1 SchemePBAC

This scheme is only applicable to symmetric groups of prime orders.

1.1 Setup() \rightarrow (mpk, msk)

```
q \leftarrow \|\mathbb{G}\|
g \leftarrow 1_{\mathbb{G}_1}
generate s, \alpha \in \mathbb{Z}_r randomly
H_1 : \{0,1\}^* \to \mathbb{G}_1
H_2 : \{0,1\}^* \to \mathbb{G}_1
H_3 : \mathbb{G}_T^2 \times \{0,1\}^{\lambda} \to \mathbb{Z}_r
H_4 : \{0,1\}^* \to \{0,1\}^{\lambda}
H_5 : \{0,1\}^* \to \mathbb{G}_1
H_6 : \{0,1\}^* \to \mathbb{G}_1
\hat{g} \leftarrow g^s
mpk \leftarrow (g, \hat{g}, H_1, H_2, H_3, H_4, H_5, H_6)
msk \leftarrow (s, \alpha)
\mathbf{return} \ (mpk, msk)
```

$1.2 \quad ext{SKGen}(id_S) ightarrow ek_{id_S}$

```
ek_{id_S} \leftarrow H_1(id_S)^{\alpha}

return ek_{id_S}
```

1.3 $\mathrm{RKGen}(id_R) o dk_{id_R}$

```
\begin{aligned} dk_{id_R,1} &\leftarrow H_2(id_R)^{\alpha} \\ dk_{id_R,2} &\leftarrow H_2(id_R)^s \\ dk_{id_R} &\leftarrow (dk_{id_R,1}, dk_{id_R,2}) \\ \mathbf{return} \ dk_{id_R} \end{aligned}
```

1.4 $\operatorname{Enc}(\boldsymbol{ek_{id_1}}, \boldsymbol{id_2}, m) \to C$

```
generate \eta_1, \eta_2 \in \mathbb{G}_T randomly r \leftarrow H_3(\eta_1, \eta_2, m)
C_1 \leftarrow g^r
C_2 \leftarrow \eta_1 \cdot e(\hat{g}, H_2(id_2)^r)
C_3 \leftarrow \eta_2 \cdot e(ek_{id_1}, H_2(id_2))
C_4 \leftarrow m \oplus H_4(\eta_1) \oplus H_4(\eta_2)
S \leftarrow H_5(id_2||C_1||C_2||C_3||C_4)^r
C \leftarrow (C_1, C_2, C_3, C_4, S)
return C
```

1.5 $\mathsf{PKGen}(ek_{id_2},dk_{id_2},id_1,id_2,id_3) o rk$

```
generate N_1 \in \{0, 1\}^{\lambda} randomly
generate N_2 \in \{0, 1\}^{\lambda} randomly
K_1 \leftarrow e(dk_{id_2, 2}, H_2(id_3))
K_2 \leftarrow e(ek_{id_2}, H_2(id_3))
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```
rk_1 \leftarrow (N_1, H_6(K_1||id_2||id_3||N_1) \cdot dk_{id_2,2})
rk_2 \leftarrow (N_2, H_6(K_2||id_2||id_3||N_2) \cdot dk_{id_2,1})
rk \leftarrow (id_1, id_2, rk_1, rk_2)
return rk
```

1.6 $\operatorname{ProxyEnc}(ct, rk) \rightarrow CT$

```
\begin{split} h &\leftarrow H_5(id_2||C_1||C_2||C_3||C_4) \\ \text{if } e(h,C_1) &= e(g,S) \text{then} \\ \text{generate } t &\in \mathbb{Z}_r \text{ randomly} \\ C_2' &\leftarrow C_2 / \frac{e(C_1,rk_{1,2} \cdot h^t)}{e(g^t,S)} \\ C_3' &\leftarrow C_3 / e(H_1(id_1),rk_{2,2}) \\ CT &\leftarrow (id_1,C_1,C_2',C_3',C_4,rk_{1,1},rk_{2,1}) \\ \text{else} \\ CT &\leftarrow \bot \\ \text{end if} \\ \text{return } CT \end{split}
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1.7 $\operatorname{Dec}_1(\operatorname{dk}_{id_2}, \operatorname{id}_2, \operatorname{id}_1, \operatorname{ct}) \to m$

```
h \leftarrow H_5(id_2||C_1||C_2||C_3||C_4) generate t \in \mathbb{Z}_r randomly \eta_1 \leftarrow C_2 / \frac{e(C_1, dk_{id_2, 2} \cdot h^t)}{e(g^t, S)} \eta_2 \leftarrow C_3 / e(dk_{id_2, 1}, H_1(id_1)) m \leftarrow C_4 \oplus H_4(\eta_1) \oplus H_4(\eta_2) r \leftarrow H_3(\eta_1, \eta_2, m) if S \neq h^r \lor C_1 \neq g^r then m \leftarrow \bot end if \mathbf{return} \ m
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1.8 $\operatorname{Dec}_2(\operatorname{\textit{dk}}_{\operatorname{\textit{id}}_3},\operatorname{\textit{id}}_3,\operatorname{\textit{id}}_2,\operatorname{\textit{CT}}) \to m'$

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\begin{split} K_1' &\leftarrow e(dk_{id_3,2}, H_2(id_2)) \\ K_2' &\leftarrow e(dk_{id_3,1}, H_1(id_2)) \\ \eta_1' &\leftarrow C_2' \cdot e(C_1, H_6(K_1'||id_2||id_3||N_1)) \\ \eta_2' &\leftarrow C_3' \cdot e(H_6(K_2'||id_2||id_3||N_2), H_1(id_1)) \\ m' &\leftarrow C_4 \oplus H_4(\eta_1') \oplus H_4(\eta_2') \\ r' &\leftarrow H_3(\eta_1', \eta_2', m') \\ \text{if } C_1 &\neq g^{r'} \text{ then } \\ m' &\leftarrow \bot \\ \text{end if } \end{split}
```