ROB311 - Apprentissage pour la robotique

TP6 - Medical Diagnosis

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3rd Year - Robotics

1 Introduction

A hospital uses a support system for detecting lung problems. The system is designed to help in the diagnosis of tuberculosis, cancer, and bronchitis. A bayesian network is available in the figure below:

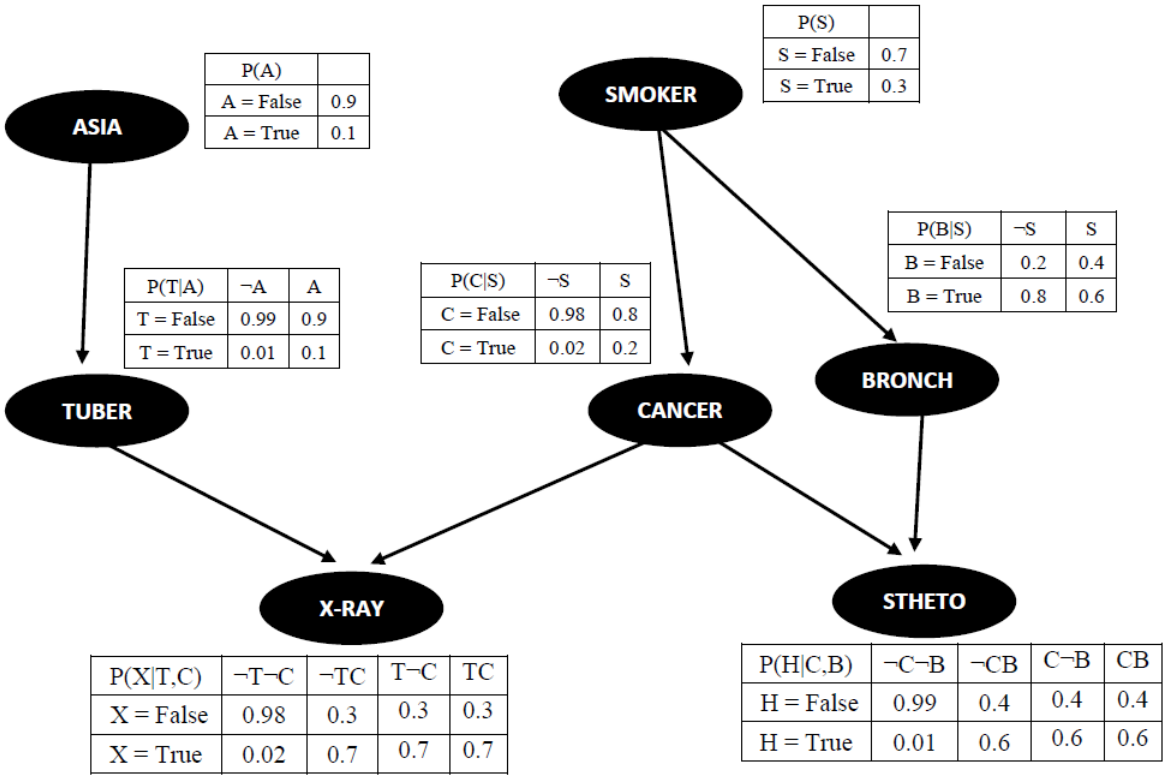

Figure 1: Bayesian network model

We are interested in the disease of a non smoker patient and has not recently visited Asia. The calculations of the probabilities of having tuberculosis, cancer or bronchitis knowing that the patient is not smoking and has not recently visited Asia, came as follows:

$$\begin{aligned} P(T/\neg A \cap \neg S) &= 0.01 \\ P(C/\neg A \cap \neg S) &= 0.02 \\ P(B/\neg A \cap \neg S) &= 0.8 \end{aligned} \tag{1}$$

Based on the calculated probabilities it is possible to infer as a first diagnosis that the patient in question has bronchitis.

2 Question 3

Bronchitis or lung cancer can be detected using the stethoscope, while the X-Ray test can detect cancer and tuberculosis. Thus, according to the disease inferred in the last question (bronchitis), the doctor decides to auscultate the patient's lungs with a stethoscope.
The stethoscope test came negative. We should calculate the new probability of each disease (lung cancer and bronchitis) knowing that the stethoscope test is negative ($\neg H$).

2.1 Lung cancer

The probability of having lung cancer disease knowing that the stethoscope test is negative and the patient doesn't smoke and didn't recently visit Asia is:

$$\begin{aligned} P(C/\neg H \cap \neg A \cap \neg S) &= \frac{P(C \cap \neg H \cap \neg A \cap \neg S)}{P(\neg H \cap \neg A \cap \neg S)} \\ &= \frac{P(\neg A) \times P(C \cap \neg H \cap \neg S)}{P(\neg A) \times P(\neg H \cap \neg S)} \\ &= \frac{P(\neg S) \times P(C/\neg S) \times P(\neg H/C)}{P(\neg S) \times P(\neg H/\neg S)} \\ &= \frac{P(C/\neg S) \times P(\neg H/C)}{P(\neg H/\neg S)} \end{aligned} \tag{2}$$

Calculating each term alone: For the numerator, we have:

$$\begin{aligned} P(C/\neg S) &= 0.02 \\ P(\neg H/C) &= P(\neg H/B \cap C) \times P(B) + P(\neg H/\neg B \cap C) \times P(\neg B) \\ &= 0.4 \times P(B) + 0.4 \times P(\neg B) \\ &= 0.4 \end{aligned} \tag{3}$$

For the denominator, we have:

$$\begin{aligned} P(\neg H/\neg S) &= \frac{P(\neg H \cap \neg S)}{P(\neg S)} \\ &= \frac{\begin{array}{c} P(\neg H \cap \neg S \cap C \cap B) + P(\neg H \cap \neg S \cap C \cap \neg B) \\ +P(\neg H \cap \neg S \cap \neg C \cap B) + P(\neg H \cap \neg S \cap C \cap B) \end{array}}{P(\neg S)} \\ P(\neg H \cap \neg S \cap C \cap B) &= P(\neg H \cap \cap C \cap B) \times P(B/\neg S) \times P(C/\neg S) \times P(\neg S) \\ &= 0.4 \times 0.8 \times 0.02 \times P(\neg S) \\ &= 0.0064 P(\neg S) \\ P(\neg H \cap \neg S \cap C \cap \neg B) &= 0.0016 P(\neg S) \\ P(\neg H \cap \neg S \cap \neg C \cap B) &= 0.314 P(\neg S) \\ P(\neg H \cap \neg S \cap \neg C \cap \neg B) &= 0.194 P(\neg S) \end{aligned} \tag{4}$$

Replacing the results in equation 4, we get:

$$\begin{aligned} P(\neg H/\neg S) &= \frac{0.0064P(\neg S) + 0.0016P(\neg S) + 0.314P(\neg S) + 00194P(\neg S)}{P(\neg S)} \\ &= 0.516 \end{aligned} \tag{5}$$

Now that we have all the terms, we replace the results in equation 2, we get:

$$\begin{aligned} P(C/\neg H \cap \neg A \cap \neg S) &= \frac{0.02 \times 0.4}{0.516} \\ &= 0.0156 \end{aligned} \tag{6}$$

2.2 Bronchitis

The probability of having bronchitis disease knowing that the stethoscope test is negative is:

$$\begin{aligned} P(B/\neg H \cap \neg A \cap \neg S) &= \frac{P(B \cap \neg H \cap \neg A \cap \neg S)}{P(\neg H \cap \neg A \cap \neg S)} \\ &= \frac{P(\neg S) \times P(\neg A) \times P(\neg H/B) \times P(B/\neg S)}{P(\neg S) \times P(\neg A) \times P(\neg H/\neg S)} \\ &= \frac{P(\neg H/B) \times P(B/\neg S)}{P(\neg H/\neg S)} \\ P(\neg H/B) &= P(\neg H/B \cap C) \times P(C) + P(\neg H/B \cap \neg C) \times P(\neg C) \\ &= 0.4 \times P(C) + 0.4 \times P(\neg C) \\ &= 0.4 \end{aligned} \tag{7}$$

Thus:

$$\begin{aligned} P(B/\neg H \cap \neg A \cap \neg S) &= \frac{0.4 \times 0.8}{0.0156} \\ &= 0.62 \end{aligned} \tag{8}$$

It is clear now that the new inferred diagnosis in this case is also bronchitis since we have $P(B/\neg H \cap \neg A \cap \neg S) > P(C/\neg H \cap \neg A \cap \neg S)$.

3 Question 4

The X-Ray test came positive. We should calculate the new probability of each disease (lung cancer and tuberculosis) knowing that the X-Ray test is positive (X) and the patient doesn't smoke and didn't recently visit Asia and knowing that also the Stethoscope test is negative.

3.1 Lung cancer

The probability of having lung cancer disease knowing that the X-Ray test is positive and the patient doesn't smoke and didn't recently visit Asia and knowing that also the Stethoscope test is negative:

$$\begin{aligned} P(C/X \cap \neg S \cap \neg A \cap \neg H) &= \frac{P(C \cap X \cap \neg S \cap \neg A \cap \neg H)}{P(X \cap \neg S \cap \neg A \cap \neg H)} \\ &= \frac{P(C/\neg S) \times P(\neg S) \times P(\neg A) \times P(X/\neg A) \times P(\neg H/C)}{P(\neg S) \times P(\neg A) \times P(X/\neg A) \times P(\neg H/C)} \\ &= P(C/\neg S) \\ &= 0.02 \end{aligned} \tag{9}$$

3.2 Tuberculosis

The probability of having tuberculosis disease knowing that the X-Ray test is positive and the patient doesn't smoke and didn't recently visit Asia and knowing also that the Stethoscope test is negative is:

$$\begin{aligned} P(T/X \cap \neg S \cap \neg A \cap \neg H) &= \frac{P(T \cap X \cap \neg S \cap \neg A \cap \neg H)}{P(X \cap \neg S \cap \neg A \cap \neg H)} \\ &= \frac{P(T/\neg A) \times P(\neg A) \times P(\neg S) \times P(\neg H/\neg S) \times P(X/T)}{P(\neg S) \times P(\neg A) \times P(X/\neg A) \times P(\neg H/\neg S)} \\ &= \frac{P(T/\neg A) \times P(X/T)}{P(X/\neg A)} \\ &= \frac{0.01 \times P(X/T)}{P(X/\neg A)} \end{aligned} \tag{10}$$

For the numerator, we have:

$$\begin{aligned} P(X/T) &= P(X/T \cap C) \times P(C) + P(X/T \cap \neg C) \times P(\neg C) \\ &= 0.7 \times P(C) + 0.7 \times P(\neg C) \\ &= 0.7 \end{aligned} \tag{11}$$

For the denominator, we have:

$$\begin{aligned} P(X/\neg A) &= \frac{P(X \cap \neg A)}{P(\neg A)} \\ &= \frac{P(X \cap \neg A \cap T) + P(X \cap \neg A \cap \neg T)}{P(\neg A)} \\ &= P(X/\neg T) \times P(\neg T/\neg A) + P(X/T) \times P(T/\neg A) \end{aligned} \tag{12}$$

With:

$$\begin{aligned} P(C) &= P(C/S) \times P(S) + P(C/\neg S) \times P(\neg S) \\ &= 0.2 \times 0.3 + 0.02 \times 0.7 \\ &= 0.074 \\ P(X/\neg T) &= P(X/C \cap \neg T) \times P(C) + P(X/\neg C \cap \neg T) \times P(\neg C) \\ &= 0.7 \times 0.074 + 0.02 \times (1 - 0.074) \\ &= 0.07032 \end{aligned} \tag{13}$$

Replacing in equation 11:

$$P(X/\neg A) = 0.08 \tag{14}$$

Replacing in equation 10:

$$\begin{aligned} P(T/X \cap \neg S \cap \neg A \cap \neg H) &= \frac{0.01 \times 0.7}{0.08} \\ &= 0.0875 \end{aligned} \tag{15}$$

It is clear now that the new inferred diagnosis in this case remains the bronchitis disease because the probabilities of having one of the two other diseases, using the X-Ray test, are very low.

4 Question 5

Given the results of question 3 and question 4, the X-ray was not necessary. The probability of having cancer or tuberculosis remain very low compared with that of having bronchitis. Since the former probabilities would have been even lower if the X-Ray scan had been negative, there was no need for doing an X-Ray scan at all.