BANA4095: Decision Models – Spring 2020
Introduction to Optimization

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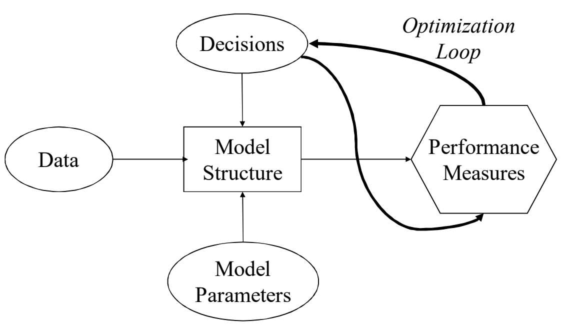
Outline

- What is Optimization?
- Key Elements of Optimization Models
- Optimization Algorithms
- Excel Solver
- Unconstrained vs. Constrained
- Linear Optimization

Model as a Laboratory for Experimentation



Optimization

- Finding an alternative that achieves the best possible result
  - » Unconstrained Optimization
  - » Constrained Optimization
- Four key elements of an optimization problem
  - » Objective function
  - » Decision variables
  - » Constraints
  - » Parameters

Optimization Elements

- Objective Function
  - » Measures the value of an alternative
  - » Basis for comparing alternatives
  - » Examples
- Decision Variables
  - » What choices/decisions affect the objective function?
  - » Values that must be chosen in order to define an alternative
  - » Consume or supply some resource

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Optimization Elements

- Constraints
  - » What resources or requirements limit the possible alternatives/decisions/choices?
  - » Determine whether a specific alternative is feasible
  - » Right-hand side: defines the amount of resource available or needed to satisfy the constraint
- Parameters
  - » Constants used in the model calculations
  - » Define relationships between decision variables, constraints, and objective function

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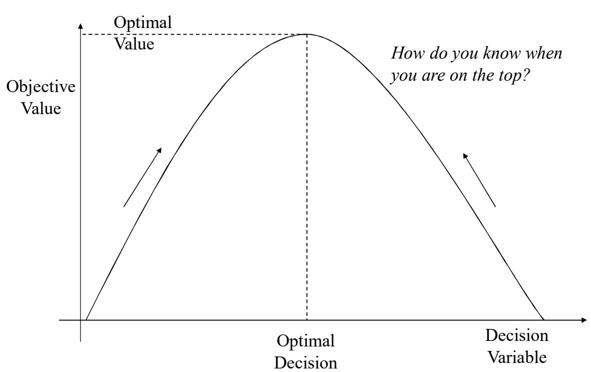
Optimization as Hill Climbing



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Example: Armstrong Bike Co.

Armstrong Bike Co. produces two new lightweight bicycle frames, the Flyer and the Razor, that are made from special aluminum and steel alloys. The cost to produce a Flyer frame is $100, and the cost to produce a Razor frame is $120. As the selling price of each frame model, P_F and P_R, increases, the weekly quantity demanded for each model, F and R, goes down linearly. ABC can produce at most 180 bikes per week.

$$F = 750 - 5P_F$$

$$R = 400 - 2P_R$$

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Example: Armstrong Bike Co.

- Define the following elements of this optimization problem
  - » Decision Variables
  - » Objective
  - » Constraints

Armstrong Bike Algebraic Formulation

$$\max \quad P_F F + P_R R - 100F - 120R$$ Maximize Total Profit

s.t. "Subject to"

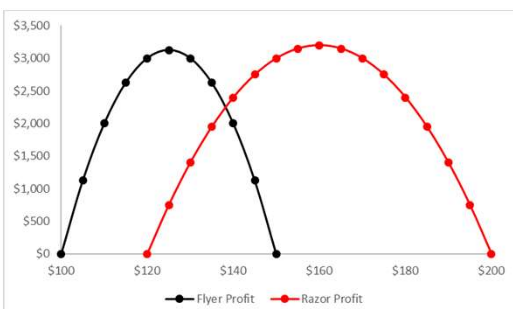
$$F = 750 - 5P_F$$
$$R = 400 - 2P_R$$ Price-Demand Curves

$$F + R \le 180$$ Capacity

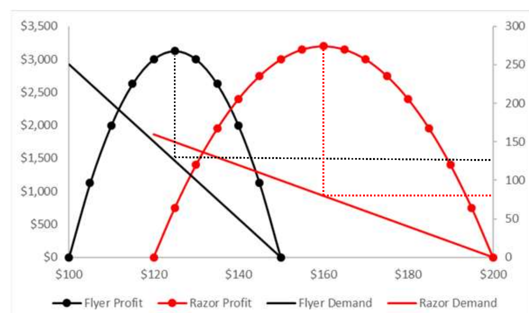
$$F, R, P_F, P_R \ge 0$$ Non-negative

This is also called a *Mathematical Programming Formulation*.

Armstrong Bike Co.



Armstrong Bike Co.



Solver

- Free Excel add-in for optimization
- PC and Mac versions
- Located on the Data tab
- Simple user interface
- Limited problem size
- Frontline Systems
  - » Developer of Solver
  - » Also offers commercial version: Analytic Solver Platform (ASP)

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Building a Solver Optimization Model

- Build base case spreadsheet model
  - » Decision variable cells
  - » Input parameter cells
  - » Objective function formula
  - » Constraint formula(s)
- Enter Solver settings
- Solve

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Solver Settings

- Set Objective (Max/Min)
  - » Select the cell containing the objective function formula
- Changing/Decision Variable Cells
  - » Select the cells containing the decision variable values
- Subject to the Constraints
  - » Add each constraint
  - » Cell Reference is cell containing the formula (left hand side)
  - » Constraint is cell containing the limiting value (right hand side)
  - » Drop down box selects the relationship between LHS and RHS
- Check box: Make unconstrained variables non-negative
- Select a Solving Method

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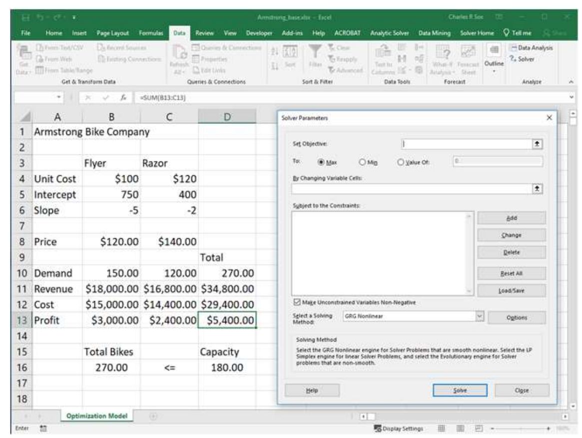

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Armstrong Bike Company

- Unconstrained Solution

	Flyer	Razor	Total
Price	$125.00	$160.00	
Quantity	125.00	80.00	205.00
Profit	$3,125.00	$3,200.00	$6,325.00

- Constrained Solution

	Flyer	Razor	Total
Price	$128.57	$163.57	
Quantity	107.14	72.86	180.00
Profit	$3,061.22	$3,174.49	$6,235.71

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Special Cases of Optimization Models

- Infeasible Model
  - » There are no feasible solutions that satisfy all of the model constraints
  - » Usually caused by one or more errors in the definition of the constraints or the formula cells referenced in the constraints
- Unbounded Solution
  - » There is no finite feasible solution that is optimal. There is always another solution that has a better objective value.
  - » Usually caused by one or more errors in the definition of the objective or the constraints or the formulas used in the objective and constraints

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Types of Optimization Models

Type	Variables	Relationships
Linear Program (LP)	Continuous	Linear
Nonlinear Program (NLP)	Continuous	Nonlinear or Linear
Integer Program (IP)	Integer	Linear
Mixed Integer-Linear (MILP)	Integer or Continuous	Linear
Mixed Integer Nonlinear (MINLP)	Integer or Continuous	Nonlinear or Linear

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What is a Linear Program

- Special class of optimization models
  - » Objective function and all constraints are linear expressions
    - *a* and *b* are scalars (numbers); *x* and *y* are variables
      - $ax + b$ OR $ax + by$
      - NOT $ax^2 + bxy$
  - » Decision variables are continuous/fractional
- Easiest class of optimization models to solve
  - » Simplex Algorithm (1947, George Dantzig)
  - » Always finds an optimal solution; can find an optimal solution relatively quickly even for very large problems.

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Example – Sidneyville Desk Mfg.

- Allocation/Product Mix Problem
- Produces two types of desk
- Using three types of wood in every desk
  (measured in board feet, b.f.)

Type	Profit/desk
Rolltop	$115
Regular	$90

	Amount Used		Amount
Wood	Rolltop	Regular	Available
Pine	10	20	200
Cedar	4	16	128
Maple	15	10	220

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Example, cont.

- What are the decision variables?
- What is the objective function?
- What are the constraints?

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Sidneyville Linear Programming (LP) Formulation

max $115x_1 + 90x_2$	Maximize Total Profit
s.t. $10x_1 + 20x_2 \le 200$	Pine
$4x_1 + 16x_2 \le 128$	Cedar
$15x_1 + 10x_2 \le 220$	Maple
$x_1, x_2 \ge 0$	Non-negative

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Feasibility

- A feasible solution is a combination of decision variable values that satisfy all of the constraints
- The feasible region is the set of all feasible solutions
- Extreme points are the "corners" of the feasible region

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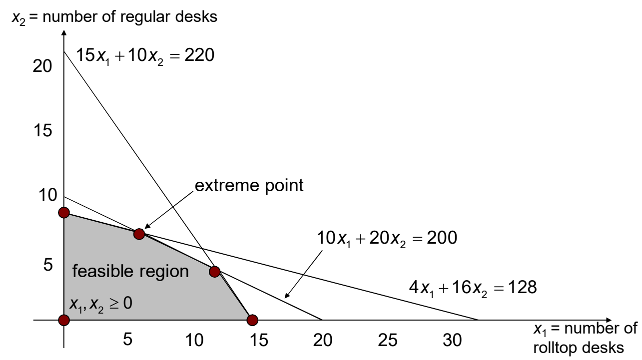
Sidneyville Feasible Region



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Optimality

- An optimal solution is a feasible solution that achieves the best possible objective function value within the feasible region
  - No other feasible solution has a better objective value
  - There may be multiple optimal solutions
- In an LP, at least one of the extreme points is an optimal solution
  - Graphical method
  - Simplex method (Excel Solver)

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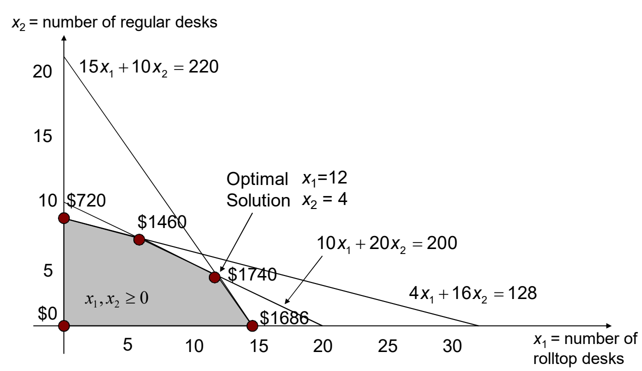
Sidneyville Graphical Solution



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Simplex Method

- Invented by George Dantzig in 1947
- The Simplex Method uses linear algebra to "pivot" from one vertex to another until it stops at an optimal vertex.
- The gradients of the objective function and the constraints are used to determine the search direction
- The gradients of the current active/binding constraints are used to compute the current vertex
- The algorithm stops when the gradient of the objective function can be expressed as a linear combination of the gradients of the active/binding constraints.

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Solver Engines/Algorithms

- Various algorithms that are used to solve different classes of optimization problems. The two basic engines in Solver are

Engine	Objective	Constraints
Simplex LP	Linear	Linear
GRG Nonlinear	Nonlinear	Nonlinear

- The specific Solver Engine is selected from the drop down list at the top of the Engine tab
- Uncheck the "Automatically Select Engine" box

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Non-Negativity

- Decision variables are frequently assumed/implied to have non-negative values. This assumption is often not explicitly given in the problem statement, but the modeler should recognize the need for this restriction even if it is not explicitly stated.

- Check box: Make Unconstrained Variables Non-Negative
  - » Constrains ALL decision variables to be non-negative
  - » If only SOME of the decision variables should be non-negative, then explicit bound constraints (>= 0) must be added to the model for the variables that require them.

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Sidneyville Manufacturing

- Construct an optimization model for Sidneyville and verify the optimal solution

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Summary

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- Unconstrained vs. Constrained
- Linear Optimization

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