Operations Research 2024
Problem set 1 (Linear Programming)

Problem 1. A company is contracted to produce two products, A and B, over the months of June, July, and August. The total production capacity (expressed in hours) varies monthly. The table below provides the basic data of the situation:

	June	July	August
Demand for A (units)	500	5000	750
Demand for B (units)	1000	1200	1200
Capacity (hours)	3000	3600	3000

The production rates in units per hour are 1.25 and 1 for products A and B, respectively. All demand must be met. However, demand for a later month may be filled from the production in an earlier one. For any carryover from one month to the next, holding costs of \$0.90 and \$0.75 per unit per month are charged for products A and B, respectively. The unit production costs for the two products are \$30 and \$28 for A and B, respectively. Develop an LP model to determine the optimum production schedule for the two products.

Problem 2. Hawaii Sugar Company produces brown sugar, processed (white) sugar, powdered sugar, and molasses from sugarcane syrup. The company purchases 4000 tons of syrup weekly and is contracted to deliver at least 25 tons weekly of each type of sugar. The production process starts by manufacturing brown sugar and molasses from the syrup. A ton of syrup produces 0.3 ton of brown sugar and 0.1 ton of molasses. White sugar is produced by processing brown sugar. It takes 1 ton of brown sugar to produce 0.8 ton of white sugar. Powdered sugar is produced from white sugar through a special grinding process that has a 95% conversion efficiency (1 ton of white sugar produces 0.95 ton of powdered sugar). The profits per ton for brown sugar, white sugar, powdered sugar, and molasses are \$150, \$200, \$230, and \$35, respectively. Formulate the problem as a linear program.

Problem 3. A product is assembled from three different parts. The parts are manufactured by two departments at different production rates and different capacity as given in the following table:

Department	Capacity (hr/wk)	Production rate (units/hr)		
		Part 1	Part 2	Part 3
1	100	10	8	12
2	120	10	12	4

Determine the maximum number of final product units that can be produced weekly.

Hint: Product units $=\min\{\text{units of part 1}, \text{units of part 2}, \text{units of part 3}\}$. Maximizing $z = \min\{x_1, x_2\}$ is equivalent to maximizing z subject to $z \leqslant x_1$ and $z \leqslant x_2$.

Problem 4. Bills in a household are received monthly (e.g., utilities and home mortgage), quarterly (e.g., estimated tax payments), semiannually (e.g., insurance), or annually (e.g., subscription renewals and dues). The table below provides the monthly bills for the next year.

Month	$	Month	$	Month	$	Month	$
JAN	800	APR	700	JUL	1500	OCT	1100
FEB	1200	MAY	600	AUG	1000	NOV	1300
MAR	400	JUN	900	SEP	900	DEC	1600

To account for these expenses, the family sets aside $1000 per month, which is the sum all payments in the next year, divided by 12. If the money is deposited in a regular savings account, it can earn 4% annual interest, provided it stays in the account for at least 1 month. The interest is paid monthly. The bank also offers 3-month and 6-month certificates of deposit that can earn 5.5% and 7% annual interest, respectively. The m-month certificate of deposit means putting money in the bank for m months, after which the bank returns the deposit and the interest revenue calculated for m months. Develop a 12-month investment schedule that will allow the family to pay all the bills and to maximize the income from the investments in the end of the next year.

Problem 5. Toolco has contracted with AutoMate to supply their automotive discount stores with wrenches and chisels. AutoMate's weekly demand consists of at least 1570 wrenches and 1250 chisels. Toolco cannot produce all the requested units with its present one-shift capacity, and must use overtime and possibly subcontract with other tool shops. The result is an increase in the production cost per unit, as shown in the table below. Market demand restricts the ratio of chisels to wrenches to at least 2:1. Formulate the problem as a linear program. How would you approach the problem if the unit production cost of wrenches for range 801-∞ would be $3.00?

Tool	Production type	Weekly production range (units)	Unit cost ($)
Wrenches	Regular	0-500	2.00
	Overtime	501-600	2.80
	Subcontracting 1	601-800	3.20
	Subcontracting 2	801-∞	3.50
Chisels	Regular	0-620	2.10
	Overtime	621-750	3.20
	Subcontracting 1	751-900	4.20
	Subcontracting 2	901-∞	4.50

Problem 6. A data scientist has a dataset $\{(x_i, y_i)\}_{i=1}^n$, where x_i is a single predictor and y_i is a response variable, and was asked to fit a strait line $f(x) = ax + b$ to this data. He is afraid that the data contain outliers and that l_2-loss function, that is used in ordinal least squares, is not robust to outliers. Therefore, he decides to use l_1-loss, and comes to the following optimization problem:

$$\sum_{i=1}^{n} |y_i - ax_i - b| \to \min_{a,b} . \tag{1}$$

Formulate an equivalent linear programming problem for solving the problem (1). Solve the resulting LP problem in order to fit the straight line to the following data

x	1	2	3	4	5	6	7	8	9	10
y	5	9	15	19	21	24	26	30	31	35

Hint: Use the fact that $\min |w|$ is equivalent to $\min z$ subject to $z \geqslant w$, $z \geqslant -w$, and $z \geqslant 0$.