UNIT- I

Regression (r = coefficient of co-relation)

1) $r = \dfrac{\sum XY}{\sqrt{\sum X^2 \sum Y^2}}$ where $X = x - \bar{x}$, $y = y - \bar{y}$

2) $\sigma x = \sqrt{\dfrac{(x-\bar{x})^2}{n}}$ $\sigma y = \sqrt{\dfrac{(y-\bar{y})^2}{n}}$

3) $b_{xy} = \dfrac{r\,\sigma x}{\sigma y}$ $b_{yx} = \dfrac{r\,\sigma y}{\sigma x}$

4) $x - \bar{x} = b_{xy}(y - \bar{y})$

5) $r = \pm\sqrt{b_{xy} \times b_{yx}}$

2) Multiple Regression

1) $y = ax_1 + bx_2 + C$

3) Cummulative distribution function

$P(X = x_i) = \sum_{i=1}^{n} P(x_i) \rightarrow \boxed{\sum_{i=1}^{n} P(x_i) = 1}$

$Mean[\mathcal{E}(x)] = \mathcal{E}(x) = \sum x \cdot P(x)$

$Var(x) = \mathcal{E}(x^2) = \mathcal{E}(x^2) - [\mathcal{E}(x)]^2$

$\hookrightarrow \mathcal{E}(x^2) = \sum x^2 P(x)$

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\star \quad F(x) = \int_{-\infty}^{x} f(x)\,dx$$

$$\text{Mean} \rightarrow \int_{-\infty}^{\infty} x f(x)\,dx$$

$$\text{Var}(x) = E(x^2) - (E(x))^2 = \int_{-\infty}^{\infty} x^2 f(x)\,dx$$

Binomial distribution

$$P(x) = {}^{n}C_{r}\, p^{r} q^{n-r}$$

p = Probability of success
q = Probability of failure $(1-p)$

$$\text{mean } (Ex) = pq$$

$$\text{variance} = npq$$

Poission Ditribution

$$P(x=r) = \frac{e^{-m}\, m^{r}}{\lfloor r}$$

m = mean

$$E(x) = m$$

$$\text{Var}(x) = m$$

$$m = np$$

n = number of items
p = Probability of occurance

UNIT - II

Exponential Distribution

$$\text{Pdf} = f(x) = \alpha e^{-\alpha x}$$

$\star$ we use integration

$$\int_a^b \alpha e^{-\alpha x}\,dx$$

$$\text{Mean} = E(x) = 1/\alpha$$

$$\text{Var} = E(x^2) - [E(x)]^2 = 1/\alpha^2$$

Normal Probability Distribution

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2}$$

μ = mean , σ = standard deviation

* standard normal variate $\rightarrow$ $\boxed{Z = \frac{x-\mu}{\sigma}}$

Gamma distribution

$$f(x) = \begin{cases} \frac{1}{\beta^\alpha \Gamma(\alpha)} x^{\alpha-1} e^{-x/\beta} , & x > 0 \\ 0 , & \text{otherwise} \end{cases}$$

* we use integration.

Mean = $\alpha\beta$

Variance = $\alpha\beta^2$

Uniform Distribution

$$f(x) = \begin{cases} \frac{1}{b-a} , & a \leq x \leq b \\ 0 , & \text{otherwise} \end{cases}$$

Joint Probability distribution

$P(x, y) = f(x) \cdot g(x)$

$Cov(X, Y) = E(X, Y) - E(X) \cdot E(Y)$

$E(X) = \Sigma x \cdot f(x)$

$E(Y) = \Sigma y f(y)$

$$\sigma_x = E(x^2) - (E(x))^2$$
$$\sigma_y = E(y^2) - (E(y))^2$$

$$E(x^2) = \sum x^2 f(x)$$
$$E(y^2) = \sum y^2 f(y)$$

UNIT – III

1) Stochastic Matrix $\rightarrow$ $PV = V$ $\rightarrow$ $\begin{bmatrix} v_1 & v_2 & v_3 \\ \vdots & \vdots & \vdots \end{bmatrix}$

& $v_1 + v_2 + v_3 = 1$

Q → In long run then use $PV = V$ find $v_1 v_2 \ldots v_n$

2) $p^{(0)} = [\; - \; - \; - \;]$

$p^{(1)} = p^{(0)} . P$

$p^{(2)} = p^{(1)} . P$

$\vdots$

Q → find probability in n^{th} task

$\therefore$ $p' = p_0 P$

$p^2 = p' P$

$\vdots$

Queuing Theory

1) $M|M|1 : \infty|FIFO$ Single server infinite Capacity

$\lambda = \frac{\rho}{\mu}$

2) $P_n = (1 - \rho)\rho^n$

3) $P_0 = 1 - \rho$ (idle or zero units)

4) n or more units in the queue ρ^n

5) $W_s = \frac{1}{\mu - \lambda}$

6) $W_q = \frac{\lambda}{\mu(\mu - \lambda)}$

7) $L_s = \frac{\lambda}{\mu - \lambda}$

8) $L_q = \frac{\lambda^2}{\mu(\mu-\lambda)}$

9) arrival have to wait for more than w min $= \left(\frac{\lambda}{\mu}\right) e^{(\lambda-\mu)w}$ in system

10) arrival have to wait more than v in queue $= e^{(\lambda-\mu)v}$

Single Server finite capacity $(M|M|1)$ $(K|FIFO)$

K = no of pple i

λ_e = λ effective

1) $\rho = \frac{\lambda_e}{\mu}$

2) $P_0 = \frac{1-\rho}{1-\rho^{K+1}}$

3) $\lambda_e = \mu(1 - P_0)$

4) $L_s = \frac{\rho}{1-\rho} - \frac{(K+1)\rho^{K+1}}{1-\rho^{K+1}}$

5) $L_q = L_s - \frac{\lambda_e}{\mu}$

6) $W_s = \frac{L_s}{\lambda_e}$

7) $W_q = \frac{L_q}{\lambda_e}$

★ when $\rho = 1 \rightarrow L_s = \frac{K}{2}$, $P_0 = \frac{1}{K+1}$

$\Rightarrow M|M|S : \infty|FIFO \Rightarrow$ Multiservers with infinite Capacity

S = no of servers.

$\rho = \frac{\lambda}{\mu}$

P_0 = Probability of idle

$$\Rightarrow P_0 = \frac{1}{\sum_{n=1}^{s-1} \frac{\rho^n}{\lfloor n} + \frac{\rho^s}{\lfloor s\left(1-\frac{\rho}{s}\right)}}$$

$$\Rightarrow L_q = \frac{\rho^{s+1} P_0}{s \lfloor s \left(1-\frac{\rho}{s}\right)^2}$$

$$\Rightarrow L_s = L_q + \frac{\lambda}{\mu}$$

$$\Rightarrow W_s = \frac{L_s}{\lambda}$$

$$\Rightarrow W_q = \frac{L_q}{\lambda}$$

$\Rightarrow$ Probability arrival has to wait for service

$$P(N \geq s) = \frac{\rho^s P_0}{\lfloor s\left(1-\frac{\rho}{s}\right)}$$

$\Rightarrow$ Probability arrival enters service without waiting

$$\Rightarrow 1 - \frac{\rho^s P_0}{\lfloor s\left(1-\frac{\rho}{s}\right)}$$

★ M|G|1 Queuing Systems

★ $\rho = \frac{\lambda}{\mu}$

★ $L_q = \frac{\lambda^2 \sigma^2 + \rho^2}{2(1-\rho)}$

★ $L_s = L_q + \frac{\lambda}{\mu} \Rightarrow L_q + \rho$

$$W_s = \frac{L_s}{\lambda}$$

$$W_q = \frac{L_q}{\lambda}$$

UNIT – IV

1) H test for large samples

(Z_α 5% = 1.96, Z_α 1% = 2.58)

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$\bar{x}$ = Mean of Sample
μ = Mean of population
n = no of items in sample
σ = Standard deviation

Confidence limits → $\bar{x} - Z_\alpha\left(\frac{\sigma}{\sqrt{n}}\right) \leq \mu \leq \bar{x} + Z_\alpha\left(\frac{\sigma}{\sqrt{n}}\right)$

$|Z| < Z_\alpha$ → Accept Hypotheses

$|Z| > Z_\alpha$ → Reject —"—

Let 'x' be the observed number of success in a sample of size n, and $\mu = np$ be the expected no. of success. The associated standard normal variate z is defined as

$$Z = \frac{x - np}{\sqrt{npq}}$$

Probable limits ⇒ $p \pm Z_\alpha\sqrt{\frac{pq}{n}}$

E: If $\overline{x_1}$ & $\overline{x_2}$ are means of 2 sample sizes n_1, n_2 and σ is the standard deviation of the population then $z = \dfrac{\overline{x_1} - \overline{x_2}}{\sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$

* If $\overline{x_1}$ & $\overline{x_2}$ are means of samples of sizes n_1, n_2 respectively, and σ_1, σ_2 are the standard deviations of the above samples then $z = \dfrac{\overline{x_1} - \overline{x_2}}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$

T- Test (Small samples)

t_α at $1\% = 2.581$

t_α at $5\% = 2.160$

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n-1}}$$

$\bar{x}$ = Mean of sample
μ = Mean of population
n = no of items in sample
s = standard deviation
$n-1$ = degree of freedom

Given two independent samples $x_1, x_2, \ldots, x_{n_1}$
$y_1, y_2, \ldots, y_{n_2}$
with means $\bar{x}$ & $\bar{y}$ and standard deviations s_x and s_y respectively from a normal population with same variance, we have to test the hypothesis that the population means μ_1 & μ_2 are same.
The test statistic 't' is given by,

$$t_\alpha = \frac{\bar{x} - \bar{y}}{\sigma_s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$n_1 + n_2 - 2$ = degree of freedom

$$\sigma_s = \frac{1}{n_1 + n_2 - 2}\left((n_1 - 1)s_x^2 + (n_2 - 1)s_y^2\right)$$

F-test

Let $(x_1, x_2, \ldots\ldots, x_{n_1})$ & $(y_1, y_2, \ldots\ldots, y_{n_2})$ be the values of two independent random samples drawn from 2 normal populations having variances σ^2. The test statistic 'F' is denoted by

$$F = \frac{s_1^2}{s_2^2}$$

$$s_1^2 > s_2^2$$

where $s_1^2 = \frac{1}{n_1 - 1} \sum (x - \bar{x})^2$

$s_2^2 = \frac{1}{n_2 - 1} \sum (y - \bar{y})^2$

The degrees of freedom are $\nu = (n_1 - 1, n_2 - 1)$

Use table to find f at $f_{(n_1-1,\, n_2-1)}$ ☆

χ^2-test

(at 5% = 9.485, at 1% =

If $O_1, O_2, O_3, \ldots\ldots, O_n$ be a set of observed frequencies and $E_1, E_2, E_3, \ldots\ldots, E_n$ are the set of expected frequencies then the test statistic χ^2 is given by,

$$\chi^2 = \frac{(O_1 - E_1)^2}{E_1} + \frac{(O_2 - E_2)^2}{E_2} + \cdots\cdots + \frac{(O_n - E_n)^2}{E_n}$$

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

where degrees of freedom $\nu = n - 1$

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

O_i = observation frequency

E_i = Estimated frequency

Expected frequency = Pdf for the distribution as given in question function × Total observation frequency

χ^2 calculated < χ^2 tabulated
Ho : accept.

NOTE: For testing of attributes,

	Y	N	
Y	a	b	a+b
N	c	d	c+d
	a+c	b+d	

$N = (a+b) + (c+d)$
(or)
$(a+c) + (b+d)$

$$\chi^2 = \frac{N(ad-bc)^2}{(a+b)(c+d)(a+c)(b+d)}$$

here $N = a+b+c+d$