von KARMAN INSTITUTE FOR FLUID DYNAMICS

PROJECT
Development of low-order shock-capturing scheme for discontinuous Galerkin method

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13th December 2018

Evaluation Team Meeting

von Karman Institute for Fluid Dynamics
Aeronautics & Aerospace department

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[4] *Senior Research Engineer in the Aeronautics and Aerospace Department, VKI*

OUTLINE

Introduction
- Atmospheric re-entry
- General methodology to tackle a problem

Current tools available & background
- Main tools available
- The DG method

Aim of the project

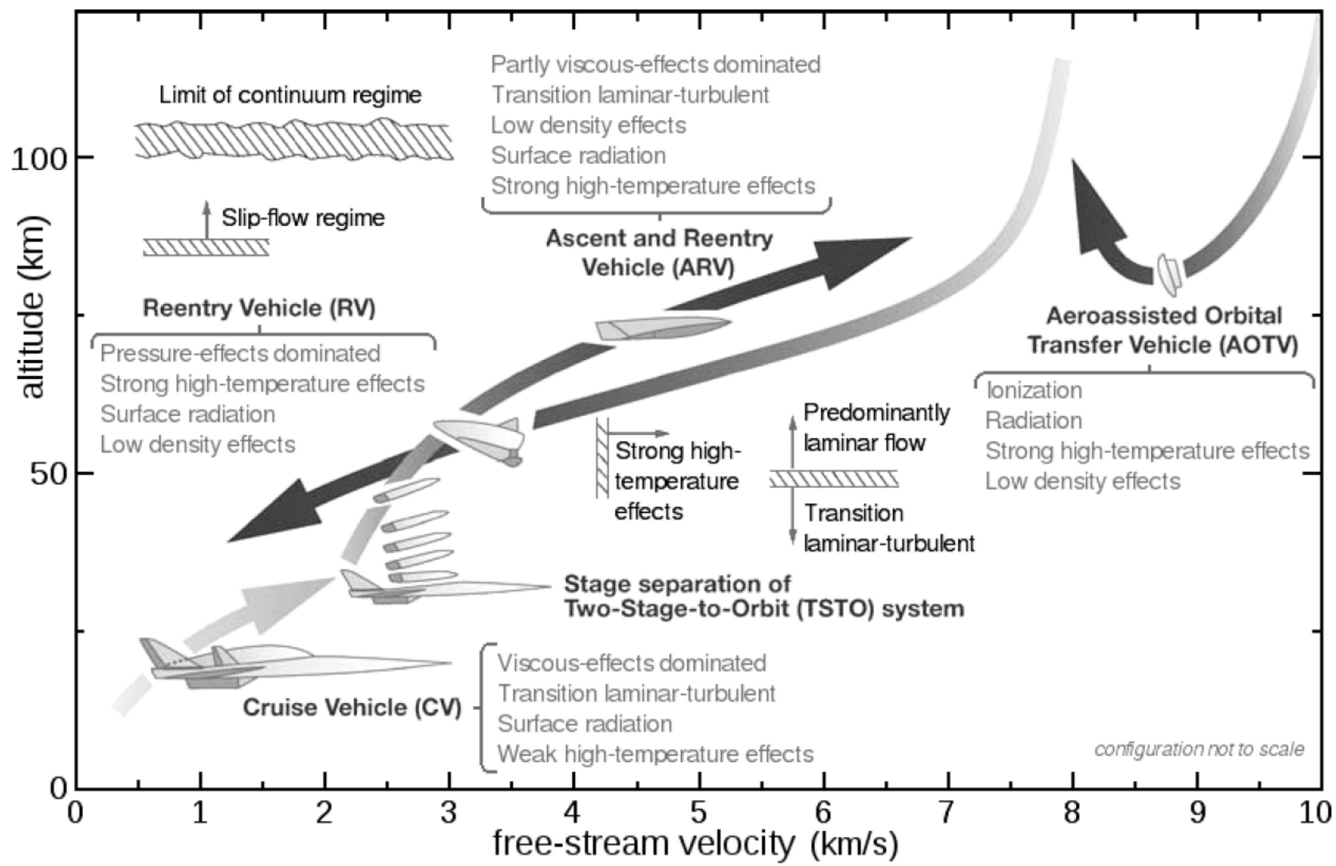
The Argo platform
- Code structure
- Shock treatment for high order method
- Interior Penalty method for the diffusive term

Methodology followed

Gantt Chart

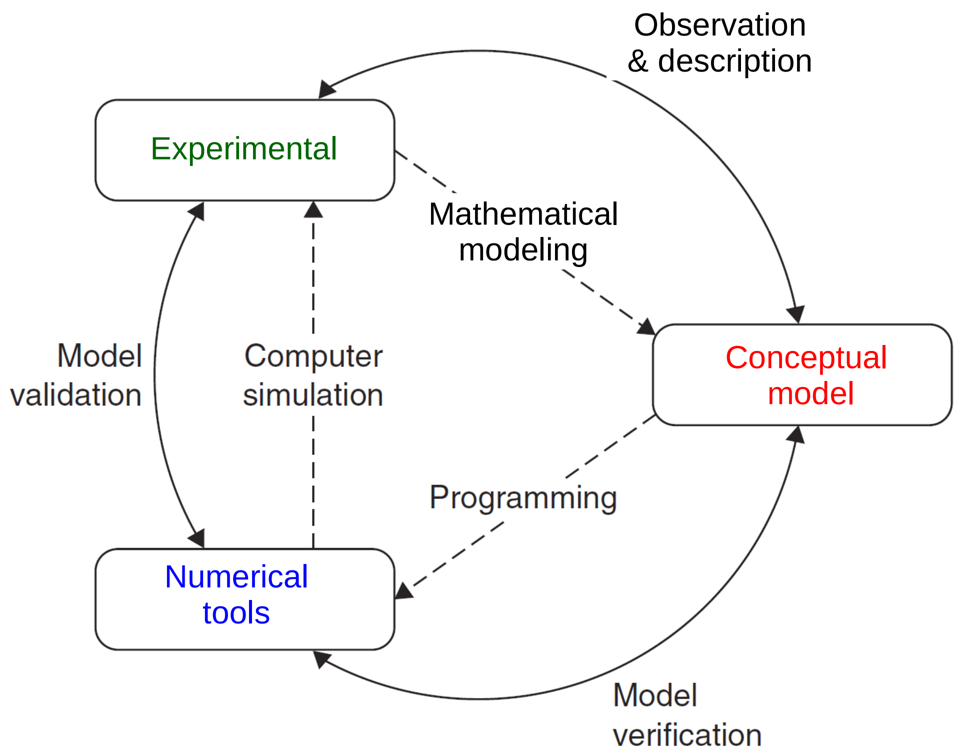
CHALLENGES

- **Measure the aerodynamic forces and heat flux on a body**: → Experiments in wind tunnels
  → Simulation by means of CFD
- **Hypersonic flow**: → Multi-physics phenomena
  → High temperature and compressible effects



Major classes of hypersonic vehicles and some characteristic aerothermodynamic phenomena (Hirschel, 2005)

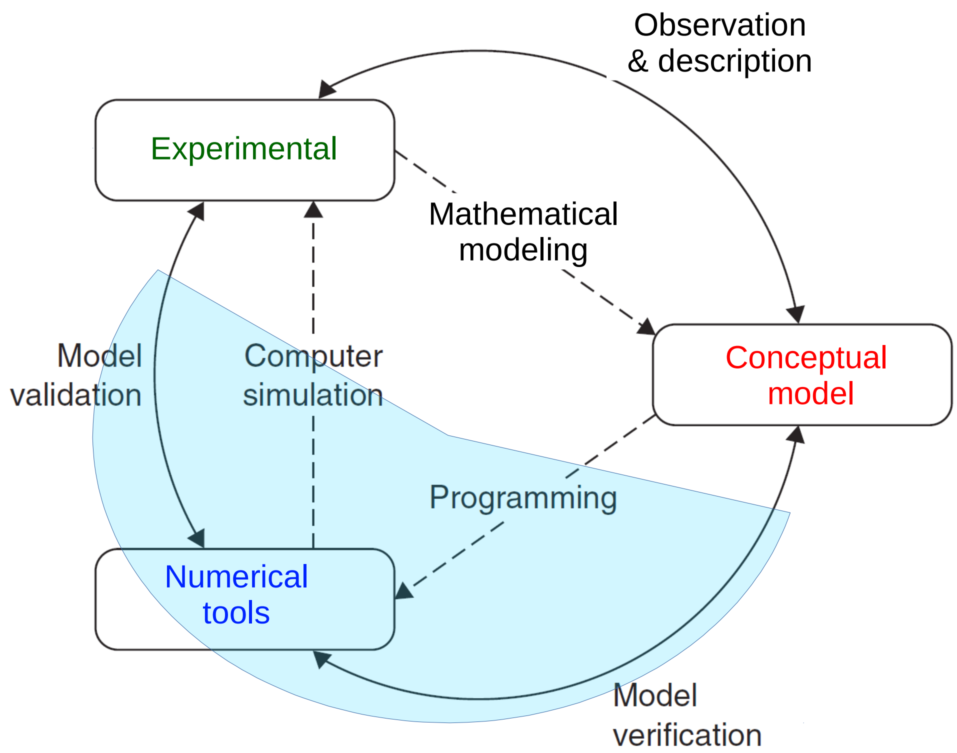
Introduction



Develop & use numerical tools for simulating the flow over a body in a facility in order to :

- provide a better understanding of the physical phenomena inside the facility
- assist the design of new components of the facility
- be a key component in fully specifying the facility's test flow conditions

Introduction



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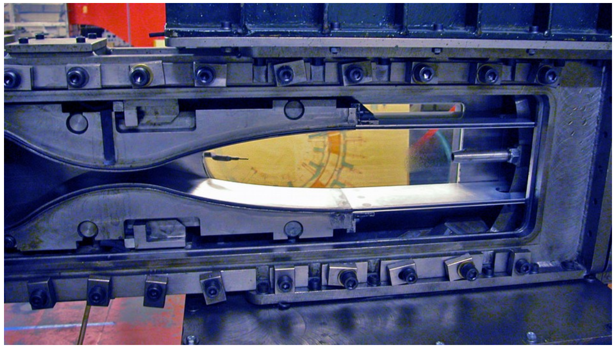
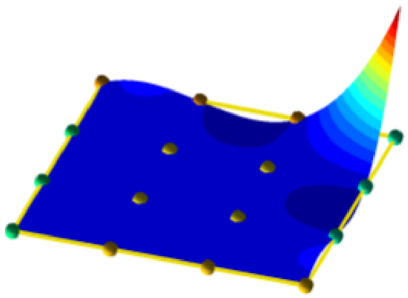
Current tools available & background



Tools available at the VKI



experimental

numerical

Wind tunnel testing facilities

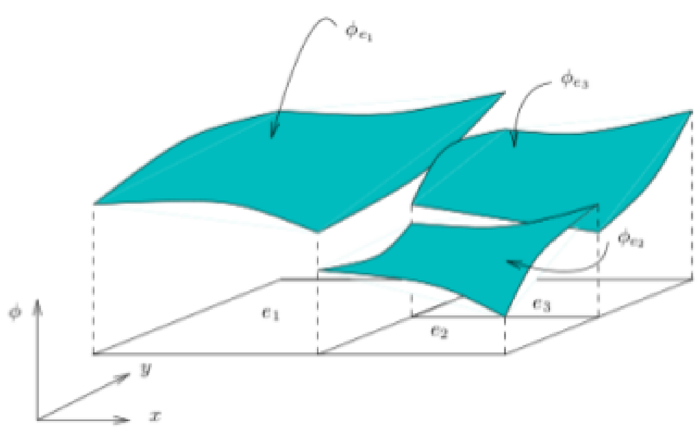
- Supersonic Wind Tunnel S-4 (Mach 3.5)
- Hypersonic Wind Tunnel H-3 (Mach 6)
- Longshot Hypersonic Gun Tunnel (Mach 10-20)

Platform

- **ARGO** Multiphysics, high resolution and high performance platform based on Discontinuous Galerkin method (DG) developed at Cenaero, a research centre in Belgium.

Why does the platform Argo use the DG method as main discretization space method for the Navier Stokes equations?

Name of the method	Geometry Complexity	Order of the method & hp-adaptability	Shock capturing at high Mach number
Finite difference (FD)	✗	✓	✓
Finite volume (FV)	✓	✗	✓
Finite element (FE)	✓	✓	✗
Discontinuous Galerkin (DG)	✓	✓	?



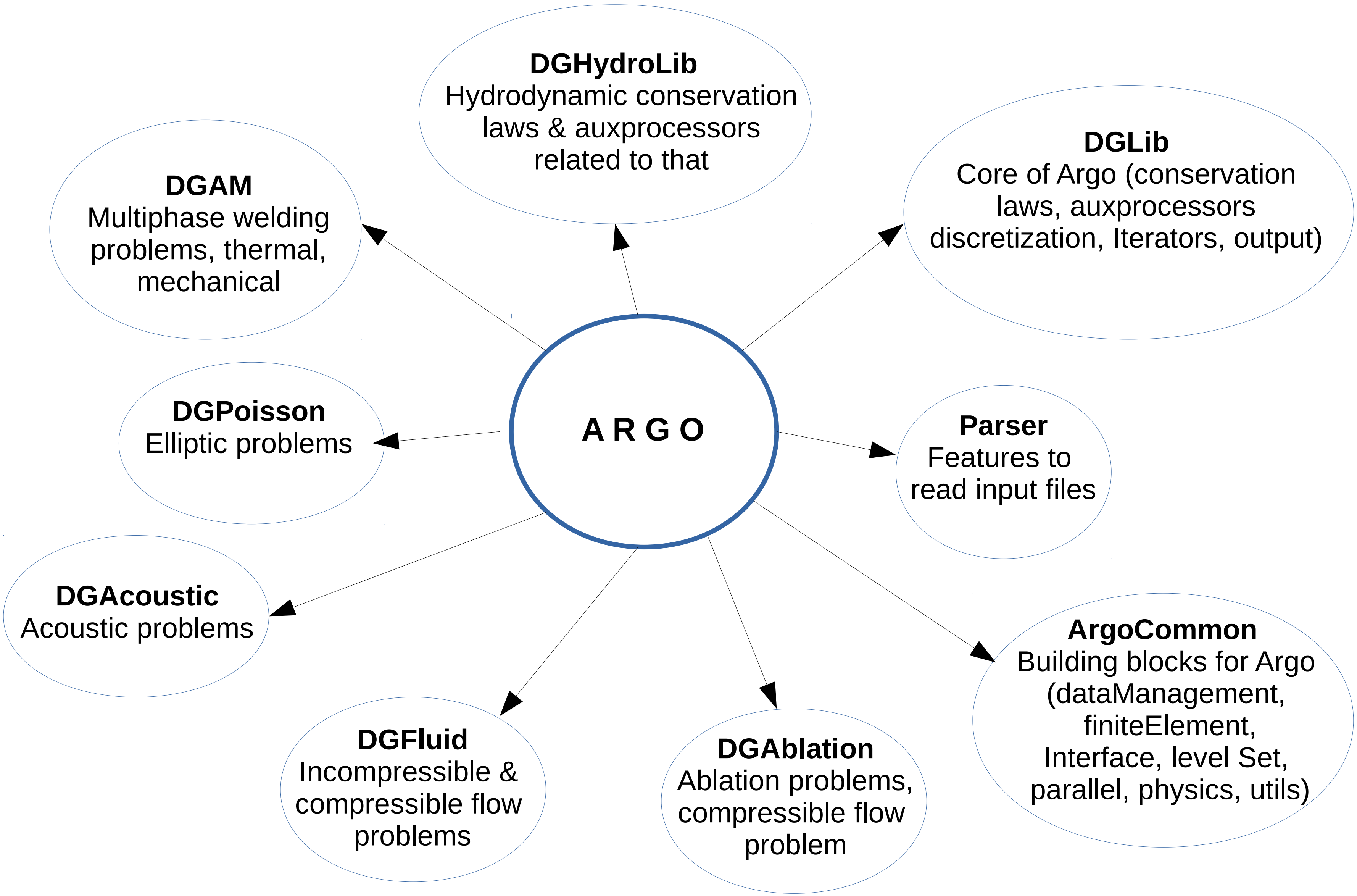
Aim of the project

Develop a robust shock capturing scheme for the DG method

Main tasks

1. Review the hypersonic flow challenges for **high order numerical simulation**
2. Identify the limit of the **artificial viscosity** into the Argo platform
3. Optimize and adapt a hybrid solver for hypersonic applications which uses a **degraded order scheme** (BR2) in the shock region and a high order method (DG) everywhere else
4. Validation on a specific test case

The Argo platform



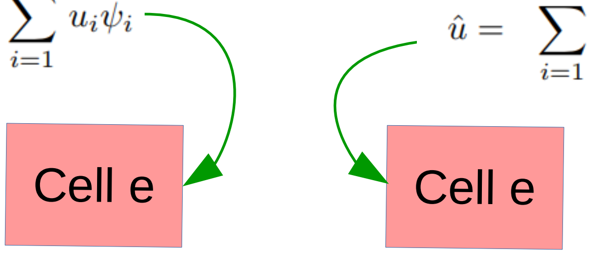
Shock detector and artificial viscosity

Shock detectors are part of most shock capturing techniques and have for purpose to identify the location of discontinuities in the computational domain

Artificial viscosity makes shocks broader, so that they can be resolved over several grid cells.

Method:

$$u = \sum_{i=1}^{N(p)} u_i \psi_i \qquad \hat{u} = \sum_{i=1}^{N(p-1)} u_i \psi_i$$



IF(the polynomials shapes are similar**)**
Then the solution on the cell e is smooth
Else the solution contains discontinuities

Sensor of the discontinuity on the cell e:

$$S_e = \frac{(u - \hat{u}, u - \hat{u})_e}{(u, u)_e}$$

$(\cdot,\cdot)_e$ inner product in $L_2(\Omega_e)$

Inputs defined by the user:

- The threshold $s_0 \sim 1/p^4$
- The interval κ
- The amount of artificial viscosity $\varepsilon_0 \sim h/p$

Artificial viscosity for each cell e:

$$\varepsilon_e = \begin{cases} 0 & \text{if} \quad s_e < s_0 - \kappa \\ \frac{\varepsilon_0}{2}\left(1 + \sin \frac{\pi(s_e - s_0)}{2\kappa}\right) & \text{if} \quad s_0 - \kappa \le s_e \le s_0 + \kappa \\ \varepsilon_0 & \text{if} \quad s_e > s_0 + \kappa \end{cases}$$

with $s_e = \log_{10} S_e$

Main weakness of sensors

- The quantity needs to be compared to a threshold defined arbitrarily
- The method can not be used for p=0

<u>Discretisation of the **diffusive term**</u> : The Interior penalty method limitation

Gouverning PDE's expressed in conservative and hypervectorial form:

$$\frac{\partial \mathbf{U}}{\partial t} + \nabla \cdot \mathbf{F}(\mathbf{U}) = \nabla \cdot \mathbf{F}_v(\mathbf{U}, \nabla \mathbf{U}) + \mathbf{S}(\mathbf{U}, \nabla \mathbf{U})$$

<u>Weak form of the equation (DG method)</u>:

$$\sum_{\Omega_e} \int_{\Omega_e} v \frac{\partial u}{\partial t} d\Omega_e + \sum_{\Omega_e} \int_{\Omega_e} v \frac{\partial F(u)}{\partial x} d\Omega_e = \sum_{\Omega_e} \int_{\Omega_e} v \frac{\partial}{\partial x}\left(D \frac{\partial u}{\partial x}\right) d\Omega_e$$

$$[v] = v^- n^- + v^+ n^+$$

$$\langle \bullet \rangle = \frac{1}{2}(\bullet^- + \bullet^+)$$

<u>Rewritting the diffusive term as</u>:

$$\boxed{\begin{aligned}\sum_{\Omega_e} \int_{\Omega_e} v \frac{\partial}{\partial x}\left(D \frac{\partial u}{\partial x}\right) d\Omega_e &= \sum_{\Omega_e} \int_{\Omega_e} \frac{\partial v}{\partial x}\left(D \frac{\partial u}{\partial x}\right) d\Omega_e \\ &- \sum_{\partial\Omega_e} \int_{\partial\Omega_e} \left\langle D(u) \frac{\partial u}{\partial x} \right\rangle [v] dS - \theta \sum_{\partial\Omega_e} \int_{\partial\Omega_e} \left\langle D(u) \frac{\partial v}{\partial x} \right\rangle [u] dS + \alpha \sum_{\partial\Omega_e} \int_{\partial\Omega_e} [v][u] dS\end{aligned}}$$

The **interior penalty coefficient θ** can take the value -1, 0 or 1.

Discretisation of the **diffusive term** : The Interior penalty method limitation

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Rewritting the diffusive term as:

The Interior Penalty method is inconsistent at p = 0

$$\sum_{\Omega_e} \int_{\Omega_e} v \frac{\partial}{\partial x} \left(D \frac{\partial u}{\partial x} \right) d\Omega_e = \cancel{\sum_{\Omega_e} \int_{\Omega_e} \frac{\partial v}{\partial x} \left(D \frac{\partial u}{\partial x} \right)} d\Omega_e$$
$$- \cancel{\sum_{\partial\Omega_e} \int_{\partial\Omega_e} \left\langle D(u) \frac{\partial u}{\partial x} \right\rangle} [v] dS - \theta \sum_{\partial\Omega_e} \cancel{\int_{\partial\Omega_e} \left\langle D(u) \frac{\partial v}{\partial x} \right\rangle} [u] dS + \alpha \sum_{\partial\Omega_e} \int_{\partial\Omega_e} [v][u] dS$$

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Rewritting the diffusive term as:

Recent tool on Argo: The second scheme of Bassi and Rebay (BR2)

[Bram van Leer, Marcus Lo, "A Venerable Family of Discontinuous Galerkin Schemes for Diffusion Revisited", University of Michigan]

Methodology followed

TASKS	METHODOLOGY	
1. Review the hypersonic flow challenges for high order numerical simulation	Literature Study Key words: *Shock detection for high order scheme, limiters, artificial viscosity, BR2*	⌛
2. Identify the limit of the artificial viscosity into the Argo platform	1. Installation of the Argo platform ➢ *Installation of the required libraries (Madlib, Boost, ParMetis...)* ➢ *Installation of the software useful to open the outputs (Gmsh, Paraview)* 2. Assess the artificial viscosity by running several test cases*	⌛
3. Optimize and adapt a hybrid solver for hypersonic applications which uses a degraded-order scheme (BR2) in the shock region and a high order method (DG) everywhere else.	1. Assess the BR2 scheme in the Argo platform 2. Develop a robust hybrid scheme	✗
4. Validation on a specific test case	1. For a specific test case, comparison with experimental data from the facilities 2. Activation of the physico-chemical phenomena (Mutation++ Library)	✗

* The choice of the test cases will be based on the work of M. Cruellas Bordes in his MSc thesis **[2]**

Methodology followed

TASKS

3. Optimize and adapt a hybrid solver for hypersonic applications which uses a degraded-order scheme in the shock region and a high order method (DG) everywhere else

METHODOLOGY

1. **Assess the BR2 scheme in the Argo platform**
   a) Detect the shock wave to refine the mesh
   b) Study the effect on a structured/ unstructured mesh
2. **Develop a robust hybrid scheme**
   a) Define a method to reduce the order of the scheme (P0) around the shock wave area without loosing the advantage of the DG method of the domain
   b) Study and analyze the convergence and the robustness of the solver

T A S K S

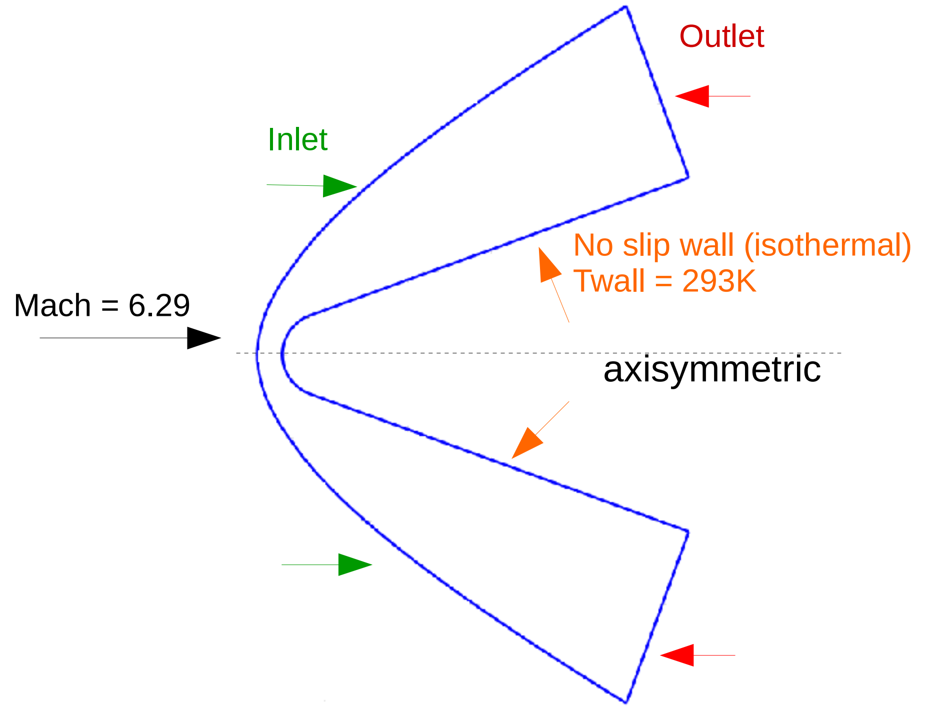
4. Validation on a specific test case

Physical model:
Study Domain and boundary conditions

The following axisymmetric blunted cone can be tested with:

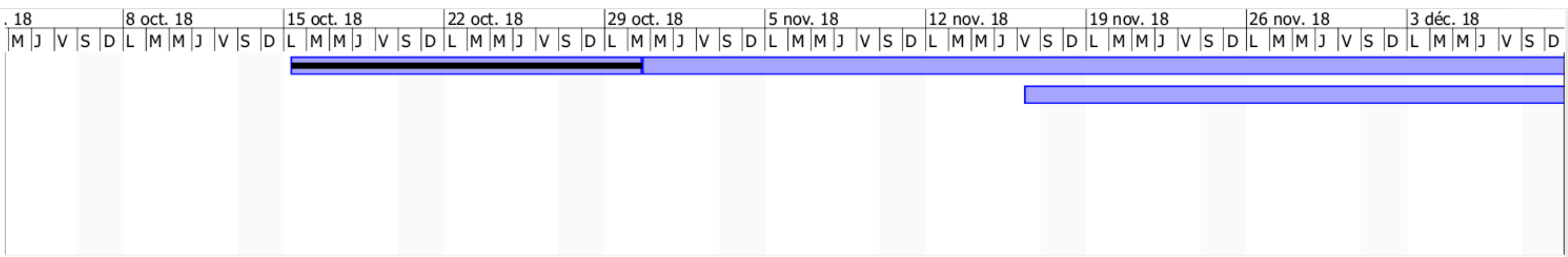
- H3 facility (calorically perfect gas)
- HEG facility (thermally perfect gas)

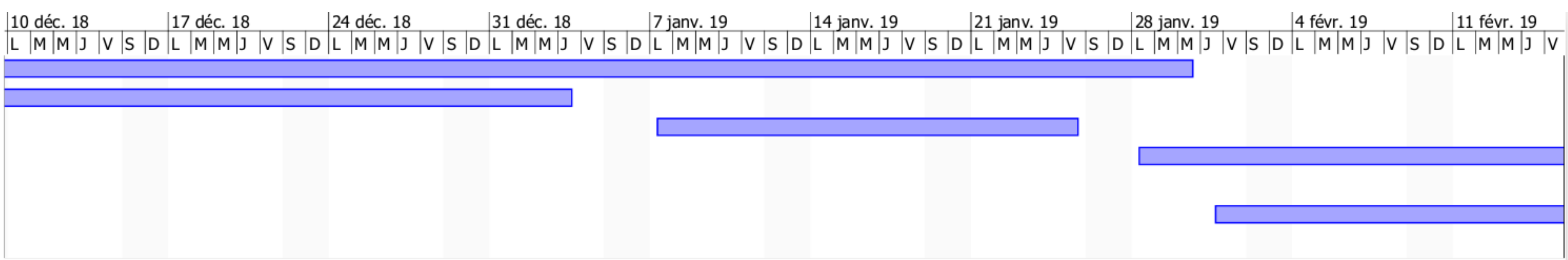
Flow Field: T = 1192K p = 6880Pa u = 4244m/s v = 0m/s



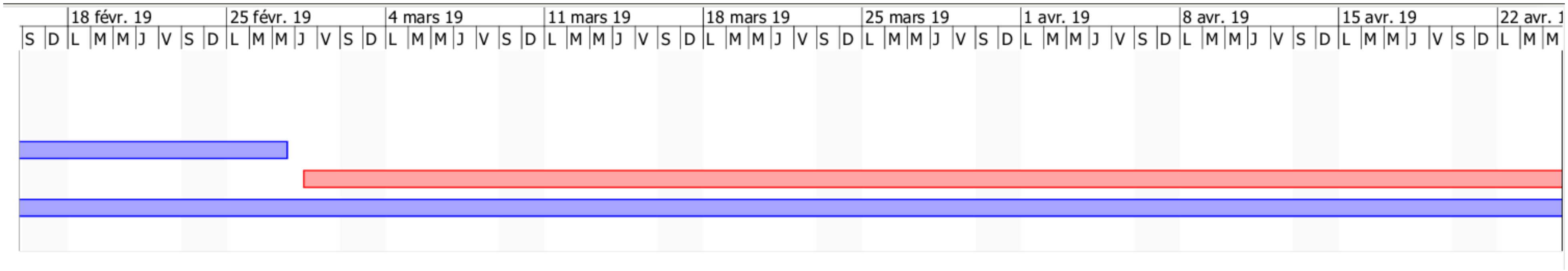
Gantt Chart

		Nom	Durée	Début	Fin	Prédécesseur
1		Literature study (+bibliogr...	78 jours	15/10/18 08:00	30/01/19 17:00	
2		Installation Argo platform	34,875 jours	16/11/18 08:00	03/01/19 16:00	
3		Run test cases AV	15 jours	07/01/19 08:00	25/01/19 17:00	2
4		Assess BR2 scheme	23 jours	28/01/19 08:00	27/02/19 17:00	3
5		Develop the Hybrid scheme	51,875 jours	28/02/19 09:00	10/05/19 17:00	4
6		Report	86,125 jours	31/01/19 16:00	31/05/19 17:00	2
7		verification/ validation	22 jours	13/05/19 08:00	11/06/19 17:00	5





Gantt Chart



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11 mars 19
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1 avr. 19
8 avr. 19
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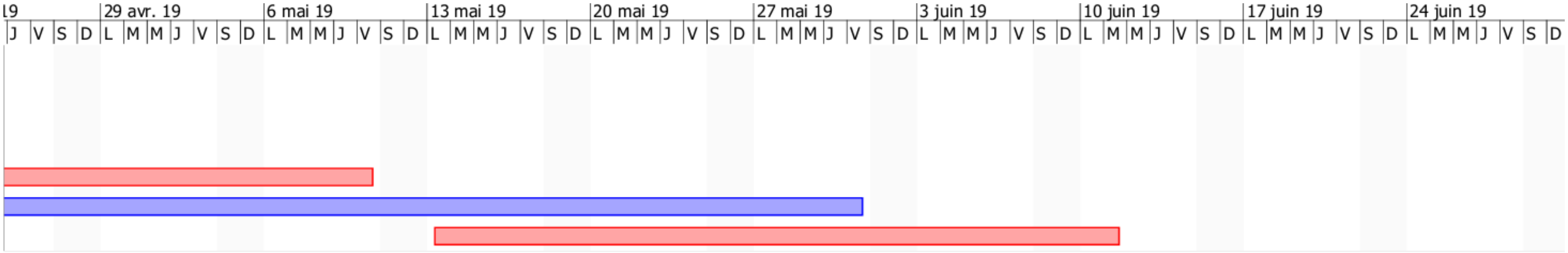

29 avr. 19
6 mai 19
13 mai 19
20 mai 19
27 mai 19
3 juin 19
10 juin 19
17 juin 19
24 juin 19

- https://www.vki.ac.be/
- Course introduction, Argo introduction, P. Schrooyen
- A study of an artificial viscosity technique for high-order discontinuous Galerkin methods, MSc Thesis, M. Cruellas Bordes