

1 Protocol

1.1 $\text{Setup}(l) \rightarrow (mpk, msk)$

generate $g \in \mathbb{G}_1$ randomly
generate $\alpha, b_1, b_2 \in \mathbb{Z}_p^*$ randomly
generate $g_2, g_3 \in \mathbb{G}_2$ randomly
generate $h_1, h_2, \dots, h_l \in \mathbb{G}_2$ randomly (Note that the indexes in implementations are 1 smaller than those in theory)
 $g_1 \leftarrow g^\alpha$
 $\bar{g} \leftarrow g^{b_1}$
 $\tilde{g} \leftarrow g^{b_2}$
 $\bar{g}_3 \leftarrow g_3^{\frac{1}{b_1}}$
 $\tilde{g}_3 \leftarrow g_3^{\frac{1}{b_2}}$
 $mpk \leftarrow (g, g_1, g_2, g_3, \bar{g}, \tilde{g}, \bar{g}_3, \tilde{g}_3, h_1, h_2, \dots, h_l)$
 $msk \leftarrow (g_2^\alpha, b_1, b_2)$

1.2 $\text{KGen}(ID_k) \rightarrow sk_{ID_k}$

generate $r \in \mathbb{Z}_p^*$ randomly
 $HI \leftarrow h_1^{I_1} h_2^{I_2} \dots h_k^{I_k}$
 $sk_{ID_k} \leftarrow (g_2^{\frac{\alpha}{b_1}} \cdot HI^{\frac{r}{b_1}} \cdot \bar{g}_3^r, g_2^{\frac{\alpha}{b_2}} \cdot HI^{\frac{r}{b_2}} \cdot \tilde{g}_3^r, g^r, h_{k+1}^{\frac{r}{b_1}}, h_{k+2}^{\frac{r}{b_1}}, \dots, h_l^{\frac{r}{b_1}}, h_{k+1}^{\frac{r}{b_2}}, h_{k+2}^{\frac{r}{b_2}}, \dots, h_l^{\frac{r}{b_2}}, h_{k+1}^{\frac{r}{b_1^{-1}}}, h_{k+2}^{\frac{r}{b_1^{-1}}}, \dots, h_l^{\frac{r}{b_1^{-1}}}, h_{k+1}^{\frac{r}{b_2^{-1}}}, h_{k+2}^{\frac{r}{b_2^{-1}}}, \dots, h_l^{\frac{r}{b_2^{-1}}})$

1.3 $\text{DerivedKGen}(sk_{ID_{k-1}}, ID_k) \rightarrow sk_{ID_k}$

generate $t \in \mathbb{Z}_p^*$ randomly
 $sk_{ID_k} \leftarrow (a_0 \cdot c_{0,k}^{I_k} \cdot (f_0 \cdot d_{0,k}^{I_k} \cdot \bar{g}_3)^t, a_1 \cdot c_{1,k}^{I_k} \cdot (f_1 \cdot d_{1,k}^{I_k} \cdot \tilde{g}_3)^t, b \cdot g^t, c_{0,k+1} \cdot d_{0,k+1}^t, c_{0,k+2} \cdot d_{0,k+2}^t, \dots, c_{0,l} \cdot d_{0,l}^t, c_{1,k+1} \cdot d_{1,k+1}^t, c_{1,k+2} \cdot d_{1,k+2}^t, \dots, c_{1,l} \cdot d_{1,l}^t, d_{0,k+1}, d_{0,k+2}, \dots, d_{0,l}, d_{1,k+1}, d_{1,k+2}, \dots, d_{1,l}, f_0 \cdot c_{0,k}^{I_k}, f_1 \cdot c_{1,k}^{I_k})$

1.4 $\text{Enc}(ID_k, M) \rightarrow CT$

generate $s_1, s_2 \in \mathbb{Z}_p^*$ randomly
 $CT \leftarrow (e(g_1, g_2)^{s_1+s_2} \cdot M, \bar{g}^{s_1}, \tilde{g}^{s_2}, (h_1^{I_1} h_2^{I_2} \dots h_k^{I_k} \cdot g_3)^{s_1+s_2})$

1.5 $\text{Dec}(CT, sk_{ID_k}) \rightarrow M$

$$M \leftarrow \frac{e(b, D) \cdot A}{e(B, a_0) \cdot e(C, a_1)}$$