From time series analysis to a modified ordinary differential equation

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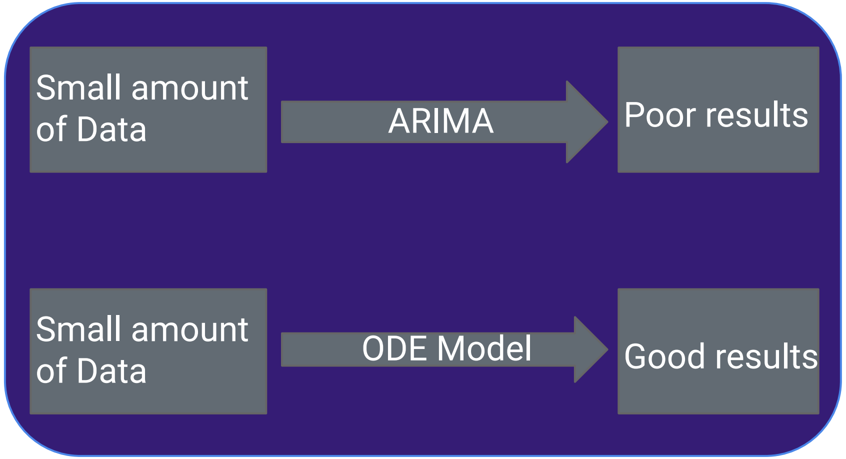
Objective

- Examine a modified ODE approach
- Compare it with TSA in modelling the price movements
- Study computational properties of the ODE with different forms of polynomials and periodic functions



ARIMA and ODE models

ARIMA(p,d,q) is defined as follows:

$$\left(1-\sum_{i=1}^{p}\varphi_i B^i\right)\Delta^d y_t=\left(1-\sum_{i=1}^{q}\theta_i B^i\right)\varepsilon_t$$

$$B^k y_t = y_{t-k}, \quad k=0,1,2,\ldots$$

$$\Delta^d y_t=(1-B)^d y_t, \quad d=0,1,2,\ldots$$

For stationary data, ARIMA(p,q) is used, otherwise the initial form takes place in the modelling and prediction.

Model comparison metrics: AIC and BIC

ODE method

Cauchy initial value problem

$$y' = ay, \quad y(t_0)=y_0$$

Particular solution

$$y = y_0 e^{a(t-t_0)}$$

Solving for ‘a’ yields:

$$a=\frac{1}{t_1-t_0}\ln\left(\frac{y_1}{y_0}\right)$$

In general case:

$$a=\frac{1}{t_i-t_{i-1}}\ln\left(\frac{y_i}{y_{i-1}}\right)$$

Comparison of the TSA and the existing ODE model

Metrics to compare TSA and ODE models

Absolute Percentage Error

$$APE_i = \frac{|\widehat{y}_i - y_i|}{y_i}$$

Mean Absolute Percentage Error

$$MAPE = \frac{1}{n}\sum_{i=1}^{n} APE_i$$

Comparison of the TSA and the existing ODE model

Table 1. Comparing ODE and TSA of petroleum price (2013).

APE	ODE	TSA
[0, 5%)	249	250
[5%, 7.5%)	1	0
[7.5%, 10%)	0	0
[10%, 1)	0	0
MAPE	1.2597%	0.8817%

APE: absolute percentage error; ODE: ordinary differential equation; TSA: time series analysis.

Table 2. Comparing ODE and TSA of petroleum price (2014).

APE	ODE	TSA
[0, 5%)	235	248
[5%, 7.5%)	9	1
[7.5%, 10%)	4	0
[10%, 1)	2	1
MAPE	1.7445%	1.0670%

APE: absolute percentage error; ODE: ordinary differential equation; TSA: time series analysis.

Table 3. Comparing ODE and TSA of petroleum price (2015).

APE	ODE	TSA
[0, 5%)	198	227
[5%, 7.5%)	30	18
[7.5%, 10%)	15	5
[10%,1)	7	0
MAPE	3.4100%	2.2922%

APE: absolute percentage error; ODE: ordinary differential equation; TSA: time series analysis.

Table 4. Comparing ODE and TSA of bank share values.

	Barclays bank		Lloyds bank	
APE	ODE	TSA	ODE	TSA
[0, 5%)	623	694	626	706
[5%, 7.5%)	90	54	82	39
[7.5%, 10%)	39	14	41	19
[10%, 1)	20	10	22	8

APE: absolute percentage error; ODE: ordinary differential equation; TSA: time series analysis.

Modification of ODE model

Generalized model with period and polynomial function

$$y' = g(t, y) = \left(\sum_{i=0}^{M} a_i t^i\right) y + b_0 + \sum_{j=1}^{N} b_j \sin\left(\frac{2\pi j y}{\theta} + c_j\right)$$

$$y_{n+1} = y_n + \frac{1}{6}h(k_1 + 4k_2 + k_3),$$

$$k_1 = g(t_n, y_n), \qquad k_2 = g\left(t_n + \frac{1}{2}h, y_n + \frac{1}{2}hk_1\right),$$

$$k_3 = g\left(t_n + \frac{1}{2}h, y_n + \frac{1}{2}hk_2\right), \qquad h = t_{n+1} - t_n,$$

$$k_4 = g(t_n + h, y_n + hk_3)$$

Table 5. Possible forms of $f(x)$.

$\alpha\sin x + \beta$	$\alpha e^x + \beta$
$\alpha x + \beta$	$\frac{\alpha x}{\beta + x}$
$\alpha x^2 + \beta x + \gamma$	$\alpha \cdot 2^{-x/\beta}$
$\alpha\ln x + \beta$	$\alpha\beta^x$
$\alpha\sin x + \beta x + \gamma$	$\alpha + \beta\gamma^x$
$\alpha\sin x + \beta\ln x + \gamma$	$e^{\alpha + \beta\gamma^x}$
$\alpha\ln x + \beta x + \gamma$	$\frac{1}{\alpha + \beta\gamma^x}$
$\alpha\sin x + \beta x + \gamma\ln x + \delta$	$\alpha e^{\beta x} + \gamma$
...	...

Empirical Analysis

Table 6. MAPE of petroleum (2013) according to equation (12).

MAPE		N: 1	2	3
M	0	0.89524%	0.88784%	0.88896%
	1	Singular	0.89059%	0.88722%
	2	Singular	Singular	Singular
	3	Singular	Singular	Singular

Table 7. MAPE of petroleum (2014) according to equation (12).

MAPE		N: 1	2	3
M	0	1.08986%	1.11213%	1.10441%
	1	Singular	Singular	1.09418%
	2	Singular	Singular	Singular
	3	Singular	Singular	Singular

MAPE: mean absolute percentage error.

Table 8. MAPE of petroleum (2015) according to equation (12).

APE		N: 1	2	3
M	0	2.27096%	2.26665%	2.30960%
	1	2.25025%	2.26416%	2.26109%
	2	Singular	Singular	Singular
	3	Singular	Singular	Singular

Table 9. MAPE of Barclays bank according to equation (12).

APE		N: 1	2	3
M	0	2.23898%	Singular	Singular
	1	2.25212%	Singular	Singular
	2	Singular	Singular	Singular
	3	Singular	Singular	Singular

APE: absolute percentage error.

Empirical Analysis

Table 12. The estimated parameters.

	Petroleum			Bank		
Parameters	2013	2014	2015	Barclays	Lloyds	RBS
a_0	−0.03031	0.01215	−0.02185	−0.00947	−0.00638	−0.00914
a_1	0.00001		−0.00004		−0.00001	−0.00002
b_0	2.90619	−1.30755	1.26420	2.36196	0.43602	4.88964
b_1	0.24353	−0.22307	0.16355	1.31137	−0.07560	0.82936
θ	0.49755	0.74135	0.46282	0.50023	0.49999	0.49879
c_1	−4.50499	398.30817	12.81797	1.56126	49.13539	−8.37850
b_2	−0.17368				0.18431	0.84405
c_2	−11.26577				1.52255	−5.84867
b_3	0.11570					
c_3	−15.31979					

Empirical Analysis

Ordinary differential equation (2)

$$y' = ay, \quad y(t_0) = y_0$$

Ordinary differential equation (12)

$$y' = g(t, y) = \left(\sum_{i=0}^{M} a_i t^i\right) y + b_0 + \sum_{j=1}^{N} b_j \sin\left(\frac{2\pi j y}{\theta} + c_j\right)$$

Table 13. MAPE compared with different y'.

	Model given in equation (2)	Model given in equation (12)
Petroleum 2013	1.2597%	0.8872%
Petroleum 2014	1.7445%	1.0809%
Petroleum 2015	3.4100%	2.2471%
Barclays	3.2526%	2.2390%
Lloyds	3.1877%	2.2855%
RBS	3.0739%	2.2127%

Conclusion

- The authors studied a variety of right-hand side solutions to the problem

$$y' = \mu(t, y)$$

- The proposed modified ODE equation (12) provides good understanding of the trend and the dynamics of the price movement and has obtained similar results as the ARIMA model for predicting and forecasting petroleum and two bank stock prices
- Numerical methods for ODEs require fewer historical market data
- The work is anticipated to lead to a systematic and an accessible way of forecasting the dynamic market

$$\begin{aligned} y' &= g(t, y) \\ &= \left(\sum_{i=0}^{M} a_i t^i\right) y + b_0 + \sum_{j=1}^{N} b_j \sin\left(\frac{2\pi j y}{\theta} + c_j\right) \end{aligned}$$

Equation 12.