The Prisoners' Dilemma & Repeated Games

Revisit a Prisoners' Dilemma game. In the one-time game, the dominant strategy is (L, L).

		2 L	2 H
1	L	$(54, 54)$	$(72, 47)$
	H	$(47, 72)$	$(60, 60)$

Now, we play under **finite repeated interaction** (T times).
But, there is no fear of retribution in the last T^{th} game and by induction each game will produce the same outcome.

Next, we play under **infinite repeated interaction**.
"Each player fears that one instance of defecting will lead to a collapse of cooperation in the future. If the value of future cooperation is large and exceeds what can be gained in the short term by defecting, then the long-term individual interests of the players can automatically and tacitly keep them from defecting, without the need for any additional punishments or enforcement by third parties" (Page 379).
We consider two **trigger strategies**:
1. Grim Trigger Strategy (GTS): Play cooperatively, but if the other player ever does not cooperate then never play cooperatively again.
2. Tit-for-Tat Trigger Strategy (TFT): Do to the other player what they did to you in the last period.
Cooperate or Defect vs. GTS:
Cooperate: 60, 60, 60, ...
Defect: 72, 54, 54, ...

Note (Present Value (PV) / Discounting)
Invest $\$1$ today -> Get $\$(1 + r)$ next year
Want $\$1$ next year -> Invest $\$(\frac{1}{1+r})$ today
Annuity: Every year on a date you get a specified amount.
Ex. You get $\$1$ today and $\$1$ each year thereafter.

PV = 1 + $\frac{1}{r}$, r interest rate
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60 + $\frac{60}{r}$ > 72 + $\frac{54}{r}$ $\implies$ $r < \frac{1}{2}$.
So, for values of r less than $\frac{1}{2}$ it is better to cooperate than defect.
Cooperate or Defect vs. TFT:
Cooperate: 60, 60, 60, ...

Defect: 72, 47, 60, 60, 60, ...

Since everything after the second period is the same, we only look at PV of first and second periods. We have $1 + r$ instead of r because of this.

60 + $\frac{60}{1+r}$ > 72 + $\frac{47}{1+r}$ $\implies$ r < $\frac{1}{12}$.

So, for values of r less than $\frac{1}{12}$ it is better to cooperate than defect.