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ECE 203 Assessment 1 Formula Sheet

Complex Numbers and Euler Relationships

Euler representation: $e^{j\theta} = \cos\theta + j\sin\theta \Rightarrow \cos\theta = \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right), \sin\theta = \frac{1}{2j}\left(e^{j\theta} - e^{-j\theta}\right)$

$\cos\theta = \mathrm{Re}\left\{e^{j\theta}\right\}, \sin\theta = \mathrm{Im}\left\{e^{j\theta}\right\}$

$z = a + jb = re^{j\theta} \Rightarrow a = r\cos\theta,\ b = r\sin\theta.$ complex conjugate: $z^* = a - jb = re^{-j\theta}$

magnitude: $|z| = r = \sqrt{a^2 + b^2}$, phase: $\theta = \tan^{-1}(b/a)$

$e^{j\theta}$: points on the uni-circle in the complex plane $\Rightarrow j = e^{j\pi/2}, \quad -1 = j^2 = e^{j\pi} = e^{-j\pi}, \quad -j = e^{j3\pi/2} = e^{-j\pi/2}.$

Sinusoidal Signals

$x(t) = A\cos(2\pi f_0 t + \phi) = A\cos(\omega_0 t + \phi) = A\cos(\omega_0(t - t_0))$

frequency: f_0 Hz, period $T = 1/f_0$, angular frequency: $\omega_0 = 2\pi f_0$ radians

Phasor Representation

$x(t) = \sum_{k=1}^{N} A_k \cos(\omega_0 t + \phi_k) = A\cos(\omega_0 t + \phi) \Rightarrow Ae^{j\phi} = \sum_{k=1}^{N} A_k e^{j\phi_k}$

Amplitude Modulation

$x(t) = v(t)\cos(2\pi f_c t) \Rightarrow X(f) = \frac{1}{2}V(f - f_c) + \frac{1}{2}V(f + f_c)$ where $X(f), V(f)$ are spectrum of $x(t), v(t)$

If $v(t) = 2A\cos(2\pi f_a t) \Rightarrow x(t) = A\cos\left(2\pi(f_c - f_a)t\right) + A\cos\left(2\pi(f_c + f_a)t\right)$

Let the duration be T seconds, and the frequency separation be $B = 2f_a$ Hz. Detection of two distinct sinusoids requires $T \cdot B > 1$.

Frequency Modulation

$x(t) = A\cos(\psi(t)) = \mathrm{Re}\left\{Ae^{j\psi(t)}\right\} \Rightarrow$ Instantaneous frequency $f_i(t) = \frac{1}{2\pi} \cdot \frac{d}{dt}\psi(t)$

Chirp, or Linear Swept Frequency: $\psi(t) = 2\pi\mu t^2 + 2\pi f_0 t + \phi$

Trigonometry Identities

$\cos(-\theta) = \cos(\theta), \quad \sin(-\theta) = -\sin(\theta), \quad \cos^2(\theta) + \sin^2(\theta) = 1$

$\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta), \quad \cos(2\theta) = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1$

$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta), \quad \sin(2\theta) = 2\sin\theta\cos\theta$

$\cos(\alpha)\cos(\beta) = \frac{1}{2}\left(\cos(\alpha + \beta) + \cos(\alpha - \beta)\right), \quad \sin(\alpha)\sin(\beta) = \frac{-1}{2}\left(\cos(\alpha + \beta) - \cos(\alpha - \beta)\right)$

$\sin(\alpha)\cos(\beta) = \frac{1}{2}\left(\sin(\alpha + \beta) + \sin(\alpha - \beta)\right), \quad \cos(\alpha)\sin(\beta) = \frac{1}{2}\left(\sin(\alpha + \beta) - \sin(\alpha - \beta)\right)$

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