MATHS FINAL PROJECT

Group 9: Aashna Rungta, Sandhya N, Eshna Reza, Sneha Batchu, Sony Kumari

Part I : Maximizing The Number of Exposures of Ads

Questions to be answered :

- Determine the decision variables
- Formulate the optimization model with all the constraints
- Coding in R, find the optimal number of advertisements to run in each media that maximize the expected number of exposures while satisfying all the constraints
    1. What is the optimal profit value?
    2. What are the optimal values for the variables?

Variables taken are defined as follows :

- tv = No. of TV Ads
- m = No. of Magazine Ads
- n = No. of Newspaper Ads

Below are the target segments for the ads :

- y = No. of people viewing in the young age range 18 - 30 years
- ma = No. of people viewing in middle aged range 30 - 55 years

Constraints that are considered :

- tv >=0
- m >=0, and
- n >=0

Framing the equation for the Ad Costs :

$300,000 * tv + $150,000 * m + 100,000 * n <= $4,000,000

Framing the equation for the Planning Costs :

$90,000 * tv + $30,000 * m + 40,000 * n <= $1,000,000

Available Spots for TV Commercials :

tv <= 5

Expected Exposures are as below:

- y >= 5M
- ma >= 5M

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Using Table 2, we translated to expected exposures based on the optimal ads information,

- 1.2M*tv + 0.2M*m >= 5M (For y)
- 0.5M*tv + 0.2M*m + 0.2M *n >= 5M (For ma)

We know the expected exposures for each medium from Table 1 as follows :

- tv * (1.2M*y + 0.5M*ma) >= 1.3M
- m * (0.2M*y + 0.2M*ma) >= 0.6M
- n * (0.2M*ma) >= 0.5M

Thus,
Expected Total Exposures = 1.3M * tv + 0.6M * m + 0.5M * n

Cost of promotions can be formulated as below :
$40,000*m + $120,000*n = $1,490,000

Maximizing the equation -> 1.3M * tv + 0.6M * m + 0.5M * n

Which is subject to :

- $40,000*m + $120,000*n = $1,490,000
- 1.2M*tv + 0.2M*m >= 5M
- 0.5M*tv + 0.2M*m + 0.2M *n >= 5M and,
- 0<=tv<=5 , m>=0, n>=0

Using a two phase method, we found the initial value for optimization with dummy variables. The initial values we found were - 3.0245999, 8.886762, 9.4544127, which we finally used for the actual optimization.

The optimal values for the TV, Magazine and Newspaper Ads are respectively:
2.3181818, 17.6818182, 6.5227273

We used R to come up with the values of the framed equations and the same has been submitted for reference.

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Part II : Maximizing the Profits

Using the optimal values we got from Part 1 for the number of advertisements:

tv = the number of TV commercials = 2.318182

m = the number of magazine ads = 17.681818

n = the number of newspaper ad 6.522727

Questions to be answered:

- *Using your results from Part I, formulate the total profit (as defined by Vijay).*

  As per Vijay, Approximation of profit ~ Exposures = 1300000*tv + 600000*m + 500000*n

- *Use R (or any other language) to determine the optimal number of ads to run in each medium to maximize the total profit while satisfying all the constraints in Part I.*

b) What are the optimal values for the variables?

tv = the number of TV commercials = 2.318182

m = the number of magazine ads = 17.681818

n = the number of newspaper ad = 6.522727

We will use the fractions itself

a) What is the optimal profit value?

We calculated the first time visits, using the given equations:

#First time visits after watching TV Ads

tv_first = -0.1*(tv^2)+ 1.13*tv - 0.04

#First time visits after viewing Magazine Ads

m_first = -0.002*(m^2) + 0.124*m + 0.14

#First time visits after viewing Newspaper Ads n_first = -0.0321*(n^2) + 0.706*n - 0.09

first_visit=(tv_first+m_first+n_first)*1000000

Given that, Profit earned per customer is $5, hence,
Total Profit = first_visit * 5

From Part 1, we have the costs for running the Advertisements and Planning.
ad_cost = 300000*tv + 150000*m + 100000*n
planning_cost = 90000*tv + 30000*m + 40000*n

Since these costs are not included in the Total Profit, we need to account for them to get the Net Profit earned by Amber India.

Net_profit = Total Profit - Ad Costs - Planning Costs

$29,493,603 is the Net Profit based on the detailed calculations in R using the optimal values of variables we achieved in Part 1.

c) *Based on your calculation, what can you conclude about the accuracy of Vjay's approximation in Part I (i.e., finding the optimal variables by using the expected number of exposures)?*

Comparing Vijay's approximation of Profit: The expected number of exposures is a rough approximation for profit. The expected exposures is given in Part 1:

Exposures = 1300000*tv + 600000*m + 500000*n = 16884090.9

Plugging in the values, we get 16884090.9, which would mean approximate profit is $16,884,091

The $16.8M profit is way off from the $29.5M optimal profit we got in part 2a. Thus, while the approximation may work for a small number, its error gets magnified when we are dealing with numbers in millions.

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Part III: Managing Demand for Amour du jour

a) Formulate the profit function for Vijay's ordering problem.
b) What is the decision variable?
c) Determine the optimal number of gallons of Amour du jour for which Amber must order raw materials each week to maximize the restaurant's profit. What is the optimal profit?

a) Formulate the profit function for Vijay's ordering problem.

Decision variables:
q = order qty of Amour du jour (in gallons)
X = customer demand of of Amour du jour (in gallons)
a = lower bound of weekly customer demand = 200
b = upper bound of weekly customer demand = 500

Profit Function:
Profit = P = Revenue - Cost

P(X,q) = 12*25*X - 75*q; when X <= q
= 12*25*q - 75*q; when X > q

Given that we have a uniform distribution of the demand, the probability density function of the demand X is f(X). Using the formula

f(X) = 1/(b-a) = 1/(500-200) = 1/300, if 200 <= X <= 500
0, otherwise

Below, we calculate the expectation of profit, E(P)

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$$E(P) = \int_{200}^{q} (12*25*X - 75*q)*f(X)\,dX + \int_{q}^{500} (12*25*q - 75*q)*f(X)\,dX$$

$$E(P) = \int_{200}^{q} (300*X - 75*q)*\frac{1}{300}\,dX + \int_{q}^{500} (225*q)*\frac{1}{300}\,dX$$

$$E(P) = \int_{200}^{q} (X - \frac{q}{4})\,dX + \int_{q}^{500} (q*\frac{3}{4})\,dX$$

Solving the integral, we get, $E(P) = \frac{-1}{2}*q^2 + 425*q - \frac{1}{2}*200^2$

To get maximum profit, $\frac{dE(P)}{dX} = 0$

$$\frac{dE(P)}{dq} = -q + 425 = 0$$

$$q = 425$$

Using the above quantity (425 gallons/week), we get the following optimal profit value:

$$p = 12*25*X - 75*q$$

$$p = 12*25*[\frac{200+425}{2}\int_{200}^{425} f(X)dX + 425*\int_{425}^{500} f(X)dX] - 75*425$$

$$p = 300*[312.5*(\frac{425}{300} - \frac{200}{300}) + 425*(\frac{500}{300} - \frac{425}{300})] - 31875$$

$$p = 102187.5 - 31875 = \$70312.5$$

The above equations show that the optimal number of gallons is **425 gallons per week.**

Therefore, optimal profit by Amour du Jour = $70312 per week