Algorithm Analysis and Design

Exercises, Ch 1

(Introduction)

1. Design an algorithm for computing $\sqrt{n}$ for any positive integer n. Besides assignment and comparison, your algorithm may only use the four basic arithmetical operations.

2. Design an algorithm to find all the common elements in two sorted lists of numbers. For example, for the lists 2, 5, 5, 5 and 2, 2, 3, 5, 5, 7, the output should be 2, 5, 5.What is the maximum number of comparisons your algorithm makes if the lengths of the two given lists are m and n, respectively?

3. What does Euclid's algorithm do for a pair of integers in which the first is smaller than the second?

4.
   a. What is the minimum number of divisions made by Euclid's algorithm among all inputs $1 \le m, n \le 10$?
   b. What is the maximum number of divisions made by Euclid's algorithm among all inputs $1 \le m, n \le 10$?

5. Which of the following formulas can be considered an algorithm for computing the area of a triangle whose side lengths are given positive numbers a, b, and c?
   a. $S = \sqrt{(p(p - a)(p - b)(p - c))}$, where $p = (a + b + c)/2$
   b. $S = \frac{1}{2} bc \sin A$, where A is the angle between sides b and c
   c. $S = \frac{1}{2} ah_a$, where h_a is the height to base a.

6. Consider the following algorithm for finding the distance between the two closest elements in an array of numbers.

ALGORITHM *MinDistance(A[0..n − 1])*

    //Input: Array $A[0..n − 1]$ of numbers

    //Output: Minimum distance between two of its elements

    dmin←∞

      for *i* ←0 **to** *n* − 1 **do**

for $j \leftarrow 0$ **to** $n - 1$ **do**

      if $i \neq j$ **and** $|A[i] - A[j]| < dmin$

      $dmin \leftarrow |A[i] - A[j]|$

return $dmin$

Make as many improvements as you can in this algorithmic solution to the problem. If you need to, you may change the algorithm altogether; if not, improve the implementation given.