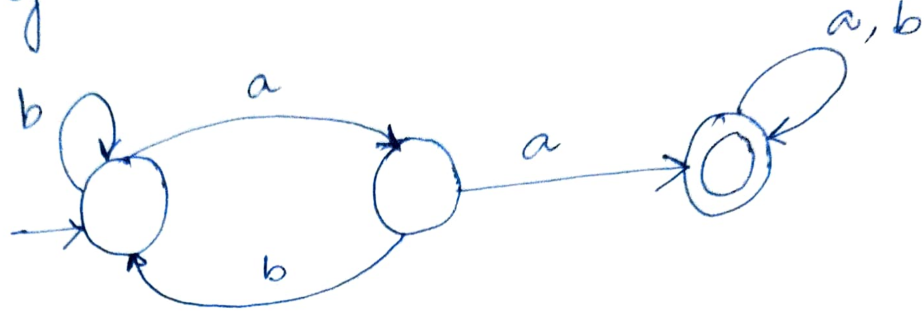
1) Regular Expressions for L_1 and L_2

$$r_1 = (a+b)^* aa (a+b)^*$$

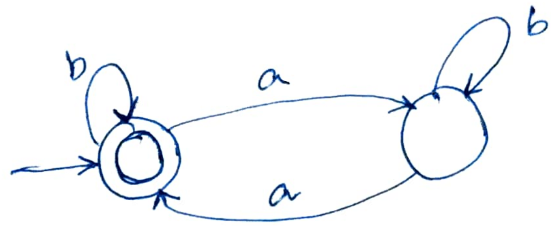
$$r_2 = b^* (ab^* ab^*)^*$$

2) Corresponding Finite Automata are:

M_1 :



M_2 :



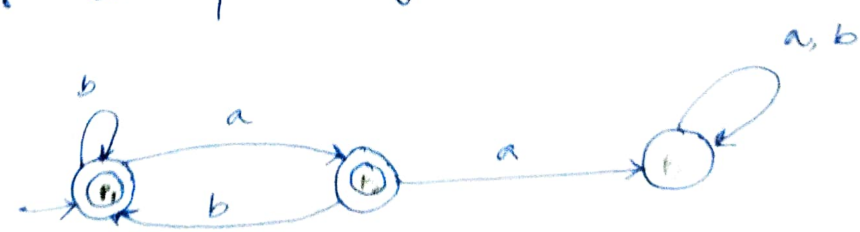
Complement Languages are:

L_1' = all strings that do not contain the substring aa

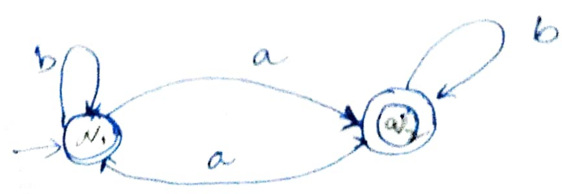
L_2' = all strings having an odd number of a's.

Hence their corresponding finite automata are

M_1' :



M_2' :



iii) In case of developing the automata M such that $L(M) = L(M') \cup L(M_2')$, we use the concept of product automata construction of M_1' and M_2'.

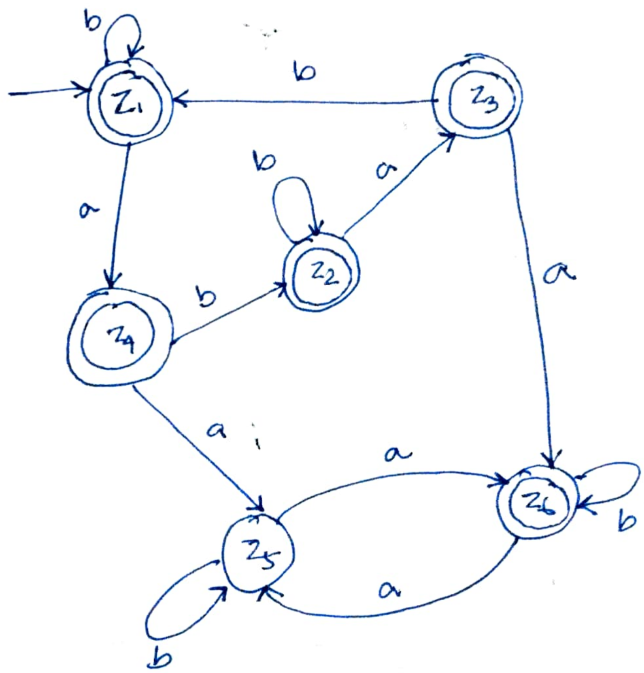
Since, M_1' and M_2' have 3 and 2 states respectively, the product of these two automata has 6 possible combination of states, which are:

$z_1 = (p_1, q_1)$ → words ending here accepted by M_1'
$z_2 = (p_1, q_2)$ → " " " " " M_1' and M_2'
$z_3 = (p_2, q_1)$ → " " " " " M_1'
$z_4 = (p_2, q_2)$ → " " " " " M_1' & M_2'
$z_5 = (p_3, q_1)$ → NOT final
$z_6 = (p_3, q_2)$ → " " " " M_2'

Hence the transition table is:

	a	b
z_1	z_4	z_1
z_2	z_3	z_2
z_3	z_6	z_1
z_4	z_5	z_2
z_5	z_6	z_5
z_6	z_5	z_6

The union automata M is :-



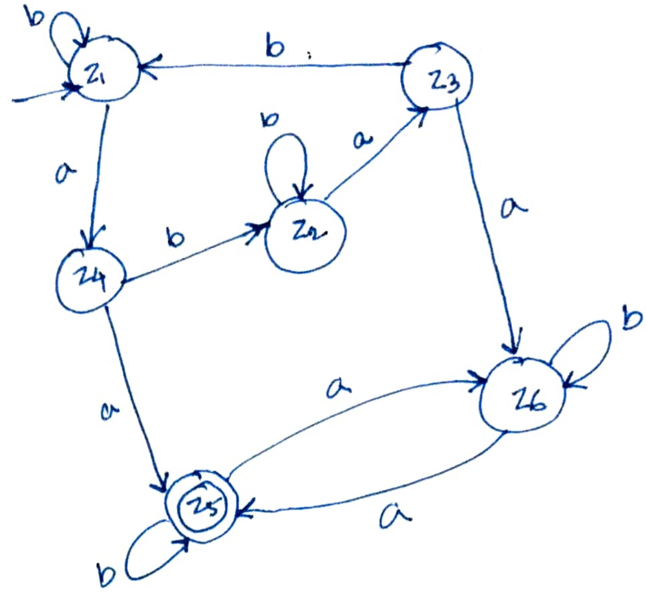
Among them final states are : $\{z_1, z_2, z_3, z_4, z_6\}$

& start states are: $\{z_1\}$

~~{ z1, z2, z3 }~~

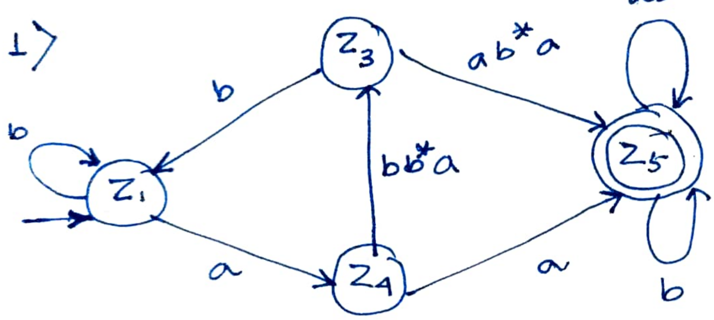
complement of M is :-

M' :



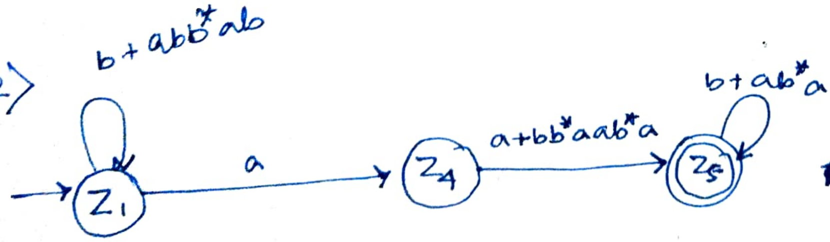
iv) Corresponding Regular Expression can be generated as follows :- <u>Generalized</u>

1)



- Path from z_4 to z_2 to z_3 has been replaced with an edge labeled bb^*a
- Edge from z_3 to z_6 has been removed and z_3 has now a direct edge to z_5 with label ab^*a
- z_6 has been replaced by a loop at z_5 labeled ab^*a.

2)



- path from z_4 to z_5 via z_3 has been replaced by an edge labeled $bb^*a\,ab^*a$, and z_3 has been removed.
- Path from z_1 to z_1 via z_4 and z_3 has been replaced by a loop with labeled abb^*ab

3) This whole automata finally reduces to the regular expressions—

$$\boxed{(b+abb^*ab)^*a(a+bb^*aab^*a)(b+ab^*a)^*}$$

Q2.) L_1 and L_2 are regular

$$L_1 \ominus L_2 = (L_1 \cup L_2) \cap (\overline{L_1 \cap L_2})$$

We know regular languages are closed under union, intersection, and complementation.

Hence $L_1 \ominus L_2$ is also regular.

Q3) $L = \{a^n b^n \mid n \geq 0\}$. $L^2 = LL$ (concatenation of two L)

~~We know~~ a ~~context-free~~ language ~~L has a context-free grammar~~ G ~~such that~~ L ~~(G) = L'~~.

Remember, a language L is said to be context-free <u>if and only if</u> there is a context-free grammar (CFG) G such that $L(G) = L$

Hence, generating a CFG for L^2 is sufficient to show that L^2 is context-free.

The CFG that recognizes L^2 is $G = (\{S, S_1\}, \{a, b\}, S, P)$ with P as

$$S \rightarrow S_1 S_1$$
$$S_1 \rightarrow a S_1 b \mid \epsilon$$