$z = z_2(r, \theta)$

$z = z_1(r, \theta)$

$r(\theta)$

(r, θ)

① $g(r, \theta) = \int_{z_1(r,\theta)}^{z_2(r,\theta)} f(x(r,\theta), y(r,\theta), z) \, \mathrm{d}z$

② $h(\theta) = \int_0^{r(\theta)} g(r, \theta) r \, \mathrm{d}r$

$\quad = \int_0^{r(\theta)} \left[\int_{z_1(r,\theta)}^{z_2(r,\theta)} f(x(r,\theta), y(r,\theta), z) \, \mathrm{d}z \right] r \mathrm{d}r$

③ $\iiint_\Omega f(x, y, z) \, \mathrm{d}x \mathrm{d}y \mathrm{d}z = \int_0^{2\pi} h(\theta) \, \mathrm{d}\theta$

$\quad = \int_0^{2\pi} \left[\int_0^{r(\theta)} \left[\int_{z_1(r,\theta)}^{z_2(r,\theta)} f(x(r,\theta), y(r,\theta), z) \, \mathrm{d}z \right] r \mathrm{d}r \right] \mathrm{d}\theta$

$\quad = \int_0^{2\pi} \mathrm{d}\theta \int_0^{r(\theta)} \left[\int_{z_1(r,\theta)}^{z_2(r,\theta)} f(x(r,\theta), y(r,\theta), z) \, \mathrm{d}z \right] r \mathrm{d}r$