Assignment 4

This assignment is due October 16th, marks will be deducted for late homework. Where needed, please add figures, formulae and comments to explain your answer. Please include references and names of students in your study group.

1. Resize the image, moon, to 512x512 and convert to grayscale. Design the following low pass filters:
   (a) Ideal lowpass filter
   (b) Gaussian lowpass filter
   (c) Butterworth lowpass filter
   Demonstrate your filters by showing the effect of different cut-off frequencies (say 10, 20, 40, 60, 100) for the 3 types filters and filter orders for Butterworth filter (say 1, 4, 8). Display and comment on the results of applying the filters to the image. *(10 points)*

2. Design a series of highpass filters in Fourier domain (Ideal, Gaussian, Butterworth) for the same cutoff frequencies and filter orders chosen in the previous question. Apply these filters on moon.jpg. Show the effect of filter order and cut-off frequencies on the same image that was used in question 1. *(5 points)*

3. Choose a pair of low pass filtered and high pass filtered results (for the same cut-off frequency and order) and add them (For example, choose the results filtered by a Gaussian high pass filter and low pass filter with cut-off frequency 20). What is the result obtained? Compare this result with the expected result. *(5 points)*

4. Cascading filters. Use the resized moon image. *(20 points)*
   (a) Design a Gaussian lowpass filter, *Ha*, with cutoff frequency 70.
   (b) Design a Gaussian highpass filter, *Hb*, with cutoff frequency 20.
   (c) Perform filtering defined by the schematics in Figure 1, comment on and compare the results.
   Ha: filter in Fourier domain designed in part (a)
   Hb: filter in Fourier domain designed in part (b)
   $Ha \cdot Hb$: multiplication of *Ha* and *Hb*
   $\mathbf{I}_{in}$: input image
   $\mathbf{I}_{out}$: output image

(a) (b)
Figure 1: Filtering schematics.

   (e) Display the filter *Ha.Hb*, what is such a filter called? Mathematically derive the relationship between the two schematics in Figure 1.

5. a) Design an 11x11 Gaussian filter, $h_g = e^{\frac{-r^2}{2\sigma^2}}$ where $r = \sqrt{x^2 + y^2}$ and σ =2. Display the result.
b) Filter the image of the moon with h_g using convolve2d. Display the result.
c) Obtain the fourier transform of *hg*, *Hg*, and filter the image with *Hg* (convolution in time domain is multiplication in frequency domain). Display the filtered result.
d) Compare the results obtained in b) and c). Display the difference between the two results. How much of a disparity do you observe? Try to minimize the differences observed.
(20 points)