1 SchemeIBBME

This scheme is applicable to symmetric and asymmetric groups of prime orders.

1.1 Setup() \rightarrow (mpk, msk)

```
generate g, v \in \mathbb{G}_1 randomly generate h \in \mathbb{G}_2 randomly generate \vec{r}_1 = (r_{1,0}, r_{1,1}, \cdots, r_1, l) \in \mathbb{Z}_r^{l+1} randomly generate \vec{r}_2 = (r_{2,0}, r_{2,1}, \cdots, r_2, l) \in \mathbb{Z}_r^{l+1} randomly generate t_1, t_2, \beta_1, \beta_2, \alpha, \rho, b, \tau \in \mathbb{Z}_r randomly \vec{r} = (r_0, r_1, \cdots, r_l) \leftarrow \vec{r}_1 + b\vec{r}_2 = (r_{1,0} + br_{2,0}, r_{1,1} + br_{2,1}, \cdots, r_{1,l} + br_{2,l}) t \leftarrow t_1 + bt_2 \beta \leftarrow \beta_1 + b\beta_2 \vec{R} \leftarrow g^{\vec{r}} = (g^{r_0}, g^{r_1}, \cdots, g^{r_l}) T \leftarrow g^t H_0: \{0,1\}^* \to \mathbb{G}_1 H_2: \{0,1\}^* \to \mathbb{G}_1 H_2: \{0,1\}^* \to \mathbb{Z}_r H_3: \mathbb{G}_T \to \mathbb{Z}_r mpk \leftarrow (v, v^{\rho}, g, g^b, \vec{R}, T, e(g, h)^{\beta}, h, h^{\vec{r}_1}, h^{\vec{r}_2}, h^{t_1}, h^{t_2}, g^{\tau\beta}, h^{\tau\beta_1}, h^{\tau\beta_2}, h^{1/\tau}, H_0, H_1, H_2, H_3) msk \leftarrow (h^{\beta_1}, h^{\beta_2}, \alpha, \rho) return (mpk, msk)
```

$1.2 \quad ext{EKGen}(id^*) ightarrow ek_{id^*}$

$$ek_{id^*} \leftarrow H_1(id^*)^{\alpha}$$

return ek_{id^*}

$1.3 \quad \mathrm{DKGen}(\mathit{id}) o \mathit{dk}_{\mathit{id}}$

```
generate z \in \mathbb{Z}_r randomly

generate rtags = (rtag_1, rtag_2, \cdots, rtag_l) \in \mathbb{Z}_r^l randomly

dk_1 \leftarrow H_0(id)^{\rho}

dk_2 \leftarrow H_0(id)^{\alpha}

dk_3 \leftarrow H_0(id)

dk_4 \leftarrow h^{\beta_1}(h^{t_1})^z

dk_5 \leftarrow h^{\beta_2}(h^{t_2})^z

dk_6 \leftarrow h^z

dk_{7,j} \leftarrow ((h^{t_1})^{rtag_j}h^{r_{1,j}}/(h^{r_{1,0}})^{H_2(id)^j})^z, \forall j \in \{1, 2, \cdots, l\}

dk_{8,j} \leftarrow (dk_1, dk_2, \cdots, dk_8, rtags)

return dk_{id}
```

1.4 $\operatorname{Enc}(S, ek_{id^*}, m) \to ct$

Compute
$$y_0, y_1, y_2, \dots y_n$$
 that satisfy $\forall x \in \mathbb{Z}_r$, we have $F(x) = \prod_{i d_j \in S} (x - H_2(id_j)) = y_0 + \sum_{i=1}^n y_i x^i$

$$\vec{y} \leftarrow (y_0, y_1, \cdots, y_n, y_{n+1}, y_{n+2}, \cdots, y_l) = (y_0, y_1, \cdots, y_n, 0, 0, \cdots, 0)$$
generate $s, d_2, ctag \in \mathbb{Z}_r$ randomly
$$C_0 \leftarrow m \cdot e(g, h)^{\beta s}$$

$$C_1 \leftarrow g^s$$

$$C_2 \leftarrow g^{bs}$$

$$C_3 \leftarrow \left(T^{ctag} \prod_{i=0}^n (g^{r_i})^{y_i}\right)^{d_2 s}$$

$$C_4 \leftarrow v^s$$

$$V_{id_i} \leftarrow H_3(e(H_0(id_i), ek_{id^*} \cdot g^{bs} \cdot v^{\rho s})), \forall id_i \in S$$
Compute $\vec{b} \leftarrow (b_0, b_1, b_2, \cdots b_n)$ that satisfy $\forall y \in \mathbb{Z}_r$, we have $g(y) = \prod_{V_{id_k} \in V_{id}} (y - V_{id_k}) + d_2 = b_0 + \sum_{k=1}^n b_k y^k$

$$ct \leftarrow (C_0, C_1, C_2, C_3, C_4, ctag, \vec{y}, \vec{b}) \mathbf{return} ct$$

$$\mathbf{1.5} \quad \mathbf{Dec}(S, \mathbf{dk_{id_i}}, \mathbf{id}^*, \mathbf{ct}) \rightarrow m$$

$$\begin{split} &V(id_i) \leftarrow H_3(e(dk_{i,3},C_2)e(dk_{i,2},H_1(id^*))e(dk_{i,1},C_4)) \\ &\mathbf{d}_2 \leftarrow g(V_{id_i}) = b_0 + \sum_{j=1}^n b_j V_{id_i}^j \\ &rtag \leftarrow \sum_{i=1}^l y_i rtags_i \\ &\mathbf{if} \ rtag = ctag \ \mathbf{then} \\ &m \leftarrow \bot \\ &A \leftarrow e(C_1, \prod_{j=1}^l dk_{7,j}^{y_j})e(C_2, \prod_{j=1}^l dk_{8,j}^{y_j})/e(C_3^{1/d_2}, dk_6) \quad \mathbf{B} \leftarrow e(C_1, dk_4) \cdot e(C_2, dk_5) \\ &m \leftarrow C_0 \cdot A^{1/(rtag-ctag)} \cdot B^{-1} \\ &\mathbf{end} \ \mathbf{if} \end{split}$$

 $\mathbf{return}\ m$