

SUBSET – SUM PROBLEM

PROBLEM & GOAL

- We are given n items $\{1, \dots, n\}$, and each has a given nonnegative weight w_i (for $i = 1, \dots, n$).
- We are also given a bound W .
- We would like to select a subset S of the items so that $\sum_{i \in S} w_i \leq W$ and, subject to this restriction, $\sum_{i \in S} w_i$ is as large as possible.
- We will call this the Subset Sum Problem.

DESIGNING THE ALGORITHM

- Let us consider an optimal solution.
- $\text{OPT}(n, W) = \max$ profit subset of items $1, \dots, n$ with weight limit W .
- There can be 2 cases if we consider an n th item as follows
 - **Case 1:** OPT does not select item n i.e. $n \notin \text{OPT}$
 - OPT selects best of $\{ 1, 2, \dots, n-1 \}$ using weight limit W .
 - **Case 2:** OPT selects item n . (i.e. $n \in \text{OPT}$)
 - New weight limit $= W - w_n$
 - OPT selects best of $\{ 1, 2, \dots, n-1 \}$ using **this new weight limit**.

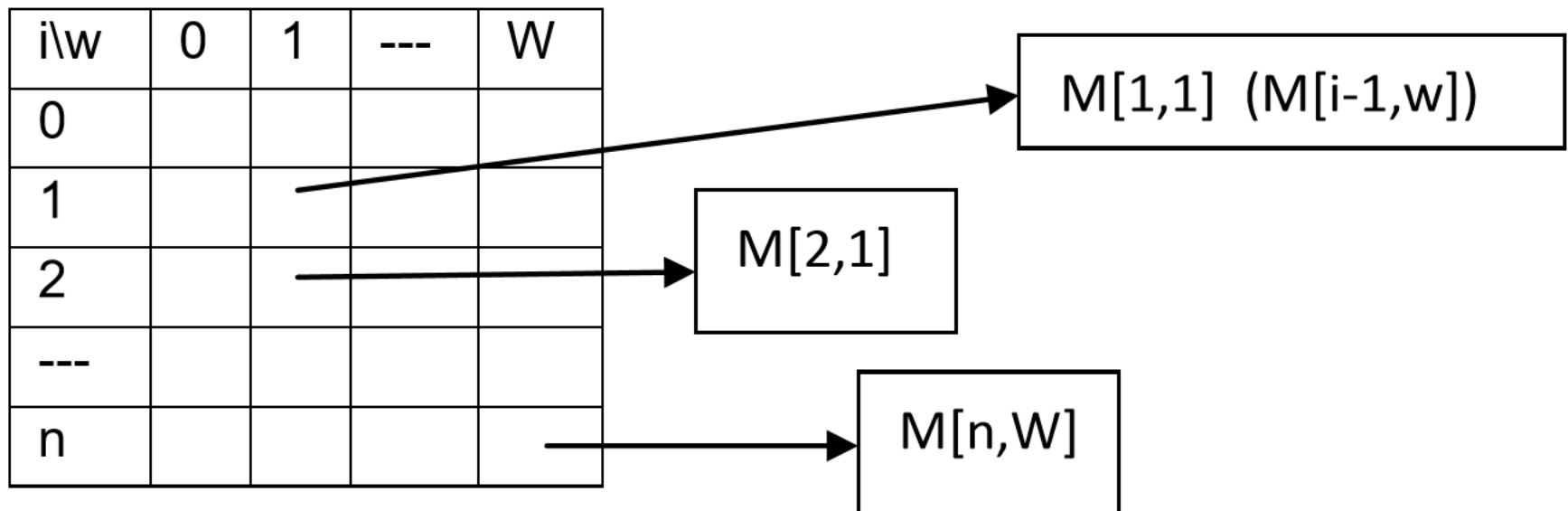
DESIGNING THE ALGORITHM

- Recurrence Relation

$$OPT(n, W) = \begin{cases} 0 & \text{if } n = 0 \\ OPT(n-1, W) & \text{if } w_n > W \\ \max\{OPT(n-1, W), w_n + OPT(n-1, W - w_n)\} & \text{otherwise} \end{cases}$$

DESIGNING THE ALGORITHM

- We need two dimensional table of (n, W) to fill in the values in order to get the maximum subset of weights of the items.



SUBSET – SUM ALGORITHM

//Purpose: To find the maximum weight from the given n items and their weights w_i

//Input: A set of items $1, 2, \dots, n$, with, w_1, \dots, w_N , capacity W

//Output: Max weight $M[n, W]$

```
for w = 0 to W
    M[0, w] = 0
for i = 0 to n
    M[i, 0] = 0
for i = 1 to n                // n items
    for w = 1 to W            // weights from 1 to max cap W
        if ( $w_i > w$ )
            M[i, w] = M[i-1, w]
        else
            M[i, w] = max {M[i-1, w],  $w_i + M[i-1, w-w_i]$  }
    endfor
endfor
return M[n, W]
```

SUBSET – SUM EXAMPLE

ITEMS	WEIGHTS	$W = 5$
1	2	
2	1	
3	3	

	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0					
2	0					
3	0					

SUBSET – SUM EXAMPLE

i	wi	w	wi > w	M [i, w]
1	2	1	2 > 1 TRUE	$= M[i-1, w]$ $= M[0, 1]$ $= 0$
		2	2 > 2 FALSE	$= \text{Max}\{ M[i-1, w], w_i + M[i-1, w-w_i] \}$ $= \text{Max} (M [0 , 2] , 2 + M [0 , 0])$ $= \text{Max}(0,2)$ $= 2$

	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	0	2			
2	0					
3	0					

SUBSET – SUM EXAMPLE

i	w _i	w	w _i > w	M [i, w]
1	2	3	2 > 3 FALSE	$= \text{Max}\{ M[i-1, w], w_i + M[i-1, w-w_i] \}$ $= \text{Max} (M [0 , 3] , 2 + M [0 , 1])$ $= \text{Max} (0 , 2)$ $= 2$
		4	2 > 4 FALSE	$= \text{Max}\{ M[i-1, w], w_i + M[i-1, w-w_i] \}$ $= \text{Max} (M [0 , 4] , 2 + M [0 , 2])$ $= \text{Max} (0 , 2)$ $= 2$

	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	0	2	2	2	
2	0					
3	0					

SUBSET – SUM EXAMPLE

i	w _i	w	w _i > w	M [i, w]
1	2	5	2 > 5 FALSE	$= \text{Max}\{ M[i-1, w], w_i + M[i-1, w-w_i] \}$ $= \text{Max} (M [0 , 5] , 2 + M [0 , 3])$ $= \text{Max} (0 , 2)$ $= 2$

	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	0	2	2	2	2
2	0					
3	0					

SUBSET – SUM EXAMPLE

i	w _i	w	w _i > w	M [i, w]
2	1	1	1 > 1 FALSE	$= \text{Max}\{ M[i-1, w], w_i + M[i-1, w-w_i] \}$ $= \text{Max} (M [1, 1], 1 + M [1, 0])$ $= \text{Max} (0, 1)$ $= 1$
		2	1 > 2 FALSE	$= \text{Max}\{ M[i-1, w], w_i + M[i-1, w-w_i] \}$ $= \text{Max} (M [1, 2], 1 + M [1, 1])$ $= \text{Max} (2, 1)$ $= 2$

	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	0	2	2	2	2
2	0	1	2			
3	0					

SUBSET – SUM EXAMPLE

i	w _i	w	w _i > w	M [i, w]
2	1	3	1 > 3 FALSE	$= \text{Max}\{ M[i-1, w], w_i + M[i-1, w-w_i] \}$ $= \text{Max} (M [1, 3], 1 + M [1, 2])$ $= \text{Max} (2, 3)$ $= 3$
		4	1 > 4 FALSE	$= \text{Max}\{ M[i-1, w], w_i + M[i-1, w-w_i] \}$ $= \text{Max} (M [1, 4], 1 + M [1, 3])$ $= \text{Max} (2, 3)$ $= 2$

	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	0	2	2	2	2
2	0	1	2	3	3	
3	0					

SUBSET – SUM EXAMPLE

i	w _i	w	w _i > w	M [i, w]
2	1	5	1 > 5 FALSE	$= \text{Max}\{ M[i-1, w], w_i + M[i-1, w-w_i] \}$ $= \text{Max} (M [1, 5], 1 + M [1, 4])$ $= \text{Max} (2, 3)$ $= 3$

	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	0	2	2	2	2
2	0	1	2	3	3	3
3	0					

SUBSET – SUM EXAMPLE

i	wi	w	wi > w	M [i, w]
3	3	1	3 > 1 TRUE	$= M[i-1, w]$ $= M [i - 1 , w]$ $= M [3 - 1 , 1]$ $= 1$
		2	3 > 2 TRUE	$= M[i-1, w]$ $= M [i - 1 , w]$ $= M [3 - 1 , 2]$ $= 2$

	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	0	2	2	2	2
2	0	1	2	3	3	3
3	0	1	2			

SUBSET – SUM EXAMPLE

i	w _i	w	w _i > w	M [i, w]
3	3	3	3 > 3 FALSE	$= \text{Max}\{ M[i-1, w], w_i + M[i-1, w-w_i] \}$ $= \text{Max} (M [2, 3], 3 + M [2, 0])$ $= \text{Max} (3, 3)$ $= 3$
		4	3 > 4 FALSE	$= \text{Max}\{ M[i-1, w], w_i + M[i-1, w-w_i] \}$ $= \text{Max} (M [2, 4], 3 + M [2, 1])$ $= \text{Max} (3, 4)$ $= 4$

	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	0	2	2	2	2
2	0	1	2	3	3	3
3	0	1	2	3	4	

SUBSET – SUM EXAMPLE

i	w _i	w	w _i > w	M [i, w]
3	3	3	3 > 5 FALSE	$= \text{Max}\{ M[i-1, w], w_i + M[i-1, w-w_i] \}$ $= \text{Max} (M [2 , 5] , 3 + M [2 , 2])$ $= \text{Max} (3 , 5)$ $= 5$

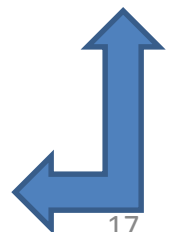
	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	0	2	2	2	2
2	0	1	2	3	3	3
3	0	1	2	3	4	5

SUBSET – SUM EXAMPLE

ITEMS	WEIGHTS	$W = 5$
1	2	
2	1	
3	3	

	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	0	2	2	2	2
2	0	1	2	3	3	3
3	0	1	2	3	4	5

SOLUTION



THANK YOU