

1 SchemeAAIBME

1.1 Setup(n, d) \rightarrow (mpk, msk)

$g \leftarrow 1_{\mathbb{G}_1}$
 generate $\alpha, \beta, t_1, t_2, t_3, t_4 \in \mathbb{Z}_r$ randomly
 generate $g_2, g_3 \in \mathbb{G}_1$ randomly
 generate $\mathbf{T} \leftarrow (\mathbf{T}_0, \mathbf{T}_1, \dots, \mathbf{T}_n) \in \mathbb{G}_1^{n+1}$ randomly
 generate $\mathbf{T}' \leftarrow (\mathbf{T}'_0, \mathbf{T}'_1, \dots, \mathbf{T}'_n) \in \mathbb{G}_1^{n+1}$ randomly
 generate $\mathbf{u} \leftarrow (\mathbf{u}_0, \mathbf{u}_1, \dots, \mathbf{u}_n) \in \mathbb{G}_\mu^{n+1}$ randomly
 generate $\mathbf{u}' \leftarrow (\mathbf{u}'_0, \mathbf{u}'_1, \dots, \mathbf{u}'_n) \in \mathbb{G}_1^{n+1}$ randomly
 $H_1 : \{0, 1\}^* \rightarrow \mathbb{G}_1$
 $g_1 \leftarrow g^\alpha$
 $g'_1 \leftarrow g^\beta$
 $Y_1 \leftarrow e(g_1, g_2)^{t_1 t_2}$
 $Y_2 \leftarrow e(g_3, g)^\beta$
 $v_1 \leftarrow g^{t_1}$
 $v_2 \leftarrow g^{t_2}$
 $v_3 \leftarrow g^{t_3}$
 $v_4 \leftarrow g^{t_4}$
 $mpk \leftarrow (g_1, g'_1, g_2, g_3, Y_1, Y_2, v_1, v_2, v_3, v_4, \mathbf{u}, \mathbf{T}, \mathbf{u}', \mathbf{T}', H_1)$
 $msk \leftarrow (g_2^\alpha, \beta, t_1, t_2, t_3, t_4)$
return (mpk, msk)

1.2 EGen(ID_A, S) $\rightarrow ek_{ID_A}(S)$

$g \leftarrow 1_{\mathbb{G}_1}$
 $H : (\mathbf{u} \leftarrow (\mathbf{u}_0, \mathbf{u}_1, \dots, \mathbf{u}_n), ID \leftarrow (ID_1, ID_2, \dots, ID_n)) \rightarrow \mathbf{u}_0 \prod_{j \in [1, n]} \mathbf{u}_j^{ID_j}$
 generate $\vec{r} = (r_1, r_2, \dots, r_n) \in \mathbb{Z}_r^n$ randomly
 generate a $(d-1)$ degree polynomial $q(x)$ s.t. $q(0) = \beta$ randomly
 $ek_{ID_{A_i}} \leftarrow (g_3^{q(i)} [H(\mathbf{u}', ID_A) T_i^{r_i}], g^{r_i}), \forall i \in \{1, 2, \dots, n\}$
 generate $ek_{ID_A}(S) \subset ek_{ID_A}$ s.t. $\|ek_{ID_A}(S)\| = d$ randomly
return $ek_{ID_A}(S)$

1.3 DGen(id_B) $\rightarrow dk_{ID_B}$

$g \leftarrow 1_{\mathbb{G}_1}$
 $H : (\mathbf{u} \leftarrow (\mathbf{u}_0, \mathbf{u}_1, \dots, \mathbf{u}_n), ID \leftarrow (ID_1, ID_2, \dots, ID_n)) \rightarrow \mathbf{u}_0 \prod_{j \in [1, n]} \mathbf{u}_j^{ID_j}$
 generate $\vec{k}_1 = (k_{1,1}, k_{1,2}, \dots, k_{1,n}) \in \mathbb{Z}_r^n$ randomly
 generate $\vec{k}_2 = (k_{2,1}, k_{2,2}, \dots, k_{2,n}) \in \mathbb{Z}_r^n$ randomly
 $dk_{ID_{B_i}} \leftarrow (g^{k_{1,i} t_1 t_2 + k_{2,i} t_3 t_4} g_2^{-\alpha t_2} [H(\mathbf{u}, ID_B) T_i]^{k_{1,i} t_2} g_2^{-\alpha t_1} [H(\mathbf{u}, ID_B) T_i]^{k_{1,i} t_1} [H(\mathbf{u}, ID_B) T_i]^{k_{2,i} t_4} [H(\mathbf{u}, ID_B) T_i]^{k_{2,i} t_3})$
 $\{1, 2, \dots, n\}$
 generate $dk_{ID_B}(S') \subset dk_{ID_B}$ s.t. $\|dk_{ID_B}(S')\| = d$ randomly
return $dk_{ID_B}(S')$

1.4 $\text{Enc}(ek_{ID_A}(S), ID_A, ID_B, S, M) \rightarrow CT$

$g \leftarrow 1_{\mathbb{G}_1}$
 $H : (\mathbf{u} \leftarrow (\mathbf{u}_0, \mathbf{u}_1, \dots, \mathbf{u}_n), ID \leftarrow (ID_1, ID_2, \dots, ID_n)) \rightarrow \mathbf{u}_0 \prod_{j \in [1, n]} \mathbf{u}_j^{ID_j}$
 generate $S'' \subset [1, n]$ s.t. $\|S''\| = d$ randomly
 generate $s \in \mathbb{Z}_r$ randomly
 generate $\vec{s}_1 = (s_{1,1}, s_{1,2}, \dots, s_{1,n})$ randomly
 generate $\vec{s}_2 = (s_{2,1}, s_{2,2}, \dots, s_{2,n})$ randomly
 generate a $(d-1)$ degree polynomial $q(x)$ s.t. $q(0) = s$ randomly
 $K_s \leftarrow Y_1^s$
 $K_l \leftarrow Y_2^s$
 $C \leftarrow M \cdot K_s \cdot K_l$
 $C_{1,i} \leftarrow [H(\mathbf{u}, ID_B)T_i]^{q(i)}, \forall i \in S''$
 $C_{2,i} \leftarrow v_1^{q(i)-s_{1,i}}, \forall i \in S''$
 $C_{3,i} \leftarrow v_2^{s_{1,i}}, \forall i \in S''$
 $C_{4,i} \leftarrow v_3^{q(i)-s_{2,i}}, \forall i \in S''$
 $C_{5,i} \leftarrow v_4^{s_{2,i}}, \forall i \in S''$
 generate $\vec{z} = (z_1, z_2, \dots, z_n) \in \mathbb{Z}_r^d$ randomly
 generate $\vec{z}' = (z'_1, z'_2, \dots, z'_n) \in \mathbb{Z}_r^d$ randomly
 $C_{6,i} \leftarrow g^{z'_i}, \forall i \in S$
 $C_{7,i} \leftarrow (ek_{ID_{A_{i,2}}}(S) \cdot g^{z_i})^s, \forall i \in S$
 $C_{8,i} \leftarrow ek_{ID_{A_{i,1}}}(S) \cdot [H(\mathbf{u}', ID_A)T'_i]^{s \cdot z_i} \cdot H_1(C \| C_{1,i} \| C_{2,i} \| C_{3,i} C_{4,i} \| C_{5,i} \| C_{6,i} \| C_{7,i}), \forall i \in S$
 $I \leftarrow S \cap S''$
if $\|I\| \leq d$ **then**
 generate $I^* \subset I$ randomly
 $CT \leftarrow (S'', I^*, C, \vec{C}_1, \vec{C}_2, \vec{C}_3, \vec{C}_4, \vec{C}_5, \vec{C}_6, \vec{C}_7, \vec{C}_8)$
return CT

1.5 $\text{Dec}(dk_{ID_B}(S'), ID_B, ID_A, CT) \rightarrow M$

$CT_i \leftarrow C \| C_{1,i} \| C_{2,i} \| C_{3,i} \| C_{4,i} \| C_{5,i} \| C_{6,i} \| C_{7,i}, \forall i \in \{1, 2, \dots, n\}$
 $K'_l \leftarrow \prod_{i \in I^*} \left(\frac{e(C_{8,i}, g)}{e([H(\mathbf{u}', ID_A)T'_i]e(H_1(CT_i), C_{6,i}))} \right)^{\Delta(i, I, 0)}$
 $K'_s \leftarrow \prod_{i \in I} ()^{\Delta(i, j, 0)}$
if $|S \cap S'| \leq d \wedge |S' \cap S''| \leq d$ **then**
 $\text{quad}M \leftarrow C \cdot K'_s \cdot K'_l$
else
 $M \leftarrow \perp$
end if
return M