1 Scheme

1.1 Setup$(l) \to (\boldsymbol{mpk}, \boldsymbol{msk})$

generate $g \in \mathbb{G}_1$ randomly
generate $\alpha, b_1, b_2 \in \mathbb{Z}_p^*$ randomly
generate $s_1, s_2, \cdots, s_l, a_1, a_2, \cdots, a_l \in \mathbb{Z}_p^*$ randomly
generate $g_2, g_3 \in \mathbb{G}_2$ randomly
generate $h_1, h_2, \cdots, h_l \in \mathbb{G}_2$ randomly (Note that the indexes in implementations are 1 smaller than those in theory)
$H_1 : \mathbb{Z}_p^* \to \mathbb{G}_1$
$H_2 : \mathbb{Z}_p^* \to \mathbb{G}_2$
$\hat{H} : \{0,1\}^* \to \{0,1\}^\lambda$
$g_1 \leftarrow g^\alpha$
$A \leftarrow e(g_1, g_2)$
$\bar{g} \leftarrow g^{b_1}$
$\tilde{g} \leftarrow g^{b_2}$
$\bar{g}_3 \leftarrow g_3^{\frac{1}{b_1}}$
$\tilde{g}_3 \leftarrow g_3^{\frac{1}{b_2}}$
$mpk \leftarrow (g, g_1, g_2, g_3, \bar{g}, \tilde{g}, \bar{g}_3, \tilde{g}_3, h_1, h_2, \cdots, h_l, H_1, H_2, \hat{H}, A)$
$msk \leftarrow (g_2^\alpha, b_1, b_2, s_1, s_2, \cdots, s_l, a_1, a_2, \cdots, a_l)$
return (mpk, msk)

1.2 EKGen$(\boldsymbol{ID}_k) \to \boldsymbol{ek_{ID_k}}$

$A_k = \prod\limits_{j=1}^{k} a_j$
$ek_{1,i} \leftarrow H_1(I_i)^{s_i A_k}, \forall i \in \{1, 2, \cdots, k\}$
$ek_{2,i} \leftarrow s_{k+i} A_k, \forall i \in \{1, 2, \cdots, l-k\}$
$ek_3 \leftarrow (a_{k+1}, a_{k+2}, \cdots, a_l)$
$ek_{ID_k} \leftarrow (ek_1, ek_2, ek_3)$
return ek_{ID_k}

1.3 DerivedEKGen$(\boldsymbol{ek_{ID_{k-1}}}, \boldsymbol{ID}_k) \to \boldsymbol{ek_{ID_k}}$

$ek'_{1,i} \leftarrow ek_{1,i}^{a_k}, \forall i \in \{1, 2, \cdots, k-1\}$
$ek'_{1,k} \leftarrow H_1(I_k)^{ek_{2,1}}$
$ek'_1 \leftarrow ek'_1 || \langle ek'_{1,k} \rangle$
$ek'_{2,i} \leftarrow ek_{2,i} \cdot a_k, \forall i \in \{2, 3, \cdots, l-k+1\}$
$ek'_3 \leftarrow (a_{k+1}, a_{k+2}, \cdots, a_l)$
$ek_{ID_k} \leftarrow (ek'_1, ek'_2, ek'_3)$
return ek_{ID_k}

1.4 DKGen$(\boldsymbol{ID}_k) \to \boldsymbol{dk_{ID_k}}$

generate $r \in \mathbb{Z}_p^*$ randomly
$HI \leftarrow h_1^{I_1} h_2^{I_2} \cdots h_k^{I_k}$
$a_0 \leftarrow g_2^{\frac{\alpha}{b_1}} \cdot HI^{\frac{r}{b_1}} \cdot \bar{g}_3^r$

$a_1 \leftarrow g_2^{\frac{\alpha}{b_2}} \cdot HI^{\frac{r}{b_2}} \cdot \tilde{g}_3^r$

$A_k \leftarrow \prod_{j=1}^{k} a_j$

$dk_1 \leftarrow (a_0, a_1, g^r, h_{k+1}^{\frac{r}{b_1}}, h_{k+2}^{\frac{r}{b_1}}, \cdots, h_l^{\frac{r}{b_1}}, h_{k+1}^{\frac{r}{b_2}}, h_{k+2}^{\frac{r}{b_2}}, \cdots, h_l^{\frac{r}{b_2}}, h_{k+1}^{b_1^{-1}}, h_{k+2}^{b_1^{-1}}, \cdots, h_l^{b_1^{-1}}, h_{k+1}^{b_2^{-1}}, h_{k+2}^{b_2^{-1}}, \cdots, h_2^{b_1^{-1}}, HI^{\frac{1}{b_1}}, I$

$dk_{2,i} \leftarrow H_2(I_i)^{s_i A_k}, \forall i \in \{1, 2, \cdots, k\}$

$dk_{3,i} \leftarrow s_{k+i} A_k, \forall i \in \{1, 2, \cdots, l-k\}$

$dk_4 \leftarrow (a_{k+1}, a_{k+2}, \cdots, a_l)$

$dk_{ID_k} \leftarrow (dk_1, dk_2, dk_3, dk_4)$

return dk_{ID_k}

1.5 DerivedDKGen($dk_{ID_{k-1}}, ID_k$) → dk_{ID_k}

generate $t \in \mathbb{Z}_p^*$ randomly

$a_0' \leftarrow a_0 \cdot c_{0,k}^{I_k} \cdot (f_0 \cdot d_{0,k}^{I_k} \cdot \bar{g}_3)^t$

$a_1' \leftarrow a_1 \cdot c_{1,k}^{I_k} \cdot (f_1 \cdot d_{1,k}^{I_k} \cdot \tilde{g}_3)^t$

$dk_1' \leftarrow (a_0', a_1', b \cdot g^t, c_{0,k+1} \cdot d_{0,k+1}^t, c_{0,k+2} \cdot d_{0,k+2}^t, \cdots, c_{0,l} \cdot d_{0,l}^t, c_{1,k+1} \cdot d_{1,k+1}^t, c_{1,k+2} \cdot$

$d_{1,k+2}^t, \cdots, c_{1,l} \cdot d_{1,l}^t, d_{0,k+1}, d_{0,k+2}, \cdots, d_{0,l}, d_{1,k+1}, d_{1,k+2}, \cdots, d_{1,l}, f_0 \cdot c_{0,k}^{I_k}, f_1 \cdot$

$c_{1,k}^{I_k})$

$dk_{2,i}' \leftarrow dk_{2,i}^{a_k}, \forall i \in \{1, 2, \cdots, k-1\}$

$dk_{2,k}' \leftarrow H_2(I_k)^{dk_{3,1}}$

$dk_2' \leftarrow dk_2' || \langle dk_{2,k}' \rangle$

$dk_{3,i}' \leftarrow dk_{3,i} \cdot a_k, \forall i\{2, 3, \cdots, l-k+1\}$

$dk_4' \leftarrow (a_{k+1}, a_{k+2}, \cdots, a_l)$

$dk_{ID_k} \leftarrow (dk_1', dk_2', dk_3', dk_4')$

return dk_{ID_k}

1.6 Enc(ek_{ID_S}, ID_{Rev}, M) → CT

generate $s_1, s_2, \eta \in \mathbb{Z}_p^*$ randomly

$T \leftarrow A^{s_1+s_2}$

If $m = n$:

$K \leftarrow \prod_{i=1}^{n} e(g^\eta \cdot ek_{1,i}, H_2(I_i'))$

If $m > n$:

$A_n \leftarrow \prod_{i=1}^{n} a_i$

$B_n^m \leftarrow \prod_{i=n+1}^{m} a_i$

$K \leftarrow (\prod_{i=1}^{n} e(ek_{1,i}, H_2(I_i')) \cdot \prod_{i=n+1}^{m} e(H_1(I_n), H_2(I_i'))^{\alpha_i A_n})^{B_n^m} \cdot e(g^\eta, \prod_{i=1}^{m} H_2(I_i'))$

If $m < n$

$K \leftarrow \prod_{i=1}^{m} e(ek_{1,i}, H_2(I_i')) \prod_{i=m+1}^{n} e(ek_{1,i}, H_2(I_m')) e(g^\eta, \prod_{i=1}^{m} H_2(I_i'))$

$C_1 \leftarrow M \oplus \hat{H}(T) \oplus \hat{H}(K)$

$C_2 \leftarrow \bar{g}^{s_1}$

$C_3 \leftarrow \tilde{g}^{s_2}$

$C_4 \leftarrow (h_1^{I_1} h_2^{I_2} \cdots h_n^{I_n} \cdot g_3)^{s_1+s_2}$

$C_5 \leftarrow g^\eta$

$CT \leftarrow (C_1, C_2, C_3, C_4, C_5)$
return CT

1.7 $\mathbf{Dec}(\boldsymbol{dk_{ID_R}}, \boldsymbol{ID_{Rev}}, \boldsymbol{ID_{Snd}}, \boldsymbol{CT}) \rightarrow M$

$T' = \dfrac{e(C_2, dk_{1,1})e(C_3, dk_{1,2})}{e(dk_{1,3}, C_4)}$

If $m = n$:

$K' \leftarrow \prod\limits_{i=1}^{n} e(H_1(I_i), dk_{2,i}) \cdot e(C_5, \prod\limits_{i=1}^{n} H_2(I_i'))$

If $m > n$:

$K' \leftarrow \prod\limits_{i=1}^{n} e(H_1(I_i), dk_{2,i}) \cdot \prod\limits_{i=n+1}^{m} e(H_1(I_n), dk_{2,i}) \cdot e(C_5, \prod\limits_{i=1}^{m} H_2(I_i'))$

If $m < n$

$A_m \leftarrow \prod\limits_{i=1}^{m} a_i$

$B_n^m \leftarrow \prod\limits_{i=m+1}^{n} a_i$

$K' \leftarrow (\prod\limits_{i=1}^{m} e(H_1(I_i), dk_{2,i}) \cdot \prod\limits_{i=m+1}^{n} e(H_1(I_i), H_2(I_m'))^{\alpha_i A_m})^{B_m^n} \cdot e(C_5, \prod\limits_{i=1}^{m} H_2(I_i'))$

$M \leftarrow C_1 \oplus \hat{H}(T') \oplus \hat{H}(K')$

return M