MArefSetup

## MArefSetup

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### 1 [Quadratic equations](#QQ2-1-5)

A quadratic equation is an equation with the form where represents an unknown and , and are known numbers with .

#### 1.1 [Solutions to a quadratic equation](#QQ2-1-6)

A solution to a quadratic equation is a value of such that the equation balances. The solutions to quadratic equations can be found by using the quadratic formula:

|  |  |
| --- | --- |
|  | (1) |

Example.  
For instance, the solutions to are:

Hence, or .

#### 1.2 [The discriminant](#QQ2-1-7)

Definition 1.1 (Discriminant).  
The **discriminant** of a quadratic equation with coefficients is:

Remark 1.2.  
Note that this is the expression beneath the square root symbol in the quadratic formula ([1](#x1-6001r1)).

We can use the discriminant to determine the number of real roots of a quadratic equation. The number depends on the value of as in table [1](#x1-70031).

|  |  |
| --- | --- |
|  |  |
| Value of | Real roots |
|  |  |
|  | Two, distinct |
|  | One, repeated |
|  | Zero |
|  |  |
|  |  |

Table 1: Number of real roots of a quadratic equation, given the discriminant

Figure [1](#x1-70051) shows an example of each possibility[1](#fn1x0).

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Figure 1: Examples of quadratic functions with zero, one and two real roots.

### 2 [The scalar product](#QQ2-1-10)

Consider two vectors and drawn so their tails are at the same point.

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Figure 2: Two vectors with angle between them.

We define the scalar product of and as follows.

Definition 2.1 (Scalar product).  
The **scalar product** of and is

where

* is the modulus of ,
* is the modulus of , and
* is the angle between and .

Remark 2.2.  
It is important to use the dot symbol for the scalar product (also called the dot product). You must not use a symbol as this denotes the vector product which is defined differently.

Example 2.3.  
Let

The angle between these vectors is . Then and . So,

Note that the result is a scalar, not a vector.

#### 2.1 [Vectors in cartesian form](#QQ2-1-12)

When vectors are given in cartesian form there is an alternative formula for calculating the scalar product.

Proposition 2.4.  
If and then

Proof.  Consider the vector . The modulus of this is

Note from figure [3](#x1-90023) that the vectors , and form a triangle:

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Figure 3: A triangle is formed by two vectors and their difference.

Let denote the angle between and . Then, the cosine rule yields:

|  |  |
| --- | --- |
|  | (2) |

Substituting the definition of the scalar product of and into equation [2](#x1-9003r2) gives:

Rearranging:

Writing this in terms of components produces:

as required. □

Example 2.5.  
Consider again the vectors

Calculating the scalar product using the components:

Note that if we are given vectors in this form, the scalar product may be used to calculate the angle between them. Since and we have:

Hence,

Rearranging:

### 3 [Using Matlab](#QQ2-1-14)

Two calculate the scalar product in Matlab the dot function is used.

Create two vectors:

> A = [4 -1 2];   
> B = [2 -2 -1];

Calculate the scalar product:

> C = dot(A,B)   
   
    C = 8

[1](#fn1x0-bk)The image is due to Olin, CC-BY-AS 3.0 downloaded from [Wikimedia Commons](https://commons.wikimedia.org/wiki/File:Quadratic_eq_discriminant.svg)