Arrange the given linear transformations in a hierarchal order, with the most general case listed first (Affine, Euclidean, Similarity, Projective). For each of these transformations, list two permissible operations, one preserved quantity, and the degrees of freedom (in both 2D and 3D).

I'm listing the order of the four transformation with decreasing generality: Project > Affine > Similarity > Eucidean

For Projective transformation,it includes rotation,scaling and so on.It's the most general transformation among these four,other transformation like affine transformation is just the special case of projective transformation. One preserved quantity is that line after projective transformation is also line, but no parallelism is guaranteed.Moreover,lengths and angles might be distorted too. A 2D projective transformation has 8 degrees of freedom.In 3D,there are 15 degree of freedom.

For Affine transformation,two permissible operations are translation and scaling. The preserved quantity during affine transformation is that the points in the same line before transformation will still be in the same line afterward.Parallel lines are still parallel lines,and straight line remain straight.However,Angles are not preserved. In 2D,the degree of freedom is 6. While in 3D, it's 12.

Similarity transformation is a subset of Affine transformation, it includes rotation and translaiton. Angles between lines, ratio of two lengths, ratio of areas are invariant under similarity transformation. Generally, it preserves the shape of the object.In 2D image processing,a similarity transformation has 4 degree of freedom.While in 3D, it's 7 degree.

The last case is Euclidean Transformations,and it's a subset of Similarity transformation.It describes the shape of object rigidly. Two permissible operation include rotation and translation. Under Euclidean transformations,angles between lines, length between two points, and area are invariant.Parallism is guarenteed. In 2D, it has 3 degree of freedom. And in 3D, there dof is 6 degree.

With reference to the above equation, derive a T matrix (3X2) to perform the following operations:

a) Scale an image by 1.7 times

```
In [1]: %pylab inline
        import IP
```

```
Populating the interactive namespace from numpy and matplotlib
```

```
In [3]: I = IP.data.coffee()
        shape(I)
```

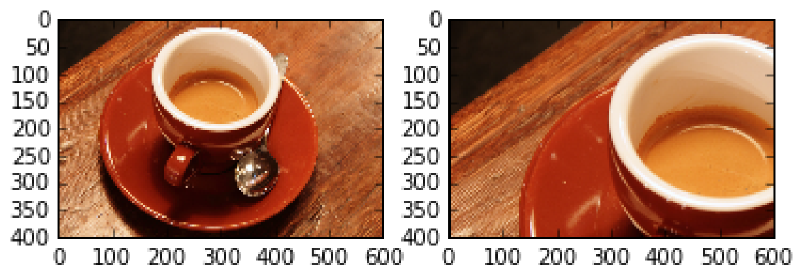
```
Out[3]: (400, 600, 3)
```

```
In [31]: T_scaling = array([[1.7,0,0],[0,1.7,0],[0,0,1]])
         T_scaling
```

```
Out[31]: array([[ 1.7,  0. ,  0. ],
                [ 0. ,  1.7,  0. ],
                [ 0. ,  0. ,  1. ]])
```

```
In [22]: subplot(121);imshow(I,cmap='gray');
         I_scaling = IP.warp(I,T_scaling)
         subplot(122);imshow(I_scaling,cmap='gray')
```

```
Out[22]: <matplotlib.image.AxesImage at 0x111e5b358>
```

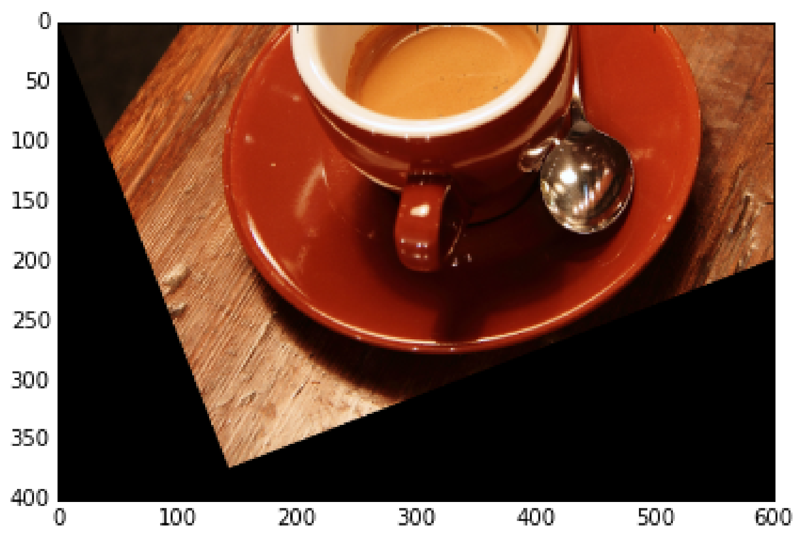


In this way, new coordinate (u,v) can be expressed in this way: u = 1.7x, v = 1.7y. Thus scaling the image by 1.7 times

b) Rotate an image by 21。

```
In [6]: T_rotation = array([[cos(radians(21)),-sin(radians(21)),0],[sin(radians(21)),cc
        I_rotation = IP.warp(I,T_rotation)
        imshow(I_rotation,cmap='gray')
        T_rotation
```

```
Out[6]: array([[ 0.93358043, -0.35836795,  0.        ],
               [ 0.35836795,  0.93358043,  0.        ],
               [ 0.        ,  0.        ,  1.        ]])
```



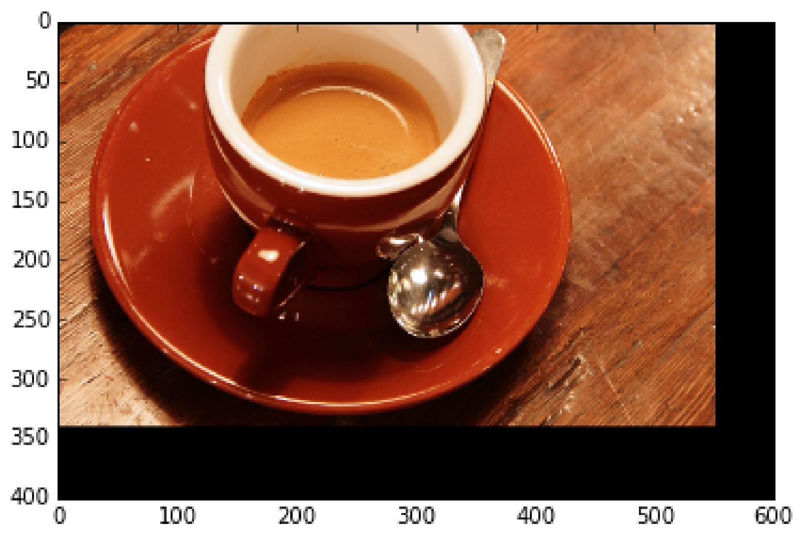
u = xcos(α)-ysin(α); v = xsin(α)+ysin(α);And here α=21 degree

c) Translate an image by (50, 60) pixels in (x, z) directions respectively.

```
In [23]: T_translation = array([[1,0,50],[0,1,60],[0,0,1]])
```

```
In [16]: I_translation = IP.warp(I,T_translation)
         imshow(I_translation,cmap='gray')
```

```
Out[16]: <matplotlib.image.AxesImage at 0x10de77e48>
```

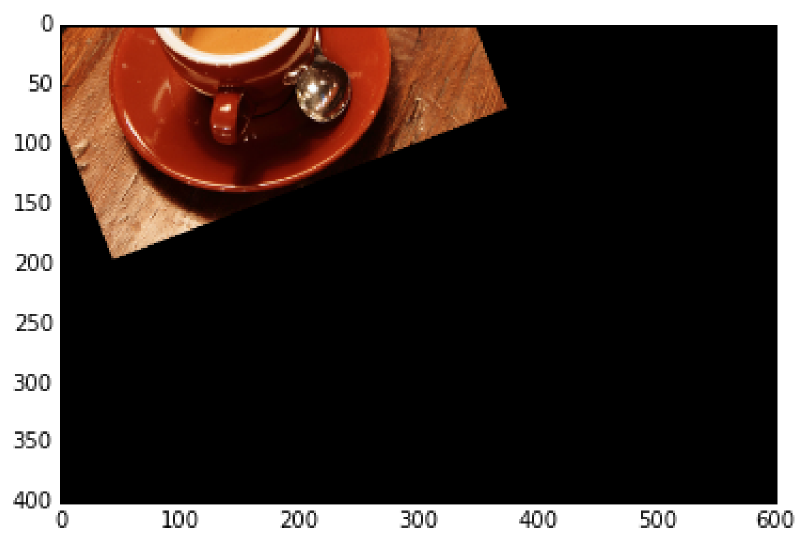


In this way, u = x + 50 , v = y + 60.Actually when I am doing to matrix operation, I think the ‘50’and '60' should be in the third row of the matrix. But it just doesn't work. The distance of translation has to be in the third column.

d) All the above together

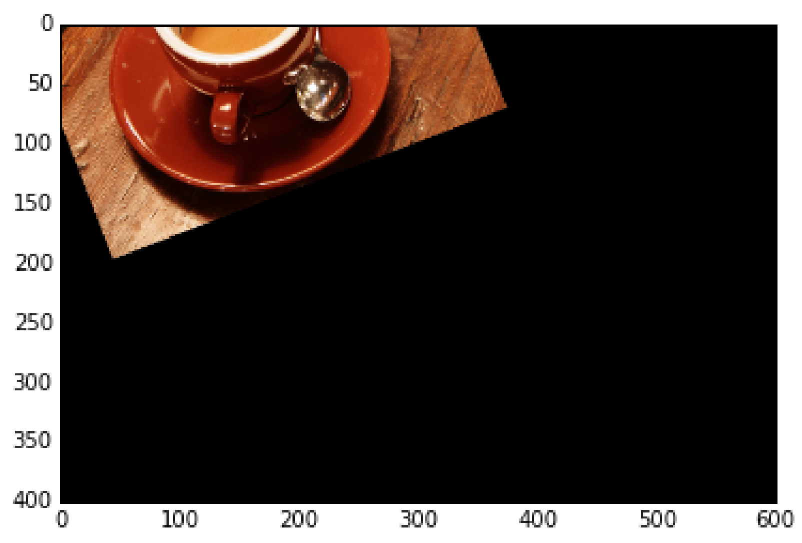
```
In [29]: T_all = array([[1.7*cos(radians(21)),-1.7*sin(radians(21)),50],[1.7*sin(radians
         I_all = IP.warp(I,T_all)
         imshow(I_all,cmap='gray')
         T_all
```

```
Out[29]: array([[  1.58708673,  -0.60922551,  50.        ],
                [  0.60922551,   1.58708673,  60.        ],
                [  0.        ,   0.        ,   1.        ]])
```



```
In [30]: T1 = IP.AffineTransform(translation=(50,60),rotation=radians(21),scale=(1.7,1.7
         I1 = IP.warp(I,T1)
         imshow(I1,cmap='gray')
```

```
Out[30]: <matplotlib.image.AxesImage at 0x1154610f0>
```



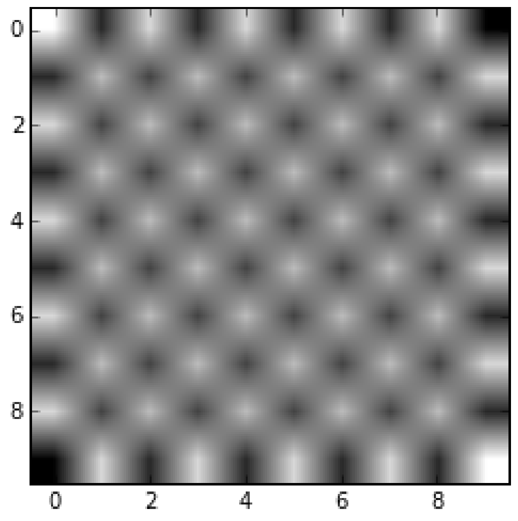
Compared the result with the result we get from the built_in function, we can see that the effect is the same.

Polynomial transfrom One

First We build a 10*10 checkerboard

```
In [2]: checkerboard = zeros([100,100])
        subboard = ones([10,10])
        for i in range(0,10):
            if i % 2 == 0:
                for j in range(0,10,2):
                    checkerboard[(i*10):(i*10+10),(j*10):(10*j+10)]=subboard
            elif i % 2 ==1:
                for j in range(1,10,2):
                    checkerboard[(i*10):(i*10+10),(j*10):(10*j+10)]=subboard
```

```
In [3]: from skimage.transform import PolynomialTransform
        II = checkerboard
        II.shape
        II = IP.imresize(II,(10,10))
        imshow(II,cmap='gray')

Out[3]: <matplotlib.image.AxesImage at 0x10760da58>
```



First put the coefficient of those two equation into $2 * N$ array, where $N * 2 = (order + 1) * (order + 2)$.So here $N = (2 + 1) * (2 + 2)/2 = 6$. And these six pairs of points are determined by:

$$X = \sum_{j=0}^{2} \sum_{i=0}^{j} (a_{ji} * x^{(j-i)} * y^{i})$$

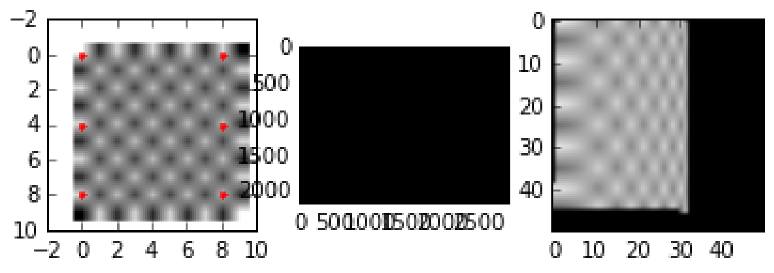
$$Y = \sum_{j=0}^{2} \sum_{i=0}^{j} (b_{ji} * x^{(j-i)} * y^{i})$$

In other words, the variables inside the function is choosen in the order: $a_{00}, a_{10}x, a_{11}y, a_{20}x^2, a_{21}xy, a_{22}y^2, a_{30}x^3, a_{31}x^2y, a_{32}xy^2, a_{33}y^3$ .Based on this order, I can choose the coefficient matrix that later I will put inside the funciton 'PolynomialTransform'

As I have resize the image to 10*10, the size of the new image will be decided by the given equation,so I substitute x = 10 and y = 10 to the equation below and get 2155 and 2929

```
In [4]: T_coefficient = array([[5,99],[2,2],[3,1],[0,9],[21,7],[0,12]])
        T_coefficient.T

Out[4]: array([[ 5,  2,  3,  0, 21,  0],
               [99,  2,  1,  9,  7, 12]])
```

```
In [5]: src = array([[0,0],[0,20],[0,40],[30,0],[30,20],[30,40]])
        dst = array([[0,0],[0,4],[0,8],[8,0],[8,4],[8,8]])

        # The three lines below is to do transformation on the whole image. As only 10(
        # is expanded to the size about 2155*2929,those points which don't have value a
        T_polynomial = PolynomialTransform(params=T_coefficient.T)
        I_polynomial = IP.warp(II,T_polynomial,output_shape=[2155,2929])
        imshow(I_polynomial,cmap='gray')


        # Unlike the method above, this mathod only choose certain points(dst) in the c
        # And the 'scr' refers to the coordinate of new image.
        T_polynomial1 = PolynomialTransform(params=T_coefficient.T)
        T_polynomial1.estimate(src,dst,order=2)
        I_polynomial1 = IP.warp(II,T_polynomial1,output_shape=[50,50])

        subplot(131);imshow(II,cmap='gray');plot(dst[:,0],dst[:,1],'.r')
        subplot(132);imshow(I_polynomial,cmap='gray')
        subplot(133);imshow(I_polynomial1,cmap='gray')

Out[5]: <matplotlib.image.AxesImage at 0x1076bea20>
```



Polynomial transfrom Two

Polynomial transform Two

I decided to use taylor series to expand sin( )/cos( )/log( )at first:

$$sin(x) = x - \frac{x^3}{3!}$$
$$cos(x) = 1 - \frac{x^2}{2!}$$
$$log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3}$$
$$log(1-y) = -y - \frac{y^2}{2} - \frac{y^3}{3}$$

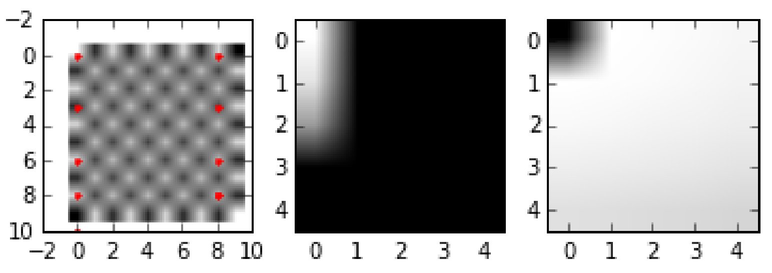
Therefore, we can express [u,v] in this way:

$$u = 0.01 - 0.5x + 0.5x^2 - \frac{5}{12}x^3 - \frac{y^2}{400}$$
$$v = 0.08 + 0.1x - \frac{1}{60}x^3 + y + \frac{12}{25}y^2 + \frac{y^3}{3}$$

This time, 10 pairs of points are chosen.

```
In [6]:  T_coefficient2 = array(([0.01,0.08],[-0.5,0.1],[0,1],[0.5,0],[0,0],[-1/400,12/2
         shape(T_coefficient2.T)

Out[6]:  (2, 10)
```

```
In [10]: src2 = array([[0,0],[0,2],[0,4],[0,6],[0,8],[8,0],[8,2],[8,4],[8,6],[8,8]])
         dst2 = array([[0,0],[0,3],[0,6],[0,8],[0,10],[8,0],[8,3],[8,6],[8,8],[8,8]])

         # Create a transformation matrix and set the coefficient, which has stored in T
         T_polynomial2 = PolynomialTransform(params=T_coefficient2.T)
         I_polynomial2 = IP.warp(II,T_polynomial2,output_shape=[5,5])

         # Unlike the methods above,which do the transformation to the whole image
         # the method below select certain points to do the transformaiton.
         T_polynomial3 = PolynomialTransform(params=T_coefficient2.T)
         T_polynomial3.estimate(src,dst,order=3)
         I_polynomial3 = IP.warp(II,T_polynomial3,output_shape=[5,5])

         subplot(131);imshow(II,cmap='gray');plot(dst2[:,0],dst2[:,1],'.r')
         subplot(132);imshow(I_polynomial2,cmap='gray')
         subplot(133);imshow(I_polynomial3,cmap='gray')

Out[10]: <matplotlib.image.AxesImage at 0x124526da0>
```

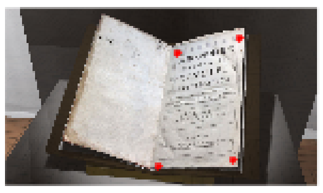


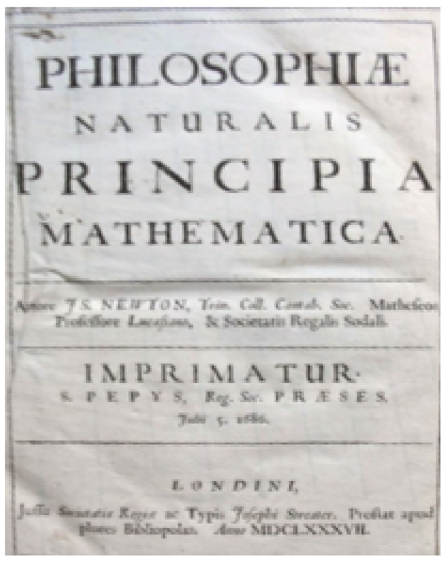
Projective transformation

```
In [111]: from skimage.transform import ProjectiveTransform
          shape(book)

Out[111]: (571, 1002, 3)
```

In [115]:
```
book = IP.imread('Projective_book.jpg')

# Specify the coordinate that we would like to take projection from(origin),anc
# we would like to place our projection(destinaiton)

origin = np.array(((0, 0),(0, 400),(300, 400),(300, 0)))
destinaiton = np.array(((550, 140),(490, 500),(730, 480),(750, 90)))

# Do projective transformaiton on the input image,as we only need to project th
# So we use 'estimate' to specify the parameters.
T_projection = ProjectiveTransform()
T_projection.estimate(origin,destinaiton)
I_pro = IP.warp(book, T_projection,output_shape=(380, 300))

# Show the image after projection and point out the area we take projection frc
fig, (ax1, ax2) = plt.subplots(1,2, figsize=(8, 10))
ax1.imshow(book)
ax1.plot(destinaiton[:, 0],destinaiton[:, 1],'.r')
ax1.axis('off')
ax2.imshow(I_pro)
ax2.axis('off')
```

Out[115]:
```
(-0.5, 299.5, 379.5, -0.5)
```





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Julii 5. 1686.

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In [ ]: