Duality in LPP :

Assosiated with every LPP (maximisation or minimisation) there always exists another LPP which is based upon the same data & having the same solution. The original problem is called the primal problem, while the assosiated problem is called its duel problem.

It is important to note that either of the two LPP's can be treated as primal & its dual. Thus the 2 problems constitute a primal dual pair.

Eye for an Eye

General primal - Dual pair :

Based on the standard form of primal, there are 2 important primal dual pair

(Type 1) Max $\Rightarrow \leq$

Definatn 1 : Standard form of primal problem is

$$\text{Maximize } Z = c_1 x_1 + c_2 x_2 + \dots + c_n x_n$$

Subject to constraints are

$$a_{i1} x_1 + a_{i2} x_2 + \dots + a_{in} x_n = b_i$$

Where $x_j \geq 0$

Dual problem

$i = 1, 2, \dots m$

$j = 1, 2 \dots n$

$$\text{Minimize } Z^* = b_1 y_1 + b_2 y_2 + \dots + b_m y_m$$

Subject to constraints are

$$a_{1j} y_1 + a_{2j} y_2 + \dots + a_{mj} y_m \geq c_j$$

$j = 1, 2, \dots n$

$i = 1, 2, \dots m$

all y_i's are unrestricted variables.

Here $x_1, x_2, \ldots x_n$ are primal variables & $y_1, y_2, \ldots y_m$ are the dual variables

Type 2: Standard primal problem is

Min

$\geq$

Minimize $z = c_1 x_1 + c_2 x_2 + \ldots + c_n x_n$

subject to constraints are

$a_{i1} x_1 + a_{i2} x_2 + \ldots + a_{in} x_n = b_i$

where $x_j \geq 0$

&

$i = 1, 2, \ldots m$

$j = 1, 2, \ldots n.$

Dual problem

Maximize $z^* = b_1 y_1 + b_2 y_2 + \ldots + b_m y_m$

subject to constraints are

$a_{1j} y_1 + a_{2j} y_2 + \ldots + a_{mj} y_m \leq c_j$

where $j = 1, 2, \ldots n$

$i = 1, 2, \ldots m$

all y_i's are unrestricted.

Here $x_1, x_2, \ldots x_n$ are primal variables & $y_1, y_2 \ldots y_n$ are dual variables

Note:

i) Identify the variables to be used in the dual problem, the no. of these (dual) variables equals to the no. of constraints in the primal problem.

ii) If the primal problem is of maximisation type the dual will be minimization type & vice versa.

iii) The objective funct^n of dual problem is constructing using the RHS constants of the primal constraints.

iv) The column of coefficient of primal constraints become the row coefficients of dual constraints. & the coefficients of primal objective function becomes the RHS constants of the dual constraints.

Problem

1) Formulate the Dual of following LPP.

Maximize $Z = 5x_1 + 3x_2$.

Subject to constraints are ~~$5x_1 + 2x_2 \le 10$~~

$$3x_1 + 5x_2 \le 15$$
$$5x_1 + 2x_2 \le 10 \quad \forall\ x_1, x_2 \ge 0.$$

Sol By introducing the slack variables, $S_1, S_2 \ge 0$ in the constraints of the given LPP

the standard primal LPP is

Maximize $Z = 5x_1 + 3x_2 + 0S_1 + 0S_2$

Subject to constraints are

$$3x_1 + 5x_2 + S_1 = 15$$
$$5x_1 + 2x_2 + S_2 = 10, \quad x_1, x_2 \ge 0.$$

Let y_1 & y_2 be the Dual ~~problem~~ variables corresponding to the primal constraints

Then the ~~dual~~ objective function of dual problem will be

Minimize $Z^* = 15y_1 + 10y_2$

Subject to constraints are

$3y_1 + 5y_2 \geq 5$

$5y_1 + 2y_2 \geq 3$

coefficient of S_1 => $1y_1 + 0.y_2 \geq 0$ => $y_1 \geq 0$

coefficient of S_2 => $0y_1 + 1.y_2 \geq 0$ => $y_2 \geq 0$.

Here y_1 & y_2 are unrestricted.

2) Write the Duality of following LPP.

Minimize $Z = 4x_1 + 6x_2 + 18x_3$

Subject to the constraints are $x_1 + 3x_2 \geq 3$

$x_2 + 2x_3 \geq 5$, $x_1, x_2, x_3 \geq 0$.

By introducing the surplus variables, $S_1, S_2 \geq 0$ in the constraints of given LPP the standard primal LPP is

Minimize $Z = 4x_1 + 6x_2 + 18x_3 + 0.S_1 + 0.S_2$

Subject to constraints are

$x_1 + 3x_2 + \cancel{0.x_3} - S_1 + 0.S_2 = 3$

$0.x_1 + x_2 + 2x_3 + 0.S_1 - S_2 = 5$

Let y_1 & y_2 be the 2 dual variable corresponding to the primal constraints

The objective function of dual problem is

Maximize $Z^* = 3y_1 + 5y_2$

Subject to constraints are

$y_1 + 0.y_2 \leq 3$ => $y_1 \leq 3$ & $0y_1 - y_2 \leq 0$

$3y_1 + y_2 \leq 6$ => $y_2 \geq 0$

$0.y_1 + 2y_2 \leq 18$ => $y_2 \leq 9$

$-y_1 + 0.y_2 \leq 0$ => $y_1 \geq 0$ & y_1 & y_2 are unrestricted