DISCRETE PROBABILITY DISTRIBUTION

CONTENT

BINOMIAL DISTRIBUTION

POISSON DISTRIBUTION

INTRODUCTION TO STATISTICAL PROBABILITY DISTRIBUTION

VARIABLE

The characteristics of the population of interest.

For example: Monthly income, respondents' age, gender, level of education, type of house etc.

QUANTITATIVE (NUMERICAL)

- → Measured on numerical scale
- → Yields numerical response
- → For example: How tall are you?

DISCRETE

- → Numerical response arises from a counting process
- → For example: How many siblings do you have?

CONTINUOUS

- → Numerical response arises from a measuring process
- → For example: What is your weight?

IT IS RELATED TO RANDOM VARIABLE

PROBABILITY DISTRIBUTION FUNCTION

PROBABILITY DISTRIBUTION FUNCTION

If X is a random variable , the function given by $P(X = x_i) = f(x)$ for each X within the range of x_i is called probability distribution function of X.

PROBABILITY DISTRIBUTION FUNCTION

where the value of $P(X = x_i) = f(x)$ should be satisfy the conditions

(i) $P(X = x_i) \geq 0$

(ii) $\sum_{x_i \in X} P(X = x_i) = 1$

MEAN AND VARIANCE OF A DISCRETE RANDOM VARIABLE

MEAN (EXPECTED VALUE)

Let X be a **discrete random variable** with **probability function** $\Pr\left(X = x_i\right)$
Then, the **expected value** of X denoted by $\mathrm{E}\left(X\right)$, defined by

$$\mu = \mathrm{E}\left(X\right) = \sum_{x_i \in X} x_i \cdot \Pr\left(X = x_i\right)$$

VARIANCE

The **variance** of a **discrete random variable** X is defined by

$$\sigma^2 = \mathrm{Var}\left(X\right) = \mathrm{E}\left(X^2\right) - \left(E\left(X\right)\right)^2 = \left(\sum_{x_i \in X} x_i^2 \cdot \Pr\left(X = x_i\right)\right) - \mu^2$$

SOME PROPERTIES OF MEAN AND VARIANCE

Let a and b be a constants. Then the following results hold:

(i) $\mathrm{E}\left(a\right) = a$ (ii) $\mathrm{E}\left(aX\right) = a\,\mathrm{E}\left(X\right)$

(iii) $\mathrm{Var}\left(aX + b\right) = a^2\,\mathrm{Var}\left(X\right)$. In particular, $\mathrm{Var}\left(aX\right) = a^2\,\mathrm{Var}\left(X\right)$, $\mathrm{Var}\left(b\right) = 0$.

BINOMIAL DISTRIBUTION

BINOMIAL DISTRIBUTION

PROBABILITY DISTRIBUTIONS

A *discrete random variable*, X is said to follow *binomial probability distribution* with parameters n and p, denoted by $bin(x;n,p)$ if

$$f(x) = P(X = x) = \binom{n}{x}(p)^x(1-p)^{n-x}, \quad x = 0, 1, 2, \ldots, n$$

where p is *probability of success* and $q = 1 - p$ is *probability of failure* on *a single trial*.

MEAN AND VARIANCE

If X is *binomial random variable* with parameters n and p, then

Mean, $\mathrm{E}(X) = \mu = np$

Variance, $\mathrm{Var}(X) = \sigma^2 = np(1-p)$

EXAMPLE 1

It is believed that 20% of Malaysians do not have any health insurance. Suppose that this is true and let a random variable, X represents the number with no health insurance in a random sample of 10 Malaysians.

(i) How is X distributed?

(ii) Give the mean, variance and standard deviation of X.

(iii) Find $P(X \geq 5)$

SOLUTION

EXAMPLE 2

Let p equal the proportion of all college and university students who would say yes to the question, " Would you drink from the same glass as your friend if you suspected that this friend were an AIDS virus carrier?" . Assume that $p = 0.10.$ Let a random variable, X represents the number of students out of a random sample of size nine who would say yes to this question.

(i) How is X distributed?

(ii) Give the mean, variance and standard deviation of X.

(iii) Find $P(X = 2)$ and $P(X \geq 2)$

EXAMPLE 3

Suppose that, in manufacturing, the probability of a certain item being defective is $p = 0.05$. Suppose further that the quality of an item is independent of the quality of the other manufactured items. An inspector selects five items at random. Let a random variable, X represents the number of defective items in the sample.

(i) How is X distributed?

(ii) Determine the values of $\mathrm{E}(X)$ and $\mathrm{Var}(X)$.

(iii) Find $P(X = 0), P(X \leq 1)$ and $P(X \geq 2)$

EXAMPLE 4

A student majoring in project management is trying to decide upon the number of firms to which she should apply. With her excellent academic qualification and working experience, she believes that 50% of the firms will offer her a place. Wanting save time, the student applies to only five firms. Assuming the student's estimate is correct, find the probability that the student receives the following.

(i) No offers.
(ii) At most two offers.
(iii) Between two and four offers.

POISSON DISTRIBUTION

POISSON DISTRIBUTION

A *discrete random variable*, X is said to follow *Poisson probability distribution* with parameters, $\lambda > 0$, denoted by $Po(x;\ \lambda)$ if

$$f(x) = P(X = x) = \frac{e^{-\lambda}\lambda^{x}}{x!}, \quad x = 0,\ 1,\ 2,\ \ldots$$

MEAN AND VARIANCE

If X Is a *Poisson random variable* with parameters λ, then

Mean, $\mathrm{E}(X) = \mu = \lambda$

Variance, $\mathrm{Var}(X) = \sigma^2 = \lambda$

EXAMPLE 1

Let a random variable, X have a Poisson distribution with a mean of 4. Find

(i) $P(2 \le X \le 5)$ (ii) $P(X \ge 3)$ (iii) $P(X \le 3)$

EXAMPLE 2

The average number of trucks arriving on any one day at a truck depot in a certain city is known to be 12. What is the probability that on a given day fewer than nine trucks will arrive at this depot?

EXAMPLE 3

Customer arrive at a travel agency at a mean rate of 11 per hour. Assuming that the number of arrivals per hour has a Poisson distribution, determine the probability that more than 10 customers arrive in any given hour.

EXAMPLE 4

In a certain desert region the number of persons who become seriously ill each year from eating a certain poisonous plant is a random variable having a Poisson distribution with average 5.2. Find the probabilities of

(i) three such illness in a given year.
(ii) at least 10 such illness in a given year.
(iii) anywhere from four to six (inclusive) such illness in a given year.

THANK YOU