

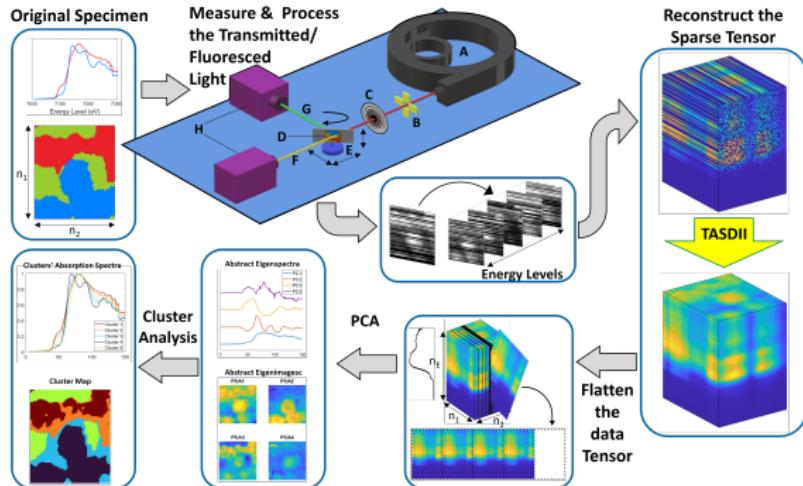
Low Rank Tensor Completion for X-Ray Spectromicroscopy

Oliver Townsend

Joint work with Silvia Gazzola (U. of Bath), Sergey Dolgov (U. of Bath), and Paul Quinn (Diamond Light Source)

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Experiment Pipeline and proposed solution



- Typically scan over a **100 × 100** grid and at **150** energy levels.
- Experiments take **several hours**.

Proposed Solution

Undersample the measurements of a specimen and use low rank completion to recover the missing entries.

Low rank matrix completion

- For $A \in \mathbb{R}^{n_E \times n_1 n_2}$, define the **sampling pattern** $\Omega \subset [n_E] \times [N]$ (subset of known entries) with **undersampling ratio** $p = |\Omega| / n_E n_1 n_2$.
- Define **sampling operator** as $\mathcal{P}_\Omega : \mathbb{R}^{n_E \times n_1 n_2} \rightarrow \mathbb{R}^{n_E \times n_1 n_2}$,

$$(\mathcal{P}_\Omega(A))_{ij} = A_{ij} \quad \text{if } (i, j) \in \Omega, \quad (\mathcal{P}_\Omega(A))_{ij} = 0 \text{ otherwise.}$$

- Use completion algorithms like ASD and LoopedASD to solve, for $X \in \mathbb{R}^{n_E \times r}$, $Y \in \mathbb{R}^{r \times n_1 n_2}$:

Definition (Robust Low Rank Completion Problem)

$$\min_{X, Y} f(X, Y), \quad \text{where } f(X, Y) = \frac{1}{2} \|D - \mathcal{P}_\Omega(XY)\|_F^2$$

Nimbus

- Use Nimbus to create diagrams of completion errors for different r , p ,

$$e_c = \frac{\|A - A_c\|_F}{\|A\|_F} \quad \text{where } A_c = \text{LoopedASD}(D, r, p)$$

- MATLAB, with Parallel Computing toolbox
- HB120rs v3 partitions, which feature 120 AMD EPYC 7V73X CPU cores, 448 GB of RAM and clock frequencies up to 3.5 GHz
- Problem: Node Failures

Slurm Job_id=67204119 Name=RealDataTASDImageTest Ended, Run time 04:44:34, NODE_FAIL, ExitCode 0



User to run slurmd <slurm@nimbus-1-scheduler.localdomain>

To Oliver Townsend

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Comparison of ASD and Looped ASD

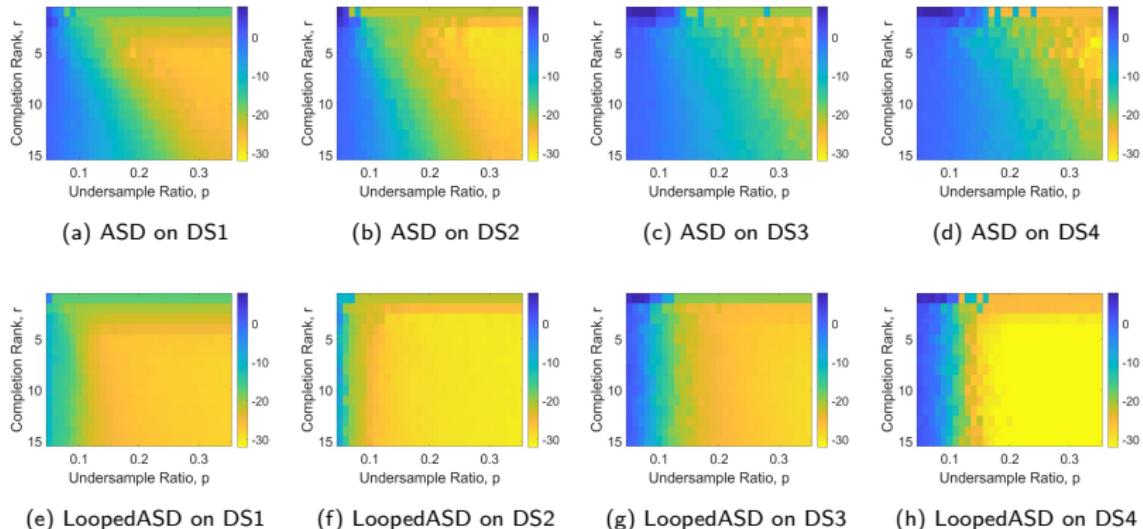
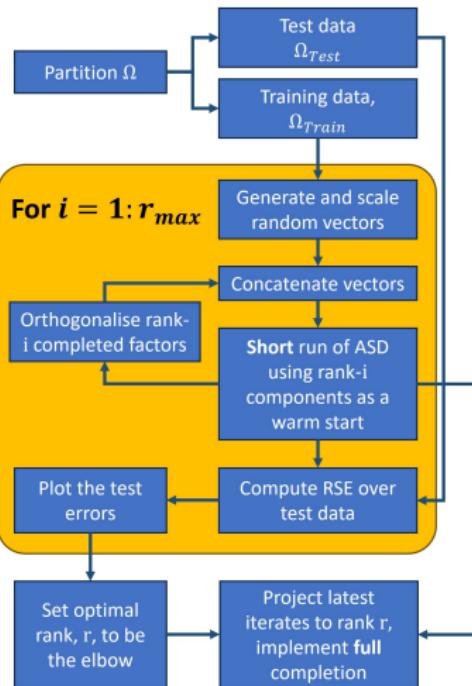


Figure 1: Diagram comparing ASD and LoopedASD on x-ray spectromicroscopy data (DS1 - DS4). We vary the undersampling ratio, p , the x-axis and the completion rank, r , along the y-axis. The colour indicates the mean relative square error in dB; **yellow means a lower completion error**.

Rank Estimation and LoopedASD

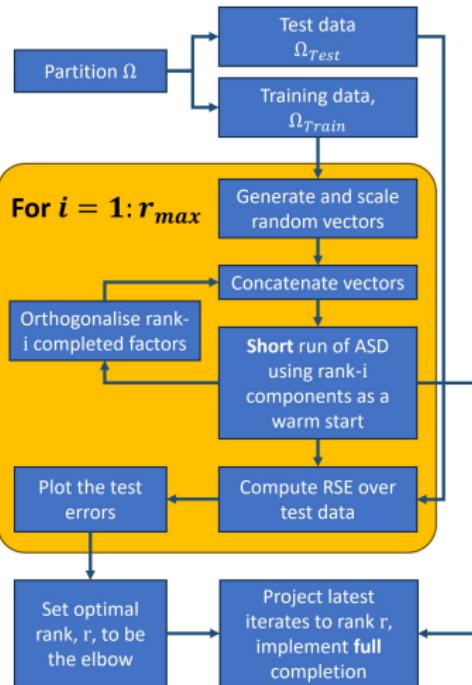
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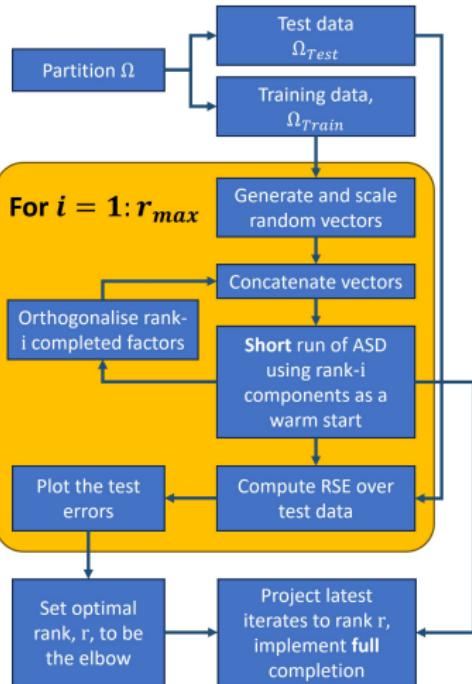
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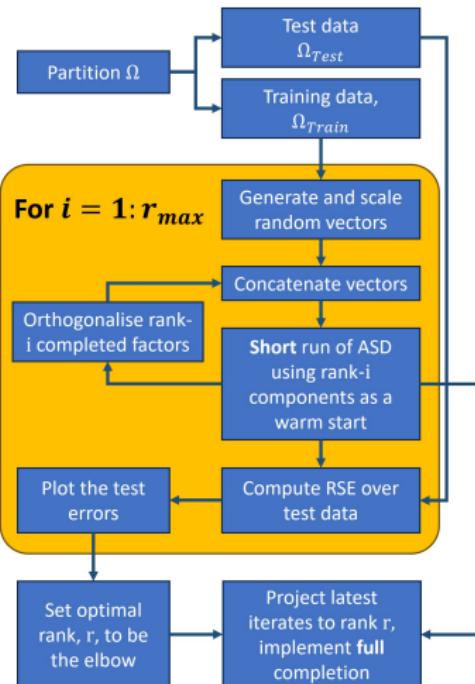
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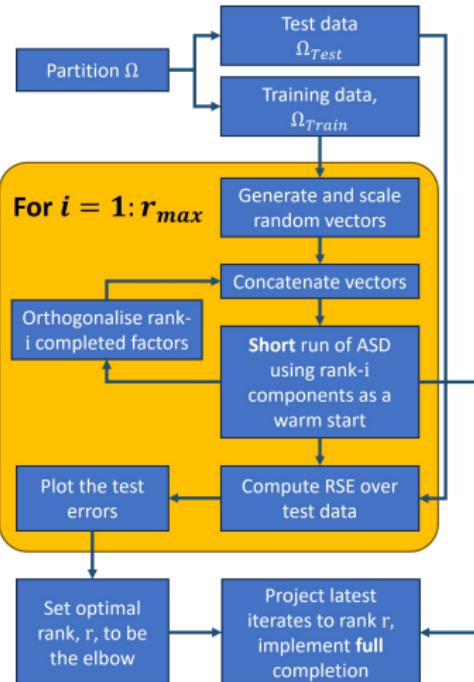
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- The optimal completion rank is set to be the elbow of the test errors.

