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Combinatorics

Pigeon-hole principle:

Thm 1:

~~If n+1 pe~~ If $n+1$ pegions are placed in n holes, then atleast one pigeon hole will have atleast 2 pigeons.

Eg: In a group of 27 english words, there must be atleast 2 words, that begin with same letter.

Eg: A busy airport sees 1500 takeoffs. Prove that atleast 2 planes ~~with~~ within a minute difference.

Sol:

no of min $= 24 \times 60 = 1440$ (holes)

no of takeoffs $= 1500$ (pegions)

$\therefore$ atleast 2 planes will take off in a certain min difference

(Q1) Minimum no of students in a class such that ~~atea~~ atleast 6 will have same grade is ____. Consider there are 5 different grades.

Sol:

First ~~dist~~ consider

there are 5 student of each grade (worst case seceneno)

no of stu so far $= 5 \times 5 = 25$

Now adding new student to any grade will be the 6th of the grade.

$\therefore$ min no of students $= 25 + 1 = 26$

Q2 What is min no of cards to be drawn ~~to ensure~~ from a standard deck to ensure there will be atleast 3 cards from same suit? 4

Sol:

$2 \times 4 + 1 = 8 + 1 = 9$

S_1	S_2	S_3	S_4
2	2	2	1

(or) (or) (or)

1

Note :

* If there are $kn+1$ ~~pegions~~ pigeons and n pegion holes then atleast one pigeon hole contains $k+1$ pegions.

* If there are n pigeon holes
  min no of pigeons required to ensure that there will k pigeons in atleast one ~~vertex~~ hole is $(k-1)n+1$

Thm 2 :

If N objects are placed in k holes, then atleast one hole ~~box~~ contains $\left\lceil \frac{N}{K} \right\rceil$ objects. and some hole contains atmost $\left\lfloor \frac{N}{K} \right\rfloor$ objects.

P/47 $(9-1)(12)+1$

$96+1 = 97$

P/48 5 balls of all colors $+1$

$5 \times 5 + 1 = 26$

P/49 ~~First~~ 8 balls of all colors $+1$

But there are not 8 balls in colors

$$\therefore \left(\overset{R}{6}+\overset{B}{8}+\overset{G}{8}+\overset{W}{8}+\overset{Y}{8}\right)+1$$

$$38+1=39$$

P/50

Red - 12, Blue - 7, Green - x

Min no of ball required to ensure 6 ~~ee~~ balls of same color = 15

$$\left(5+5+\underset{\uparrow \text{ no of green balls choosen}}{a}\right)+1=15$$

$$5+5+a=14$$

$$\boxed{a=4}$$

There total no of green balls itself, i.e must be 4

$$\therefore x=4$$

P/51

$S=\{0,1,2,\cdots 9\}$

Consider

$0 + 9 = 9$

$1 + 8 = 9$

$2 + 7 = 9$

$3 + 6 = 9$

$4 + 5 = 9$

set1   set2

we need to select elements from set1 & ~~ste~~ set2 such that 2 distinct sets appear.

In worst case we may select 5 elements from set1.

Now we need to select atleast 2 elements from set2.

$\therefore$ Min no of elements $= 5+2=7$

Q3) Min no of elements to be taken from $\{1, 2, 3 \cdots 10\}$ such there exist two elements in the drawn set such that sum of elements is 10.

Sol:

1		9
2		8
3		7
4		6
	5	
	10	

Min no of elements is 7

$\{1, 2, 3, 4, 5, 10\}$ and 1 element from $\{6, 7, 8, 9\}$

Q4) Min no of elements to be drawn from set $S = \{1, 2 \cdots 200\}$ so ~~thex~~ that there exist two element x & y and $x|y$ (or) y/x.

Sol:

In the worst case it is possible to choose

101 to 200 all the number

none of them divides none.

Adding any element from 1 to 100 will satisfy the condition.

$\therefore$ we need 101 elements

Q5) How many times do we need to roll a single die in order to get the same score

a) at least twice $- 1(6)+1 = 7$

b) atleast thrice $- 2(6)+1 = 13$

c) atleast n times $- (n-1)6+1 = 6n-5$

P/45

S1: $\left\lceil \frac{410}{50} \right\rceil = \lceil 8.\_ \rceil = 9$

$\therefore$ S1 is true

S2: $\left\lfloor \frac{410}{50} \right\rfloor = \lfloor 8.\_ \rfloor = 8.$

$\therefore$ S2 is true

from S1 we can conclude S5

from S2 we can conclude S6

P/46

S1: $\left\lceil \frac{2000}{30} \right\rceil = \lceil 66.\_ \rceil = 67$

$\therefore$ S1 is true

S2: $\left\lfloor \frac{2000}{30} \right\rfloor = \lfloor 66.\_ \rfloor = 66$

$\therefore$ Some bus will carry atmost 66 students

$\Rightarrow$ some bus will have atleast 14 empty seats

P/52 Consider we choose two numbers ending with same digit then their difference will be divisible by 10

Now consider the case without two ending with samedigit

0

1 9

2 8

3 7

4 6

5

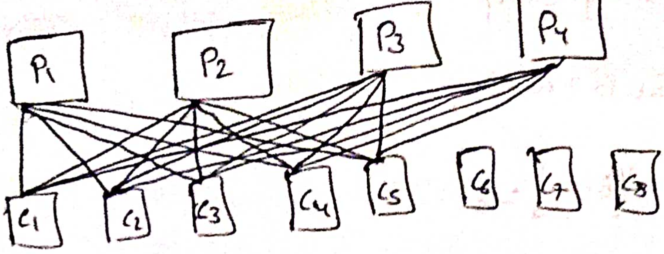
In worst case we may choose

$\{1, 2, 3, 4, 0, 5\}$

we need one more element for sum to be 10

$\therefore$ min elements req = 6+1 = 7

P/53



$5 \times 4 = 20$ cables are enough

Reason: In worst case we may chose C_6, C_7, C_8.
Now choosing any computer from C_1 to C_5 will satisfy the condition

13/05/20

Euler totient funtion (ϕ):

→ totient function $\phi(n)$ is no of co-primes to n that are less than n.

→ Consider

$\phi(3) = 2$ $\quad$ $\phi(4) = 2$

1, 2, 3 $\quad$ 1 2 3 4

$\phi(12) = 4$

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12

Now $\phi(12) = \phi(3 \times 4)$

$= \phi(3) \times \phi(4)$

$= 2 \times 2$

$= 4$

$\boxed{\phi(AB) = \phi(A) \times \phi(B) \Leftrightarrow \text{if } A, B \text{ are relatively prime}}$

→ $\phi(7) = 6$

1, 2, 3, 4, 5, 6, 7.

$\boxed{\phi(P) = P - 1, \text{ if } P \text{ is a prime number}}$

→ Consider $\phi(49)$

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21

22 23 49

$\phi(49)$ = no of relative primes = total − no of non-relative prime

$$= 49 - \frac{49}{7}$$

$$= 49\left(1 - \frac{1}{7}\right)$$

$$\boxed{\phi(7^2) = 7^2\left(1 - \frac{1}{7}\right)}$$

from this we conclude

$$\boxed{\phi(p^a) = p^a - \frac{p^a}{p} = p^a\left(1 - \frac{1}{p}\right)} \boxed{= p^a\left(\frac{p-1}{p}\right)}$$

→ Consider $\phi(n)$

Let $n = p_1^{k_1} p_2^{k_2} \cdots p_m^{k_m}$

$$\phi(n) = \phi\left(p_1^{k_1} p_2^{k_2} \cdots p_m^{k_m}\right)$$

$$= \phi\left(p_1^{k_1}\right) \cdot \phi\left(p_2^{k_2}\right) \cdots \phi\left(p_m^{k_m}\right)$$

$$= p_1^{k_1}\left(\frac{p_1 - 1}{p_1}\right) \cdot p_2^{k_2}\left(\frac{p_2 - 1}{p_2}\right) \cdots p_m^{k_m}\left(\frac{p_m - 1}{p_m}\right)$$

$$= p_1^{k_1} p_2^{k_2} \cdots p_m^{k_m}\left(\frac{p_1 - 1}{p_1}\right)\left(\frac{p_2 - 1}{p_2}\right) \cdots \left(\frac{p_m - 1}{p_m}\right)$$

$$\boxed{\phi(n) = n \cdot \left(\frac{p_1 - 1}{p_1}\right)\left(\frac{p_2 - 1}{p_2}\right) \cdots \left(\frac{p_m - 1}{p_m}\right)}$$

$$\boxed{\phi(n) = n \cdot \frac{(p_1 - 1)(p_2 - 1) \cdots (p_m - 1)}{p_1 \cdot p_2 \cdots p_m}}$$

$\rightarrow \phi(180) = ?$

$180 = 2 \times 2 \times 3 \times 3 \times 5$

$= 2^2 \times 3^2 \times 5$

$\phi(180) = 180\left(\frac{(2-1)(3-1)(5-1)}{2 \cdot 3 \cdot 5}\right)$

$= 6(1 \cdot 2 \cdot 4)$

$\phi(180) = 48$

$\rightarrow \phi(3528)$

$3528 = 2 \times 2 \times 2 \times 441$

$= 2 \times 2 \times 2 \times 3 \times 3 \times 7 \times 7$

$= 2^3 \times 3^2 \times 7^2$

$\phi(3528) = 3528\left(\frac{(1)(2)(\cancel{6})}{\cancel{2} \cdot \cancel{3} \cdot 7}\right)$

$= 1008$

$\rightarrow \phi(210)$

$210 = 2 \times 3 \times 5 \times 7$

$\phi(210) = 210\left(\frac{1 \cdot 2 \cdot 4 \cdot 6}{\cancel{2} \cdot \cancel{3} \cdot \cancel{5} \cdot \cancel{7}}\right)$

$= 48$

$\rightarrow \phi(317)$

317 is prime

$\therefore \phi(317) = 317 - 1$

$= 316$

GATE 2005

Let $n = p^2 q$ where p and q are distinct prime numbers. 11

How many numbers m satisfy $1 \le m \le n$ and $\gcd(m, n) = 1$?

a) $p(q-1)$ b) pq c) $(p^2-1)(q-1)$ d) $p(p-1)(q-1)$

Sol:

$$\phi(n) = \phi(p^2 q)$$

$$= n \cdot \frac{(p-1)(q-1)}{p \cdot q}$$

$$= p^2 q \frac{(p-1)(q-1)}{pq} = p(p-1)(q-1)$$

P/57

$$\phi(p^k) = p^k \frac{(p-1)}{p}$$

$$= p^{k-1}(p-1)$$

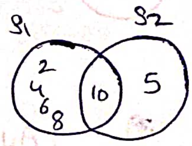
15/05/20

Inclusion - Exclusion:

Eg: How many elements are divisible by 2 or 5 from the set $S = \{1, 2, 3, 4, \ldots 10\}$

Sol:

n_1 = no of elements divisible by 2 = $\left\lfloor \frac{10}{2} \right\rfloor = 5$



n_2 = no of element divisible by 5 = $\left\lfloor \frac{10}{5} \right\rfloor = 2$.

1 2 3 4 5 6 7 8 9 10

2 2 5 2 2 2 5

Ans: $7 - 1 = 6$ Here 10 is counted twice. So we need to subtract 1. This called Exclusion.

$$\boxed{|A \cup B| = |A| + |B| - |A \cap B|}$$ (Exclusion)

Now no of elements not divisible by 2 or 5

Total − divisible by 2 or 5

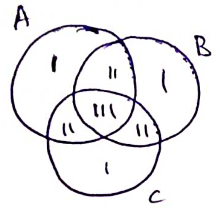
$10 - 6 = 4$

Note:

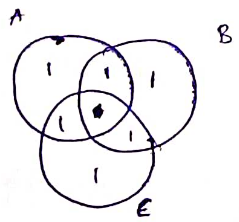
→ $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |C \cap A| + |A \cap B \cap C|$

→ $|A \cup B \cup C \cup D| = |A| + |B| + |C| + |D| - |A \cap B| - |A \cap C| - |A \cap D| - |B \cap C| - |B \cap C|$
$- |C \cap B| + |A \cap B \cap C| + |A \cap B \cap D| + |A \cap C \cap D| + |B \cap C \cap D|$
$- |A \cap B \cap C \cap D|$

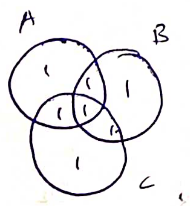
Proof:



$|A| + |B| + |C|$



$-|A \cap B| - |A \cap C| - |B \cap C|$



$+|A \cap B \cap C|$

$\therefore |A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$

→ Slly this can be extended to any number of variables.

$\rightarrow |A_1 \cup A_2 \cup \cdots \cup A_n| = \Sigma|A_i| - \Sigma|A_i \cap A_j| + \Sigma|A_i \cap A_j \cap A_k| + \cdots$

$+ (-1)^{n+1} \Sigma |A_1 \cap A_2 \cap \cdots \cap A_n|$

GATE 2017 — The number of integers b/w 1 & 500 (both inclusive) that are divisible by 3 or 5 or 7 is ____

Sol:

$|D_3| = \left\lfloor \frac{500}{3} \right\rfloor = 166$

$|D_5| = 100$

$|D_7| = 71$

$|D_3 \cap D_5| = \left\lfloor \frac{500}{15} \right\rfloor = 33$

$|D_3 \cap D_7| = \left\lfloor \frac{500}{21} \right\rfloor = 23$

$|D_5 \cap D_7| = \left\lfloor \frac{500}{35} \right\rfloor = 14$

$|D_3 \cap D_5 \cap D_7| = \left\lfloor \frac{500}{105} \right\rfloor = 4$

$(166+100+71) - (33+23+14) + 4$

$337 - (70) + 4$

$341 - 70 = 271$

P/3 First find divisible by 5 or 6 or 8

i.e, $(80+66+50) - (13+10+16) + (3)$

$196 - 39 + 3 = 199 - 39 = 160$

not divisible by 5 (or) 6 (or) 8 $= 400 - 160 = 240$

Derangement:

No of possibilities such that no element is in its correct position.

Eg:

1	2	3	
1	2	3	✗
1	3	2	✗
2	1	3	✗
2	3	1	✓ Derangements
3	1	2	✓ Derangements
3	2	1	✗

Derangement of n elements is represented as D_n

→ $D_1 = 0$, $D_2 = 1$, $D_3 = 2$, $D_4 = 9$, $D_5 = 44$, $D_6 = 265$, $D_7 = 1854$

→ Consider

$D_3 = 3! - $ (atleast 1 element at right position)

$D_4 = 4! - $ (atleast 1 element at right position)

let

A_i represent no of ways such that i^{th} position element is in right position

$\Rightarrow |A_1| = \underline{\checkmark}\ \underbrace{\_\ \_\ \_}\ = 3!$ ways

$|A_2| = 3!$ ways

$\Rightarrow$ ~~Atleast~~ Cases where atleast 1 element at right position is

$|A_1 \cup A_2 \cup A_3 \cup A_4|$

Now $|A_1 \cap A_2|$ represents no of cases in which 1^{st} & 2^{nd} elements are correctly placed

$\Rightarrow |A_1 \cap A_2| = 2!$

$\Rightarrow$ Total no of cases in which atleast 2 elements are correctly placed is $4C_2 \times 2!$

Now $\Rightarrow$ Slly total case where atleast 3 elements are correctly placed is $4C_3 \times 1!$

$$|A_1 \cup A_2 \cup A_3 \cup A_4| = \Sigma|A_i| - \Sigma|A_i \cap A_j| + \Sigma|A_i \cap A_j \cap A_k| - \Sigma|A_1 \cap A_2 \cap A_3 \cap A_4|$$

$$= 4C_1 \times 3! - 4C_2 \times 2! + 4C_3 \times 1! - 1$$

$$D_4 = 4! - |A_1 \cup A_2 \cup A_3 \cup A_4|$$

$$= 4! - \{4 \times 3! - 4C_2 \times 2! + 4C_3 \times 1! - 1\}$$

$$= 4! - 4! + 4C_2 \times 2! - 4C_3 + 1$$

$$= 4C_2 \cdot 2! - 4C_3 + 1$$

$$= 4C_2 \cdot 2! - 4C_3 + \frac{4!}{4!}$$

$$= \frac{4!}{2!\,\cancel{2!}}\cancel{2!} - \frac{4!}{3!\,1!} + \frac{4!}{4!}$$

$$D_4 = 4!\left(\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}\right)$$

Slly this formula can be generalized as

$$\boxed{D_n = n!\left(\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \dots + (-1)^n \frac{1}{n!}\right)}$$

$\rightarrow$ Consider e^x

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \infty$$

put $x = -1$

$$e^{-1} = 1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} + \dots$$

$$e^{-1} = \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \dots \infty$$

for sufficiently large n

$$e^{-1} \cong \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \dots + \frac{(-1)^n}{n!}$$

Now

$$n!\,e^{-1} \cong n!\left[\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5} + \dots + \frac{(-1)^n}{n!}\right]$$

$$\Rightarrow \boxed{D_n = n!\,e^{-1}} \Rightarrow \boxed{D_n = n! \times 0.368}$$

$$\Rightarrow \boxed{\frac{D_n}{n!} = e^{-1} = 0.368}$$

i.e., probability that the arrangement is a derangement is 0.368 (but remember n should be sufficiently large) (around $n \geq 5$)

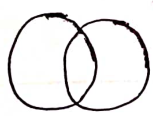
Note:

→ For sufficiently large n, probability ~~is same~~ that the arrangement is derangement is same (almost) for every n.

→ $\boxed{D_{n+1} = (n+1)D_n + (-1)^{n+1}}$ (or) $\boxed{D_n = nD_{n-1} + (-1)^n}$

P/1 Total = 50

$|F \cap G| = 11$



Total − $|F \cup G| = 8$

$|F \cup G| = 50 - 8 = 42$

$\cancel{|F \cup G| = |F| + |G| - |F \cap G|}$

$\cancel{42 = |F| + |G| - 11}$

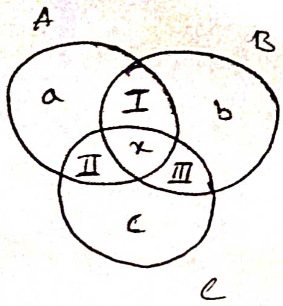
$\cancel{|F| + |G| = 53}$

People who can speak atleast 1 lang = 42

Who can speak both = 11

Who can speak exactly one = 42 − 11 = 31

P/2 ~~31 = (20+16+8)~~



$A \cap B \cap \bar{C}$ — I

$A \cap \bar{B} \cap C$ — II

$\bar{A} \cap B \cap C$ — III

$$|A|+|B|+|C| = (a+I+II+x)+(b+I+III+x)+(c+II+III+x)$$

$$20+16+8 = (a+b+c)+2(I+II+III)+3x$$

$$44 = (a+b+c+I+II+III+x)+(I+II+III)+2x$$

$$44 = (31)+(3+4+2)+2x$$

$$44 = 40+2x \Rightarrow x=2$$

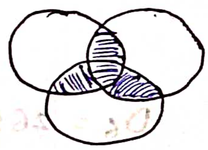
P/4

$|A_1| = 45 \quad |A_2| = 50 \quad |A_3| = 50$

$|A_1 \cap A_2 \cap A_3| = 15 \qquad |A_1 \cup A_2 \cup A_3| = 80$

All of them are skilled in atleast 1 area $\Rightarrow |\bar{A_1} \cap \bar{A_2} \cap \bar{A_3}| = 0$

a) atleast two skills



$$|(A_1 \cap A_2) \cup (A_2 \cap A_3) \cup (A_1 \cap A_3)|$$

Consider

$$|A_1 \cup A_2 \cup A_3| = |A_1|+|A_2|+|A_3| - (|A_1 \cap A_2| + |A_2 \cap A_3| + |A_3 \cap A_1|) + |A_1 \cap A_2 \cap A_3|$$

$$80 = 45+50+50 - (|A_1 \cap A_2| + |A_2 \cap A_3| + |A_3 \cap A_1|) + 15$$

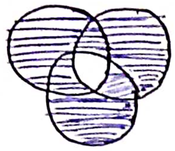
$$|A_1 \cap A_2| + |A_2 \cap A_3| + |A_3 \cap A_1| = 80$$

~~$|A_1 \cap A_2$~~

$$|(A_1 \cap A_2) \cup (A_2 \cap A_3) \cup (A_3 \cap A_1)| = |A_1 \cap A_2| + |A_2 \cap A_3| + |A_3 \cap A_1| - 3|A_1 \cap A_2 \cap A_3| + |A_1 \cap A_2 \cap A_3|$$

$= |A_1 \cap A_2| + |A_2 \cap A_3| + |A_3 \cap A_1| - 2|A_1 \cap A_2 \cap A_3|$

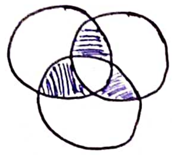
$= 80 - 2(15) = 50$

b) 

Atmost 2 = total - exactly 3

$= 80 - 15$

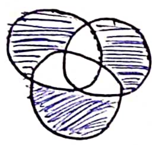
$= 65$

c) 

Exactly 2 = atleast 2 - exactly 3

$= 50 - 15$

$= 35$

d) 

exatly 1 = Total - atleast 2

$= 80 - 50$

$= 30$

P/5

$(42 + 36 + 28 + 24) - 6 \cdot 12 + 4 \cdot 8 - 4$

$130 + 32 - 72 - 4$

$= 86$

P/58

$D_6 = 265$

P/59

$$n! \sum_{i=2}^{n} \frac{(-1)^i}{i!}$$

$$= \left[n! \sum_{i=2}^{n} \frac{(-1)^i}{i!} \right] + \frac{n!}{0!} - \frac{n!}{1!}$$

$$= n! \sum_{i=0}^{n} \frac{(-1)^i}{i!}$$

$$= \sum_{i=0}^{n} (-1)^i \frac{n!}{i!}$$

P/60 (i) $D_5 = 44$

(ii) Aleast one correct = Total − no correct

$= 5! - 44$

$= 76$

(iii) Here we select two correctly placed letters and we need to find derangement for the rest.

$= {}^5C_2 \times D_3$

$= 10 \times 2 = 20$

(iv) D_5 + exatly 1 is correct

$44 + {}^5C_1 \times D_4$

$44 + 45 = 89$

(v) atleast 1 wrong = total − no wrong

$= 120 - 1$

$= 119$

(vi) exactly 1 wrong = exactly 4 correct

i.e., not possible

$\therefore 0$

P/61 (i) $D_5 \times D_5 = 44 \times 44 = 1936$

(ii) ~~$D_5 \times D_5 = 44 \times 44 = 1936$~~

$\underbrace{\_\ \_\ \_\ \_\ \_}_{\text{every place is wrong here for f, g, h, i, j}} \quad \underbrace{\_\ \_\ \_\ \_\ \_}_{\text{every place is wrong here for a, b, c, d, e}}$

$\therefore 5! \times 5!$

$120 \times 120 = 14400$

P/62 $4! \times D_4 = 24 \times 9 = 216$

giving 1st time is arrangement

2nd time its derangement

23/06/20

Binomial Coefficients:

Consider

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

Here we are interested in coefficients because they are the ones that remain same even if values of a & b changes.

Consider this multiplication as choosing a's or b's from below boxes. Now we determine the coefficients as shown below.

$\boxed{a+b}$ $\boxed{a+b}$ $\boxed{a+b}$

Now the coefficient of a^3 is nothing but no of way we can choose 3 a's

i.e., a a a

$\therefore$ 1 way $= 3C_0 = 3C_1$

Coefficient of a^2b is no of way we can choose two a's and one b

i.e., a a b

a b a

b a a

$\therefore$ 3 ways i.e., $3C_2 = 3C_1$

Slly coefficient of $ab^2 = 3C_2 = 3C_1$

" " " $b^3 = 1 = 3C_0 = 3C$

$\therefore (a+b)^3 = {}^3C_0 a^3 b^0 + {}^3C_1 a^2 b + {}^3C_2 ab^2 + {}^3C_3 a^0 b^3$

Now we can generalize this concept to find $(a+b)^n$

$$\boxed{\begin{aligned} (a+b)^n &= {}^nC_0 a^n b^0 + {}^nC_1 a^{n-1} b^1 + {}^nC_2 a^{n-2} b^2 + \dots + {}^nC_n a^0 b^n \\ (a+b)^n &= \sum_{i=0}^{n} \underbrace{{}^nC_i}_{\text{Binomial coefficient.}} a^{n-i} b^i \end{aligned}}$$

* Now in the above expansion put $a=1$, $b=1$

$$(1+1)^n = \sum_{i=0}^{n} {}^nC_i (1)^{n-i} (1)^i$$

i.e., $\boxed{\sum_{i=0}^{n} {}^nC_i = 2^n}$

i.e., $\boxed{{}^nC_0 + {}^nC_1 + \dots + {}^nC_n = 2^n}$

* Now consider $a=1$ $b=-1$

$$(1-1)^n = \sum_{i=0}^{n} {}^nC_i (1)^{n-i} (-1)^i$$

i.e., $0 = \sum_{i=0}^{n} {}^nC_i (1) (-1)^i$

i.e., ${}^nC_0 - {}^nC_1 + {}^nC_2 - {}^nC_3 + \dots + (-1)^i\, {}^nC_n = 0$

$\Rightarrow$ ${}^nC_0 + {}^nC_2 + {}^nC_4 + \dots = {}^nC_1 + {}^nC_3 + {}^nC_5 + \dots$

$\therefore$ $\boxed{{}^nC_0 + {}^nC_2 + {}^nC_4 + \dots = {}^nC_1 + {}^nC_3 + {}^nC_5 + \dots = 2^{n-1}}$

* Now consider $a=1$ $b=2$

$$(1+2)^n = \sum_{i=0}^{n} {}^nC_i \, (1)^{n-i} \, 2^i$$

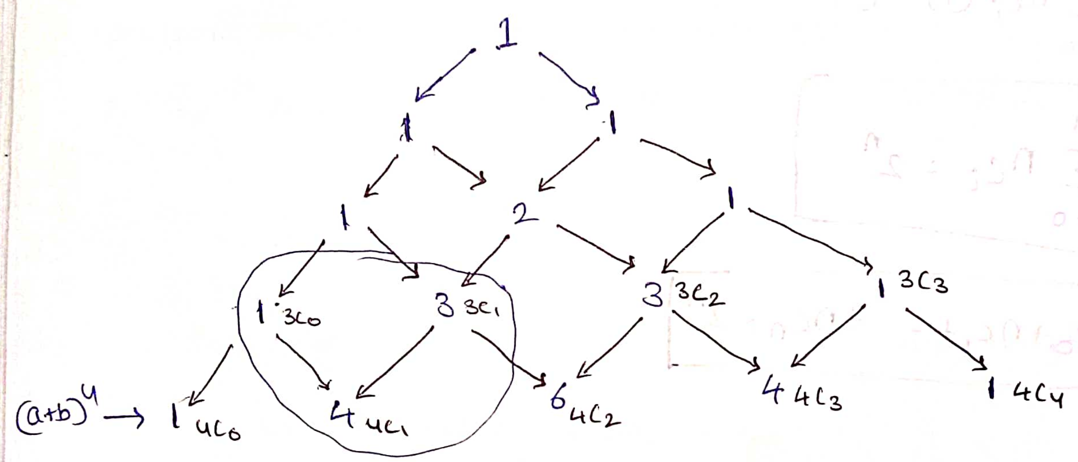
$$\boxed{3^n = \sum_{i=0}^{n} {}^nC_i \, 2^i}$$

i.e., $\boxed{{}^nC_0 + 2^1\,{}^nC_1 + 2^2\,{}^nC_2 + \dots + 2^n\,{}^nC_n = 3^n}$

Pascal's Triangle :

Pascal's triangle shows the coefficients of binomial expansion.

It is constructed as shown below



$$\therefore (a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$

From the circled part of the pascal's triangle we have

$$^3C_0 + {}^3C_1 = {}^4C_1$$

we can generalize this as

$$\boxed{{}^nC_i + {}^nC_{i+1} = {}^{n+1}C_{i+1}}$$