Assignment 3

This assignment is due on October 2, marks will be deducted for late homework. Where needed, please add drawings, formulae, and comments to explain your answers. **Please include all references, and the names of all students in your study group.**

1. Consider an infinite checkerboard image grid (infinite in both coordinate directions).
   (a) Assume that the grid elements are 1×1 mm squares. What is the sampling rate (in samples/mm) required to avoid aliasing?
   (b) Assume that the grid elements are 2×1 mm. What should be the sampling rate in each coordinate direction to avoid aliasing?
   (c) What would be your answer to part (b) if you can use a **single** sampling frequency for both coordinate directions?
2. In this problem you will develop and test a 2-D median filtering algorithm.
   (a) Develop a 2-D median filtering algorithm. **Do not** use the inbuilt 1D/2D median filter functions from python. Draw a block diagram explaining the filtering algorithm.
   (b) Describe the problem at image boundaries when performing the median filtering. Describe how your algorithm in part (a) treats the boundaries.
   (c) Compare your algorithm and python's function **median** (from skimage.filters) for the following cases (use python image **checkerboard**):
   - Median filtering performed with a 3×3 mask
   - Median filtering performed with a 7×7 mask
   - Median filtering performed with a 15×15 mask
   - Median filtering performed with a 21×21 mask
3. Use an image of your choice to demonstrate the effect of repeatedly applying a 3×3 low-pass spatial filter to a digital image. You may ignore border effects.
4. Certain Fourier transform relations are very important in image processing.
   (a) Describe what a separable function is and give a few examples of separable functions. Explain the benefit(s) of dealing with separable functions in image processing.
   (b) Prove the following Fourier transform relation: $\mathcal{F}\{rect(x)rect(y)\} = sinc(f_x)sinc(f_x)$.
   (c) Demonstrate the relation in part (b) using python.
   (d) Describe the Gibbs phenomenon.