Operations Research 2024
Home assignment 2
Deadline: 11th March 2024, 22:00

Guidelines:

1. This home assignment covers integer programming therefore solutions must be based on this technique only.
2. Each solution must consist of two parts: mathematical model and program.
3. Mathematical model must contain clear definition of all the elements and the logics behind the objective and constraints. Any ambiguity will be used against you.
4. The correctness of any usages of artificial integer variables for extending modeling capabilities, that were not covered explicitly in the lectures, must be proven.
5. Programs must be either in AMPL, or in Python.
6. Programs in Python must only use the following packages: numpy, pandas, amplpy, collections.

Problem 1 (10 points) SmartMac manufactures four types of details on five machines, which are rented. Different machines can manufacture different types of details. The company may choose which machines to rent, taking into account different monthly rental costs as well as unit production costs. The machines also have capacity limits for the total number of details they can manufacture in a single month. All the data on the machines is presented in the table 1. The company's planned monthly production rates for the details are 200, 100, 50, and 100, respectively. Determine optimal renting and usage of the machine with MILP.

Table 1: The data for the problem 1.

Machine	Details produced	Rental cost ($)	Unit production cost ($)	Total production capacity
1	1, 2, 3	2000	17	250
2	1, 3, 4	2500	15	200
3	2, 4	1500	20	100
4	3, 4	1400	18	300
5	1, 4	1700	14	150

Grading: Mathematical model – 5 points; program – 5 points.

Problem 2 (30 points) United Carpets of Fanetton uses three machines to manufacture carpets in two designs – Aladdin tale and persian night. Each machine can manufacture both types of design, but each design requires special resettings of the machines which results in additional costs and decreased time available for production. Therefore, the company follows the policy, that such resetting, if required, occurs only at the beginning of the month, so that in each month each machine is used to manufacture only one design. The company is contracted to deliver a certain number of the carpets in the next 5 months, according to the data in table 2. The company can use inventory to store overproduced carpets at the cost of $5 per month per carpet. In the beginning of month 1 the company already has 20 Aladdin tale and 15 persian night carpets in the inventory.

I_0

Table 2: Demand data for problem 2.

Design	Required number of carpets				
	Month 1	Month 2	Month 3	Month 4	Month 5
Aladdin tale	30	20	20	20	30
Persian night	10	30	35	15	10

The three machines differ in resetting cost and duration, as well as unit production times and unit production costs for different designs according to data in tables 3 and 4

Table 3: Resetting cost and times data for problem 2.

	Machine 1	Machine 2	Machine 3
Aladdin tale $\to$ Persian night			
Resetting cost ($)	100	90	110
Resetting time (hours)	20	15	18
Persian night $\to$ Aladdin tale			
Resetting cost ($)	150	180	120
Resetting time (hours)	15	10	12

Table 4: Production times and costs data for problem 2.

	Machine 1	Machine 2	Machine 3
Unit production time (hours)			
Aladdin tale	10	12	8
Persian night	12	14	16
Unit production cost ($)			
Aladdin tale	90	80	120
Persian night	120	110	130

Develop a model for determining optimal production schedule for United Carpets of Fanetton, assuming that the company has 160 working hours for each machine in each month and in the beginning of month 1 all the machines are set to produce Aladdin tale design. Solve the model with AMPL.

Grading: Mathematical model – 15 points; program – 15 points.

Problem 3 (60 points) Côte d'Or extracts gold from ore. One of the stages is grinding the ore with ball mills. For this stage the company has 3 ball mills. Each ball mill can process at most 1000 tons of ore per month. To run the next stages of gold extraction smoothly, the mills must process between 2000 and 3000 tons of ore per month. Based on the forecasted prices of gold and gold content in ore, the production planning department forecasts the profit from a ton of ore, processed by the mills, for the next 6 months. These forecasts are presented in table 5.

$process_i \le 1000$

$\sum_{i\in I} process_i \le 3000$ $\sum_{i\in I} process_i \ge 2000$

Table 5: Profit forecasts for problem 3.

	Month 1	Month 2	Month 3	Month 4	Month 5	Month 6
Profit ($/ton)	100	120	110	140	90	100

The mills require maintenance. For simplification, assume that the decisions on whether to do a maintenance for the mills are made at the beginning of month. A decision to do maintenance for a mill leads to maintenance costs as well as decreased mill processing capacity in the month in which the maintenance happens. Both the maintenance costs and decrease in processing capacity depend on the accumulated amount of processed ore and are summarized in table 6. Notice, that the decrease in processing capacity is expressed as the percentage by which the full capacity is decreased, and each mill can process no more than 4000 tons without maintenance. Each month Côte d'Or can do maintenance for no more than one mill.

→ variable

→ шб три variables надо.

Table 6: Maintenance costs and capacity decrease data for problem 3.

Accumulated amount of processed ore	Maintenance cost ($)	Processing capacity decrease (%)
0 – 1500 tons	1000	20%
1500 – 2500 tons	2000	30%
2500 – 4000 tons	4000	50%

Develop MILP for finding optimal processing and maintenance schedule for the grinding stage assuming that at the beginning of month 1 the accumulated amounts of processed ore for the mills are 600, 400, 1000 tons.

Grading: Mathematical model – 40 points; program – 20 points.

$x \leq 1500 y_i \longrightarrow x - 1500 y_1 \leq 0$

if $x \leq 1500 \longrightarrow y_1 = 1$, $y_2 = 0$

if x

Problem 1:

Decision Variables:

x_{ip} – production at machine i of detail p $\qquad i = \overline{1,5} = I$
$\qquad p = \overline{1,4} = P$

y_i – binary variable indicating decision renting machine i

Objective Function:

minimize Total Cost

Total Cost = Total Rental Cost + Production Cost

$$\text{Total Rental Cost} = \sum_{i \in I} r_i * y_i$$

where r_i is rental cost at machine i

$$\text{Production Cost} = \sum_{p \in P} \sum_{i \in Y} x_{ip} * pr_i$$

where pr_i is production cost at machine i.

Subject to:

Demand Condition

- $\sum_{i \in I} x_{ip} * bin_{ip} = D_p \qquad \forall p$

where bin_{ip} is an indicator that machine i can produce detail p.

Capacity limit $\wedge$ binary variable activation

- $\sum_{p \in P} x_{ip} \leq C_i y_i \qquad \forall i$

where c_i is the capacity of machine i

Sanity Condition

- $x_{it} \geq 0 \quad \forall i \forall t$
- $y_i \in \{0, 1\} \quad \forall i$

Problem 2:

Decision V's:

I_{it} - inventory of good i at the beginning of month t — $i = \overline{1,2} = C$, $t = \overline{1,5} = T$

x_{ijt} - amount of good i produced by machine j in month t. — $i \in C$, $j = \overline{1,3} = N$, $t \in T$

S_{ijt} - state indicator $\{0,1\}$ shows which carpet type is produced by j'th machine in t'th time period — $i \in C$, $j \in N$, $t \in T$

$switch_{ijt}$ - indicator variable $\{0,1\}$ shows whether there's been a switch to i'th carpet type by j'th machine in beginning time t. — $i \in C$, $j \in N$, $t \in T$

Objective f'n:

min cost = prod.cost + reset.cost + inv.cost

$$\text{prod. cost} = \sum_{t \in T} \sum_{j \in M} \sum_{i \in C} x_{ijt} * pc_{ij}$$

where pc_{ij} is production cost of good i by machine j.

$$\text{reset. cost} = \sum_{t \in T} \sum_{j \in N} \left(switch[1,j,t] * rcpa[j] + switch[2,j,t] * rcap[j] \right)$$

$rcap_j$ is the reset cost of machine j to switch from Alladin to Persian.

$rcpa_j$ is the reset cost of machine j to switch from Persian to Alladin.

$$\text{inv. cost} = \sum_{t \in T} \sum_{i \in C} inv\_c \times I_{it}$$

where inv_c is the inventory cost

Subject to:

Inventory Balance:

- $$I_{i,t+1} = I_{i,t} + \sum_{j \in M} X_{ijt} - d_{it} \quad \forall\, i,t$$

where d_{it} is demand for carpet i in time t.

State update:

$$S_{1,j,t} = S_{1,j,t-1} + switch_{1,j,t} - switch_{2,j,t}$$
$$S_{2,j,t} = S_{2,j,t-1} - switch_{1,j,t} + switch_{2,j,t} \qquad \forall\, i,j,t$$

Switch balance:

$$switch[1,j,t] + switch[2,j,t] \leq 1 \quad \forall\, j,t$$

State Balance:

$$S_{1,j,t} + S_{2,j,t} = 1 \qquad \forall\, j,t$$

Production limit

$$x_{ijt} * pt_{ij} \leq 160 - rtpa_j * switch_{1,j,t} - rtap_j * switch_{2,j,t} \quad \forall\, j,t$$

$$x_{ijt} \leq 160\, S_{ijt}$$

pt_{ij} - production time of carpet i by machine j.
rtpa - reset time from P to A
rtap - reset time from A to P

Initial Conditions:

- $I[1,1] = 20$
- $I[2,1] = 10$
- $S[1,j,0] = 1 \quad \forall j$
- $S[2,j,0] = 0 \quad \forall j.$

Sanity conditions:

- $I_{it} \geq 0$
- $X_{it} \geq 0$
- $S_{ijt} \in \{0,1\}$
- $Switch_{ijt} \in \{0,1\}$

Problem 3:

Decision Variables:

I_{it} = accumulated amount of processed ore for mill i at the start of time t $\quad i = (\overline{1,3}) = M$ $\quad t = (\overline{1,6}) = T$

x_{it} = processed amount of ore by mill i at the end time t $\quad i = (\overline{1,3}) = M$ $\quad t \in T$

y_{itk} = decision to maintenance mill i in start time t using method k $\quad i \in M$ $\quad t \in T$

$$k = \begin{cases} 1 & 0 \leq I_{it} \leq 1499 \\ 2 & 1500 \leq I_{it} \leq 2499 \\ 3 & 2499 \leq I_{it} \leq 4000 \end{cases} \quad K$$

Objective:

max profit = TR − TC

$$TR = \sum_{t \in T} \sum_{i \in N} x_{it} * fp_t$$

where fp_t is forecasted price in month t.

$$TC = \sum_{t \in T} \sum_{i \in M} \sum_{k \in K} y_{itk} * mc_i$$

where mc_i are the respective maintenance costs.

Subject to

One mill maintenance:

- $\sum_{i \in M} y_{itk} \leq 1 \qquad \forall t, k$

One maintenance decision:

- $\sum_{k \in K} y_{itk} \leq 1 \qquad \forall i, t$

Maintenance activation

- $y_{it1} \le 1500 - I_{it}$
- $y_{it2} \le 2500 - I_{it}$
- $y_{it3} \le 4000 - I_{it}$

$\forall\, i,t$

If $x_{it} > k^{th}$ upper bound $\rightarrow$ y_{itk} blocked

Mill capacity balance:

- $I_{i,t+1} \le 4000\left(1 - \sum_{k \in K} y_{itk}\right)$ $\begin{cases} I_{it} = 0 & \text{if } \sum y_{it} = 1 \\ \text{True} & \text{if } \sum y_{itk} = 0 \end{cases}$ $\forall i,t$

- $I_{i,t+1} \ge I_{i,t} + x_{it} - 10000\left(\sum_{k \in K} y_{itk}\right)$
- $I_{i,t+1} \le I_{i,t} + x_{it} + 10000\left(\sum_{k \in K} y_{itk}\right)$

$\forall i,t$ $\begin{cases} \text{If } \sum_{k \in K} y_{itk} = 1 \text{ then conditions are true} \\ \text{If } \sum_{k \in K} y_{it} = 0 \text{ then } I_{i,t+1} = I_{i,t} + x_{it} \end{cases}$

These ensure that $I_{it} = \begin{cases} 0 & \text{if } \sum_{k \in K} y_{i,t-1,k} = 1 \\ I_{i,t-1} + x_{i,t-1} & \text{if } \sum_{k \in K} y_{i,t-1,k} = 0 \end{cases}$

Upper ∧ lower production balance:

- $\sum_{i \in N} x_{it} \ge 2000 \quad \forall t$
- $\sum_{i \in N} x_{it} \le 3000 \quad \forall t$

Production capacity:

- $x_{it} \le 1000 - \sum_{k=1}^{3} pcd_k \times y_{itk} \quad \forall i,t$

pcd_k – processing capacity decrease of decision k.

Initial Conditions

- $I_{11} = 600$
- $I_{21} = 400$
- $I_{31} = 1000$

Sanity conditions

- $I_{it} \ge 0$
- $x_{it} \ge 0$ $\longrightarrow$ Норм решилось.
- y_{itk} – binary