Operations Research 2024
Home assignment 1
Deadline: 19th February 2023, 22:00

Guidelines:

1. This home assignment covers linear programming and networks, therefore solutions must be based on these techniques only.

2. Each solution must consist of two parts: mathematical model and script.

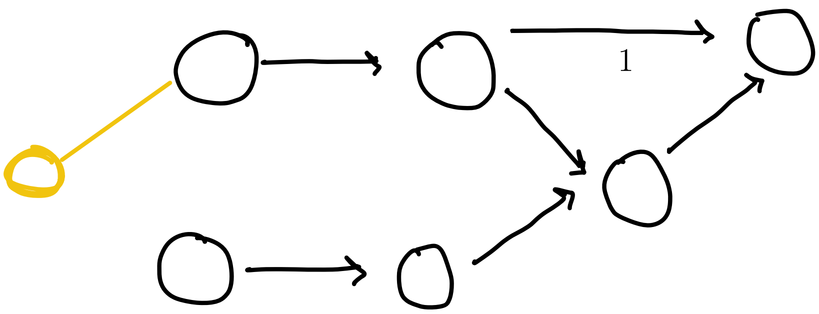
3. Mathematical model must contain clear definition of all the elements and the logics behind the objective and constraints. Any ambiguity will be used against you.

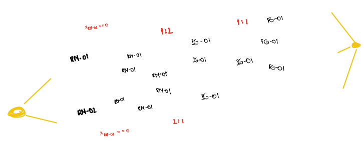
4. Scripts must be either in AMPL, or in Python.

5. Scripts in Python must only use the following external packages: numpy, pandas, networkx, amplpy.

6. Use GanttProject for building Gantt charts.

Problem 1 (60 points). BevCo has a pretty sophisticated production and distribution chain for their beverage, which is known under code name FG-01. The chain is shown in figure 1. The beverage is made from two secret ingredients, RM-01 supplied by supplier S01, and RM-02 supplied by supplier S02. The suppliers deliver the ingredients to two warehouses, WH01 and WH02, each dedicated to only one of the two ingredients. Each of these warehouses can send the ingredients directly to one of the two factories, F01 and F02. Both of these two warehouses can send the ingredients to the intermediate warehouse WH03 capable of storing both ingredients and sending them to the factories. The factories F01 and F02 produce intermediate product under code name IG-01. Factory F01 can send the produced intermediate product to factory F03 as well as to warehouse WH04. Factory F02 can send its production only to warehouse WH04. Factory F03 produces the beverage from the intermediate product and sends it to distribution centers DC01 and DC03, as well as to warehouse WH05, which then sends the beverage to the distribution centers DC01 and DC02. Distribution center DC01 can send part of its product to distribution center DC02, which can send the beverage to distribution center DC03.





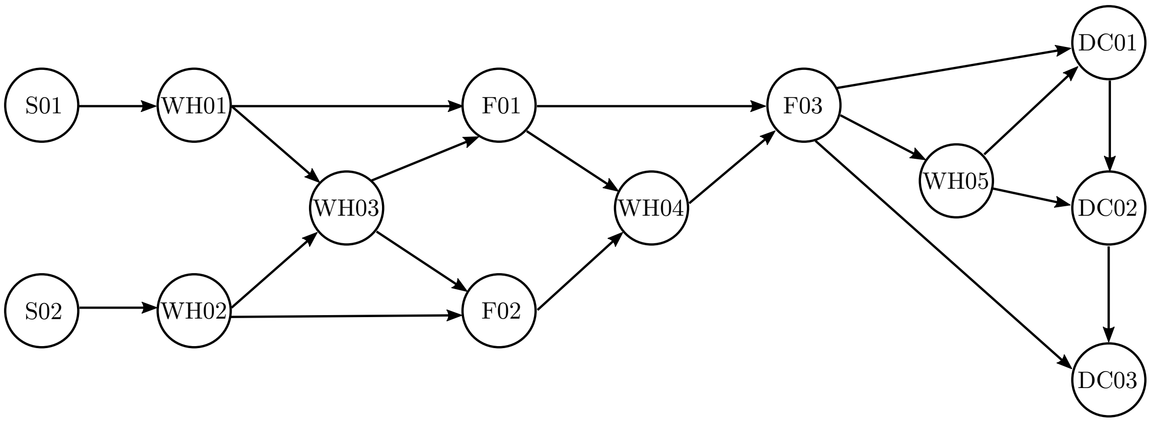


Figure 1: Production and distribution scheme for problem 1.

The sales department has forecast for the demand at each of the three distribution centers for the next 10 weeks presented in HA1_P1_Data.xlsx, sheet Demand.

The factories use different production technologies. The factory F01 requires 1 unit of RM-01 and 2 units of RM-02 in order to produce a unit of IG-01. The factory F02 requires 2 units of RM-01 and 1 unit of RM-01 in order to produce a unit of IG-01. The factory F03 requires 2 units of IG-01 in order to produce 1 unit of FG-01. This data also collected in HA1_P1_Data.xlsx, sheet BOM. BOM stands for bill of materials. The first columns in the sheet, Source ID, is an ID for production technology. The last column, Component Quantity, gives the amount of component specified in column Component ID needed to produce a single unit of output product specified in column Product ID.

All the warehouses and distribution centers can keep products in inventory. The factories do not have inventories. The inventories have limited space and different unit holding costs. The data on storage capacity and holding costs is presented in HA1_P1_Data.xlsx, sheet Inventory. If an inventory can store several products, the storage capacity is applied to the total amount of all the products kept in the inventory. At the beginning of planning horizon BevCo has some amount of products in the inventories. The data on starting inventory levels is summarized in HA1_P1_Data.xlsx, sheet StartingInventory.

Each node with both incoming and outgoing arcs places orders to nodes connected by incoming arcs (upstream nodes) and sends the product according to the orders placed by the nodes connected by the outgoing arcs (downstream nodes). The total amount of product ordered by all the downstream nodes cannot exceed the amount available at the upstream node at the time when the order is placed. The distribution centers additionally have to satisfy all the forecasted demand. Each arc has transportation capacity (applied to total amount of all the products sent over the arc), transportation cost and a lead time. A lead time for an arc (u, v) stands for a time needed for the product to move from node u to node v. For example, if lead time for the arc from S01 to WH01 is 2 weeks, then an order for RM-01 placed by warehouse WH01 to supplier S01 at time t will be delivered at time $t + 2$. All these arc attributes are presented in HA1_P1_Data.xlsx, sheet Transportation.

Develop linear programming model for determining optimal production and distribution plan which guarantees full demand satisfaction and minimizes the transportation and inventory cost. Solve the model with AMPL.

Hints:

1) Good and convenient notation is super crucial for this problem. We have considered variables indexed by at most two indices, but really nobody prohibits considering variables indexed by three and more indices. Any advanced optimization software will convert all the multidimensional sets of variables into one-dimensional set.

2) I would recommend using string indices, instead of numeric ones, if you index node names or products. Say, you may find useful to have a set W of all the warehouses, which contains 5 strings – from "WH01" to "WH05", and consider using index variable i to loop over items of that set, assuming that i can take values "WH01" to "WH05" instead of from 1 and 5.

3) You may find function itertools.product useful for iterating over multiple sets of indices.

Grading: Mathematical model – 30 points; program – 30 points.

Problem 2 (15 points). A central library in a town N has varying demand for the computer consultants, depending on the day of week and the time of day. The estimated required number of computer consultants is summarized in the table 1.

Table 1: Required number of consultants for problem 2.

Time period	Day of week						
	Mon	Tue	Wed	Thu	Fri	Sat	Sun
8:00 AM – 10:00 AM	2	1	1	1	1	1	1
10:00 AM – 12:00 PM	2	1	1	1	1	2	3
12:00 PM – 2:00 PM	2	2	2	2	2	4	5
2:00 PM – 4:00 PM	2	2	2	2	2	4	5
4:00 PM – 6:00 PM	3	3	3	3	5	8	8
6:00 PM – 8:00 PM	4	3	3	3	4	6	6

The library can hire full-time and part-time consultants. The contract fixes five consecutive days, for which the consultant is in the library, as well as the working time period. The full-time consultants work five consecutive days for one of the two 8-hr. shifts – either from 8:00 AM to 4:00 PM or from 12:00 PM to 8:00 PM. The full-time wage is $1000 per month. The part-time consultants work five consecutive days for one of the five 4-hr. shifts – from 8:00 AM to 12:00 PM, from 10:00 AM to 2:00 PM, etc. The wage of half-time consultant is $600 per month. Develop and solve a linear programming problem for determining optimal number of consultants and their schedule.

Grading: Mathematical model – 10 points; program – 5 points.

Problem 3 (10 points). Jim lives in Denver, Colorado, and likes to spend his annual vacation in Yellowstone National Park in Wyoming. Being a nature lover, Jim tries to drive a different scenic route each year. After consulting the appropriate maps, Jim has represented his preferred routes between Denver (D) and Yellowstone (Y) by the network in figure 2. Nodes 1 through 14 represent intermediate cities. Although driving distance is not an issue, Jim stipulation is that selected routes between D and Y do not include any common cities. Determine all the distinct routes available to Jim by modifying the maximal flow LP model to determine the maximum number of unique paths between D and Y.

Grading: Mathematical model – 5 points; program – 5 points.

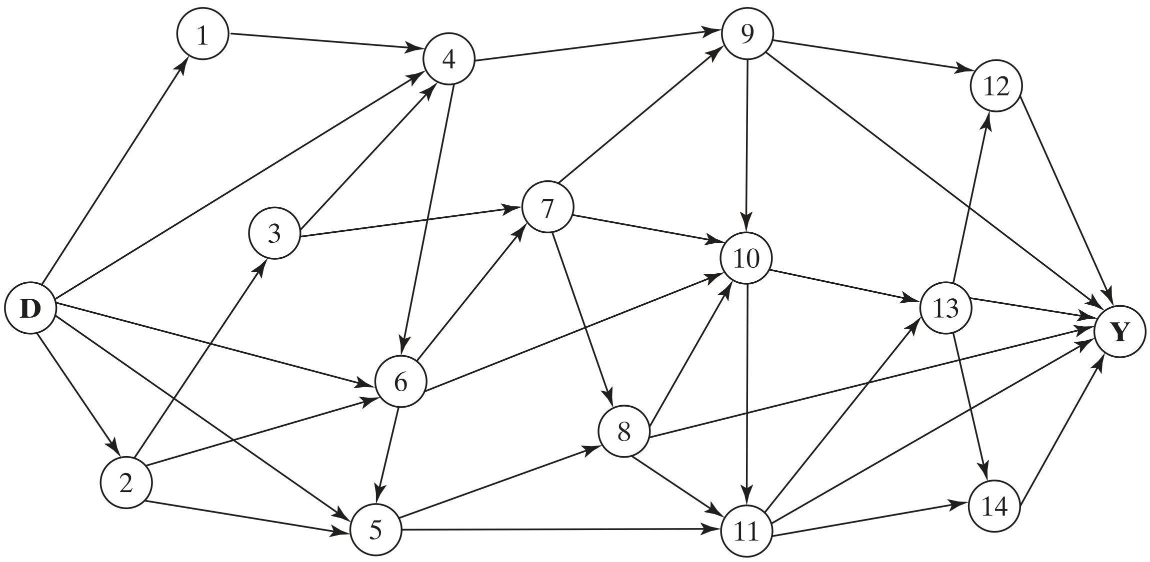


Figure 2: Network for the problem 3.

Problem 4 (15 points). Three jobs, J1, J2, and J3, are processed on 3 machines, M1, M2, and M3, according to the following sequences (processing times are shown in parentheses):

J1: M3(3) – M1(4) – M2(6)

J2: M2(1) – M3(5) – M1(9)

J3: M3(2) – M2(8) – M1(7)

The order in which the jobs are processed on the different machines is predetermined as:

M1: J1 – J2 – J3

M2: J2 – J3 – J1

M3: J3 – J1 – J2

Represent the problem as a CPM network for which the critical path determines the make span of all three jobs. Develop Gantt chart for scheduling of the jobs, assuming that each operation is scheduled at its earliest start time.

Grading: Mathematical model – 10 points; program – 5 points.

Problem 1:

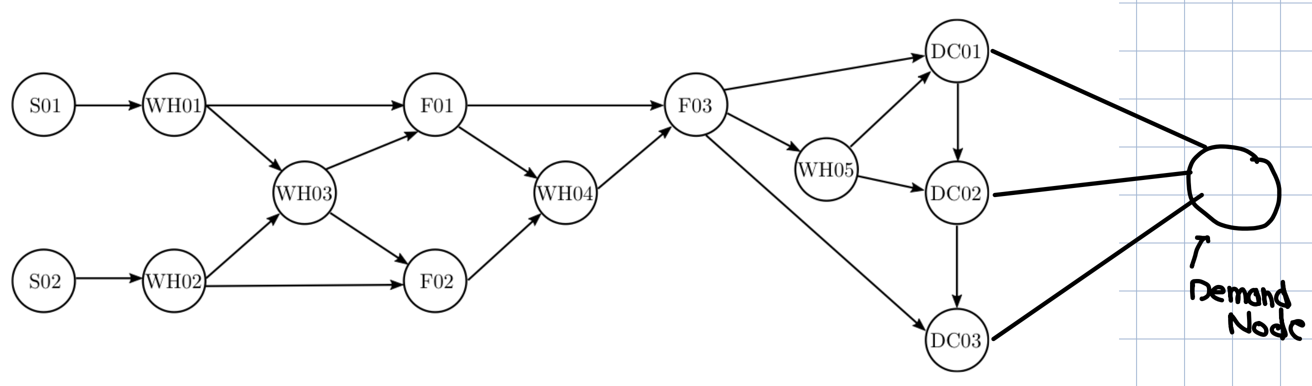


Figure 1: Production and distribution scheme for problem 1.

Decision Variables:

Part 1: • $X_{i,j,t}$ where $i = (u, v) \in A$

total flow from node u to node v
of substance j
at time t.

$A = \{$all edges in the graph (edges)$\}$
$j \in \{RM01, RM02, IG01, FG01\}$
$t = (\overline{1, 10})$

• $I_{i,j,t}$

amount of good i stored in location j at time t

$i \in \{RM01, RM02, IG01, FG01\} = P$
$j \in \{Nodes \setminus \{F01, F02\}\} = J$
$t = (\overline{1, 10}) = T$

Objective function:

minimize total cost = Inventory Cost + Transportation Cost

$$\text{Inventory Cost} := \sum_{j \in J} HC_j * \sum_{i \in P} \sum_{t \in T} I_{i,j,t}$$

where HC_j - holding cost in location j.

$$\text{Transportation Cost} := \sum_{i=(u,v)\in A} \sum_{t\in T} (TrC_i \times X_{i,j,t})$$

where TrC_i – transport cost from node $(u,v)=i$.

<u>constraints:</u>

i) conversion balances

$$\sum_{u\in\{WH01,\,WH02,\,WH03\}} (2\times X_{(u,F01),RM01,t} + X_{(u,F01),RM02,t}) = \sum_{v\in\{WH04\}} X_{(F01,v),IG01,t} \quad \forall t\in T$$

$$\sum_{u\in\{WH01,\,WH02,\,WH03\}} (X_{(u,F02),RM01,t} + 2X_{(u,F02),RM02,t}) = \sum_{v\in\{F03,\,WH04\}} X_{(F02,v),IG01,t} \quad \forall t\in T$$

$$\sum_{u\in\{WH01,\,WH02,\,WH03\}} X_{(u,F03),IG01,t} = \sum_{v\in\{DC01,\,WH05,\,DC03\}} X_{(F03,v),FG01,t} \quad \forall t\in T$$

ii) inventory balance

$$I_{p,w,t-1} + \sum_{i=(u,w)\in A} X_{i,p,t-l} = I_{p,w,t} + \sum_{i=(w,v)\in A} X_{i,p,t} \quad \forall p,t$$

where l is lead time.

iii) route capacity

$$\sum_{j\in P} X_{i,j,t} \le cap_i \quad \text{where } i=(u,v)\in A$$

cap_i – capacity of edge i.

iv) storage limit

$$\sum_{j\in P} I_{i,j,t} \le lim_i \quad \forall t$$

lim_i – storage limit of node i.

v) sanity conditions

$$X_{(u,v),p,t} \ge 0 \quad \forall (u,v),p,t$$

$$I_{p,j,t} \ge 0 \quad \forall p,j,t$$

vi) demand satisfaction

$$X_{d,\,demand\text{-}node,\,t} \geq demand_{d,t} \qquad \forall d, t$$

$d \in \{DC01, DC02, DC03\}$

* demand-node is an extraneous node

Problem 2 :

① Decision Variables

FC_{ij} - full time consultants working shift i ($i = \overline{1,2}$) and starting job on weekday j.

PC_{ij} - part-time consultants working shift i ($i = \overline{1,5}$) and starting job on weekday j.

② Objective: Minimize $1000 \times \sum_j \sum_i FC_{ij} + 600 \times \sum \sum PC_{ij}$

③ Constraints :

i) From 8AM to 10AM

$$\sum_{e=0}^{e=5} (FC_{1,j-e} + PC_{1,j-e}) \geq d_{1,j} \qquad \forall j$$

ii) 10AM - 12AM

$$\sum_{e=0}^{e=5} (FC_{1,j-e} + PC_{1,j-e} + PC_{2,j-e}) \geq d_{2,j} \qquad \forall j$$

iii) 12AM - 2PM

$$\sum_{e=0}^{e=5} (FC_{1,j-e} + FC_{2,j-e} + PC_{2,j-e} + PC_{3,j-e}) \geq d_{3,j} \quad \forall j$$

iv) 2PM - 4PM

$$\sum_{e=0}^{e=5} (FC_{1,j-e} + FC_{2,j-e} + PC_{3,j-e} + PC_{4,j-e}) \geq d_{4,j} \quad \forall j$$

v) 4PM - 6PM

$$\sum_{\ell=0}^{\ell=5} (FC_{1,j-\ell} + PC_{4,j-\ell} + PC_{5,j-\ell}) \geq d_{5,j} \quad \forall j$$

vi) 6PM - 8PM

$$\sum_{\ell=0}^{\ell=5} (FC_{1,j-\ell} + PC_{5,j-\ell}) \geq d_{6,j} \quad \forall j$$

* d_{ij} - demand in shift i, day j. $\quad i = \overline{(1,6)} \quad j = \overline{(1,7)}$
- where $FC_{t,j}$, $PC_{t,j}$ where $t \leq 0 \rightarrow t = t+7$
- common sense : $FC_{ij}, PC_{ij} \geq 0 \quad \forall i,j$

Problem 3:

- Decision variables :

x_{ij} - flow from node i to node j. $((i,j) \in A)$
A are arcs in the network.

- Objective
  - total outflow from source $\rightarrow$ max
  - $\sum_{j:(s,j)\in A} x_{sj} \rightarrow \max$, where s is the source node.

- Constraints
  - capacity $x_{ij} \leq c_{ij} \quad ((i,j) \in A)$
    where c_{ij} is the capacity of each arc.

  - conservation of flow
    - total inflow = total outflow
    - $\sum_{i:(i,k)\in A} x_{ik} = \sum_{j:(k,j)\in A} x_{kj} \quad (k \in \text{Nodes} \setminus \{s,t\})$
  - common sense : $x_{ij} \geq 0 \quad ((i,j) \in A)$

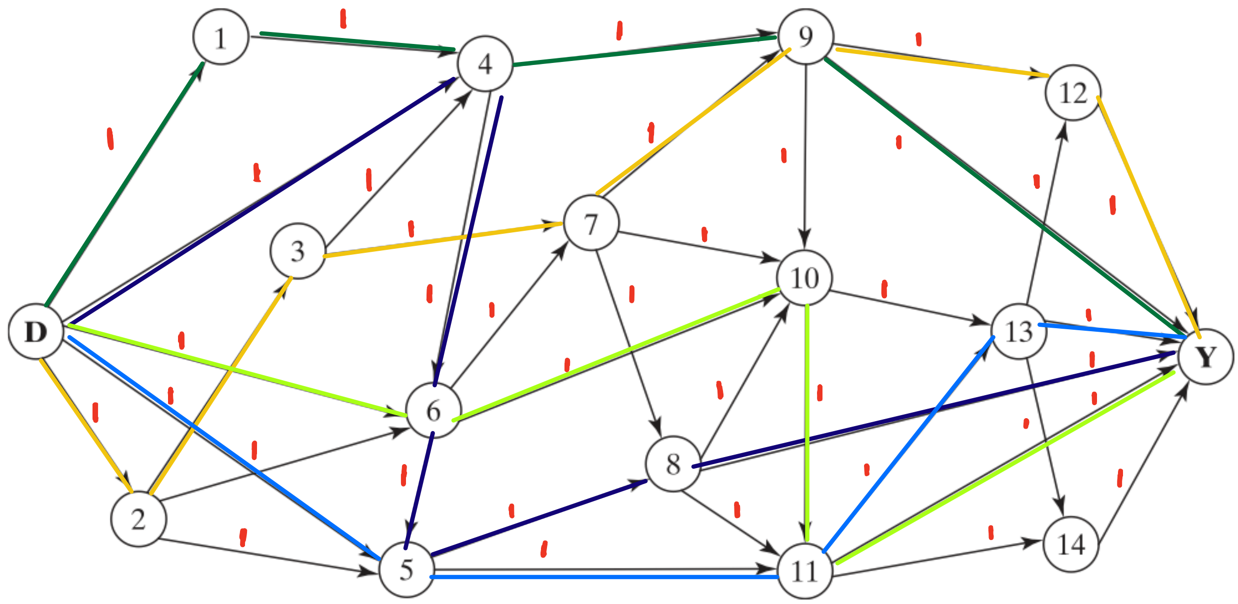


Figure 2: Network for the problem 3.

* Figure showing distinct paths identified by algorithm

Problem 4:

	Predecessor	Duration
M1J1	M3J1	4
M1J2	M3J2; M1J1	9
M1J3	M1J2; M2J3	7
M2J1	M1J1; M2J3	6
M2J2	—	1
M2J3	M3J3; M2J2	8
M3J1	— ; M3J3	3
M3J2	M2J2; M3J1	5
M3J3	—	2

GANT CHART