12. Evolutionary Games

Def. A **selection** is the changing of distribution of types (e.g. phenotype, genotype).

Note We call the equilibrium the Evolutionary Stability Solution (ESS).

Prisoners' Dilemma Style Game

	Defect	Cooperate
Defect	$(54, 54)$	$(72, 47)$
Cooperate	$(47, 72)$	$(60, 60)$

Let x be the proportion of cooperators in the population.
We consider the fitness levels of each type:
Defector: Exp(Π) = 54(1 - x) + (72)(x) = 18x + 54
Cooperator: Exp(Π) = 47(1 - x) + (60)(x) = 13x + 47
It is more fit to be a cooperator if:
13x + 47 > 18x + 54 $\implies$ x < $\frac{-7}{5}$.
Here, in practice, it is never better to be a cooperator.

The ESS is x = 0. So, we end with a **monotone distribution** since only defectors will remain.

At this point, imagine a **mutation** enters the population, a cooperator in this case. The mutation will not survive.

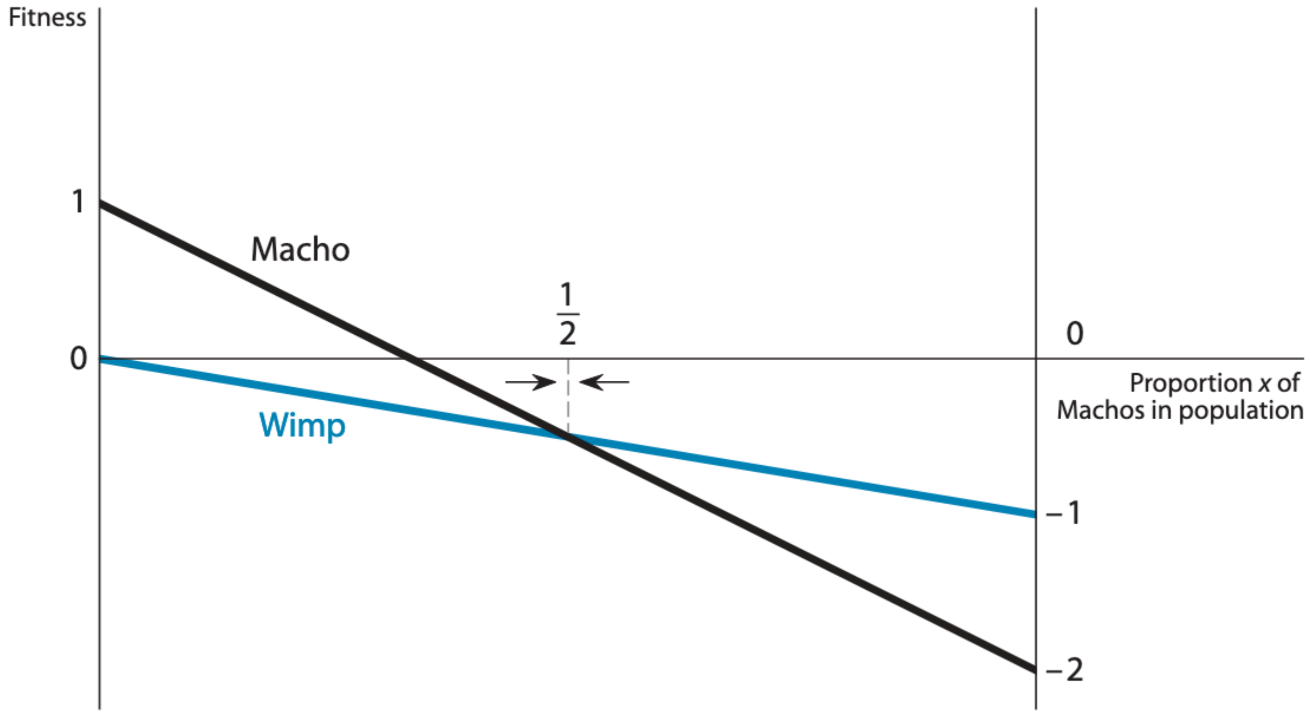
Chicken Style Game

	W	M
W	$(0, 0)$	$(-1, 1)$
M	$(1, -1)$	$(-2, -2)$

Let x be the proportion of M in the population.
We consider the fitness levels of each type:
W: Exp(Π) = (0)(1 - x) + (-1)(x) = -x
M: Exp(Π) = (1)(1 - x) + (-2)(x) = -3x + 1
It is more fit to be M if:
-3x + 1 > -x $\implies$ x < $\frac{1}{2}$.

The ESS is x = $\frac{1}{2}$. So, we do not have a one-type solution.

Here, a mutation is of either type.



Hawk & Dove Game

	H	D
H	$(\frac{V-C}{2}, \frac{V-C}{2})$	$(V, 0)$
D	$(0, V)$	$(\frac{V}{2}, \frac{V}{2})$

Let Value be V.
Imagine we have two types come together:
Two Hawks: Equal chance of getting V or enduring injury C
Two Doves: They share V
One of Each: Hawk gets all, Dove gets nothing

Consider V = 10, C = 4 so V > C.
Let x be the proportion of Doves in the population.
Fitness levels:
Hawk: $(\frac{10-4}{2})(1 - x) + (10)(x) = 7x + 3$
Dove: $(0)(1 - x) + (\frac{10}{2})(x) = 5x$
It is more fit to be a Dove if:
$7x + 3 < 5x \implies x < \frac{-3}{2}$.
So, ESS is x = 1 and hence it is always better to be a Hawk.

Consider V = 10, C = 12 so V < C.
Let x be the proportion of Doves in the population.
Fitness levels:
Hawk: $(\frac{10-12}{2})(1 - x) + (10)(x) = 11x - 1$
Dove: $(0)(1 - x) + (\frac{10}{2})(x) = 5x$

It is more fit to be a Dove if:
11x - 1 < 5x $\implies$ x < $\frac{1}{6}$.
So, it is better to be a Dove if x < $1/6$.
Then, ESS is x = $\frac{1}{6}$.