BANA4095: Decision Models – Spring 2020
Linear Optimization, Part 2

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Review

- Optimization
- Excel Solver
- Types of Optimization Problems
- Linear Optimization/Programming (LP)

Outline

- Review HW4
- Binding/Non-Binding Constraints
- Marginal Value
- Sensitivity Analysis
- Modeling Tips

Example – Sidneyville Desk Mfg.

- Allocation/Product Mix Problem
- Produces two types of desk
- Using three types of wood in every desk
  (measured in board feet, b.f.)

Type	Profit/desk
Rolltop	$115
Regular	$90

	Amount Used		Amount
Wood	Rolltop	Regular	Available
Pine	10	20	200
Cedar	4	16	128
Maple	15	10	220

Sidneyville Linear Programming (LP) Formulation

$$\max \quad 115x_1 + 90x_2$$ Maximize Total Profit

s.t. $$10x_1 + 20x_2 \le 200$$ Pine

$$4x_1 + 16x_2 \le 128$$ Cedar

$$15x_1 + 10x_2 \le 220$$ Maple

$$x_1, x_2 \ge 0$$ Non-negative

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Binding vs. Non-Binding Constraints

- Example: Sidneyville
- Binding Constraint
  - » Left Hand Side = Right Hand Side
- Non-Binding Constraint
  - » Left Hand Side not equal to Right Hand Side
  - » Slack = RHS – LHS
- Marginal Value of a Resource
  - » If a constraint is non-binding then the marginal value of its corresponding resource is . . . ?

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Marginal Value / Shadow Price

- The change in the optimal objective value for one additional unit of a resource.
  - » Increases for maximization problems
  - » Decreases for minimization problems
  - » For LPs the marginal value is constant over a range
- What is the “economic value” of obtaining an additional amount of this resource?
- What would you be willing to pay for an additional amount of this resource?
- Example: Sidneyville

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LP Optimization Sensitivity Report

- Provides sensitivity information for the optimal solution
  - » Decision Variables,
  - » Objective Coefficients, and
  - » Constraints
- Decision Variable (DV) - Reduced Cost (RC)
  - » The RC for a DV is the shadow price associated with the non-negativity constraint for the DV. RC = 0 indicates the DV may be positive in an optimal solution; RC > 0 indicates the DV must be 0 in an optimal solution.
  - » If a DV = 0 and its RC = 0, there may be multiple optimal solutions.
    - – Methods to find alternate solutions

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LP Optimization Sensitivity Report

- Objective Coefficient – Allowable Increase/Decrease
  - » Indicates the range of coefficient values over which the current solution remains optimal
- Constraint – Shadow Price
  - » The marginal value or marginal cost of the right-hand-side (RHS) of the constraint
- Constraint – Allowable Increase/Decrease
  - » Indicates the range of RHS values over which the current shadow price remains constant
- Any change outside of an allowable range requires re-optimization of the model

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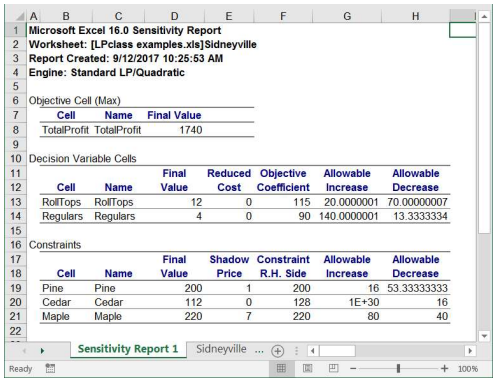
LP Optimization Sensitivity Report



Microsoft Excel 16.0 Sensitivity Report
Worksheet: [LPclass examples.xls]Sidneyville
Report Created: 9/12/2017 10:25:53 AM
Engine: Standard LP/Quadratic

Objective Cell (Max)

Cell	Name	Final Value
TotalProfit	TotalProfit	1740

Decision Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
RollTops	RollTops	12	0	115	20.0000001	70.00000007
Regulars	Regulars	4	0	90	140.0000001	13.3333334

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
Pine	Pine	200	1	200	16	53.33333333
Cedar	Cedar	112	0	128	1E+30	16
Maple	Maple	220	7	220	80	40

Sensitivity Report 1 | Sidneyville ...

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LP Optimization Sensitivity Report

- Answer the following questions using only the optimization sensitivity report on the previous slide. How does the optimal solution change when . . .

1. The profit for each Regular desk is increased by $20?
2. The sales price for each Rolltop desk increases by $25?
3. There is 15 additional board feet of Pine?
4. There is 100 additional board feet of Cedar?
5. We can purchase 40 additional board feet of Maple at a cost of $3 per board foot.

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Modeling Tip: SUMPRODUCT( ) Function

- Very useful for LP modeling
- SUMPRODUCT(A1:A3, B1:B3) =A1*B1+A2*B2+A3*B3

A1	B1
A2	B2
A3	B3

- SUMPRODUCT(A1:C1, A2:C2) = A1*A2+B1*B2+C1*C2

A1	B1	C1
A2	B2	C2

- SUMPRODUCT(A1:B2,C3:D4) = A1*C3+B1*D3+A2*C4+B2*D4

A1	B1
A2	B2

C3	D3
C4	D4

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Modeling Tip: Range Constraints

- Can use a range of cells for left-hand and/or right/hand side of a constraint to model multiple constraints
  - » D10:D15 >= E5
    - – Each cell in the range D10:D15 must be >= E5
  - » D10:D15 >= E10:E15
    - – Each cell in the range D10:D15 must be >= the corresponding cell in the range E10:E15

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Modeling Tip: Designing LP Spreadsheets

- Organize the LP model in a series of rows
- Each column of the model corresponds to a Decision Variable
- Decision Variables and Objective Coefficients at the top
- Constraints in a separate section below the DVs and Objective
- List similar constraints together

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Modeling Tip: Debugging

- Debugging optimization models can be difficult
- Read the error message carefully
  - » It may provide a clue about which model element(s) are generating the error
- Audit all the optimization model settings
- Are all the cell addresses/ranges accurate and complete?
  - » Min/Max? Assume Non-negative?
- Review/audit all the formulas in the spreadsheet
  - » Are they correctly computing the appropriate values?

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Summary

- Review HW4
- Binding/Non-Binding Constraints
- Marginal Value
- Sensitivity Analysis
- Modeling Tips

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