Derivative of $V(t) = V_{max}(1 - e^{-t/RC})$

- $V(t)$ is the voltage across the capacitor at time t,
- V_{max} is the maximum (final) voltage
- R is the resistance in Ohms (Ω),
- C is the capacitance in farads (F).

All these are constants.

Thus, $$\frac{dV}{dt} = \frac{d}{dt}\left[V_{max}\left(1 - e^{-t/RC}\right)\right]$$

$$= V_{max} \cdot \frac{d}{dt}\left(1 - e^{-t/RC}\right) \Rightarrow \frac{d}{dt}(1) = 0$$

Let $u(t) = -t/RC$, so $e^{-t/RC} = e^{u(t)}$

$$\frac{d}{dt}\left(e^{u(t)}\right) = e^{u(t)} \cdot \frac{du}{dt} \Rightarrow \frac{du}{dt} = \frac{d}{dt}\left(-\frac{t}{RC}\right) = -\frac{1}{RC}$$

$$\Rightarrow e^{u(t)} \cdot \frac{du}{dt} = e^{-t/RC} \cdot \frac{-1}{RC}$$

Thus, $$\frac{dV}{dt} = V_{max} \cdot \left(0 - \left(-\frac{1}{RC} \cdot e^{-t/RC}\right)\right)$$

$$\Rightarrow \frac{dV}{dt} = V_{max} \cdot \frac{1}{RC} \cdot e^{-t/RC}$$

$$\boxed{\therefore \frac{dV}{dt} = \frac{V_{max}}{RC} \cdot e^{-t/RC}}$$

NOTE: $RC = \tau$ (tau)