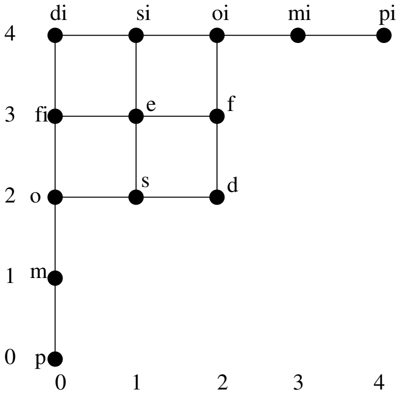
Geometrical / quantitative representation:



Allen's intervals: temporal reasoning

Qualitative representation: lattice:



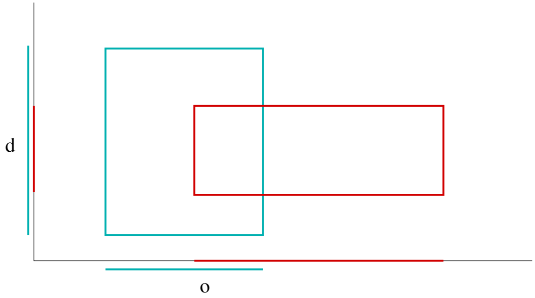


⇒ conceptual neighborhoods (Freksa)

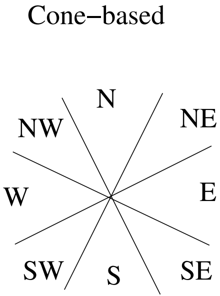
Allen's intervals: temporal reasoning

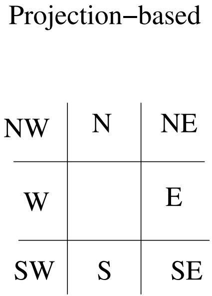
Extensions: rectangle, cube algebra

- Allen's interval in each direction
- 2D (rectangles): $13^2 = 169$ relations
- 3D (cubes): $13^3 = 2197$ relations
- $\Rightarrow$ high complexity, and fixed shaped objects



Cardinal directions (Frank, Egenhofer, Ligozat)
Qualitative directions: N, NE, E, SE, S, SW, W, NW



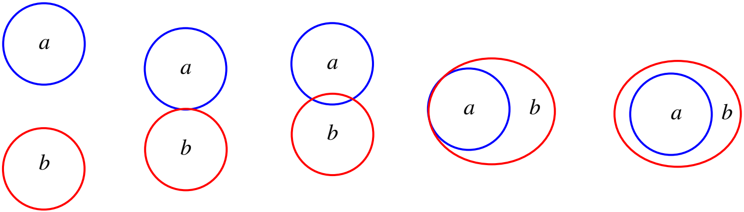


How to deal with complex shapes?
Only few compositions can be exactly determined.

RCC: Region Connection Calculus (Randell, Cui, Cohn - Vieu...)

- Spatial entities, defined in a qualitative way.
- No reference to points.
- Connection predicate C.
- Parthood predicate P:

$$P(x,y) : \forall z, C(z,x) \rightarrow C(z,y)$$

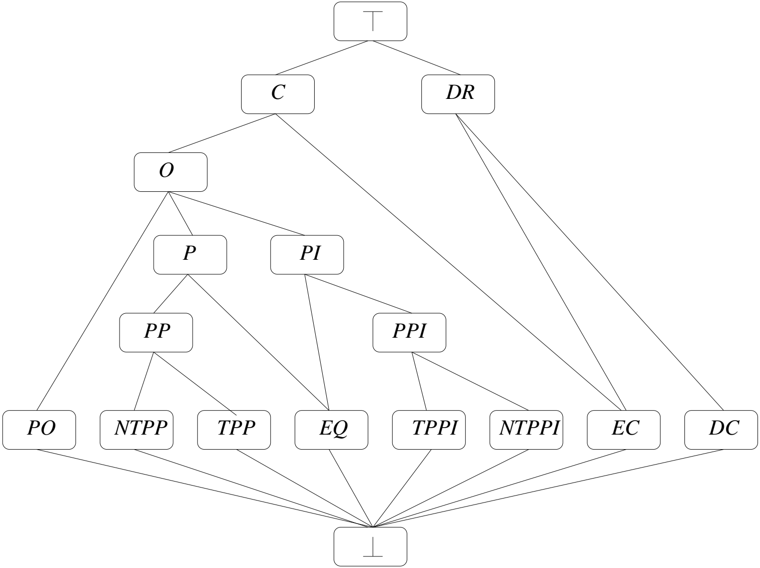


DC(a,b) *EC(a,b)* *PO(a,b)* *TPP(a,b)* *NTPP(a,b)*

RCC: Region Connection Calculus (Randell, Cui, Cohn - Vieu...)

$DC(x,y)$	x is disconnected from y	$\neg C(x,y)$
$P(x,y)$	x is a part of y	$\forall z, C(z,x) \rightarrow C(z,y)$
$PP(x,y)$	x is a proper part of y	$P(x,y) \wedge \neg P(y,x)$
$EQ(x,y)$	x is identical with y	$P(x,y) \wedge P(y,x)$
$O(x,y)$	x overlaps y	$\exists z, P(z,x) \wedge P(z,y)$
$DR(x,y)$	x is discrete from y	$\neg O(x,y)$
$PO(x,y)$	x partially overlaps y	$O(x,y) \wedge \neg P(x,y) \wedge \neg P(y,x)$
$EC(x,y)$	x is externally connected to y	$C(x,y) \wedge \neg O(x,y)$
$TPP(x,y)$	x is a tangential proper part of y	$PP(x,y) \wedge \exists z[EC(z,x) \wedge EC(z,y)$
$NTPP(x,y)$	x is a non tangential proper part of y	$PP(x,y) \wedge \neg\exists z[EC(z,x) \wedge EC(z,y)]$

RCC: Region Connection Calculus (Randell, Cui, Cohn - Vieu...)

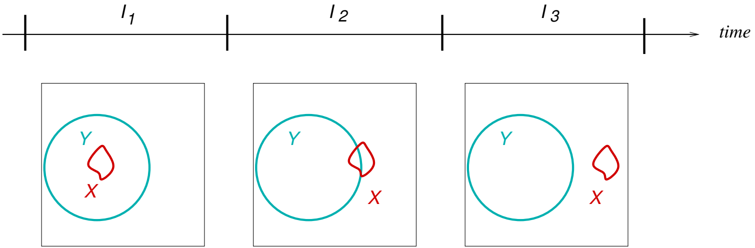


Qualitative trajectory calculus (Cohn et al.)

- Extension of RCC to take time into account (dynamic scenes).
- RCC + Allen
- Example:
  - X, Y objects
  - I_i time intervals

$$(P(X,Y), I_1) \wedge (PO(X,Y), I_2) \wedge (DR(X,Y), I_3)$$

$$\wedge meet(I_1, I_2) \wedge meet(I_2, I_3) \wedge before(I_1, I_3)$$



Adding shape

- Varzi: predicates for part, hole, fill, convex hull.
- Dugat: mereogeometry based on spheres (congruence, distance, order...).

Topology:

- $\Box A$: A is locally true (A is true at point x iff A is true in a neighborhood of x).
- $\Diamond A = \neg\Box\neg A$: A is true at x iff A is true at least one point of the neighborhood of x.
- Reasoning axioms and inference rules of S4:
  - $A \rightarrow (B \rightarrow A)$
  - $(A \rightarrow (B \rightarrow C)) \rightarrow ((A \rightarrow B) \rightarrow (A \rightarrow C))$
  - $(\neg A \rightarrow \neg B) \rightarrow (B \rightarrow A)$
  - $\Box(A \rightarrow B) \rightarrow (\Box A \rightarrow \Box B)$
  - $\Box A \rightarrow A$
  - $\Box A \rightarrow \Box\Box A$

Example (Van Benthem et al.)

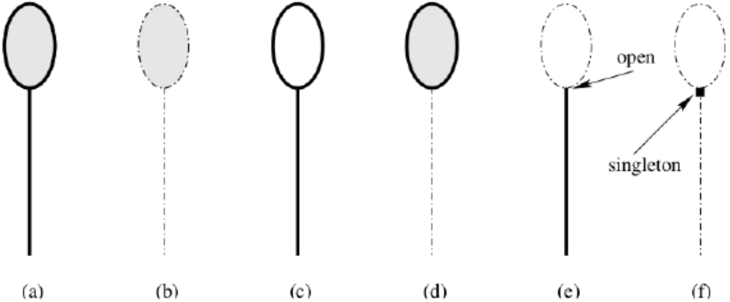


Figure 1.1. Each modal formula identifies a region in a topological space. (a) A spoon, p. (b) The container part of the spoon, $\Box p$. (c) The boundary of the spoon, $\Diamond p \wedge \Diamond \neg p$. (d) The container part of the spoon with its boundary, $\Diamond \Box p$. (e) The handle of the spoon, $p \wedge \neg \Diamond \Box p$. In this case the handle does not contain the junction handle-container point. (f) The junction handle-container point of the spoon, $\Diamond \Box p \wedge \Diamond (p \wedge \neg \Diamond \Box p)$: a singleton in the topological space.

Other spatial logics:

- Translation of RCC into modal logics.
- Logics of places ($\Box$ = everywhere, $\Diamond$ = somewhere).
- Modal logics of proximity ($\Box A$ = everywhere close to A).
- Modal logics of distance ($\Box^{\leq a}$ = everywhere in a neighborhood of radius a).
- Logics of inclusion and contact (inference in GIS).
- Modal logics of geometry, with modalities for points and for lines (affine, projective, parallelism...).

RCC-8 and modal logic

- $\Box X$: X is valid at any point.
- $\Diamond X$: there exists a point where X is valid.
- $C(X,Y) \equiv \Diamond(X \wedge Y)$
- $DC(X,Y) \equiv \Box(\neg X \vee \neg Y)$
- $P(X,Y) \equiv \Box(X \rightarrow Y)$
- $O(X,Y) \equiv \Diamond(i(X) \wedge i(Y))$ (i = interior)
- $TP(X,Y) \equiv \Box(X \rightarrow Y) \wedge \Diamond(X \wedge c(\neg Y))$ (c = closure)
- ...

A few important issues

- Context
- Representation issues
- Reasoning (inference, satisfiability, decidability, CSP...)
- Complexity
- Applications

State of the art:

- Very few applications
- Focus on topology
- Almost nothing on metric relations
- Almost nothing on uncertainty

Composition tables

Allen intervals:

.	p	m	o	F	D	s	e	S	d	f	O	M	P
p	(p)	(p)	(p)	(p)	(p)	(p)	(p)	(p)	(pmosd)	(pmosd)	(pmosd)	(pmosd)	*full*
m	(p)	(p)	(p)	(p)	(p)	(m)	(m)	(m)	(osd)	(osd)	(osd)	(Fef)	(DSOMP)
o	(p)	(p)	(pmo)	(pmo)	(pmoFD)	(o)	(o)	(oFD)	(osd)	(osd)	*concur*	(DSO)	(DSOMP)
F	(p)	(m)	(o)	(F)	(D)	(o)	(F)	(D)	(osd)	(Fef)	(DSO)	(DSO)	(DSOMP)
D	(pmoFD)	(oFD)	(oFD)	(D)	(D)	(oFD)	(D)	(D)	*concur*	(DSO)	(DSO)	(DSO)	(DSOMP)
s	(p)	(p)	(pmo)	(pmo)	(pmoFD)	(s)	(s)	(seS)	(d)	(d)	(dfO)	(M)	(P)
e	(p)	(m)	(o)	(F)	(D)	(s)	(e)	(S)	(d)	(f)	(O)	(M)	(P)
S	(pmoFD)	(oFD)	(oFD)	(D)	(D)	(seS)	(S)	(S)	(dfO)	(O)	(O)	(M)	(P)
d	(p)	(p)	(pmosd)	(pmosd)	*full*	(d)	(d)	(dfOMP)	(d)	(d)	(dfOMP)	(P)	(P)
f	(p)	(m)	(osd)	(Fef)	(DSOMP)	(d)	(f)	(OMP)	(d)	(f)	(OMP)	(P)	(P)
O	(pmoFD)	(oFD)	*concur*	(DSO)	(DSOMP)	(dfO)	(O)	(OMP)	(dfO)	(O)	(OMP)	(P)	(P)
M	(pmoFD)	(seS)	(dfO)	(M)	(P)	(dfO)	(M)	(P)	(dfO)	(M)	(P)	(P)	(P)
P	*full*	(dfOMP)	(dfOMP)	(P)	(P)	(dfOMP)	(P)	(P)	(dfOMP)	(P)	(P)	(P)	(P)

full=(pmoFDseSdfOMP) and concur=(oFDseSdfO)

From `http://www.ics.uci.edu/~alspaugh/cls/shr/allen.html`

Composition tables

RCC-8 :

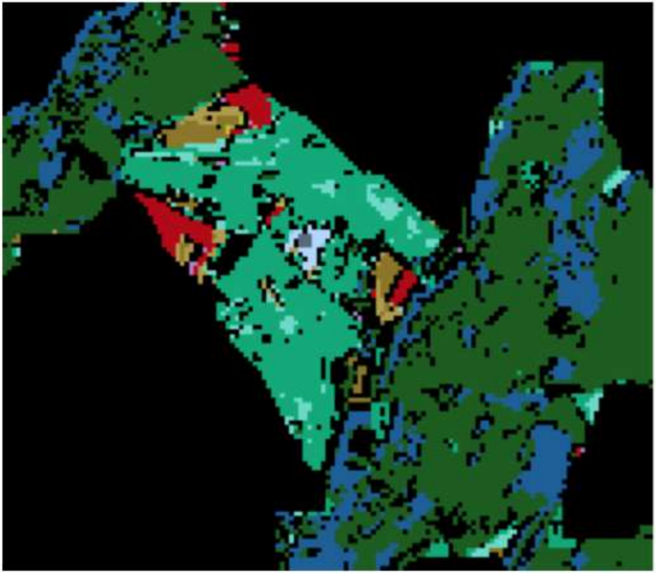
o	DC	EC	PO	TPP	NTPP	TPPi	NTPPi	EQ
DC	*	DC,EC,PO,TPP,NTPP	DC,EC,PO,TPP,NTPP	DC,EC,PO,TPP,NTPP	DC,EC,PO,TPP,NTPP	DC	DC	DC
EC	DC,EC,PO,TPPi,NTPPi	DC,EC,PO,TPP,TPPi,EQ	DC,EC,PO,TPP,NTPP	EC,PO,TPP,NTPP	PO,TPP,NTPP	DC,EC	DC	EC
PO	DC,EC,PO,TPPi,NTPPi	DC,EC,PO,TPPi,NTPPi	*	PO,TPP,NTPP	PO,TPP,NTPP	DC,EC,PO,TPPi,NTPPi	DC,EC,PO,TPPi,NTPPi	PO
TPP	DC	DC,EC	DC,EC,PO,TPP,NTPP	TPP,NTPP	NTPP	DC,EC,PO,TPP,TPPi,EQ	DC,EC,PO,TPPi,NTPPi	TPP
NTPP	DC	DC	DC,EC,PO,TPP,NTPP	NTPP	NTPP	DC,EC,PO,TPP,NTPP	*	NTPP
TPPi	DC,EC,PO,TPPi,NTPPi	EC,PO,TPPi,NTPPi	PO,TPPi,NTPPi	PO,TPP,TPPi,EQ	PO,TPP,NTPP	TPPi,NTPPi	NTPPi	TPPi
NTPPi	DC,EC,PO,TPPi,NTPPi	PO,TPPi,NTPPi	PO,TPPi,NTPPi	PO,TPPi,NTPPi	PO,TPP,NTPP,TPPi,NTPPi,EQ	NTPPi	NTPPi	NTPPi
EQ	DC	EC	PO	TPP	NTPP	TPPi	NTPPi	EQ

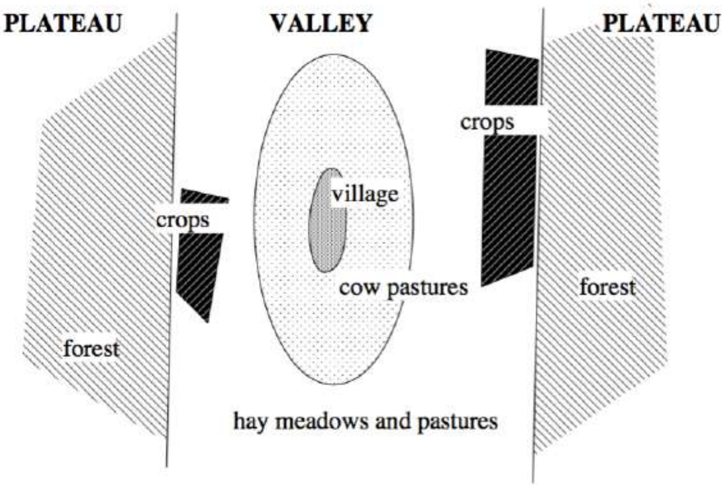
From wikipedia

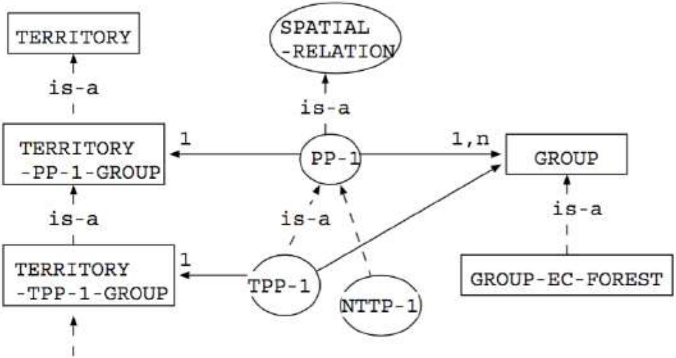
Other approaches

- Ontologies and description logics.
- Graph-based reasoning.
- Grammars.
- Formal concept analysis.
- Decision trees.
- Constraint Satisfaction Problem.
- Relational algebras on temporal or spatial configurations.
- Graphical models.
- ...

Example using RCC: region identification (Le Ber et al.)




PLATEAU
VALLEY
PLATEAU
crops
village
crops
cow pastures
forest
forest
hay meadows and pastures


TERRITORY
is-a
SPATIAL
-RELATION
is-a
TERRITORY
-PP-1-GROUP
1
PP-1
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GROUP
is-a
is-a
is-a
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-TPP-1-GROUP
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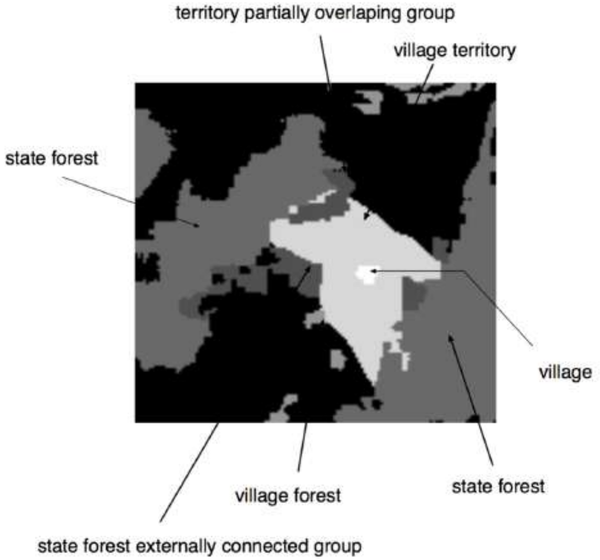
TPP-1
NTTP-1
GROUP-EC-FOREST



F-ILOT - F-ILOTLIM - F-ILOTECDOM - F-POCOM - F-ECDOM - F-ENTREDOM

Semi-quantitative spatial reasoning: fuzzy approaches

- Limitations of purely qualitative reasoning
- Interest of adding semi-quantitative extension to qualitative value for deriving useful and practical conclusions
- Limitations of purely quantitative representations in the case of imprecise statements, knowledge expressed in linguistic terms, etc.
- Integration of both quantitative and qualitative knowledge using semi-quantitative (or semi-qualitative) interpretation of fuzzy sets
- Freeman (1975): fuzzy sets provide computational representation and interpretation of imprecise spatial constraints, expressed in a linguistic way, possibly including quantitative knowledge
- Granularity, involved in:
  - objects or spatial entities and their descriptions
  - types and expressions of spatial relations and queries
  - type of expected or potential result

Motivation: model-based recognition and spatial reasoning

- representation of imprecision
- spatial relations as structural information
  - topological relationships (set relations, adjacency)
  - distances
  - relative directional relationships
  - more complex relations (between, along...)
- two classes of relations
  - well defined in the crisp case (adjacency, distances...)
  - vague even in the crisp case (directional relationships...)
- fusion of several and heterogeneous pieces of knowledge and information

$\Rightarrow$ Fuzzy set theory, mathematical morphology

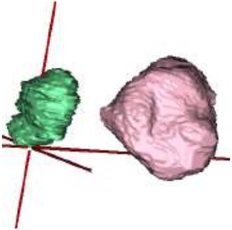
Imprecision and fuzziness

- objects (no clear boundaries, coarse segmentation...)
- relations (ex: *left of, quite close*)
- type of knowledge available (ex: *the caudate nucleus is close to the lateral ventricle*)
- question to be answered (ex: *go towards this object while remaining at some security distance*)

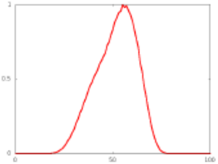
Types of representations: example of distances

- number in $\mathbb{R}^+$ (or in $[0, 1]$)
- interval
- fuzzy number, fuzzy interval
- Rosenfeld:
  - distance density: degree to which the distance is equal to n
  - distance distribution: degree to which the distance is less than n
- linguistic value
- logical formula

$\Rightarrow$ unifying framework of fuzzy set theory



$d_{min} = 17, d_{Haus} = 80$

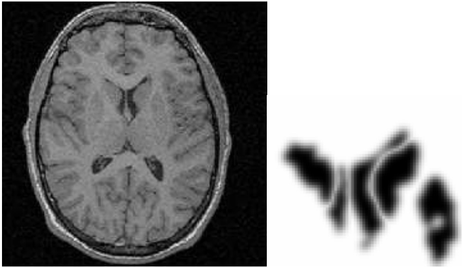


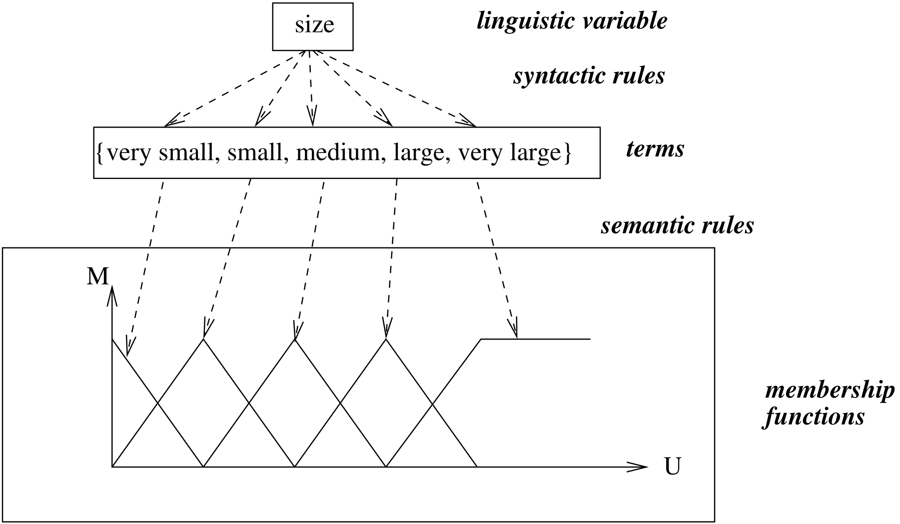
Fuzzy sets in a nutshell (Zadeh, 1965)

- Space $\mathcal{S}$ (image space, space of characteristics, etc.).
- Fuzzy set: $\mu : \mathcal{S} \rightarrow [0, 1]$ – $\mu(x) =$ membership degree of x to μ.
- Set theoretical operations: complementations, conjunctions (t-norms), disjunctions (t-conorms).
- Logic operators, aggregation and fusion operators...

Example: spatial fuzzy set

- $\mathcal{S}$: $\mathbb{R}^3$ or $\mathbb{Z}^3$ in the digital case.
- $\mu : \mathcal{S} \rightarrow [0, 1]$ - $\mu(x) =$ degree to which x belongs to the fuzzy object.




size
linguistic variable
syntactic rules
{very small, small, medium, large, very large}
terms
semantic rules
M
U
membership functions

Mathematical morphology

Dilation: operation in complete lattices that commutes with the supremum.
Erosion: operation in complete lattices that commutes with the infimum.

$\Rightarrow$ applications on sets, fuzzy sets, functions, logical formulas, graphs, etc.

Using a structuring element:

- dilation as a degree of conjunction: $\delta_B(X) = \{x \in \mathcal{S} \mid B_x \cap X \neq \emptyset\}$,
- erosion as a degree of implication: $\varepsilon_B(X) = \{x \in \mathcal{S} \mid B_x \subseteq X\}$.

A lot of other operations...

Fuzzy mathematical morphology

- Dilation as degree of intersection:

$$D_\nu(\mu)(x) = \sup\{t[\nu(y-x), \mu(y)],\ y \in \mathcal{S}\}$$

- Erosion as degree of inclusion:

$$E_\nu(\mu)(x) = \inf\{I[\nu(y-x), \mu(y)],\ y \in \mathcal{S}\}$$

  I from a t-conorm T or by residuation from the t-norm t
- Opening and closing by composition
- Similar properties as in classical mathematical morphology

Fuzzy spatial relations

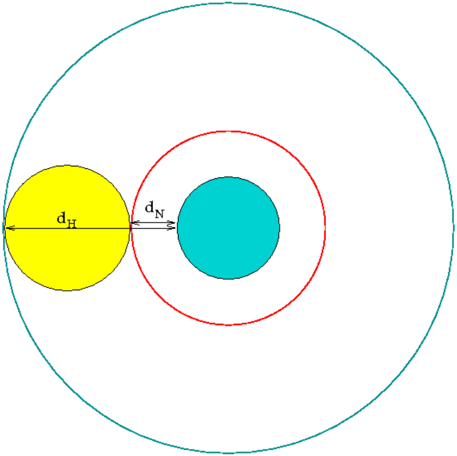
Fuzzy sets $\rightarrow$ relations become a matter of degree

- Set theoretical relations
- Topology: connectivity, connected components, neighborhood, boundaries, adjacency
- Distances
- Relative direction
- More complex relations: between, along, parallel, around...

Most of them can be defined from mathematical morphology.

Distances between fuzzy sets: morphological approach

Expression of distances (minimum, Hausdorff...) in morphological (i.e. algebraic) terms $\Rightarrow$ easy translation to the fuzzy case



Minimum (nearest point) distance distribution

$$d_N(X,Y) = \inf\{n \in \mathbb{N}, X \cap D^n(Y) \neq \emptyset\} = \inf\{n \in \mathbb{N}, Y \cap D^n(X) \neq \emptyset\}$$

Degree to which the distance between μ and μ' is less than n (distance distribution):

$$\Delta_N(\mu,\mu')(n) = f[\sup_{x\in\mathcal{S}} t[\mu(x), D^n_\nu(\mu')(x)], \sup_{x\in\mathcal{S}} t[\mu'(x), D^n_\nu(\mu)(x)]]$$

Hausdorff distance: similar equations

Minimum (nearest point) distance density

$$d_N(X, Y) = n \Leftrightarrow D^n(X) \cap Y \neq \emptyset \text{ and } D^{n-1}(X) \cap Y = \emptyset$$

$$d_N(X, Y) = 0 \Leftrightarrow X \cap Y \neq \emptyset$$

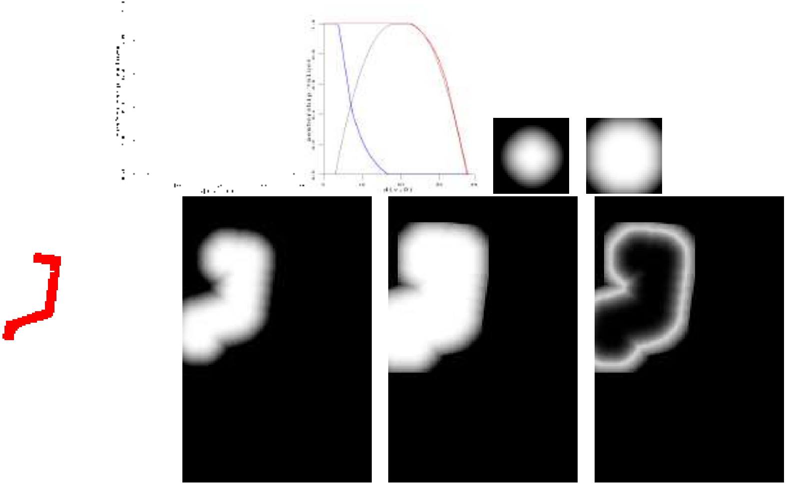
Degree to which the distance between μ and μ' is equal to n (distance density):

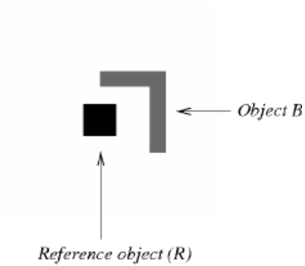
$$\delta_N(\mu, \mu')(n) = t[\sup_{x \in \mathcal{S}} t[\mu'(x), D^n_\nu(\mu)(x)], c[\sup_{x \in \mathcal{S}} t[\mu'(x), D^{n-1}_\nu(\mu)(x)]]]$$

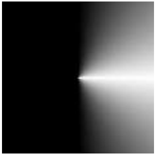
$$\delta_N(\mu, \mu')(0) = \sup_{x \in \mathcal{S}} t[\mu(x), \mu'(x)]$$

Hausdorff distance: similar equations

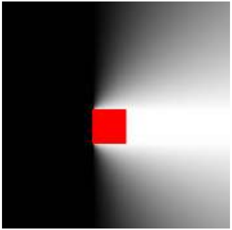
Example: spatial representation of knowledge about distance







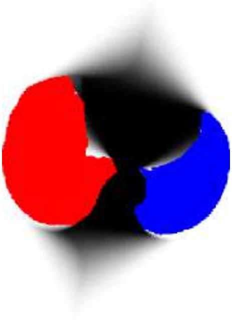
ν_{Right}



$$\mu_{Right}(R) = \delta_{\nu_{Right}}(R)$$

Complex relations

Example: the heart is between the lungs



Reasoning with mathematical morphology

- Chaining operations (image interpretation, recognition)
- Fusion of spatial relations (ex: structural recognition)
- Links with logics
  - propositional logics:
    - elegant tools for revision, fusion, abduction
    - links with mereotology, "egg-yolk" structures, logics of distances, nearness logics, linear logics, logics of convexity...
  - modal logics:
    - $(\Diamond, \Box) =$ (dilation, erosion)
    - symbolic and qualitative representations of spatial relations
  - fuzzy logic

Example: dilation and erosion of a formula

Structuring element B: relation between worlds

Dilation:

$$Mod(D_B(\varphi)) = \{\omega \in \Omega \mid B(\omega) \cap Mod(\varphi) \neq \emptyset\}$$

Erosion:

$$Mod(E_B(\varphi)) = \{\omega \in \Omega \mid B(\omega) \models \varphi\}$$

Dilation and erosion as modal operators

Structuring element B: accessibility relation $R(\omega, \omega')$ iff $\omega' \in B(\omega)$

$$\begin{aligned} \mathcal{M}, \omega \models \Box\varphi & \Leftrightarrow \forall \omega' \in \Omega,\ R(\omega, \omega') \Rightarrow \mathcal{M}, \omega' \models \varphi \\ & \Leftrightarrow \{\omega' \in \Omega \mid \omega' \in B(\omega)\} \models \varphi \\ & \Leftrightarrow B(\omega) \models \varphi \end{aligned}$$

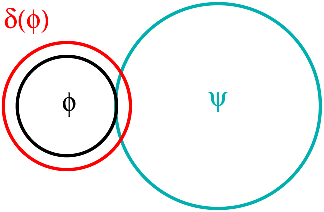
$$\begin{aligned} \mathcal{M}, \omega \models \Diamond\varphi & \Leftrightarrow \exists \omega' \in \Omega,\ R(\omega, \omega') \text{ et } \mathcal{M}, \omega' \models \varphi \\ & \Leftrightarrow \{\omega' \in \Omega \mid \omega' \in B(\omega)\} \cap Mod(\varphi) \neq \emptyset \\ & \Leftrightarrow B(\omega) \cap Mod(\varphi) \neq \emptyset \end{aligned}$$

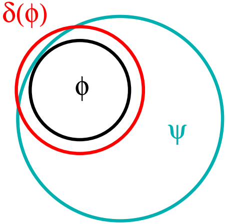
$$\boxed{\Box\varphi \equiv E_B(\varphi) \qquad \Diamond\varphi \equiv D_B(\varphi)}$$

Spatial interpretation: restriction or necessary region / extension or possible region

Example: logical expressions and links with mereotology

- Spatial entities represented as formulas.
- Structuring element: binary relationship between worlds, accessibility relation...
- Adjacency: $\varphi \wedge \phi \rightarrow \perp$ and $\delta\varphi \wedge \psi \not\rightarrow \perp$ and $\varphi \wedge \delta\psi \not\rightarrow \perp$.
- Tangential part: $\varphi \rightarrow \psi$ and $\delta\varphi \wedge \neg\psi \not\rightarrow \perp$.
- Proper tangential part in mereotopology:
  $TPP(\varphi, \psi) = P(\varphi, \psi) \wedge \neg P(\psi, \varphi) \wedge \neg P(\delta(\varphi), \psi)$.





RCC expression for $(\varphi = x, \psi = y)$:
$TPP(x, y) = (P(x, y) \wedge \neg P(y, x)) \wedge \exists z[(C(z, x) \wedge \neg(\exists z', P(z', z) \wedge P(z', x)))) \wedge (C(z, y) \wedge \neg(\exists z', P(z', z) \wedge P(z', y)))]$

Model based image understanding

Models of various types:

- acquisition properties (geometry, noise statistics...)
- shape
- appearance
- spatial relations
- ...

Important

- to use available knowledge
- to guide the image exploration, for segmentation, recognition, scene understanding
- to solve ambiguities
- to deal with imprecision
- ...

Issues:

- semantic gap
- imprecisions and uncertainties
- pathological cases
- algorithms

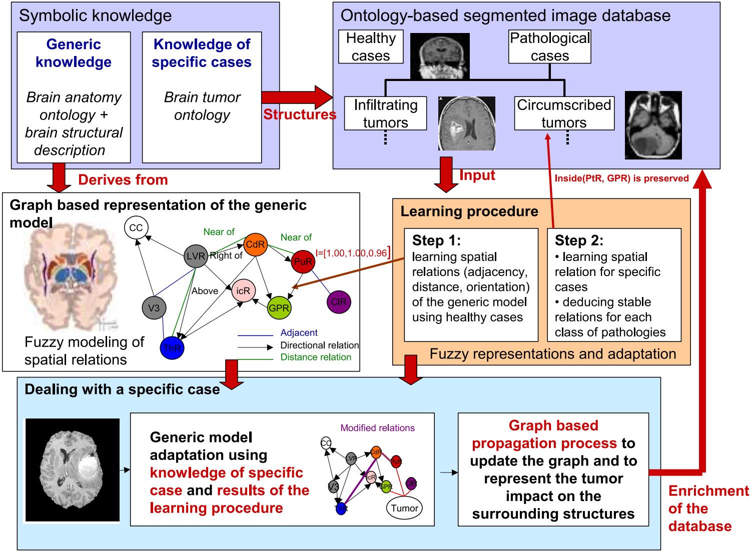
Two main questions in structural recognition in images:

- given two objects (possibly fuzzy), assess the degree to which a relation is satisfied
- given one reference object, define the area of the space in which a relation to this reference is satisfied (to some degree)

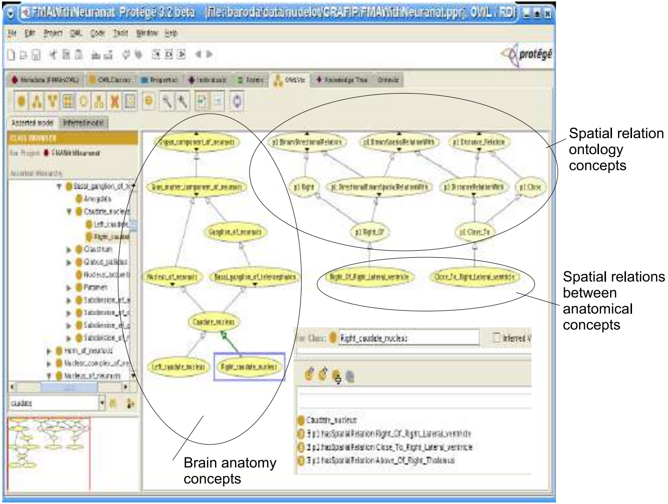
Example in brain imaging

- Concepts:
  - brain: part of the central nervous system located in the head
  - caudate nucleus: a deep gray nucleus of the telencephalon involved with control of voluntary movement
  - glioma: tumor of the central nervous system that arises from glial cells
  - ...
- Spatial organization:
  - the left caudate nucleus is inside the left hemisphere
  - it is close to the lateral ventricle
  - it is outside (left of) the left lateral ventricle
  - it is above the thalamus, etc.
  - ...
- Pathologies: relations are quite stable, but more flexibility should be allowed in their semantics

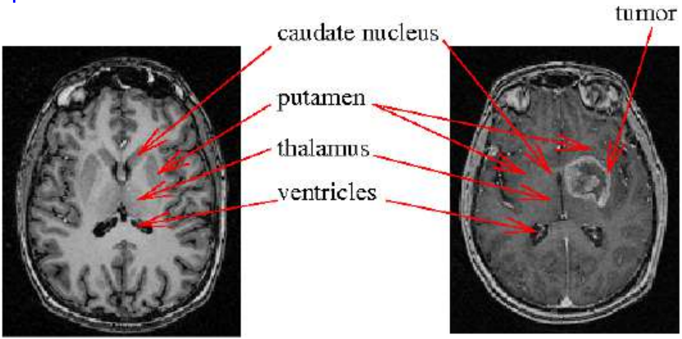
Integration of ontologies, spatial relations and fuzzy models

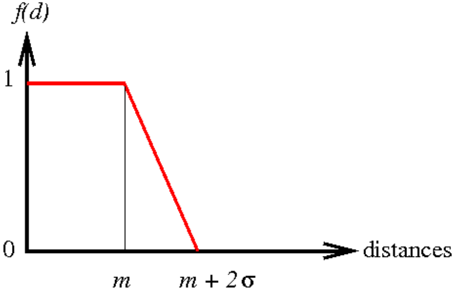


Ontology of the anatomy (FMA) enriched with an ontology of spatial relations



Learning spatial relations

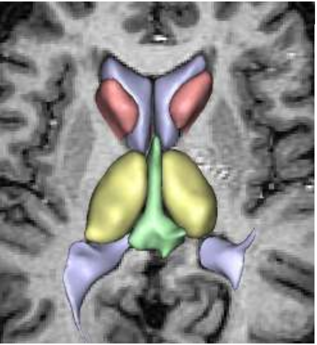


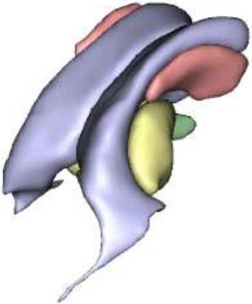


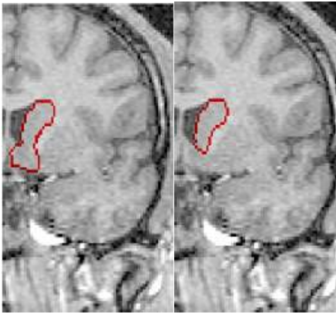
Spatial reasoning for model-based recognition

Segmentation and recognition of some internal structures on a normal case (O. Colliot et al.):

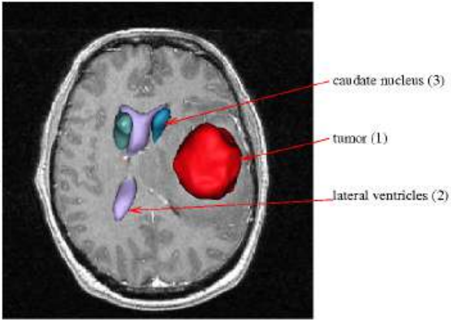
- fusion of spatial relations (given by the model) to previously recognized objects
- deformable model constrained by spatial relations



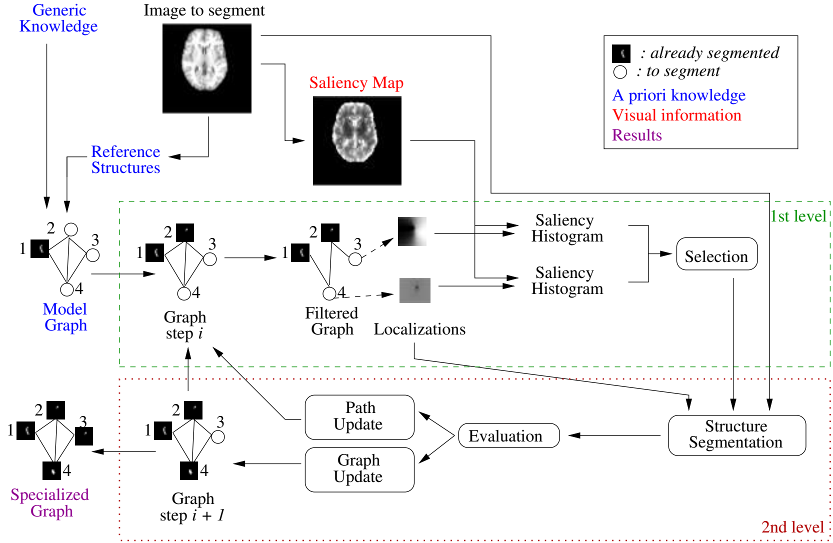




Examples in pathological cases (H. Khotanlou, J. Atif, et al.)

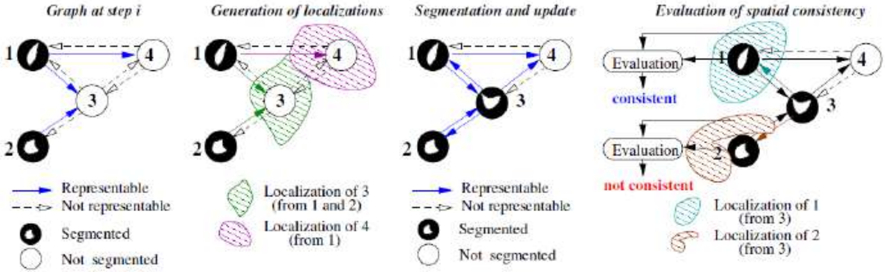


Best segmentation path (G. Fouquier et al.)



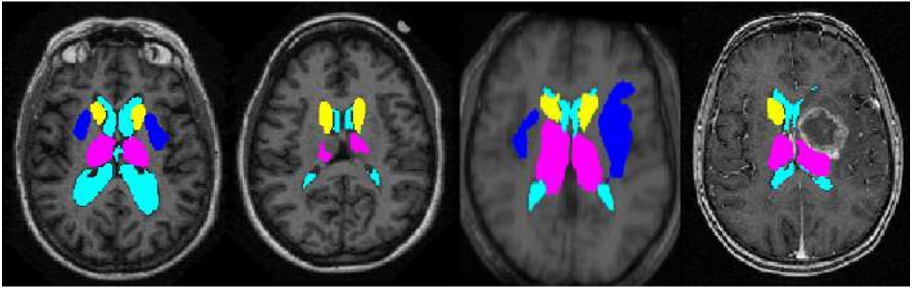
Best segmentation path (G. Fouquier et al.)

Evaluation and backtracking

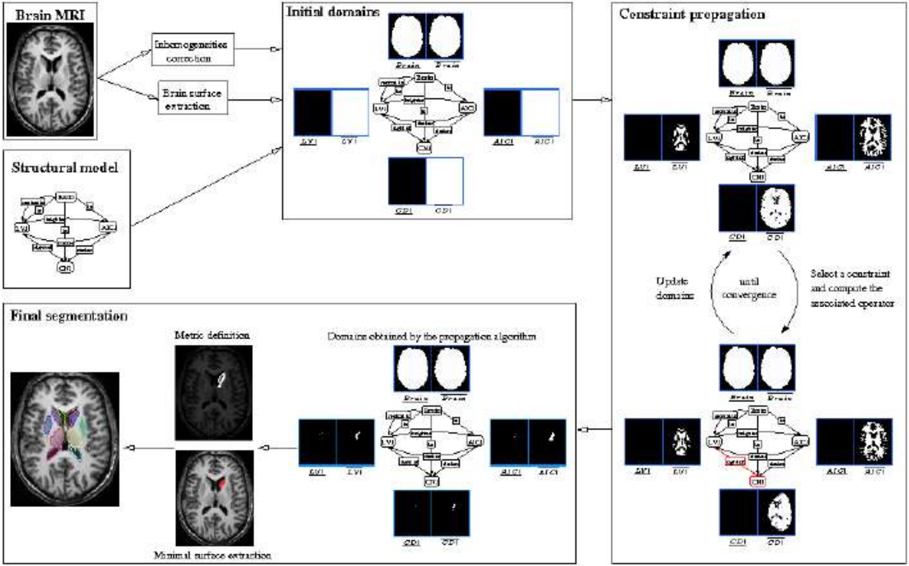


Best segmentation path (G. Fouquier et al.)

Some results



Global approach based in CSP (O. Nempont et al.)



Global approach based in CSP (O. Nempont et al.)

Constraint Satisfaction Problem (CSP):

- Constraint network $= (\chi, \mathcal{D}, \mathcal{C})$
- $\chi =$ variables
- $\mathcal{D} =$ set of associated domains
- $\mathcal{C} =$ constraints involving variables of χ, relations on the variable domains
- Propagation of constraints:
  - Locally consistent constraint if all values of the domains can satisfy the constraint.
  - Suppression of inconsistent values: $(\chi, \mathcal{D}, \mathcal{C}) \to (\chi, \mathcal{D}', \mathcal{C})$
  - Propagator $=$ operator reducing the domains according to a constraint.

Global approach based in CSP (O. Nempont et al.)

- Variables = anatomical structures.
- Domain of a variable = interval of fuzzy sets $[\underline{A}, \overline{A}]$.
- Example of constraint (1): inclusion

$$
\begin{aligned}
C^{in}_{A,B}: \quad \mathcal{D}(A) \times \mathcal{D}(B) & \rightarrow \{0, 1\} \\
(\mu_1, \mu_2) & \mapsto \begin{cases} 1 & \text{if } \mu_1 \leq \mu_2, \\ 0 & \text{otherwise.} \end{cases}
\end{aligned}
$$

- Associated propagator:

$$
\frac{\langle A, B; (\underline{A}, \overline{A}), (\underline{B}, \overline{B}); C^{in}_{A,B} \rangle}{\langle A, B; (\underline{A}, \overline{A} \wedge \overline{B}), (\underline{B} \vee \underline{A}, \overline{B}); C^{in}_{A,B} \rangle}
$$

Global approach based in CSP (O. Nempont et al.)

- Example of constraint (2): directional relation

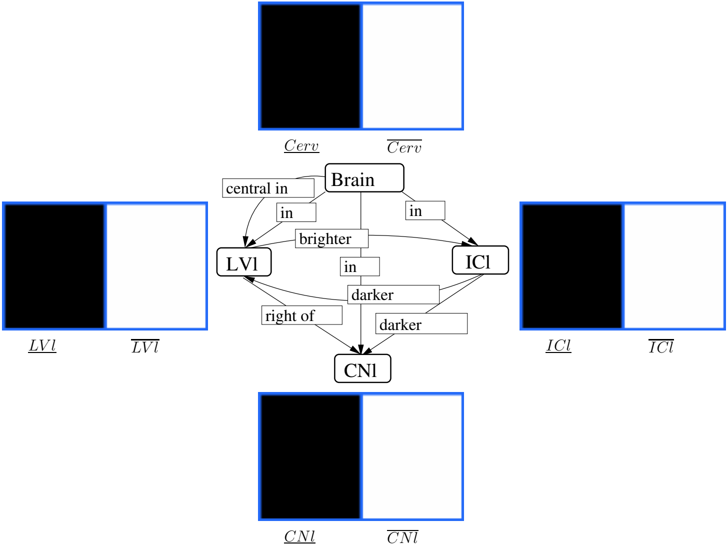
$$C_{A,B}^{dir\ \nu}: \quad \mathcal{D}(A) \times \mathcal{D}(B) \rightarrow \{0,1\}$$
$$(\mu_1, \mu_2) \mapsto \begin{cases} 1 & \text{if } \mu_2 \leq \delta_\nu(\mu_1), \\ 0 & \text{otherwise.} \end{cases}$$

- Associated propagator:

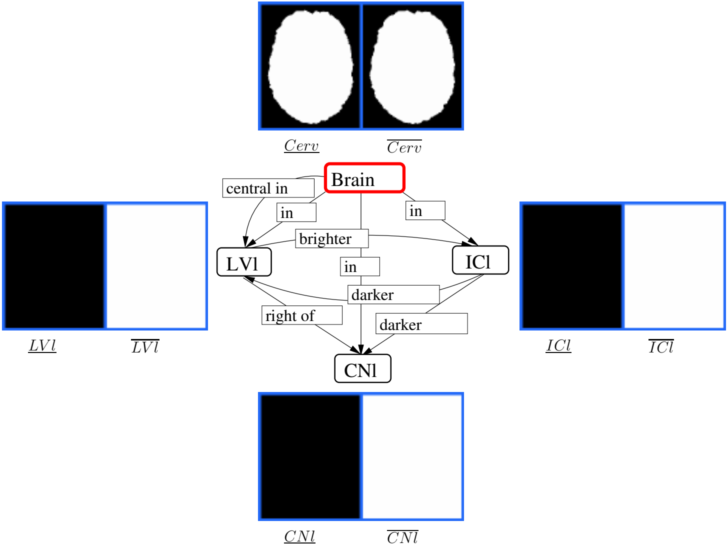
$$\frac{\langle A, B; (\underline{A}, \overline{A}), (\underline{B}, \overline{B}); C_{A,B}^{dir\ \nu} \rangle}{\langle A, B; (\underline{A}, \overline{A}), (\underline{B}, \overline{B} \wedge \delta_\nu(\overline{A})); C_{A,B}^{dir\ \nu} \rangle}$$

- Other constraints: distance, partition, connectivity, adjacency, volume, contraste...
- Ordering of the propagators and iteration application.

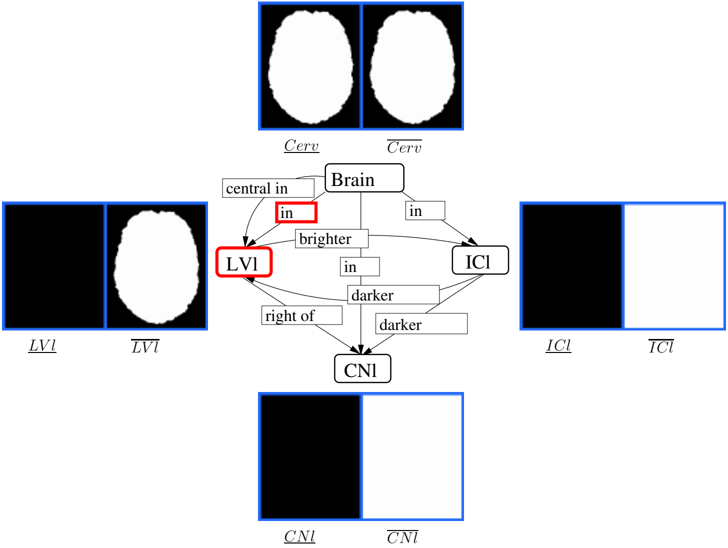
Propagation of constraint: example



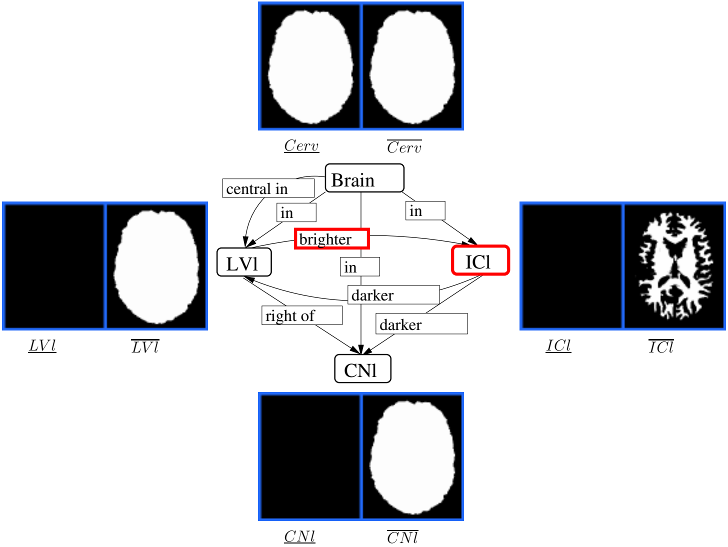
Propagation of constraint: example



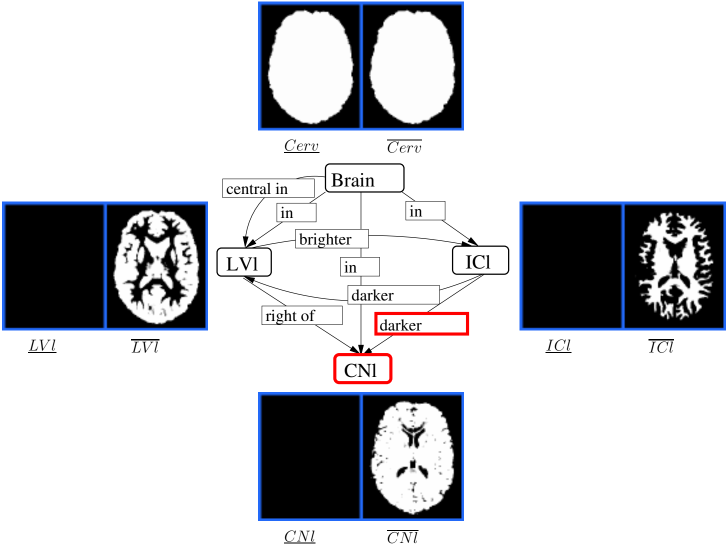
Propagation of constraint: example



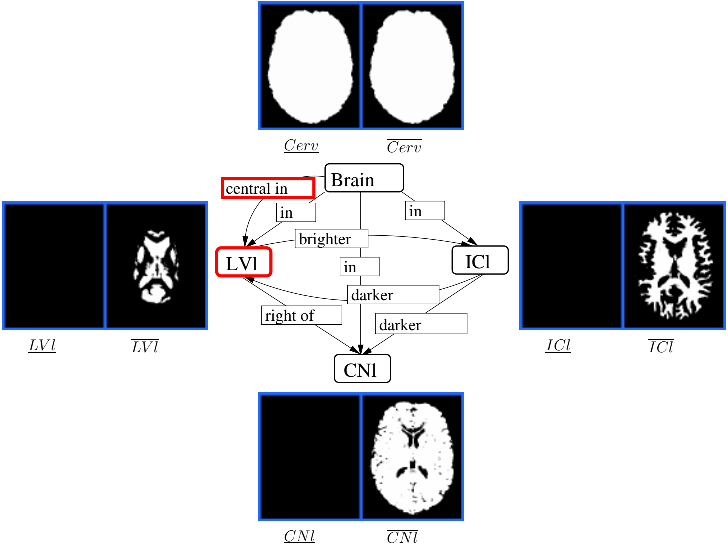
Propagation of constraint: example



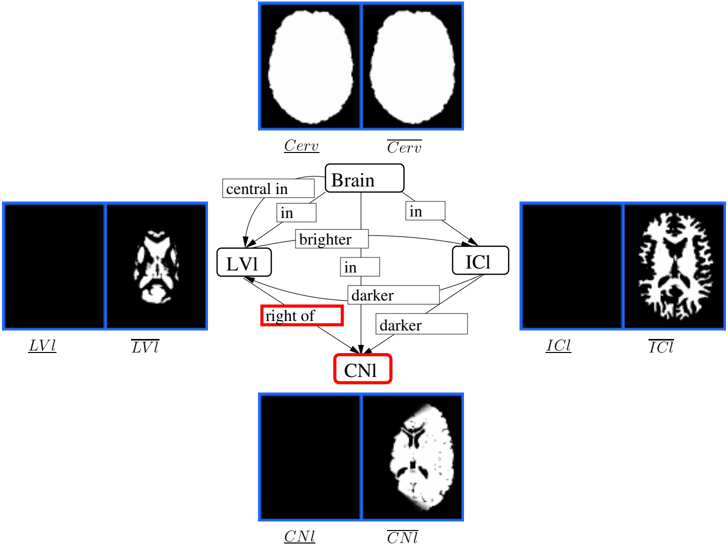
Propagation of constraint: example

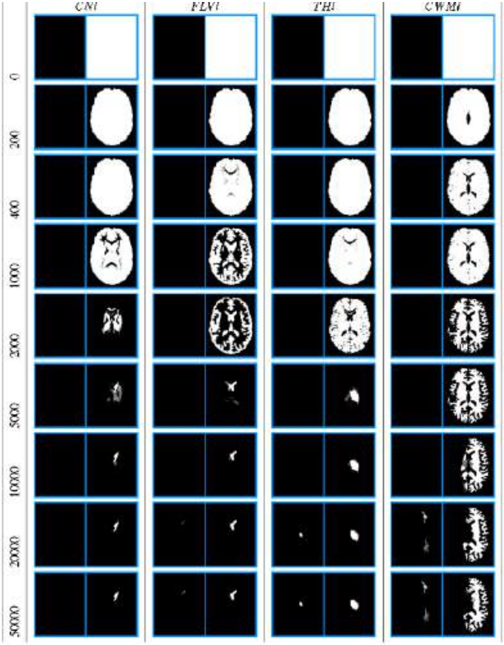


Propagation of constraint: example

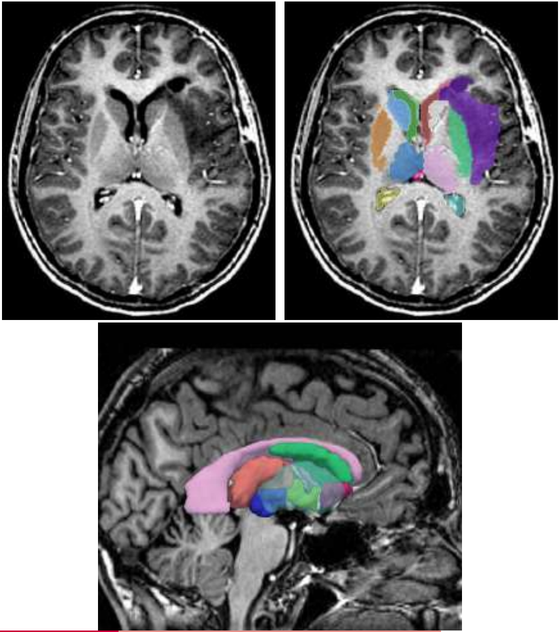


Propagation of constraint: example

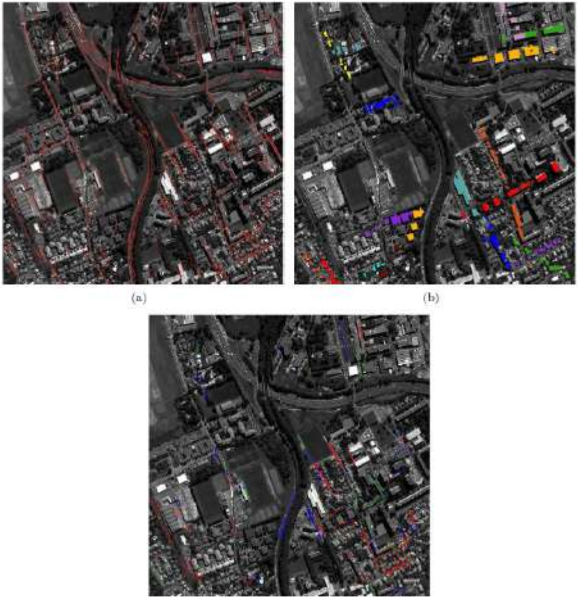



CNI
FLVI
THI
CWMI
0
200
400
1000
2000
5000
10000
20000
50000

Result: example



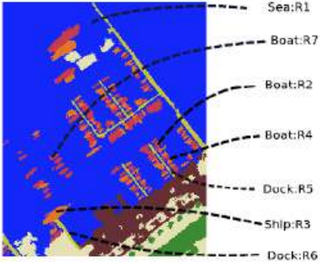
Examples in remote sensing (C. Vanegas)



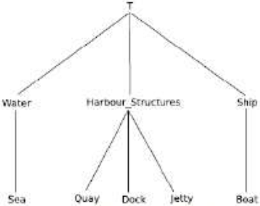
(a) (b) (c)

Examples in remote sensing (C. Vanegas)

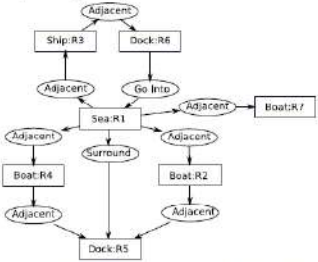
(a) Example image.



(b) Labeled image: The blue regions represent the sea, the red and orange represent ships or boats and the yellow regions represent the docks.



(c) Concept hierarchy T_C in the context of



(d) Conceptual graph representing the spatial orga-

Image understanding as an abduction problem

Formulation in DL:

- Knowledge base $\mathcal{K}$.
- TBox T, m_i concepts defined in T.
- ABox A, such $\forall a \in A$, $\mathcal{K} \not\models \neg a$.
- ABox Abduction Problem $\langle \mathcal{K}, A \rangle$: finding a set of assertions γ such that $\mathcal{K} \cup \gamma \models A$.

Link between FCA and DL: semantic context $\mathbb{K}_T = (G, M, I)$ defined as:

$$\begin{aligned} G &= \{(\mathcal{I}, d) \mid \mathcal{I} \text{ is a model of } T \text{ and } d \in \Delta^{\mathcal{I}}\} \\ M &= \{m_1, \ldots, m_n\} \\ I &= \left\{ ((\mathcal{I}, d), m) \mid d \in m^{\mathcal{I}} \right\} \end{aligned}$$

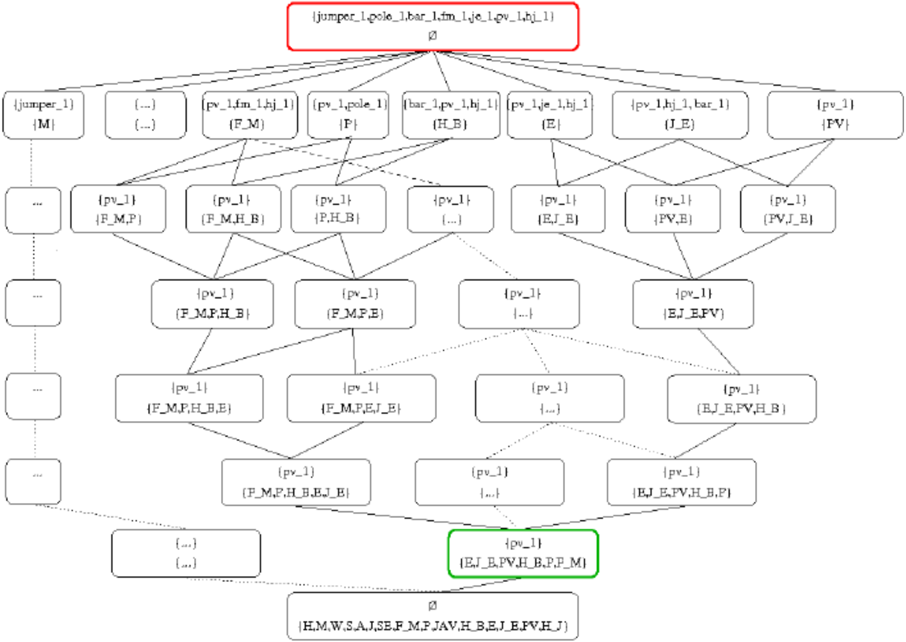
Example

Tbox:

$$
\begin{array}{rcl}
Man & \sqsubseteq & Human \\
Woman & \sqsubseteq & Human \\
Athlete & \equiv & Human \sqcap \\
 & & \exists hasProfession.Sport \\
Jumper & \sqsubseteq & Athlete \sqcap \\
 & & \exists use.SportEquipment \\
Foam\_Mat & \sqsubseteq & SportEquipmenent \\
Pole & \sqsubseteq & SportEquipment \\
Javelin & \sqsubseteq & SportEquipment \\
Horizontal\_bar & \sqsubseteq & SportEquipment \\
Jumping\_Event & \sqsubseteq & Event \sqcap \\
 & & \exists hasPart.Jumper \sqcap \\
Pole\_Vault & \sqsubseteq & Jumping\_Event \sqcap \\
 & & \exists hasPart.Pole \sqcap \\
 & & \exists hasPart.Horizontal\_Bar \sqcap \\
 & & \exists hasPart.Foam\_Mat \\
High\_Jump & \sqsubseteq & Jumping\_Event \sqcap \\
 & & \exists hasPart.Horizontal\_Bar \sqcap \\
 & & \exists hasPart.Foam\_Mat \\
 & \ldots &
\end{array}
$$

Semantic context:

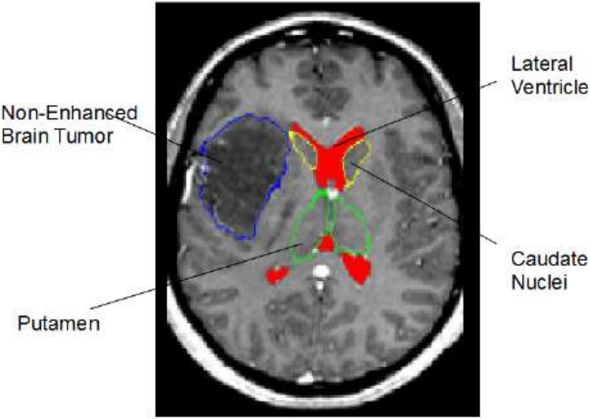
$\mathbb{K}_{athletic}$	Human	Man	Woman	Athlete	Jumper	SportEquipment	Foam_Mat	Pole	Javelin	Horizontal_bar	Event	Jumping_Event	Pole_Vault	High_Jump
m_1	X	X												
jumper_1	X	X		X	X	X								
pole_1						X		X						
bar_1						X				X				
fm_1						X	X							
je_1					X	X					X	X		
jav_1						X			X					
pv_1					X	X	X	X		X	X	X	X	
hj_1					X	X	X			X	X	X		X


{jumper_1,pole_1,bar_1,fm_1,je_1,pv_1,hj_1}
∅
{jumper_1}
{M}
{...}
{...}
{pv_1,fm_1,hj_1}
{F_M}
{pv_1,pole_1}
{P}
{bar_1,pv_1,hj_1}
{H_B}
{pv_1,je_1,hj_1}
{E}
{pv_1,hj_1, bar_1}
{J_E}
{pv_1}
{PV}
...
{pv_1}
{F_M,P}
{pv_1}
{F_M,H_B}
{pv_1}
{P,H_B}
{pv_1}
{...}
{pv_1}
{E,J_E}
{pv_1}
{PV,E}
{pv_1}
{PV,J_E}
...
{pv_1}
{F_M,P,H_B}
{pv_1}
{F_M,P,E}
{pv_1}
{...}
{pv_1}
{E,J_E,PV}
...
{pv_1}
{F_M,P,H_B,E}
{pv_1}
{F_M,P,E,J_E}
{pv_1}
{...}
{pv_1}
{E,J_E,PV,H_B}
...
{pv_1}
{F_M,P,H_B,E,J_E}
{pv_1}
{...}
{pv_1}
{E,J_E,PV,H_B,P}
{...}
{...}
{pv_1}
{E,J_E,PV,H_B,P,F_M}
∅
{H,M,W,S,A,J,SE,F_M,P,JAV,H_B,E,J_E,PV,H_J}

Abox:
$\{bar_1 : Horizontal\_Bar, fm_1 : Foam\_Mat, pole_1 : Pole, je_1 : Jumping\_Event\}$.

$\boxed{\text{An explanation } \gamma \text{ could be } \{pv_1 : Pole\_Vault\}}$

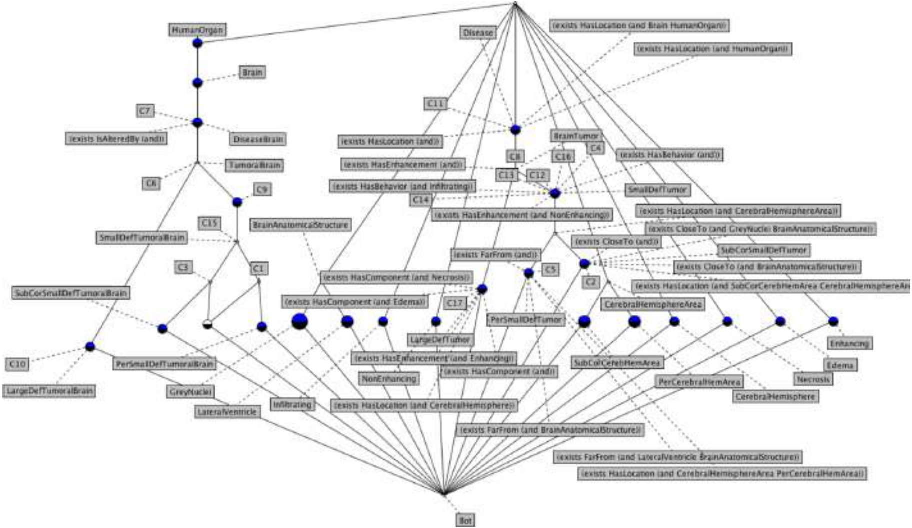
Brain image interpretation



Tbox:

$$
\begin{aligned}
\mathit{Brain} &\sqsubseteq \mathit{HumanOrgan} \\
\mathit{CerebralHemisphere} &\sqsubseteq \mathit{BrainAnatomicalStructure} \\
\mathit{PeripheralCerebralHemisphere} &\sqsubseteq \mathit{CerebralHemisphereArea} \\
\mathit{SubCorticalCerebralHemisphere} &\sqsubseteq \mathit{CerebralHemisphereArea} \\
\mathit{GreyNuclei} &\sqsubseteq \mathit{BrainAnatomicalStructure} \\
\mathit{LateralVentricle} &\sqsubseteq \mathit{BrainAnatomicalStructure} \\
\mathit{BrainTumor} &\sqsubseteq \mathit{Disease} \sqcap \exists \mathit{hasLocation}.\mathit{Brain} \\
\mathit{SmallDeformingTumor} &\equiv \mathit{BrainTumor} \sqcap \exists \mathit{hasBehavior}.\mathit{Infiltrating} \\
& \quad \sqcap \exists \mathit{hasEnhancement}.\mathit{NonEnhanced} \\
\mathit{SubCorticalSmallDeformingTumor} &\equiv \mathit{SmallDeformingTumor} \sqcap \\
& \quad \exists \mathit{hasLocation}.\mathit{SubCorticalCerebralHemisphere} \\
& \quad \sqcap \exists \mathit{closeTo}.\mathit{GreyNuclei} \\
\mathit{PeripheralSmallDeformingTumor} &\equiv \mathit{BrainTumor} \sqcap \\
& \quad \exists \mathit{hasLocation}.\mathit{PeripheralCerebralHemisphere} \\
& \quad \sqcap \exists \mathit{farFrom}.\mathit{LateralVentricle} \\
\mathit{LargeDeformingTumor} &\equiv \mathit{BrainTumor} \sqcap \\
& \quad \exists \mathit{hasLocation}.\mathit{CerebralHemisphere} \\
& \quad \sqcap \exists \mathit{hasComponent}.\mathit{Edema} \\
& \quad \sqcap \exists \mathit{hasComponent}.\mathit{Necrosis} \\
& \quad \sqcap \exists \mathit{hasEnhancement}.\mathit{Enhanced}
\end{aligned}
$$

$$
\begin{aligned}
\mathit{DiseasedBrain} &\equiv \mathit{Brain} \sqcap \exists \mathit{isAlteredBy}.\mathit{Disease} \\
\mathit{TumoralBrain} &\equiv \mathit{Brain} \sqcap \exists \mathit{isAlteredBy}.\mathit{BrainTumor} \\
\mathit{SmallDeformingTumoralBrain} &\equiv \mathit{Brain} \sqcap \exists \mathit{isAlteredBy}.\mathit{SmallDeformingTumor} \\
\mathit{LargeDeformingTumoralBrain} &\equiv \mathit{Brain} \sqcap \exists \mathit{isAlteredBy}.\mathit{LargeDeformingTumor} \\
\mathit{PeripheralSmallDeformingTumoralBrain} &\equiv \mathit{Brain} \sqcap \exists \mathit{isAlteredBy}.\mathit{PeripheralSmallDeformingTumor} \\
\mathit{SubCorticalSmallDeformingTumoralBrain} &\equiv \mathit{Brain} \sqcap \exists \mathit{isAlteredBy}.\mathit{SubCorticalSmallDeformingTumor} \\
& \dots
\end{aligned}
$$


HumanOrgan
Disease
(exists HasLocation (and Brain HumanOrgan))
(exists HasLocation (and HumanOrgan))
Brain
C11
C7
(exists IsAlteredBy (and))
DiseaseBrain
(exists HasLocation (and))
BrainTumor
C4
C8
C16
(exists HasBehavior (and))
TumoralBrain
(exists HasEnhancement (and))
C13
C12
C6
C9
(exists HasBehavior (and Infiltrating))
SmallDefTumor
C14
C15
BrainAnatomicalStructure
(exists HasEnhancement (and NonEnhancing))
(exists HasLocation (and CerebralHemisphereArea))
SmallDefTumoralBrain
(exists CloseTo (and GreyNuclei BrainAnatomicalStructure))
(exists CloseTo (and))
SubCorSmallDefTumor
(exists FarFrom (and))
C3
C1
C5
(exists CloseTo (and BrainAnatomicalStructure))
(exists HasComponent (and Necrosis))
C2
(exists HasLocation (and SubCorCerebHemArea CerebralHemisphereAr
SubCorSmallDefTumoralBrain
(exists HasComponent (and Edema))
C17
CerebralHemisphereArea
PerSmallDefTumor
LargeDefTumor
Enhancing
C10
PerSmallDefTumoralBrain
(exists HasEnhancement (and Enhancing))
SubCorCerebHemArea
Edema
(exists HasComponent (and))
NonEnhancing
Necrosis
PerCerebralHemArea
LargeDefTumoralBrain
GreyNuclei
CerebralHemisphere
Infiltrating
(exists HasLocation (and CerebralHemisphere))
LateralVentricle
(exists FarFrom (and BrainAnatomicalStructure))
(exists FarFrom (and LateralVentricle BrainAnatomicalStructure))
(exists HasLocation (and CerebralHemisphereArea PerCerebralHemArea))
Bot

Abox:

$$\begin{array}{rcl} t_1 & : & \mathit{BrainTumor} \\ e_1 & : & \mathit{NonEnhanced} \\ l_1 & : & \mathit{LateralVentricle} \\ p_1 & : & \mathit{PeripheralCerebralHemisphere} \\ (t_1, e_1) & : & \mathit{hasEnhancement} \\ (t_1, l_1) & : & \mathit{farFrom} \\ (t_1, p_1) & : & \mathit{hasLocation} \end{array}$$

Most specific concept:

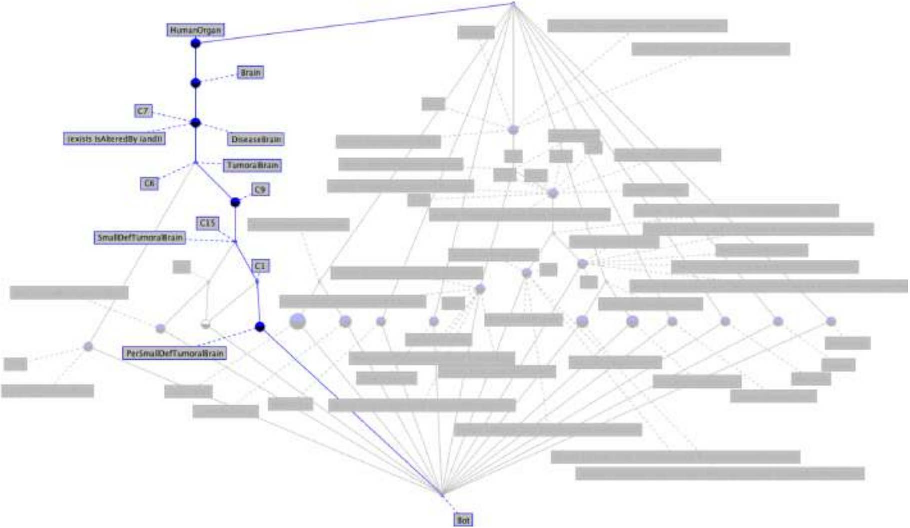
$$\begin{aligned} C \equiv \mathit{BrainTumor} \sqcap \exists \mathit{hasEnhancement}.\mathit{NonEnhanced} \sqcap \\ \exists \mathit{farFrom}.\mathit{LateralVentricle} \sqcap \\ \exists \mathit{hasLocation}.\mathit{PeripheralCerebralHemisphere} \end{aligned}$$

Concept abduction problem $\langle \mathcal{K}, C \rangle$: $\gamma \sqsubseteq_{\mathcal{K}} C$

Possible explanation set:
$\{$*DiseasedBrain*$, \exists$*isAlteredBy*$.\top$, *SmallDeformingTumoralBrain*,
PeripheralSmallDeformingTumoralBrain...$\}$.

A preferred solution with respect to some minimality criteria:

$$\boxed{\gamma \equiv \mathit{PeripheralSmallDeformingTumoralBrain}}$$


HumanOrgan
Brain
C7
(exists IsAlteredBy (and))
DiseaseBrain
TumoralBrain
C6
C9
C15
SmallDefTumoralBrain
C1
PerSmallDefTumoralBrain
Bot

Some references

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