

1 SchemeFuzzyME

This scheme is only applicable to symmetric groups of prime orders.

1.1 Setup(n, d) \rightarrow (mpk, msk)

$g \leftarrow 1_{\mathbb{G}_1}$
 generate $g_2, g_3 \in \mathbb{G}_1$ randomly
 generate $\vec{t} = (t_1, t_2, \dots, t_{n+1}) \in \mathbb{G}_1^{n+1}$ randomly
 generate $\vec{l} = (l_1, l_2, \dots, l_{n+1}) \in \mathbb{G}_1^{n+1}$ randomly
 generate $\alpha, \beta, \theta_1, \theta_2, \theta_3, \theta_4 \in \mathbb{Z}_r$ randomly
 $g_1 \leftarrow g^\alpha$
 $\eta_1 \leftarrow g^{\theta_1}$
 $\eta_2 \leftarrow g^{\theta_2}$
 $\eta_3 \leftarrow g^{\theta_3}$
 $\eta_4 \leftarrow g^{\theta_4}$
 $Y_1 \leftarrow \hat{e}(g_1, g_2)^{\theta_1 \theta_2}$
 $Y_2 \leftarrow \hat{e}(g_3, g^\beta)^{\theta_1 \theta_2}$
 $H_1 : \{0, 1\}^* \rightarrow \mathbb{G}_1$
 $mpk \leftarrow (g_1, g_2, g_3, Y_1, Y_2, \vec{t}, \vec{l}, \eta_1, \eta_2, \eta_3, \eta_4, H_1)$
 $msk \leftarrow (\alpha, \beta, \theta_1, \theta_2, \theta_3, \theta_4)$
return (mpk, msk)

1.2 EKGen(S_A) $\rightarrow ek_{S_A}$

$g \leftarrow 1_{\mathbb{G}_1}$
 $\Delta : i, S, x \rightarrow \prod_{j \in S, j \neq i} \frac{x-j}{i-j}$
 $N \leftarrow (1, 2, \dots, n+1)$
 $H : x \rightarrow g_3^{x^n} \prod_{i=1}^{n+1} l_i^{\Delta(i, N, x)}$
 generate a $(d-1)$ degree polynomial $q(x)$ s.t. $q(0) = \beta$ randomly
 generate $\vec{r} = (r_1, r_2, \dots, r_n) \in \mathbb{Z}_r^n$ randomly
 $E_i \leftarrow g_3^{q(a_i)\theta_1\theta_2} H(a_i)^{r_i}, \forall i \in \{1, 2, \dots, n\}$
 $e_i \leftarrow g^{r_i}, \forall i \in \{1, 2, \dots, n\}$
 $ek_{S_A} \leftarrow \{E_i, e_i\}_{a_i \in S_A}$
return ek_{S_A}

1.3 DKGen(id_R) $\rightarrow dk_{id_R}$

$g \leftarrow 1_{\mathbb{G}_1}$
 $\Delta : i, S, x \rightarrow \prod_{j \in S, j \neq i} \frac{x-j}{i-j}$
 $N \leftarrow (1, 2, \dots, n+1)$
 $T : x \rightarrow g_2^{x^n} \prod_{i=1}^{n+1} t_i^{\Delta(i, N, x)}$
 $H : x \rightarrow g_3^{x^n} \prod_{i=1}^{n+1} l_i^{\Delta(i, N, x)}$
 generate $\gamma \in \mathbb{Z}_r$ randomly
 generate $G_{ID} \in \mathbb{G}_1$ randomly

generate a $(d-1)$ degree polynomial $f(x)$ s.t. $f(0) = \alpha$ randomly
 generate a $(d-1)$ degree polynomial $h(x)$ s.t. $h(0) = \gamma$ randomly
 generate a $(d-1)$ degree polynomial $q'(x)$ s.t. $q'(0) = \beta$ randomly
 generate $\vec{k}_1 = (k_{1,1}, k_{1,2}, \dots, k_{1,n}) \in \mathbb{Z}_r^n$ randomly
 generate $\vec{k}_2 = (k_{2,1}, k_{2,2}, \dots, k_{2,n}) \in \mathbb{Z}_r^n$ randomly
 generate $\vec{r}'_1 = (r'_{1,1}, r'_{1,2}, \dots, r'_{1,n}) \in \mathbb{Z}_r^n$ randomly
 generate $\vec{r}'_2 = (r'_{2,1}, r'_{2,2}, \dots, r'_{2,n}) \in \mathbb{Z}_r^n$ randomly
 $dk_{S_{B_0}, i} \leftarrow g^{k_{1,i}\theta_1\theta_2 + k_{2,i}\theta_3\theta_4}, \forall i \in \{1, 2, \dots, n\}$
 $dk_{S_{B_1}, i} \leftarrow g_2^{-f(b_i)\theta_2} (G_{ID})^{-h(b_i)\theta_2} [T(b_i)]^{-k_{1,i}\theta_2}, \forall i \in \{1, 2, \dots, n\}$
 $dk_{S_{B_2}, i} \leftarrow g_2^{-f(b_i)\theta_1} (G_{ID})^{-h(b_i)\theta_1} [T(b_i)]^{-k_{1,i}\theta_1}, \forall i \in \{1, 2, \dots, n\}$
 $dk_{S_{B_3}, i} \leftarrow [T(b_i)]^{-k_{2,i}\theta_4}, \forall i \in \{1, 2, \dots, n\}$
 $dk_{S_{B_4}, i} \leftarrow [T(b_i)]^{-k_{2,i}\theta_3}, \forall i \in \{1, 2, \dots, n\}$
 $dk_{S_B} \leftarrow (dk_{S_{B_0}}, dk_{S_{B_1}}, dk_{S_{B_2}}, dk_{S_{B_3}}, dk_{S_{B_4}})$
 $dk_{P_{A_0}, i} \leftarrow g^{r'_{1,i}\theta_1\theta_2 + r'_{2,i}\theta_3\theta_4}, \forall i \in \{1, 2, \dots, n\}$
 $dk_{P_{A_1}, i} \leftarrow g_2^{-2q'(a_i)\theta_2} (G_{ID})^{h(a_i)\theta_2} H(a_i)^{-r'_{1,i}\theta_2}, \forall i \in \{1, 2, \dots, n\}$
 $dk_{P_{A_2}, i} \leftarrow g_2^{-2q'(a_i)\theta_1} (G_{ID})^{h(a_i)\theta_1} H(a_i)^{-r'_{1,i}\theta_1}, \forall i \in \{1, 2, \dots, n\}$
 $dk_{P_{A_3}, i} \leftarrow [H(a_i)]^{-r'_{2,i}\theta_4}, \forall i \in \{1, 2, \dots, n\}$
 $dk_{P_{A_4}, i} \leftarrow [H(a_i)]^{-r'_{2,i}\theta_3}, \forall i \in \{1, 2, \dots, n\}$
 $dk_{P_A} \leftarrow (dk_{P_{A_0}}, dk_{P_{A_1}}, dk_{P_{A_2}}, dk_{P_{A_3}}, dk_{P_{A_4}})$
 $dk_{S_B, P_A} \leftarrow (dk_{S_B}, dk_{P_A})$
return dk_{S_B, P_A}

1.4 Encryption(ek_{S_A}, S_A, P_B, M) $\rightarrow CT$

$g \leftarrow 1_{\mathbb{G}_1}$
 $\Delta : i, S, x \rightarrow \prod_{j \in S, j \neq i} \frac{x-j}{i-j}$
 $N \leftarrow (1, 2, \dots, n+1)$
 $T : x \rightarrow g_2^{x^n} \prod_{i=1}^{n+1} t_i^{\Delta(i, N, x)}$
 $H : x \rightarrow g_3^{x^n} \prod_{i=1}^{n+1} l_i^{\Delta(i, N, x)}$
 generate $s, s_1, s_2, \tau \in \mathbb{Z}_r$ randomly
 $K_s \leftarrow Y_1^s$
 $K_l \leftarrow Y_2^s \cdot \hat{e}(g_3, g^{-\tau})$
 $C_0 \leftarrow M \cdot K_s \cdot K_l$
 $C_1 \leftarrow \eta_1^{s-s_1}$
 $C_2 \leftarrow \eta_2^{s_1}$
 $C_3 \leftarrow \eta_3^{s-s_2}$
 $C_4 \leftarrow \eta_4^{s_2}$
 $C_{1,i} \leftarrow T(b_i)^s, \forall b_i \in P_B$
 $C_{2,i} \leftarrow H(a_i)^s, \forall a_i \in S_A$
 generate a $(d-1)$ degree polynomial $l(x)$ s.t. $l(0) = \tau$ randomly
 generate $\vec{\xi} = (\xi_1, \xi_2, \dots, \xi_n) \in \mathbb{Z}_r^n$ randomly
 generate $\vec{\chi} = (\chi_1, \chi_2, \dots, \chi_n) \in \mathbb{Z}_r^n$ randomly
 $C_{3,i} \leftarrow e_i \cdot g^{\xi_i}, \forall i \in \{1, 2, \dots, n\}$

$C_{4,i} \leftarrow g^{x_i}, \forall i \in \{1, 2, \dots, n\}$
 $C_{5,i} \leftarrow E_i^s \cdot g_3^{l(a_i)} H(a_i)^{s \cdot \xi_i} \cdot H_1(C_0 || C_1 || C_2 || C_3 || C_4 || C_{1,i} || C_{2,i} || C_{3,i} || C_{4,i})^{x_i}$
 $CT \leftarrow (C_0, C_1, C_2, C_3, C_4, \vec{C}_1, \vec{C}_2, \vec{C}_3, \vec{C}_4, \vec{C}_5)$
return CT

1.5 Decryption($dk_{S_B, P_A}, S_A, P_A, S_B, P_B, CT$) $\rightarrow M$

$W'_A \leftarrow S_A \cap P_A$
 $W'_B \leftarrow S_B \cap P_B$
if $|W'_A| \leq d \wedge |W'_B| \leq d$ **then**
 generate $W_A \subset W'_A$ s.t. $|W_A| = d$ randomly
 generate $W_B \subset W'_B$ s.t. $|W_B| = d$ randomly
 $g \leftarrow 1_{\mathbb{G}_1}$
 $\Delta : i, S, x \rightarrow \prod_{j \in S, j \neq i} \frac{x-j}{i-j}$
 $K'_s \leftarrow \prod_{b_i \in W_B} (\hat{e}(C_{1,i}, dk_{S_{B_0,i}}) \hat{e}(C_1, dk_{S_{B_1,i}}) \hat{e}(C_2, dk_{S_{B_2,i}}) \hat{e}(C_3, dk_{S_{B_3,i}}) \hat{e}(C_4, dk_{S_{B_4,i}})) \Delta(b_i, W_B, 0)$
 $CT_i \leftarrow C_0 || C_1 || C_2 || C_3 || C_4 || C_{1,i} || C_{2,i} || C_{3,i} || C_{4,i}, \forall i \in \{1, 2, \dots, n\}$
 $K'_l \leftarrow \prod_{a_i \in W_A} \left(\frac{\hat{e}(C_{1,i}, dk_{P_{A_0,i}}) \hat{e}(C_1, dk_{P_{A_1,i}}) \hat{e}(C_2, dk_{P_{A_{i,2}}})}{\hat{e}(H_1(CT_i), C_{4,i}) \cdot \hat{e}(C_{3,i}, C_{2,i})} \cdot \hat{e}(C_3, dk_{P_{A_{i,3}}}) \hat{e}(C_4, dk_{P_{A_{i,4}}}) \hat{e}(C_{5,i}, g) \right)^{\Delta(a_i, W_A, 0)}$
 $M \leftarrow C_0 \cdot K'_s \cdot K'_l$
else
 $M \leftarrow \perp$
end if
return M