Logic

Propositional statement:

It is a statement which can be either true or false. But not ~~same at~~ both at the same time.

→ two types propositional stmts:

Simple propositional statement:

It is a propositional stmt which can't be broken into two or more parts.

Eg: $x > y$

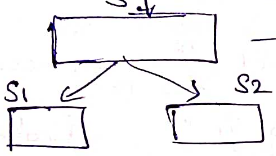
Compound propositional statements

It is a propositional stmt which can be broken into two or more parts

Eg: $x \geq y \cong x > y$ (or) $x = y$

Let

S ↓ breakable point or Connectives



→ be a compound prop. stmt

→ Truth value of compound stmt can't be determined directly.

Connectives:

-we have 4 different types of connectives:

(i) Conjunction ($\wedge$)

(ii) Disjunction ($\vee$)

(iii) Simple Implication ($\rightarrow$)

(iv) Double Implication ($\leftrightarrow$)

Modiers:

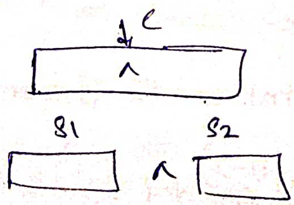
~~$\sim$ is a~~

Negation ($\sim$) is a modifier.

1) Conjunction ($\wedge$) (AND/but):

S_1	S_2	$S_1 \wedge S_2$
F	F	F
F	T	F
T	F	F
T	T	T

→ Compound propositional stmt



$F \wedge \_ \wedge \_ \wedge \_ \wedge \ldots = F$

(ii) Disjunction ($\vee$) (~~or~~ Inclusive OR):

S_1	S_2	$S_1 \vee S_2$
F	F	F
F	T	T
T	F	T
T	T	T

$T \vee \_ \vee \_ \vee \_ \vee \ldots = T$

(iii) Single Implication ($\rightarrow$)

$p \rightarrow q$

if p, then q

if p, q

q if p

q when p

q whenever p

q unless $\sim p$ ★

p implies q

Here p is called antecedent (or) premise (or) hypothesis

q is called consequent

p	q	$p \rightarrow q$
F	F	T
F	T	T
T	F	F
T	T	T

(iv) Biconditional ($\leftrightarrow$)

→ if and only if, iff,

p	q	$p \leftrightarrow q$
F	F	T
F	T	F
T	F	F
T	T	T

Logic Equivalence ($\equiv$)

$A \equiv B$

whenever A is true B is also true

whenever A is ~~true~~ false B is also false

$A = T \Rightarrow B = T$, $A = F \Rightarrow B = F$, $B = T \Rightarrow A = T$, $B = F \Rightarrow A = F$

Tautology (or) Valid:

For any truth values, if the compound prop stmt is true, then its called tautology.

Contradiction

If the compound prop stmt is false, for any truth values, then it is called contradiction.

Satisfiable:

If the compount prop stmt is true, for atleast one set of truth values, then it is said to be satisfiable.

Contingency:

It is neither ~~tautology~~ tautology nor Contradiction.

Note:

→ Every contingency is satisfiable. But reverse need not to be true.

→ Every tautology is ~~satif~~ satisfiable. But reverse need not to be true.

P/14

a) $[(p \to q) \to r] \to (p \to (q \to r))$

To p.T the above is not tautology

we need to prove that left side is ~~false~~ true &
right side is false in atleast one case

$\Rightarrow (p \to q) \to r = T \;\&\; p \to (q \to r) = F$

$\Rightarrow$ ~~$(p \to q) = T \;\&\; r = F$~~ $\qquad p = T \;\&\; q \to r = F$

$\Rightarrow \qquad p = T \;\&\; q = T \;\&\; r = F$

$p = T \;\&\; q = T \;\&\; r = F \Rightarrow (T \to T) \to F \equiv T \to F \equiv F$

∴ ~~For p = T, q = T & r = F the above Comp. prop. stmt is~~ false

i.e., whenever right side part is ~~true~~ false, left is also false

∴ opt (a) is tautology

b) $[p \to (r \vee q)] \to [(p \to r) \vee (p \to q)]$

To prove that above is not tautology

$p \to (r \vee q) = T$ $\qquad$ $(p \to r) \vee (p \to q) = F$

$p \to r = F$ & $p \to q = F$

$p = T$ & $r = F$ & $p = T$ & $q = F$

$T \to (F \vee F) = T \to F = F$

$\therefore$ b is tautology

c) $p \to (r \wedge q) = T$ & $(p \to r) \vee (p \to q) = F$

$p \to r = F$ & $p \to q = F$

$p = T$, $r = F$ & $q = F$

$T \to (F \wedge F)$

$= F$

$\therefore$ c is tautology

d) $[p \to (q \to r)] \to [(p \to q) \to r]$

$p \to (q \to r) = T$ $\qquad$ $(p \to q) \to r = F$

$\Rightarrow p \to q = T$ & $r = F$

Here possible ways are:

p	q
F	F
F	T
T	T

Consider

$p = F$ $q = F$ $r = F$

$F \to (F \to F) = T$

$\therefore$ for $p = F$, $q = F$, $r = F$

value of given comp. prop. stmt is false

$\therefore$ d is not a tautology.

GATE 2004 Let p, q, r & s be four primitive stmts. Consider following arguments.

P: $[(\sim p \vee q) \wedge (r \to s) \wedge (p \vee r)] \to (\sim s \to q)$

Q: $[(\sim p \wedge q) \wedge (q \to (p \to r))] \to \sim r$

R: $[[(q \wedge r) \to p] \wedge (\sim q \vee p)] \to r$

S: $[p \wedge (p \to r) \wedge (q \vee \sim r)] \to q$

Options
a) P & Q
b) P & R
c) P & S
d) P, Q, R, S

which of the above arguments are valid ?

To P.T a stmt is valid we need to P.T if right side is false the left side is ~~also false~~ not true.

P: $\sim s \to q = F$

$\sim s = T$ & $q = F$

$s = F$ & $q = F$

antecedent :

$(\sim p \vee F) \wedge (r \to F) \wedge (p \vee r)$

if ~~P=F~~ $p = F$ & $r = F$

~~antecedent is false~~

$\therefore$ P is tautology

$\sim p \vee F = T \Rightarrow \sim p = T \Rightarrow p = F$

$r \to F = T \Rightarrow r = F$

$p \vee r = T \Rightarrow F \vee r = T \Rightarrow F \vee F = T$

~~$\Rightarrow F \vee T$~~ $F = T$

This not possible.

Hence we can't P.T antedent is true when consequent is false

Q: $\sim r = F \Rightarrow r = T$

$(\sim p \wedge q) \wedge q \to (p \to T) = T$

$\sim p \wedge q = T$

$p = F$ & $q = T$

$q \to (p \to T)$

$T \to (F \to T)$

$T \to T = T$

~~$\therefore$ for $p = F$, $q = T$ & $r = T$~~

we can P.T that Q is false

$\therefore$ Q is not tautology

R: $r = F$

$[(q \wedge F) \to p] \wedge (\sim q \vee p)$

$(F \to p) \wedge (\sim q \vee p) \equiv T \wedge (\sim q \vee p) \equiv \sim q \vee p$

$q = F$ & $p = T$

$\therefore R$ is not valid

S: $q = F$

$p \wedge (p \to r) \wedge (q \vee \sim r)$

$p \wedge (p \to r) \wedge (F \vee \sim r)$

$p = T$ $\quad p \to r$

$T \to r$

$r = T$

$\cancel{F \to T} = T$

$F \vee F = T$

$\therefore S$ is ~~not~~ tautology

$\therefore$ P and S only

GATE 2014 which one of the following boolean expressions is not a tautology

a) $[(a \to b) \wedge (b \to c)] \to (a \to c)$

b) $[a \leftrightarrow c] \to (\sim b \to (a \wedge c))$

c) $(a \wedge b \wedge c) \to (c \vee a)$

d) $a \to (b \to a)$

Sol:

a⇒ $a \to c = F$

$a = T$ & $c = F$

$(a \to b) \wedge (b \to c)$

$(T \to b) \wedge (b \to F)$

if $b = T$ $\quad$ if $b = F$

$(T \to T) \wedge (T \to F)$ $\quad (T \to F) \wedge (F \to F)$

$T \wedge F$ $\quad F \wedge T$

$= F$ $\quad = F$

$\therefore$ opt ⓐ ~~can't~~ is a tautology

b⇒ $\sim b \to (a \wedge c) = F$

$\sim b = T$ $\quad a \wedge c = F \Rightarrow$

a	c
F	F
T	F
F	T

$b = F$

$a \leftrightarrow c = T \Rightarrow a \to c = T$ & $c \to a = T$

$\Rightarrow a = T$ & $c = T$ (or) $a = F$ & $c = F$

for $a = T, b = F, c = T$

$(T \leftrightarrow T) \rightarrow (\sim F \rightarrow (T \wedge T))$

$T \rightarrow (T \rightarrow T)$

$= T$

for $a = F, b = F, c = F$

$(F \leftrightarrow F) \rightarrow (\sim F \rightarrow (F \wedge F))$

$T \rightarrow (T \rightarrow F)$

$T \rightarrow (F)$

$= F$

$\therefore$ b is not tautology

c) $c \vee a = F$

$c = F$ & $a = F$

$\Rightarrow a \wedge b \wedge c = F$

$\therefore$ Valid

d) $b \rightarrow a = F$

$b = T \quad a = F$

$F \rightarrow (T \rightarrow F) = F \rightarrow F = T$

$\therefore$ valid

$\therefore$ b

We have a few type of question only in this chapter

Type: 1:

Check if below is tautology

$\underbrace{\qquad\qquad\qquad}_{T} \rightarrow \underbrace{\bigcirc}_{F}$

we try to prove above is false intentionally

i.e., we need to show

$T \rightarrow F$

For consequent $= F$

if we can P.T antecedent is T

then above is not tautology.

For consequent $= F$

if we can't P.T that antecedent is T

then above is tautology

P/30

Eg: $[p \wedge (p \rightarrow q) \wedge (\sim q \vee x)] \rightarrow x$

we now try to P.T above is not valid

i.e., we need to P.T

$[p \wedge (p \rightarrow q) \wedge (\sim q \vee x)] = T$ & $x = F$

Frm ~~In~~ antecendent

$p = T$ & $p \to q = T$ & $\sim q \vee x = T$

$T \to q = T$ $\quad\quad$ $\sim q \vee F = T$ (from consequent)

$\Rightarrow \underline{q = T}$ $\quad\quad$ $\sim T \vee F = T$

$F \vee F = T$

$F = T$

$\Downarrow$

This is not possible

∴ we can't P.T given stmt is false

∴ given prop. stmt is a tautology

P/28

$$\overbrace{[p \wedge (p \to q) \wedge (s \vee r) \wedge (r \to \sim q)]}^{T} \to \overbrace{(s \vee t)}^{F}$$

T T T F F F F F F F

$s = F$
$t = F$
$p = T$
$q = T$
$r = F$

∴ valid

P/29

$$\{[(\sim p \vee q) \to r] \wedge [r \to (s \vee t)] \wedge (\sim s \wedge \sim u) \wedge (\sim u \to \sim t)\} \to p$$

T T T T T T T T F F

$p = F$
$r = T$
$s = F$
$u = F$
$t = T$
$u = T$

∴ ~~not~~ valid

~~for P.T above stmt. becomes true~~

∴ ~~satisfiable but not valid~~

P/30

$$[(p \to q) \wedge (\sim r \vee s) \wedge (p \vee r)] \to \overbrace{(\sim q \to s)}^{F}$$

T T F T

∴ not valid

$q = F$
$s = F$
$r = F$
$p = T$
$q = T$

P/30

$$\left[\overset{T}{\overline{(p\to q)}} \wedge \overset{T}{\overline{(\underset{T}{\sim r}\vee s)}} \wedge \overset{T}{\overline{(p\vee r)}}\right] \to \overset{F}{\overline{(\underset{T}{\sim q}\to \underset{F}{s})}}$$

$p\to q$: $p = T$, $q = F$ $\Rightarrow$ $\underbrace{T}_{F}$

$\sim r\vee s$: $s = F$

$p\vee r$: $p = T$, $r = F$

$q = F$
$s = F$
$r = F$
$p = T$

$\therefore$ valid

P/31

$$\left[(\underset{T}{\sim p}\leftrightarrow q)\wedge(q\to r)\wedge(\sim r)\right]\to p$$

$q = T$, $q = T$, $r = T$, $r = F$

$p = F$
$q = T$
$r = T$

$\therefore$ valid

GATE 1990

Indicate which of the following a valid?

a) $\left[(P\Rightarrow Q)\wedge(Q\Rightarrow R)\right]\Rightarrow(P\Rightarrow R)$

b) $(P\Rightarrow Q)\Rightarrow(\sim P\Rightarrow\sim Q)$

c) $\left[P\wedge(\sim P\vee\sim Q)\right]\Rightarrow Q$

d) $\left[(P\Rightarrow R)\vee(Q\Rightarrow R)\right]\Rightarrow\left[(P\vee Q)\Rightarrow R\right]$

Sol:

a) $\Rightarrow$ $\left[(P\Rightarrow Q)\wedge(Q\Rightarrow R)\right]\Rightarrow\overset{F}{\overline{(\underset{T}{P}\Rightarrow \underset{F}{R})}}$

$P = T$, $R = F$

$Q = T$ $\quad$ $Q = F$

$P = T$
$\cancel{R} = F$
$Q = T$

$\therefore$ valid

b) $\Rightarrow$ $(\underset{F}{P}\Rightarrow \underset{T}{Q})\Rightarrow\overset{F}{\overline{(\sim P\Rightarrow\sim Q)}}$

$\underbrace{F \quad T}_{T}$

$P = F$
$Q = T$

$\therefore$ not valid

c) $\left[P \wedge (\underset{F}{\sim P} \vee \underset{T}{\sim Q})\right] \Rightarrow Q$

$\downarrow$ T

$Q = F$

$P = T$

$\therefore$ not valid

d) $\left[\overset{T}{(\overset{F}{P \Rightarrow R}) \vee (Q \Rightarrow R)}\right] \Rightarrow \left[\overset{F}{(P \vee Q) \Rightarrow R}\right]$

$R = F$

$P \vee Q = T$

$P = T$ & $Q = F$

$P = F$ & $Q = T$

$P = T$ & $Q = T$

Case(i):

$(T \to F) \vee (F \to F)$

$F \vee T = T$

$\therefore$ not valid

GATE 1992 which of the following is/are valid?

a) $(a \vee b) \to (b \wedge c)$

b) $(a \wedge b) \to (b \vee c)$

c) $(a \vee b) \to (b \to c)$

d) $(a \to b) \to (b \to c)$

a) $a = T$ $b = F$ $c = T$

$\therefore$ not valid

b) $a = T$ $b = T$ $c = T/F$

$\therefore$ valid

c) $a = T$ $b = T$ $c = F$

$(T \vee T) \to (T \to F) \equiv T \to F \equiv F$

$\therefore$ not valid

d) $a = T$ $b = T$ $c = F$

$\therefore$ not valid

GATE 2001 Consider two well formed formulas in propositional logic

F1: $P \Rightarrow \sim P$

F2: $(P \Rightarrow \sim P) \vee (\sim P \Rightarrow P)$

which of following statements is correct?

a) F_1 is satisfiable, F_2 is valid

b) F_1 is unsatisfiable, F_2 is satisfiable

c) F_1 is unsatisfiable, F_2 is valid

d) F_1 & F_2 are both satisfiable

P	$\sim P$	F_1: $P \Rightarrow \sim P$	$\sim P \Rightarrow P$	F_2: $(P \Rightarrow \sim P) \vee (\sim P \Rightarrow P)$
T	F	F	T	T
F	T	T	F	T

$\therefore F_1$ is satisfiable, F_2 is valid

GATE 2004

The following prop. stmt is

$$[P \to (Q \vee R)] \to ((P \wedge Q) \to R)$$

a) satisfiable but not valid

b) Valid

c) A contradiction

d) None of the above

Sol :

$$[P \to (Q \vee R)] \to \overset{F}{\overline{[\underbrace{(P \wedge Q)}_{T} \to \underbrace{R}_{F}]}}$$

P: T, Q: T, R: F

$P = T$

$Q = T$

$R = F$

$T \to (T \vee F)$

$T \to T$

$= T$

$\therefore$ not valid

for $P = T$, $Q = T$, $R = T$

$[T \to (T \vee T)] \to [(T \wedge T) \to T]$

$T \to T = T$

$\therefore$ satisfiable

To P.T the given stmt is satisfiable we P.T Consequent is true (or) we P.T both antecedent & Consequent are false at the same time.

Consider

$p \to q$

Inverse: $\sim p \to \sim q$

Converse: $q \to p$

Contrapositive: $\sim q \to \sim p$

$$\boxed{p \to q \equiv \sim q \to \sim p}$$

Laws

Identitity laws:

1) $p \wedge T \equiv p$
   $p \vee F \equiv p$ } Identity law

2) $p \vee T \equiv T$
   $p \wedge F \equiv F$ } Dominating law

3) $p \vee p \equiv p$
   $p \wedge p \equiv p$

4) $p \vee q = q \vee p$
   $p \wedge q = q \wedge p$ } Commutatic law

5) $(p \vee q) \vee r = p \vee (q \vee r)$
   $(p \wedge q) \wedge r = p \wedge (q \wedge r)$ } Associative law

6) $p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$
   $p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$ } Distributive laws

7) $\sim(p \wedge q) \equiv \sim p \vee \sim q$
   $\sim(p \vee q) \equiv \sim p \wedge \sim q$ } Demorgan's law

8) $p \vee (p \wedge q) \equiv p$
   $p \wedge (p \vee q) \equiv p$ } Absorption Law

9) $p \vee \sim p \equiv T$
   $p \wedge \sim p \equiv F$ } Negation laws

→ $p \to q \equiv \sim p \vee q$

$p \to q \equiv \sim q \to \sim p$

→ $\sim p \to q \equiv p \vee q$

→ $(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$

$(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$

Proof:

$(p \vee q) \to r$

$\sim (p \vee q) \vee r$

$(\sim p \wedge \sim q) \vee r$

$(\sim p \vee r) \wedge (\sim q \vee r)$

$(p \to r) \wedge (q \to r)$

→ $(p \to q) \vee (p \to r) \equiv p \to (q \vee r)$

$(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$

$p \Leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

GATE 2008

P and Q are two propositions. which of the following logical expressions are equivalent?

I. $P \vee \sim Q$

II. $\sim(\sim P \wedge Q)$

III. $(P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge \sim Q)$

IV. $(P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge Q)$

a) I & II b) I, II, III c) I, II, IV d) All

II) $\sim(\sim P \wedge Q) \equiv P \vee \sim Q \equiv I$

III) $\underset{T\ T}{(P \wedge Q)} \vee \underset{T\ F}{(P \wedge \sim Q)} \vee \underset{F\ F}{(\sim P \wedge \sim Q)}$

$\equiv Q \to P \equiv \sim Q \vee P \equiv I$

we can directly say it is $Q \to P \equiv \sim Q \vee P$

It is like S.O.P in DLD

IV) $\underset{T\ T}{(P \wedge Q)} \vee \underset{T\ F}{(P \wedge \sim Q)} \vee \underset{F\ T}{(\sim P \wedge Q)} \equiv P \vee Q$

$\therefore I \equiv II \equiv III$

(or)

III) $(P\wedge Q)\vee(P\wedge\sim Q)\vee(\sim P\wedge\sim Q)$

$(P\wedge Q)\vee\left[(P\vee\sim P)\wedge\sim Q\right]$

$(P\wedge Q)\vee[T\wedge\sim Q]\equiv(P\wedge Q)\vee\sim Q$

$\equiv(P\vee\sim Q)\wedge(Q\vee\sim Q)$

$\equiv(P\vee\sim Q)\wedge T$

$\equiv P\vee\sim Q$

IV) $(P\wedge Q)\vee(P\wedge\sim Q)\vee(\sim P\wedge Q)$

$\left[(P\vee\sim P)\wedge Q\right]\vee(P\wedge\sim Q)$

$Q\vee(P\wedge\sim Q)$

$(Q\vee P)\wedge(Q\vee\sim Q)\equiv P\vee Q$

P/7 S1: $[\sim r\vee\sim s]\rightarrow q$

contra positive is

$\sim q\rightarrow\sim[\sim r\vee\sim s]\equiv\sim q\rightarrow(r\wedge s)$

$\equiv q\vee(r\wedge s)$

$\therefore$ S1 is true

S2: $[\sim r\vee\sim s]\rightarrow q$

Convers: $q\rightarrow[\sim r\vee\sim s]$

$\equiv q\rightarrow\sim[r\wedge s]$

$\therefore$ S2 is true

S3: $[\sim r\vee\sim s]\rightarrow q$

inverse: $\sim[\sim r\vee\sim s]\rightarrow\sim q$

$(r\wedge s)\rightarrow\sim q$

$\therefore$ S3 is true

S_4: $(\sim r \vee \sim s) \to q$

$\equiv \sim(\sim r \vee \sim s) \vee q$

$\equiv (r \wedge s) \vee q$

Negation:

$\sim[(r \wedge s) \vee q]$

$\sim(r \wedge s) \wedge \sim q \equiv (\sim r \vee \sim s) \wedge \sim q$

$\therefore S_4$ is true

$\therefore$ opt (d)

P/9

$(\sim p \wedge q) \vee (p \wedge \sim q) \vee (p \wedge q)$

$[(\sim p \vee p) \wedge q] \vee (p \wedge \sim q)$

$q \vee (p \wedge \sim q)$

$(p \vee q) \wedge (q \vee \sim q)$

$\equiv p \vee q$

GATE 2015

Which one of the following is NOT equivalent to $p \leftrightarrow q$?

a) $(\sim p \vee q) \wedge (p \vee \sim q)$ b) $(\sim p \vee q) \wedge (q \to p)$

c) $(\sim p \wedge q) \vee (p \wedge \sim q)$ d) $(\sim p \wedge \sim q) \vee (p \wedge q)$

Sol:

$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

$\equiv (p \wedge q) \vee (\sim p \wedge \sim q)$

a) $(\sim p \vee q) \wedge (p \vee \sim q)$

$[(\sim p \vee q) \wedge p] \vee [(\sim p \vee q) \wedge \sim q]$

$[\underbrace{(\sim p \wedge p)}_{F} \vee (q \wedge p)] \vee$ ~~$\sim p \wedge q \wedge \sim q$~~ $(\sim p \wedge \sim q) \vee \underbrace{(q \wedge \sim q)}_{F}$

alternate:

$(\sim p \vee q) \wedge (p \vee \sim q)$

$(p \to q) \wedge (q \to p)$

$p \leftrightarrow q$

$\equiv (q \wedge p) \vee (\sim p \wedge \sim q) \equiv p \leftrightarrow q$

c) $(\sim p \wedge q) \vee (p \wedge \sim q)$

It is clearly not equivalent to $p \leftrightarrow q$

b) $(\sim p \vee q) \wedge (q \to p)$

$(\sim p \vee q) \wedge (\sim q \vee p)$

$[(\sim p \vee q) \wedge \sim q] \vee [(\sim p \vee q) \wedge p]$

$[(\sim p \wedge \sim q) \vee \underbrace{(q \wedge \sim q)}_{F}] \vee [\underbrace{(\sim p \wedge p)}_{F} \vee (q \wedge p)]$

$(\sim p \wedge \sim q) \vee (p \wedge q) \equiv p \leftrightarrow q$

$\therefore$ opt (c)

$p \leftrightarrow q = (p \to q) \wedge (q \to p)$

$\equiv (\sim p \vee q) \wedge (q \to p)$

GATE 2006

A logical binary relation $\odot$ is defined as follows

A	B	$A \odot B$
T	T	T
T	F	T
F	T	F
F	F	T

Let $\sim$ be the unary negation (NOT) operator, with higher precedence than $\odot$. Which one of the following is equivent to $A \wedge B$.

Sol

a) $\sim A \odot B$ b) $\sim (A \odot \sim B)$

c) $\sim (\sim A \odot \sim B)$ d) $\sim (\sim A \odot B)$

Sol :

It is clear that

$A \odot B \equiv B \to A$

a) $\sim A \odot B \equiv B \to \sim A \equiv \sim B \vee \sim A = \sim (A \wedge B)$

b) $\sim (A \odot \sim B) \equiv \sim (\sim B \to A) \equiv \sim (B \vee A) = \sim B \wedge \sim A$

c) $\sim (\sim A \odot \sim B) \equiv \sim (\sim B \to \sim A) \equiv \sim (B \vee \sim A) = \sim B \wedge A$

d) $\sim (\sim A \odot B) \equiv \sim (B \to \sim A) = \sim (\sim B \vee \sim A) = A \wedge B$

$\therefore$ opt (d)

(P/6)

P_1	P_2	P_3	Fun
F	F	F	2 way
F	F	T	2
F	T	F	2
F	T	T	2
T	F	F	2
T	F	T	2
T	T	F	2
T	T	T	2

2^8 ways

$= 256$ non-equivalent functions

(P/26) $[p \wedge (\sim r \vee q \vee \sim q)] \vee [(r \vee t \vee \sim r) \wedge \sim q]$

$[p \wedge T] \vee [T \wedge \sim q]$

$p \vee \sim q$

(P/27) $[p \vee (p \wedge q) \vee (p \wedge q \wedge \sim r)] \wedge [(p \wedge r \wedge t) \vee t]$

$[p \vee (p \wedge q \wedge \sim r)] \wedge t$

$p \wedge t$

(P/24) $[(a \wedge b) \to c] \leftrightarrow [(a \to c) \vee (b \to c)]$

Consider

$\overset{T}{[(a \wedge b) \to c]} \to \overset{F}{[\underset{F}{(a \to c)} \vee \underset{F}{(b \to c)}]}$

$a = T$
$b = T$
$c = F$

$(a \wedge b) \to c$

$T \to F = F$

$\therefore$ above is tautology

Consider

$[(a \to c) \vee (b \to c)] \to \overset{F}{[\underset{T}{(a \wedge b)} \to \underset{F}{c}]}$

T F T F

$F \vee F = F$

$a = T$
$b = T$
$c = F$

$\therefore$ tautology

Hence given is valid

alternate method

$(a \wedge b) \to c$

$\sim(a \wedge b) \vee c$

$\sim a \vee \sim b \vee c$

$(a \to c) \vee (b \to c)$

$(\sim a \vee c) \vee (\sim b \vee c)$

$\sim a \vee \sim b \vee c$

$\therefore$ valid

P/20

S_1: $\overbrace{[(a \vee b) \to c]}^{T} \Rightarrow \overbrace{[\underbrace{(a \wedge b)}_{T} \to \underbrace{c}_{F}]}^{F}$

$\underbrace{T \quad T}_{} \quad F$ → F

$a = T$

$b = T$

$c = F$

$\therefore S_1$ is valid

S_2: $\overbrace{[(a \wedge b) \to c]}^{T} \Rightarrow \overbrace{[\underbrace{(a \vee b)}_{T} \to \underbrace{c}_{F}]}^{F}$

T F F

F

T

a	b
T	T
T	F
F	T

$c = F$

$\therefore$ not valid

P/21

S_1: $[p \to (q \wedge r)] \Leftrightarrow [(p \to q) \wedge (p \to r)]$

$p \to (q \wedge r)$

$\sim p \vee (q \wedge r) \equiv (\sim p \vee q) \wedge (\sim p \vee r)$

$\equiv (p \to q) \wedge (p \to r)$

$\therefore$ tautology

S_2: $(p \vee q) \to r$

$\sim (p \vee q) \vee r$

$(\sim p \wedge \sim q) \vee r \equiv (\sim p \vee r) \wedge (\sim q \vee r)$

$\equiv (p \to r) \wedge (q \to r)$

P/22

$[p \to (q \vee r)] \leftrightarrow [(p \wedge \sim q) \to r]$

L.H.S $= p \to (q \vee r) \equiv \sim p \vee (q \vee r)$

R.H.S $=$ $(p \wedge \sim q) \to r \equiv \sim (p \wedge \sim q) \vee r$

$\equiv \sim p \vee q \vee r$

$\therefore$ valid

P/13

$P \vee (P \leftrightarrow Q) \vee Q$

$P \vee [(P \vee Q) \wedge (\sim P \vee \sim Q)] \vee Q \equiv (P \vee Q) \vee [P \leftrightarrow Q]$

for

P, Q
T F
F T
T T

$P \vee Q$ is true, For

P Q
F F

$P \leftrightarrow Q$ is true $\therefore$ valid

Q/10 $[p \wedge (\sim p \vee \sim q) \wedge (\sim p \vee q \vee r) \wedge \sim r]$

$[[(p \wedge \sim p) \vee (p \wedge \sim q)] \wedge (\sim p \vee q \vee r) \wedge \sim r]$

$(p \wedge \sim q) \wedge (\sim p \vee q \vee r) \wedge \sim r$

for $p = F$ above expression is false

$\therefore$ not valid

Now lets try to satisfy the stmt.

$r = F$

$(p \wedge \sim q) \wedge (\sim p \vee q \vee F) \wedge T$

$(p \wedge \sim q) \wedge (\sim p \vee q) \equiv p \wedge \sim q \wedge (\sim p \vee q)$

$\cancel{(q \to p) \wedge (p \to q)}$ $\quad p = T, q = F$

$\cancel{(p \leftrightarrow q)}$ $\quad \Rightarrow \sim p \vee q = F$

~~for $p = T, q = T, r = F$~~ $\quad \therefore$ not satisfiable

~~above stmt is true~~

~~$\therefore$ given stmt is a contingency.~~

GATE 2017

The statement $(\sim p) \Rightarrow (\sim q)$ is logically equivalent to which of the statements below?

I. $p \Rightarrow q$

II. $q \Rightarrow p$

III. $\sim q \vee p$

IV. $\sim p \vee q$

a) I only b) I & IV c) II only d) II & III

Sol :

$\sim p \to \sim q \equiv q \to p \equiv \sim q \vee p$

$\therefore$ II & III

GATE 2017

Let p, q, and r be propositions and the expression $(p \to q) \to r$ be a contradiction. Then, the expression $(r \to p) \to q$ is

a) a tautology b) a contradiction

c) always TRUE when p is FALSE

d) always TRUE when q is TRUE

Sol: $(p\to q)\to r \equiv F$

$\Rightarrow p\to q \equiv T$ & $r \equiv F$

p	q
F	F
F	T
T	T

given exp $(r\to p)\to q$

$(F\to p)\to q$

$T\to q$

when p is true q must be true

when if $p=T$

$T\to q \equiv T\to T \equiv T$

if $p=F$

$T\to q$

$T\to F \quad T\to T$

$F \quad T$

if $q=T$

$p=F$: $T\to q$, $T\to T$, T

$p=T$: $T\to q$, $T\to T$, T

$\therefore$ always true when p is true

(or)

always true when q is true

$\therefore$ option (d)

☆

Q (GATE 2002) ☆ "If X then Y unless Z" is represented by which of the following formulas in propositional logic?

a) $(X\wedge\sim Z)\to Y$ b) $(X\wedge Y)\to\sim Z$

c) $X\to(Y\wedge\sim Z)$ d) $(X\to Y)\wedge\sim Z$

$\boxed{\text{If } x \text{ then } Y}$ unless Z

$p\to q$

q unless $\sim p$

$\sim z\to(x\to Y)$

$z\vee(\sim x\vee y)$

☆ associativity ~~Precedence~~ of implication is from ~~left to~~ right to left ☆

a) $\Rightarrow (X\wedge\sim Z)\to Y \equiv \sim(X\wedge\sim Z)\vee Y \equiv \sim X\vee Z\vee Y$

b) $\Rightarrow (X\wedge Y)\to\sim Z \equiv \sim(X\wedge Y)\vee\sim Z \equiv \sim X\vee\sim Y\vee\sim Z$

c) $\Rightarrow \sim X\vee(Y\wedge\sim Z)$

$\boxed{\text{Precedence}(\to) > \text{Precedence}(\leftrightarrow)}$

d) $\Rightarrow (\sim X\vee Y)\wedge\sim Z$

$\therefore$ option (a)

GATE 2014

which one of the following propositional logic formulas is TRUE when exactly two of p, q and r are true?

a) $[(p \leftrightarrow q) \wedge r] \vee (p \wedge q \wedge \sim r)$

b) $[\sim(p \leftrightarrow q) \wedge r] \vee (p \wedge q \wedge \sim r)$

c) $[(p \to q) \wedge r] \vee (p \wedge q \wedge \sim r)$

d) $[\sim(p \leftrightarrow q) \wedge r] \wedge (p \wedge q \wedge \sim r)$

(i) $p = T$, $q = T$, $r = F$

ⓐ $\equiv T$ ✓

ⓑ $\equiv T$ ✓

ⓒ $\equiv T$ ✓

ⓓ $\equiv F$

(ii) $p = F$, $q = T$, $r = T$

ⓐ $\equiv F$

ⓑ $\equiv T$ ✓

ⓒ $\equiv T$ ✓

(iii) $p = T$, $q = F$, $r = T$

ⓑ $\equiv T$

ⓒ $\equiv F$

$\therefore$ opt ⓑ

07/05/20

Note:

→ Order of Precedences: $(\sim) > (\wedge) > (\vee) > (\to) > (\leftrightarrow)$

→ Associativity of '$\to$' is from left to right

→ $(p \to q) \to r$ is not equivalent to $p \to (q \to r)$

Logic	Sets	Boolean Algebra
$\wedge$	$\cap$	$\cdot$
$\vee$	$\cup$	$+$
T	U (Universal set)	1
F	$\emptyset$	0

Inference Rule:

* $p \to q$

  p – Premises (or) Argument

  q – Conclusion

→ Consider premises as true and check ~~from~~ for conclusion. If conclusion is true, it comes under inference rule.

→ premises → conclusion | $\dfrac{\text{premises}}{\therefore \text{Conclusion}}$

→ If $P_1, P_2, P_3 \ldots P_n$ are premises and Q is conclusion then we write it as

$$(P_1 \wedge P_2 \wedge P_3 \wedge \cdots \wedge P_n) \to Q$$

(or)

$$\begin{array}{c} P_1 \\ P_2 \\ \vdots \\ P_n \\ \hline \therefore Q \end{array}$$

It is means if all the premises are true, then Q must be true.

~~We~~ we can say

$(P_1 \wedge P_2 \wedge \cdots \wedge P_n) \to Q$ is a tautology if the set of premises infer Q.

Inference rules

1) $P \to q$
p
$\therefore p$
Modus ponens

2) $p \to q$
$\sim q$
$\sim p$
Modus tollens

3) $P \vee Q$
$\sim P$
$\therefore Q$
(or)
$P \vee Q$
$\sim Q$
$\therefore P$
Disjunctive Syllogism

4) $P \to Q$
$Q \to R$
$P \to R$
Hypothetical Syllogism

5) P
$\therefore P \vee Q$
Addition

6) $P \wedge Q$
$\therefore P$
Simplification

7) p
q
$p \wedge q$
Conjunction

8) $p \vee q$
$\sim q \vee r$
$p \vee r$
Resolution

Q1) Consider the following argument & check whether it is valid:

1). Rita is baking a cake.

2) If Rita is baking a cake, then she is not practicing her flute.

3) If Rita is not praticing her flute, then her father with not buy her a car.

4) Therefore Rita's father will not buy her a car.

sol:

1) P

2) $P \to \sim Q$

3) $\sim Q \to \sim R$

$\therefore \sim R$

$P \to \sim Q$

$\sim Q \to \sim R$

$P \to \sim R$

P

$\therefore \sim R$

$\therefore$ Given argument is valid

P/30

$P \to q \equiv \sim q \to \sim P$

$\sim r \vee s \equiv r \to s$ $\longrightarrow$ $\sim q \to r \to$ $\sim q \to s$

$P \vee r \equiv \sim P \to r$

$\sim q \to s$

(or)

$\sim r \vee s$

$P \vee r$

$P \vee s$ (resolution) } $q \vee s$ resolution

$P \to q \equiv \sim P \vee q$

$\sim q \to s$

GATE 2012 Consider the following logical inferences

I_1: If it rains then the ~~cricket~~ cricket match will not be played

The cricket match was played

~~I_2: If it r~~

Inference: There was no rain

I_2: If it rains the cricket match will not be played

It did not rain

Inference: The cricket match was played

which of the following is True

a) I_1 & I_2 are correct

b) I_1 is only correct

c) I_2 is only correct

d) both are not correct

Sol:

I_1:

$$\frac{\begin{array}{l} rains \rightarrow \sim play \\ play \end{array}}{\therefore \sim rain} \qquad \equiv \quad \begin{array}{l} paly \rightarrow \sim rain \\ play \\ \hookrightarrow \therefore \sim rain \end{array}$$

$\therefore I_1$ is correct

I_2:

$$\frac{\begin{array}{l} rains \rightarrow \sim play \\ \sim rain \end{array}}{\therefore play}$$

It is fallacy

It is clear that I_2 is not correct

$\therefore$ only I_1

GATE 2015

Consider following two statements

S1: If a candidate is known to be corrupt, then he will not be elected.

S2: If a candidate is kind, he will be elected.

which one of the following statements follows from S1 & S2 as per sound inference rule of logic

a) If a person is known to be corrupt, he is kind

b) If a person is not known to be corrupt, he is not kind.

c) If a person is kind, he is ~~kn~~ not known to be corrupt.

d) If a person is not kind, he is not known to be corrupt

Sol;

S1: corrupt $\rightarrow \sim$ elect

S2: kind $\rightarrow$ elect $\equiv \sim$elect $\rightarrow \sim$kind

$\therefore$ corrupt $\rightarrow \sim$kind

(or)

kind $\rightarrow \sim$corrupt

a) corrupt $\rightarrow$ kind

b) $\sim$corrupt $\rightarrow \sim$ kind

c) kind $\rightarrow \sim$ corrupt

d) $\sim$kind $\rightarrow \sim$ Corrupt

$\therefore$ option (c)

P/17

$a \rightarrow c$ (1)

$b \rightarrow d$ (2) $\equiv \sim d \rightarrow \sim b$

$c \rightarrow \sim d$ (3)

frm (1) & (3)

$a \rightarrow \sim d$ (4)

(2) & (4)

$a \rightarrow \sim b \equiv \sim a \vee \sim b$

$\therefore$ valid

Q2) Verify whether below arguments are valid or not

a) $p \to q$ ①

$q \to (r \wedge s)$ ②

$\sim r \vee (\sim t \vee u)$ ③

$p \wedge t$ ④

$\therefore u$

① & ② $\Rightarrow p \to r \wedge s$ ——— ⑤

~~③ $\Rightarrow \sim u \to (\sim r \vee \sim t)$ ——— ⑥~~

⑤ & ④ $\Rightarrow r \wedge s$ ——— ⑥

③ & ⑥ $\Rightarrow r$

t

$\sim r \vee \sim t \vee u$

$\therefore u$

b) $u \to r$

$(r \wedge s) \to (p \vee t)$

$q \to (u \wedge s)$

$\sim t$

$\therefore q \to p$

This can be seen as

$u \to r$ ①

$(r \wedge s) \to (p \vee t)$ ②

$q \to (u \wedge s)$ ③

$\sim t$ ④

q ⑤

$\therefore p$

③ & ⑤ $\Rightarrow u \wedge s$ ⑥

⑥ & ① $\Rightarrow r$ ⑦

⑥ ⑦ ② $\Rightarrow p \vee t$ ⑧

⑧ ④ $\Rightarrow p$ $\therefore$ valid

c) $p \to q$ ① $\equiv \sim q \to \sim p$

$\sim q$ ②

$\sim r$ ③

$\sim(p \vee r)$

① & ② $\Rightarrow \sim p$ ④

④ & ③ $\Rightarrow \sim p \wedge \sim r$

$\equiv \sim(p \vee r)$

e) $p \to (q \to r)$ ①

$\sim q \to \sim p$ ②

p ③

$\therefore r$

① & ③ $\Rightarrow q \to r$ ④

② $\Rightarrow p \to q$ ⑤

⑤ & ④ $\Rightarrow p \to r$ ⑥

⑥ ③ $\Rightarrow r$

d) $p \to q$

$r \to \sim q$

r

$\sim p$

$p \to q$

$r \to \sim q \equiv q \to \sim r$

$\Rightarrow p \to \sim r$

$\Rightarrow r \to \sim p$

r

$\therefore \sim p$

f) $p \wedge q$

$p \to (r \wedge q)$ — $r \wedge q$

$r \to (s \vee t)$ — $s \vee t$

$\sim s$ — t

$\therefore t$

g) $p \to (q \to r)$ ①

$p \vee s$ ②

$t \to q$ ③

$\sim s$ ④

$\sim r \to \sim t$

② ④ $\Rightarrow p$ ⑤

⑤ ① $\Rightarrow q \to r$ ⑥

⑥ ③ $\Rightarrow t \to r$

$\equiv \sim r \to t$

h) $p \vee q$

$\sim p \vee r$ — $\sim p$

$\sim r$ — q

$\therefore q$

GATE 2004

P: $[(\sim p \vee q) \wedge (r \to s) \wedge (p \vee r)] \to (\sim s \to q)$

Q: $[(\sim p \wedge q) \wedge (q \to (p \to r))] \to \sim r$

R: $[((q \wedge r) \to p) \wedge (\sim q \vee p)] \to r$

S: $[p \wedge (p \to r) \wedge (q \vee \sim r)] \to q$

which of the above arguments are valid

a) P & Q b) P & R c) P & S d) P, Q, R & S

Sol :

P:

$\sim p \vee q \equiv p \to q$

$r \to s \equiv$

$p \vee r \equiv \sim p \to r$ — $\sim p \to s$ — $\sim q \to s \cong \sim s \to q$

$\therefore \sim s \to q$

$\therefore$ P is valid

Q:

$\sim p \wedge q \Rightarrow \sim p, q$

$q \to (p \to r)$ — $p \to r$

$\sim r$

$\sim r \to \sim p$

$\sim p$

$\therefore \sim r$ this is fallacy

$\therefore$ Q is not valid

R:

$(q \wedge r) \to p \equiv \sim p \to \sim(q \wedge r)$

$\sim q \vee p \equiv q \to p \equiv \sim p \to \sim q$ — $\sim q \vee \sim r$

$\therefore r$

$\therefore$ Here we can't conclude r

$\therefore$ not valid

S:

p

$p \to r$

$q \to \sim r \equiv r \to q$ — $p \to q$ — q

$\therefore q$

$\therefore$ S is valid

$\therefore$ P & S

P/5

$\sim p \to (q \to \sim w)$
$\sim s \to q$
$\sim t$
$\sim p \vee t$

$\sim t,\ \sim p \vee t \vdash \sim p$

$\sim p \to (q \to \sim w),\ \sim p \vdash q \to \sim w$

$q \to \sim w,\ \sim s \to q \vdash \sim s \to \sim w \equiv w \to s$

$w \to s$

$\therefore S_1$ is valid

$q \to t$
$s \to r$
$\sim q \to s \equiv \sim s \to q$

$\sim s \to q,\ q \to t \vdash \sim s \to t$

$\sim s \to t,\ s \to r \vdash \sim t \to r$

$\sim t \to r$

$\therefore S_2$ is valid

P/6 a) $\sim p$
$p \to q$
$q \to r$

$\therefore \sim r$

$p \to q,\ q \to r \vdash p \to r$
$\sim p$

✗ This fallacy

c) $p \to (q \to r)$
$p \wedge q$
$\Downarrow$
p, q

$p \to (q \to r),\ p \vdash q \to r$

$q \to r,\ q \vdash r$

$\therefore$ valid

b) $p \to q$
$q \to r$
r

$\therefore p$

$p \to q,\ q \to r \vdash p \to r$
r

$\therefore p$ is fallacy

d) $\sim(p \wedge q) \equiv \sim p \vee \sim q$
$\sim p$

$\vdash$ ✗

$\therefore$ fallacy

P/12

$(a \vee b) \to c$
$c \to (d \wedge e)$ ~~$(a \vee b) \to (d \wedge e)$~~

$a \to d$

$\therefore S_1$ is valid

This is seen as

$(a \vee b) \to c$
$c \to (d \wedge e)$
a

$\therefore d$

$(a \vee b) \to c,\ c \to (d \wedge e) \vdash (a \vee b) \to (d \wedge e)$
a
$\Rightarrow d \wedge e$
$\Rightarrow d$

(or)

$a \to c$
$b \to c$
$c \to d$
$\therefore c \to d$

$\therefore a \to d$

$\boxed{\begin{array}{l} (a \vee b) \to c \\ \equiv (a \to c) \wedge (b \to c) \\ c \to (d \wedge e) \\ \equiv (c \to d) \wedge (c \to e) \end{array}}$

$p \to q$
$\sim(p \wedge q) \equiv \sim p \vee \sim q \equiv p \to \sim q$

$\therefore \sim p$

$p \to q,\ p \to \sim q \vdash \sim p$

~~Let p is true~~
~~$\Rightarrow q$~~
~~$\Rightarrow \sim(p \wedge q) = $ false~~
~~But it give $\sim(p \wedge q)$ is true~~

$\therefore$ ~~given argument is~~ valid

P/28

p
$p \to q$ } — q
$s \vee r$
$r \to \sim q$
___
$s \vee t$

$\sim r$ } — s

$\therefore s \vee t$

valid

P/29

$(\sim p \vee q) \to r \equiv \sim r \to (p \wedge \sim q)$

$r \to (s \vee t) \equiv \sim(s \vee t) \to \sim r \equiv (\sim s \wedge \sim t) \to \sim r$

$\sim s \wedge \sim u$

$\sim u \to \sim t$
___
$\therefore p$

$\sim t$ — $\sim s \wedge \sim t$ — $\sim r$ — $p \wedge \sim q$

$\therefore p$

$\therefore$ valid

P/30

$p \to q$

$\sim r \vee s \equiv r \to s$

$p \vee r \equiv \sim p \to r$
___
$\sim q \to s$

$\sim p \to s$ — $\sim q \to s$

$\therefore$ valid

P/31

$\sim p \leftrightarrow q \equiv (\sim p \to q) \wedge (q \to \sim p)$

$q \to r \equiv \sim r \to \sim q$

$\sim r$
___
$\therefore p$

q, $q \to \sim p$) $\to \sim p$

P/31

$\sim p \to q \equiv \sim q \to p$

$q \to \sim p$

$q \to r$

$\sim r$
___
$\therefore p$

$\sim q$ — p

$\therefore$ valid

08/05/20

First Order Logic (or) Predicate Logic

Open statement:

A stmt for which we can't define truth value until we provide inputs is called open statement.

Eg: x is a even number.

$P \wedge \sim q$

$\therefore P$

when i/p is provided open stmt changes into simple stmt.

Eg: if $x = 2$

2 is a even number. (simple stmt)

→ Consider a open stmt

x is even number

The set from which we take input is called Domain (or) Domain of Discourse (or) Universe of Discourse

Here Domain is integers.

Domain: set of possible choices for an open statement.

→ Consider

$\underbrace{x}_{\text{Subject (or) Predicate variable}}$ is $\underbrace{\text{even number}}_{\text{Predicate (P)}}$

$P(x)$: x is a even number

$P(1)$: 1 is a even number

$P(2)$: 2 is a even number

⋮

Here x is called predicate variable

→ Predicate is a property of perdicate variable.

Quantifiers:

1) Universal quantifier (for all) ($\forall$)
2) Existential quantifier (for some) ($\exists$)

Universal Quantifier: ($\forall$)

→ for all, for every element.

let Domain, $D: \{1, 2, 3, 4\}$

Predicate $P(x): x^2 < 19$

$\forall x\, P(x)$ is true

$\therefore$ truth value of $\forall x\, P(x)$ is true

Eg:

$D: \mathbb{Z}$

$P(x): x^2 \geq 0$

$\forall x, P(x)$ is true

> $\forall x, P(x)$ is false, iff $P(x)$ is false for atleast one element in domain

Eg:

$D: \mathbb{Z}$

$P(x): x^2 > 0$

$\forall x, P(x)$ is false

cuz $P(0)$ is false

> **Note:**
> If domain is empty,
> $\forall x\, P(x)$ is true
> $\exists x\, P(x)$ is false

Existential Quantifiers ($\exists$)

→ $\exists x\, P(x)$ (read as there exist x such that $P(x)$)

→ $\exists x\, P(x)$ is true iff atleast one element is true.

Eg:

$D: \mathbb{Z}$

$P(x): x^2 = 4$

$\exists x\, P(x)$ is true

> $\exists x, P(x)$ is false, iff $P(x)$ is false for all the elements in the domain

Note:

$$\boxed{\forall x, P(x) \rightarrow \exists x, P(x)}$$

The above implication is tautology.

Negation of Quantifier:

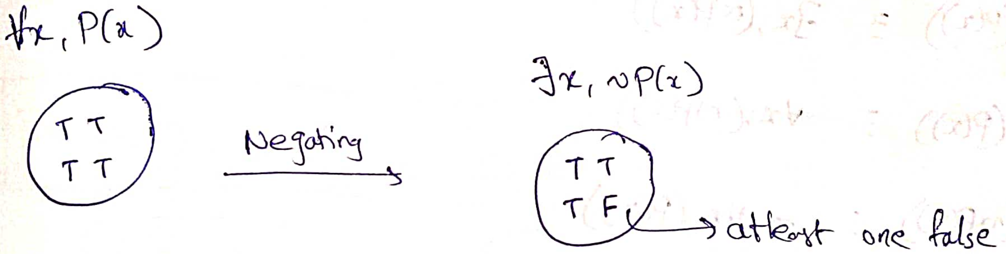
$$\sim[\forall x, P(x)] \equiv \exists x, \sim P(x)$$

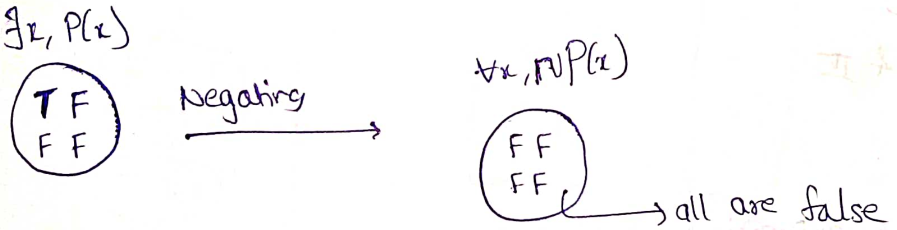
$$\sim[\exists x, P(x)] \equiv \forall x, \sim P(x)$$

$\sim \forall x, P(x)$ is considered $\sim[\forall x, P(x)]$

i.e., $\sim$ is given to whole stmt

Proof





Alternate Proof:

→ Let $D: \{x_1, x_2, x_3, x_4\}$

$$\forall x, P(x) \equiv P(x_1) \wedge P(x_2) \wedge P(x_3) \wedge P(x_4)$$

~~$\exists x, P(x) \equiv P($~~ $\exists x, P(x) \equiv P(x_1) \vee P(x_2) \vee P(x_3) \vee P(x_4)$

$$\sim[\forall x, P(x)] \equiv \sim[P(x_1) \wedge P(x_2) \wedge P(x_3) \wedge P(x_4)]$$

$$\equiv \sim P(x_1) \vee \sim P(x_2) \vee \sim P(x_3) \vee \sim P(x_4)$$

$$\equiv \exists x, \sim P(x)$$

$$\sim[\exists x, P(x)] \equiv \sim[P(x_1) \vee P(x_2) \vee P(x_3) \vee P(x_4)]$$

$$\equiv \sim P(x_1) \wedge \sim P(x_2) \wedge \sim P(x_3) \wedge \sim P(x_4) \equiv \forall x, \sim P(x)$$

GATE 2009

Consider the following well-formed formulae:

I. $\sim\forall x(P(x))$ II. $\sim\exists x(P(x))$

III. $\sim\exists x(\sim P(x))$ IV. $\exists x(\sim P(x))$

which of the above are equivalent?

a) I & II b) I & IV

c) III & II d) II & IV

Sol:

I. $\sim\forall x(P(x)) \equiv \exists x,(\sim P(x))$

II. $\sim\exists x(P(x)) \equiv \forall x,(\sim P(x))$

III. $\sim\exists x(\sim P(x)) \equiv \forall x,(P(x))$

IV. $\exists x(\sim P(x))$

$\therefore$ I & IV

Eg: find negation below expressions

i) $\forall x\,[P(x) \rightarrow Q(x)]$

$\sim\forall x(P(x)\rightarrow Q(x)) \equiv \exists x,\sim(P(x)\rightarrow Q(x)) \equiv \exists x,\ P(x)\wedge\sim Q(x)$ *

ii) $\forall x\,[P(x)\wedge Q(x)]$

$\sim\forall x[P(x)\wedge Q(x)] \equiv \exists x,[\sim P(x)\vee\sim Q(x)]$

Note:

* $\boxed{\exists x\,[P(x) \wedge Q(x)] \Rightarrow \exists x\, P(x) \wedge \exists x\, Q(x)}$ (tautology)

$$\begin{pmatrix} P(x_1) \wedge Q(x_1) \\ \vee \\ P(x_2) \wedge Q(x_2) \\ \vee \\ P(x_3) \wedge Q(x_3) \\ \vee \\ \vdots \end{pmatrix} \Rightarrow \begin{pmatrix} P(x_1) \\ \vee \\ P(x_2) \\ \vee \\ P(x_3) \\ \vee \\ \vdots \end{pmatrix} \wedge \begin{pmatrix} Q(x_1) \\ \vee \\ Q(x_2) \\ \vee \\ Q(x_3) \\ \vee \\ \vdots \end{pmatrix}$$

$\Rightarrow$ means ~~taut~~ the implication which is tautology

But vice versa is not a tautology

* $\boxed{\exists x\,[P(x) \vee Q(x)] \equiv \exists x\, P(x) \vee \exists x\, Q(x)}$

$$\begin{pmatrix} F \vee F \;\; F \\ \vee \\ F \vee T \;\; T \\ \vee \\ F \vee F \;\; F \end{pmatrix} \qquad \begin{pmatrix} F \\ \vee \\ F \\ \vee \\ F \end{pmatrix} \vee \begin{pmatrix} F \\ \vee \\ T \\ \vee \\ F \end{pmatrix}$$

$T \qquad F \vee T \equiv T$

* $\boxed{\forall x\,(P(x) \wedge Q(x)) \equiv [\forall x\, P(x)] \wedge [\forall x\, Q(x)]}$

$$\begin{pmatrix} T \wedge T \\ \wedge \\ T \wedge T \\ \wedge \\ T \wedge T \end{pmatrix} \qquad \begin{pmatrix} T \\ \wedge \\ T \\ \wedge \\ T \end{pmatrix} \wedge \begin{pmatrix} T \\ \wedge \\ T \\ \wedge \\ T \end{pmatrix}$$

* ~~$\forall x\,(P(x) \vee Q(x))$~~

* $\boxed{[\forall x\, P(x)] \vee [\forall x\, Q(x)] \Rightarrow \forall x\,[P(x) \vee Q(x)]}$

$$\begin{pmatrix} T \\ \wedge \\ T \\ \wedge \\ T \end{pmatrix} \vee \begin{pmatrix} F \\ \wedge \\ T \\ \wedge \\ F \end{pmatrix} \qquad \begin{pmatrix} T \vee F \\ \wedge \\ T \vee T \\ \wedge \\ T \vee F \end{pmatrix}$$

* $\boxed{\forall x\,[P(x) \to Q(x)] \Rightarrow [\forall x\, P(x)] \to [\forall x\, Q(x)]}$

$(T \to T) \wedge (T \to T) \wedge (F \to T) \wedge (F \to F)$ = T

$(T \wedge T \wedge F \wedge F) \to (T \wedge T \wedge T \wedge F)$

$F \to F \equiv T$

* $[\forall x\, P(x) \to \forall x\, Q(x)] \not\Rightarrow \forall x\,[P(x) \to Q(x)]$

$(T \wedge F \wedge T \wedge F) \to (T \wedge T \wedge F \wedge F)$

$F \to F \equiv T$

$(T \to T) \wedge (F \to T) \wedge (T \to F) \wedge (F \to F)$: $T \wedge T \wedge F \wedge T$ = F

∴ This is not valid

* $\boxed{\forall x\,[P(x) \leftrightarrow Q(x)] \Rightarrow \forall x\, P(x) \leftrightarrow \forall x\, Q(x)}$

$(T \leftrightarrow T) \wedge (F \leftrightarrow F) \wedge (T \leftrightarrow T)$ = T

$(T \wedge F \wedge T) \leftrightarrow (T \wedge F \wedge T)$

$F \leftrightarrow F \equiv T$

or

$(T \leftrightarrow T) \wedge (T \leftrightarrow T) \wedge (T \leftrightarrow T)$ = T

$(T \wedge T \wedge T) \leftrightarrow (T \wedge T \wedge T)$

$T \leftrightarrow T \equiv T$

* $[\forall x\, P(x)] \leftrightarrow [\forall x\, Q(x)] \not\rightarrow \forall x\, [P(x) \leftrightarrow Q(x)]$

$$(T \wedge F \wedge T) \leftrightarrow (T \wedge T \wedge F) \qquad\qquad (T \leftrightarrow T) \wedge (F \leftrightarrow T) \wedge (T \leftrightarrow F)$$

$$F \leftrightarrow F \equiv T \qquad\qquad F$$

$\therefore$ Not valid

(GATE 2013) Which one of the following is NOT logically equivalent to

$$\sim \exists x \left(\forall y (\alpha) \wedge \forall z (\beta) \right)$$

a) $\forall x \left(\exists z (\sim \beta) \rightarrow \forall y (\alpha) \right)$

b) $\forall x \left(\forall z (\beta) \rightarrow \exists y (\sim \alpha) \right)$

c) $\forall x \left(\forall y (\alpha) \rightarrow \exists z (\sim \beta) \right)$

d) $\forall x \left(\exists y (\sim \alpha) \rightarrow \exists z (\sim \beta) \right)$

Sol:

Here α, β are predicate functions over predicate variable x, y, z.

$$\sim \exists x \left(\forall y, (\alpha) \wedge \forall z (\beta) \right) \equiv \forall x \left[\sim \left(\forall y, (\alpha) \wedge \forall z (\beta) \right) \right]$$

$$\equiv \forall x \left[\exists y (\sim \alpha) \vee \exists z (\sim \beta) \right]$$

$$\equiv \forall x \left[\forall y (\alpha) \rightarrow \exists z (\sim \beta) \right] \quad (c)$$

$$\equiv \forall x \left[\forall z (\beta) \rightarrow \exists y (\sim \alpha) \right] \quad (b)$$

So both (a) & (d) are not equivalent to the given expression

Remaining ACE material problems on propositional logic:

(P/1)

$(flood \vee fire) \rightarrow pay$

a ⇒ $pay \rightarrow (flood \vee fire)$

b ⇒ $pay \rightarrow (flood \wedge fire)$

c ⇒ $\sim pay \rightarrow (\sim flood \vee \sim fire)$

d ⇒ $\sim pay \rightarrow (\sim flood \wedge \sim fire)$

$\equiv \sim pay \rightarrow \sim(flood \vee fire)$

$\equiv (flood \vee fire) \rightarrow pay$

(P/2)

S_1: $\overbrace{[(r \rightarrow (q \rightarrow p)) \wedge \sim p]}^{T} \rightarrow \overbrace{(\sim r \vee \sim q)}^{F}$

$r = T$

$q = T$

T T $p = F$

⇒ $q = F$

∴ valid

S_2:

$p \rightarrow q$
$q \rightarrow r$ } $p \rightarrow r$

$\sim q \wedge r$

r

∴ p

∴ p

This is fallacy

∴ only 'S_1'

(P/3)

$[(a \vee b) \wedge (\sim a \vee c) \wedge \sim(b \vee c)]$

a = T a = F

$(c) \wedge \sim(b \vee c)$ $b \wedge (\sim a \vee c) \wedge \sim(b \vee c)$

c = F c = T b = T b = F

F F $(\sim a \vee c) \wedge F = F$ F

∴ Contradiction

P/4 $\left[(a \vee b) \to c\right] \to \left[(a \wedge b) \to c\right]$

$\downarrow$ T, T, F ; $(a \wedge b)$ = T, c = F

$T \to F \equiv F$

a = T
b = T
c = F

$\therefore$ valid

S2: $\left[(a \wedge b) \to c\right] \to \left[(a \vee b) \to c\right]$

T ; F

$(a \wedge b)$ → F, c → F ; $(a \vee b)$ = T, F ; c = F

a	b
F	F
F	T
T	F

a	b
T	T
F	T
F	T

For these common sets premises = T & conclusion = F

$\therefore$ not valid

P/8 From table we can see that

$p * q \equiv \sim(p \to q)$

$\Rightarrow p \to q \equiv \sim(p * q)$

P/11 S1: $\left[(P \to Q) \to P\right] \to Q$

$(P \to Q)$ ↓ F ; Q ↓ F

for P = T

$(T \to F) \to T \equiv F \to T \equiv T$

$\therefore$ not valid

for P = F

~~$(F \to F) \to F$~~

~~$T \to F \equiv F$~~

S2: $P \to (Q \to (P \to Q))$

P = T ; Q ; $(P \to Q)$ ↓ T

for Q = F

$F \to (T \to F) \equiv F \to F \equiv T$

Q = T

$T \to (T \to T) \equiv T \to T \equiv T$ $\therefore$ valid

(P/11) S_1: $[(P\to Q)\to P]\to Q$

$[(\sim P\vee Q)\to P]\to Q$

$[\sim(\sim P\vee Q)\vee P]\to Q$

$[(P\wedge\sim Q)\vee P]\to Q$

$\equiv P\to Q$

S_2: $P\to(Q\to(P\to Q))$

$\sim P\vee(Q\to(P\to Q))$

$\sim P\vee(Q\to(\sim P\vee Q))$

$\sim P\vee(\sim Q\vee\sim P\vee Q))$

$\sim P\vee\sim Q\vee Q\equiv T$

Consider $S_2\Rightarrow S_1$

As S_2 is always true & S_1 is false for some value

$S_2\Rightarrow S_1$ is not a tautology

by $S_1\Rightarrow S_2$ is tautology as conclusion is true for any S_1

∴ opt ⓐ

(P/15) S_1: $\sim(p\vee q)\to p\to q$

$\sim p\wedge\sim q$

$\therefore p\to q$ ∴ valid

$\equiv\sim q\to\sim p$

S_2: $\sim(q\to\sim p)\to(p\to\sim q)$

$\sim(\sim q\vee\sim p)\equiv\sim p\wedge q$

$p\to\sim q$ $\quad$ $\therefore p\to\sim q$ is a fallacy

S_3: $\sim(p\to q)\equiv\sim(\sim p\vee q)\equiv p\wedge\sim q$

$\therefore p\vee q$ $\quad$ ∴ valid

S4: $\dfrac{p \wedge \sim q}{p \leftrightarrow q}$ $\Rightarrow p = T \quad q = F$

$\therefore$ not valid

(P/16) This is similar to no of function possible in DLD

P_1	P_2	P_3	fun
F	F	F	2 ways (T/F)
F	F	T	2 ways (T/F)
F	T	F	⋮
F	T	T	
T	F	F	
T	F	T	
T	T	F	
T	T	T	

$\underbrace{2 \times 2 \times \cdots \times 2}_{2^3 \text{ time}} = 2^{2^3} = 2^8 = 256$

(P/18) $p \rightarrow q \equiv F$

$\Rightarrow p = T \;\&\; q = F$

c) $p \vee q \vee r \equiv T$

(P/19) S1: $\overset{T}{(a \leftrightarrow b)} \rightarrow \overset{F}{(a \wedge b)}$

a	b
T	T
F	F

for $a = F \quad b = F$

above is false

$\therefore$ not valid

S2: $(a \leftrightarrow b) \leftrightarrow (a \wedge b) \vee (\sim a \wedge \sim b)$

It is clearly valid

(P/23) Dual is obtained by changing $\wedge$ to $\vee$ and $\vee$ to $\wedge$

S1: $p \rightarrow (q \wedge r) \equiv \sim p \vee (q \wedge r)$

dual is $\sim p \wedge (q \vee r)$

S2: $p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$

dual is $(p \vee q) \wedge (\sim p \vee \sim q)$

(P/25) Arg1:

ball → diff ≡ ~diff → ~ball

time → ~diff

time

∴ ~ball

~diff

~ball

∴ valid

Arg2:

Miss → fail

fail → uneducated

Miss → uneducated (1)

reads → ~ uneducated (2)

Miss ∧ reads (3)

∴ ~ uneducated

(1) & (3) ⇒ uneducated

(2) & (3) ⇒ ~uneducated

This means given premises are inconsistent. This means ~~∴ not valid~~ argument is valid.

(or)

$$[(M \to F) \wedge (F \to U) \wedge (R \to \sim U) \wedge M \wedge R] \to \sim U$$

T T T T T T T T

U & ~U can't be true at same time.

Conjunction of primises is a contradiction.

This means implication is a tautology.

(P/32) S1: $(a \wedge b) \vee c \equiv ab + c$ (Correspond this to Digital Logic Design)

$(a \vee b \vee c) \wedge (a \vee \sim b \vee c) \wedge (\sim a \vee b \vee c)$

$\equiv (a+b+c)(a+\bar{b}+c)(\bar{a}+b+c)$

0 0 0 0 1 0 1 0 0

⇒ it is true for (001, 011, 101, 110, 111)

$ab+c$ ⇒ it is true for (001, 011, 101, 110, 111) ∴ S1 is true

(or)

$(a \wedge b) \vee c \equiv (a \vee c) \wedge (b \vee c)$

$\cancel{\equiv (a \vee b \vee c)(a \vee \sim b \vee c)} \wedge ($

$\equiv (a \vee b \vee c) \wedge (a \vee \sim b \vee c) \wedge (a \vee b \vee c) \wedge (\sim a \vee b \vee c)$

$\equiv (a \vee b \vee c) \wedge (a \vee \sim b \vee c) \wedge (\sim a \vee b \vee c)$

S_2: $a \wedge (b \leftrightarrow c)$

$a \wedge ((b \wedge c) \vee (\sim b \wedge \sim c))$

$\equiv (a \wedge b \wedge c) \vee (a \wedge \sim b \wedge \sim c)$

∴ both S_1 & S_2 are true

?/33 I. $\sim R \vee U$

$\sim U \vee \sim W$ $\Big\} \;\; \sim R \vee \sim W \Big\} \;\; \sim W$

$R \vee \sim W$

$\sim W$

∴ valid

II. $(\text{able} \wedge \text{willing}) \rightarrow \text{prevent}$

$\cancel{\sim \text{prevent} \rightarrow \text{impotent}}$

$\sim \text{able} \rightarrow \text{impotent}$

$\sim \text{willing} \rightarrow \text{Malevolent}$

$\sim \text{prevent}$

$\text{exist} \rightarrow (\sim \text{impotent} \wedge \sim \text{Malevolent})$

∴ $\sim \text{exist}$

II.

a : able to prevent

w : willing to prevent

i : impotent

m : malevolent

p : prevent

e : exist

$(a \wedge w) \rightarrow p \equiv \sim p \rightarrow (\sim a \vee \sim w) \rightarrow$ 1

$\sim a \rightarrow i$ —— 2

$\sim w \rightarrow m$ —— 3

$\sim p$ —— 4

$e \rightarrow (\sim i \wedge \sim m) \equiv (i \vee m) \rightarrow \sim e \rightarrow$ 5

$\therefore \sim e$

1 & 4 $\Rightarrow$ $\sim a \vee \sim w$ 6

2 & 3 $\Rightarrow$ $(\sim a \vee \sim w) \rightarrow (i \vee m)$ 7

6 & 7 $\Rightarrow$ $i \vee m$ 8

8 & 5 $\Rightarrow$ $\sim e$

$\therefore$ valid

09/05/20

GATE 2005 Let $P(x)$ and $Q(x)$ be arbitrary predicates. which of the following statement is always true?

a) $[\forall x (P(x) \vee Q(x))] \Rightarrow \forall x P(x) \vee \forall x Q(x)$

b) $\forall x [P(x) \Rightarrow Q(x)] \Rightarrow \forall x P(x) \Rightarrow \forall x Q(x)$

c) $\forall x P(x) \Rightarrow \forall x Q(x) \Rightarrow \forall x [P(x) \Rightarrow Q(x)]$

d) $[\forall x P(x)] \Leftrightarrow [\forall x Q(x)] \Rightarrow \forall x [P(x) \Leftrightarrow Q(x)]$

Sol :

As per synopsis in notes

opt (b)

22) Consider

$P(x): x^2 - 7x + 10 = 0$

$q(x): x^2 - 2x - 3 = 0$

$r(x): x < 0$

Domain is set of integers.

~~which~~ Find whether below predicate stmts are true or false?

a) $\forall x [q(x) \to r(x)]$

b) $\exists x [q(x) \to r(x)]$

c) $\exists x [P(x) \to r(x)]$

P(x)

$x^2 - 7x + 10 = (x-5)(x-2)$

$\therefore P(5)$ & $P(2)$ are true

q(x):

$x^2 - 2x - 3 = (x-3)(x+1)$

$\therefore q(3)$ & $q(-1)$ are true

r(x):

$r(-1), r(-2), r(-3) \dots$ are true

a) $\forall x [q(x) \to r(x)]$

(To prove this is wrong, we need to atleast one value of x for which the stmt is false.)

for $x = 3$

$q(3) \to r(3)$

$T \to F \equiv F$

If atleast one is false in Universal Quantifier, the result is false

b) $\exists x [q(x) \to r(x)]$

for $x = -1$

$q(-1) \to r(-1)$

$T \to T \equiv T$

$\therefore$ True

c) $\exists x [P(x) \to r(x)]$

for $x = -1$

$P(-1) \to r(-1)$

$F \to T \equiv T$

$\therefore$ True

Q3 Let $p(x), q(x)$ & $r(x)$ denote the following open stmts.

$p(x): x^2-8x+15=0$

$q(x):$ x is odd

$r(x): x>1$

For universe of all integers, determine the truth or falsity of each of the following statements. If a statement is false, given a counter example.

a) $\forall x\,[p(x)\to q(x)]$ b) $\forall x\,[q(x)\to p(x)]$

c) $\exists x\,[p(x)\to q(x)]$ d) $\exists x\,[q(x)\to p(x)]$

e) $\exists x\,[r(x)\to p(x)]$ f) $\forall x\,[\sim q(x)\to \sim p(x)]$

g) $\exists x\,[p(x)\to (q(x)\wedge r(x))]$ h) $\forall x\,[(p(x)\vee q(x))\to r(x)]$

Sol:

$p(5), p(3)$ are true

$q(1), q(3), q(-1), q(-3)\ldots$ are true

$r(2), r(3)\ldots$ are true

a) $\forall x\,[p(x)\to q(x)]$

$p(3)\to q(3)$ T

$p(5)\to q(5)$ T ...

$\therefore$ True

b) $\forall x\,[q(x)\to p(x)]$

$q(1)\to p(1)$

$T\to F\equiv F$

$\therefore$ False

c) $\exists x\,[p(x)\to q(x)]$

$p(3)\to q(3)\equiv T$

$\therefore$ True

d) $\exists x\,[q(x)\to p(x)]$

$q(3)\to p(3)$

$\therefore$ True

e) $\exists x\,[r(x)\to p(x)]$

$r(-1)\to p(-1)$

$\therefore$ True

f) $\forall x\,[\sim q(x)\to \sim p(x)]\equiv \forall x\,[p(x)\to q(x)]$

$\therefore$ True

g) $\exists x \left[P(x) \rightarrow (q(x) \wedge r(x)) \right]$

for $x = 2$
$P(2)$ is false

$\therefore$ True

h) $\forall x \left[(P(x) \vee q(x)) \rightarrow r(x) \right]$

for ~~every x for which~~
~~$P(x) \vee q(x)$ is true~~
~~$r(x)$ is also true.~~ for $x = -1$

$\therefore$ ~~True~~ $\therefore$ False

English to logical statement:

* Consider

All mothers are female.

Domain (all humans)

MT/F
Shyam/M
Rita/F

$\underbrace{\text{All of } x}_{\forall x}$ $\underbrace{x \text{ is mother}}_{MT(x)}$ $\underbrace{\text{~~then~~ } x \text{ is female}}_{F(x)}$

$\forall x \left[MT(x) \rightarrow F(x) \right]$

This mean for every x in domain, if x is mother, then x is female for sure

$\forall x \left[M(x) \wedge F(x) \right]$

This means for every x in domain, every x is both mother and female. This seems wrong.

$$\boxed{\therefore \text{ All mothers are female } \equiv \forall x \left[MT(x) \rightarrow F(x) \right]}$$

→ Also ~~for~~ from the example domain we can ~~solve~~ verify it.

→ Also check why we can't use $\vee$ & $\leftrightarrow$.

→ Some cats are black

Domain (All animals)

cat1/white
cat2/white
Crow

$\underbrace{\text{Some of } x}_{\exists x}$, $\underbrace{x \text{ is cat}}_{C(x)}$, $\underbrace{x \text{ is black}}_{B(x)}$

~~Eg~~ Case (i): $\exists x \left[C(x) \wedge B(x) \right]$

$T \wedge F$
$\vee$
$T \wedge F$
$\vee$
$F \wedge -$
F

Case (ii) $\exists x \left[C(x) \xrightarrow{} B(x) \right]$

~~Ex~~

$T \rightarrow F$
$\vee$
$T \rightarrow F$
$\vee$
$F \rightarrow -$
T
T

→ $\exists x[C(x) \to B(x)]$ is giving true without single black cat.

$\therefore$ case (ii) is wrong representation

consider domain

Cat1/black
Cat2/white
dog

$\exists x[C(x) \land B(x)]$

$T \land F$
$\lor$
$T \land T$
$\lor$
$F \land \_$

T

This is correct representation

$$\boxed{\therefore \text{ Some cats are black} \equiv \exists x[C(x) \land B(x)]}$$

→ Also consider if domain doesn't contain any cat, ~~even~~ then also $\exists x[\underset{(F)}{C(x)} \to B(x)]$ is true.

This means the expression is not even depending on cats.

GATE 2012 Find correct representation of below stmt:

"Some real numbers are rational"

a) $\exists x(\text{real}(x) \lor \text{rational}(x))$

b) $\forall x(\text{real}(x) \to \text{rational}(x))$

c) $\exists x(\text{real}(x) \land \text{rational}(x))$

d) $\exists x(\text{rational}(x) \to \text{real}(x))$

GATE 2014 The correct formula for the sentence

"Not all rainy days are cold" is

a) $\forall d(\text{Rainy}(d) \land \sim\text{Cold}(d))$

b) $\forall d(\sim\text{Rainy}(d) \to \text{Cold}(d))$

c) $\exists d(\sim\text{Rainy}(d) \to \text{Cold}(d))$

d) $\exists d(\text{Rainy}(d) \land \sim\text{Cold}(d))$

Sol:
Given stmt can be seen as

Some rainy days are ~~cold~~ not cold

$\exists d\,[\text{rainy}(d) \wedge \sim\text{cold}(d)]$

$\therefore$ opt (d)

"not all are satisfying a property" means there are some which doesn't satisfy the property

(or)

Method : 2

not [all rainy day are cold]

negation

$\sim\left[\forall d,\ (\text{rainy}(d) \rightarrow \text{cold}(d))\right]$

$\exists d,\ \sim[\sim\text{rainy}(d) \vee \text{cold}(d)]$

$\exists d,\ [\text{rainy}(d) \wedge \sim\text{cold}(d)]$

→ Consider below statements

All graphs are connected

$\forall x\,[G(x) \rightarrow C(x)]$

not all graphs are connected

$\exists x\,[G(x) \wedge \sim C(x)]$

All graphs are not connected

$\forall x\,[G(x) \rightarrow \sim C(x)]$

No graphs are connected

$\forall x\,[G(x) \rightarrow \sim C(x)] \equiv \sim\exists x\,[G(x) \wedge C(x)]$

→ All mothers are female

$\forall x\,[MT(x) \rightarrow F(x)]$

not all mothers are female

$\sim\forall x\,[MT(x) \rightarrow F(x)] \equiv \exists x\,[MT(x) \wedge \sim F(x)]$

All mothers are not female

$\forall x\,[MT(x) \rightarrow \sim F(x)]$

No mothers are female ——— this stmt's meaning is same as above stmt.

$\forall x\,[MT(x) \rightarrow \sim F(x)]$

Note:

all	$\forall x [\ \rightarrow\ ]$
not all	$\exists x [\ \wedge \sim\ ] \equiv \sim \forall x [\ \rightarrow\ ]$
no/none	$\forall x [\ \rightarrow \sim\ ] \equiv \sim \exists x [\ \wedge\ ]$

GATE 2014

Consider the stmt

"Not all that glitters is gold"

which of the following logical formulae represents the above statement?

a) $\forall x$: glitters$(x) \Rightarrow \sim$ gold(x)

b) $\forall x$: gold$(x) \Rightarrow$ glitters(x)

c) $\exists x$: gold$(x) \wedge \sim$glitters(x)

d) $\exists x$: glitters$(x) \wedge \sim$gold(x)

GATE 2007

which of the following ~~denotes~~ does not represent "Not every graph is connected".

~~Ans: $\exists x$ [graph(x) $\wedge \sim$connected(x)]~~

a) $\sim \forall x$ (graph$(x) \Rightarrow$ connected(x))

b) $\exists x$ (Graph$(x) \wedge \sim$connected(x))

c) $\sim \forall x$ ($\sim$ graph$(x) \vee$ connected(x))

d) $\forall x$ (graph$(x) \Rightarrow \sim$Connected(x))

Sol:

given stmt is written as

$$\exists x \,[\text{graph}(x) \wedge \sim\text{connected}(x)]$$

$\equiv \sim \forall x [\sim$graph $\vee$ connected$(x)] \equiv \sim \forall x$[graph$(x) \Rightarrow$ connected(x)]

$\therefore$ d

GATE 2006

"Tigers and lions attack if they are hungry or threatned"

which of the following is correct representation?

a) $\forall x \left[(tiger(x) \wedge lion(x)) \rightarrow \left[(hungry(x) \vee threatened(x)) \rightarrow attacks(x) \right] \right]$

b) $\forall x \left[(tiger(x) \wedge lion(x)) \rightarrow \left[(hungry(x) \vee threatened(x)) \rightarrow attacks(x) \right] \right]$

c) $\forall x \left[(tiger(x) \vee lion(x)) \rightarrow \left[attacks(x) \rightarrow (hungry(x) \vee threatened(x)) \right] \right]$

d) $\forall x \left[(tiger(x) \vee lion(x)) \rightarrow \left[(hungry(x) \vee threatened(x)) \rightarrow attacks(x) \right] \right]$

Sol:

Given sentense means

if given animal is a tiger or a lion, then $\overbrace{\text{(they attack}_{q}\text{, if they are hungry or threatened}_{p}\text{)}}^{p \rightarrow q}$

$$\forall x \left[(tiger(x) \vee lion(x)) \rightarrow \left[(hungry(x) \vee threatened(x)) \rightarrow attacks(x) \right] \right]$$

∴ opt d

GATE 2009

Representation of

"Gold and silver ornaments are precious"

$G(x)$: gold
$S(x)$: silver
$P(x)$: precious

Ans: $\forall x \left[(G(x) \vee S(x)) \rightarrow precious(x) \right]$

Though it is mentioned gold 'and' silver in logic meaning it OR

Q4) For the universe of all integers, let $p(x)$, $q(x)$, $r(x)$, $s(x)$ & $t(x)$ be the following open statements 122

$p(x)$: $x > 0$

$q(x)$: x is even

$r(x)$: x is a perfect square

$s(x)$: x is (exactly) divisible by 4

$t(x)$: x is (exactly) divisible by 5

write the following statements in symbolic form

i) Atleast one integer is even

$$\exists x, q(x)$$

ii) There exists a positive integer that is even

$$\exists x, [p(x) \wedge q(x)]$$

→ Some positive integers are even.

iii) If x is even, then x is not divisible by 5

$$\forall x \, [q(x) \rightarrow \sim t(x)]$$

(iv) No even integer is divisible by 5

$$\forall x \, [q(x) \rightarrow \sim t(x)]$$

(All even integers are not divisible by 5)

(v) There exists an even integer divisible by 5.

$$\exists x \, [q(x) \wedge t(x)]$$

(vi) If x is even and x is a perfect square, then x is divisible by 4.

$$\forall x \, [(q(x) \wedge r(x)) \rightarrow s(x)]$$

Q5) Convert below english sentence into its logical equivalent

"you cannot ride the roller coaster if you are under 4 feet tall unless you are older than 16 years old"

take

q: can't ride roller coaster

r : under 4 feet tall

s : older than 16 years

Method 1 :

The sentence can be seen as

$$q \text{ if } r \text{ unless } s$$

As associativity is from left to right for $\rightarrow$

$$(q \text{ if } r) \text{ unless } s$$

$$(r \rightarrow q) \text{ unless } s$$

$$\sim s \rightarrow (r \rightarrow q) \equiv s \vee \sim r \vee q$$

$$\equiv \sim(s \vee \sim r) \rightarrow q$$

$$\equiv (r \wedge \sim s) \rightarrow q$$

Method 2 :

By the general understanding we can say that

a person can't ride the roller coaster (if he is under 4 feet and not older than 16 yrs)

$$\therefore \quad (r \wedge \sim s) \rightarrow q$$

Note :

Using method 1 is better, Sometimes method 2 may go wrong cuz of your inability to interpret the sentence correctly.

10/05/20

GATE 2016 — which one of the following well-formed formula in predicate calculus is NOT valid?

a) $[\forall x\, P(x) \Rightarrow \forall x\, Q(x)] \Rightarrow [\exists x \sim P(x) \vee \forall x\, Q(x)]$

b) $[\exists x\, P(x) \vee \exists x\, Q(x)] \Rightarrow \exists x\,(P(x) \vee Q(x))$

c) $\exists x\,(P(x) \wedge Q(x)) \Rightarrow (\exists x\, P(x)) \wedge (\exists x\, Q(x))$

d) $\forall x\,(P(x) \vee Q(x)) \Rightarrow (\forall x\, P(x)) \vee (\forall x\, Q(x))$

Sol:

a) $\overbrace{[\forall x\, P(x) \Rightarrow \forall x\, Q(x)]}^{T} \Rightarrow \overbrace{(\exists x \sim P(x)) \vee (\forall x\, Q(x))}^{F}$

$\boxed{\begin{matrix} T \\ T \end{matrix}} \Rightarrow \boxed{\begin{matrix} F \\ T \end{matrix}}$ $\qquad \underbrace{\overset{F}{\boxed{\begin{matrix} T \\ T \end{matrix}}} \qquad \overset{F}{\boxed{\begin{matrix} F \\ T \end{matrix}}}}_{F}$

T $\Rightarrow$ F

= F

$\therefore$ valid $\qquad [\forall x P(x) \Rightarrow \forall x\, Q(x)] \equiv (\sim \forall x\, P(x)) \vee \forall x\, Q(x)$

$[\exists x \sim P(x)] \vee \forall x\, Q(x)$

$\therefore$ Above are equivalent too

b) $[\exists x\, P(x) \vee \exists x\, Q(x)] \Rightarrow \exists x\,(P(x) \vee Q(x))$

These are equivalent

$\therefore \Rightarrow$ also holds

c) It is clearly true as we have derived already

d) $\forall x\,(P(x) \vee Q(x)) \Rightarrow \forall x\, P(x) \vee \forall x\, Q(x)$

$\begin{matrix} T \vee F \\ \wedge \\ F \vee T \end{matrix}$ T $\qquad \begin{matrix} T \\ F \end{matrix}$ F $\qquad \begin{matrix} F \\ T \end{matrix}$ F

$\therefore$ Not valid $\qquad$ F

Nested Quantifier: A quantifier which in scope of another quantifier is called nested quantifier.

1) $\forall x \forall y$ D: {1,2,3,4}

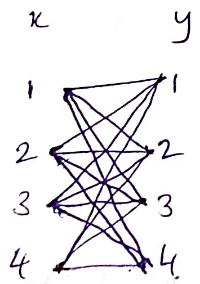
$P(x,y): x * y \leq 16$

P(x,y) is called 2-variable predicate

$\forall x \forall y \, P(x,y)$

$\forall x \forall y \, (x * y \leq 16)$

$1*1=1\leq 16$	$2\times 1=2\leq 1$	- - - -	$4\times 1=4\leq 16$
$1*2=2\leq 16$	⋮		
$1\times 3=3\leq 16$			
$1\times 4=4\leq 16$	$2\times 4=8\leq 16$	- - - -	$4\times 4=4\leq 16$

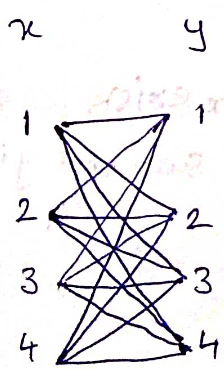


It is complete bipartite graph

$\forall x \forall y \, P(x,y)$ is false when atleast one edge is false

$\forall x \forall y \, P(x,y)$ is true when ~~graph~~ the complete bipartite's all edges are true.

$\forall y \forall x \, (x \times y \leq 16)$



It is just same as ① (i.e., $\forall x \forall y$)

$\therefore \boxed{\forall x \forall y \equiv \forall y \forall x}$ ↳ This holds only when domain of x & y are same

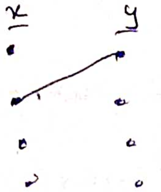
3) $\exists x \exists y$

for some value of x, some value of y

for atleast one value of x, atleast one value of y

→ $\exists x, \exists y$ is true when atleast one edge of the complete bipartite graph is true

→ $\exists x \exists y$ is false when all the edges are false



∴ $\exists x, \exists y$ is true

∴ $\exists x \exists y$ is false

4) $\exists y \exists x$

This same as $\exists x \exists y$

∴ $\boxed{\exists x \exists y \equiv \exists y \exists x}$ This holds only when x & y are same.

5) $\forall x \exists y$

Let Domain be set of integers i.e., $x \in Z, y \in Z$
This is false when there exist an x such that there is no y for that

$\forall x \exists y \, (x+y=10)$

This is x	y
1	9
2	8
20	-10
-10	20
⋮	⋮

for any x

$y = 10 - x$ exists

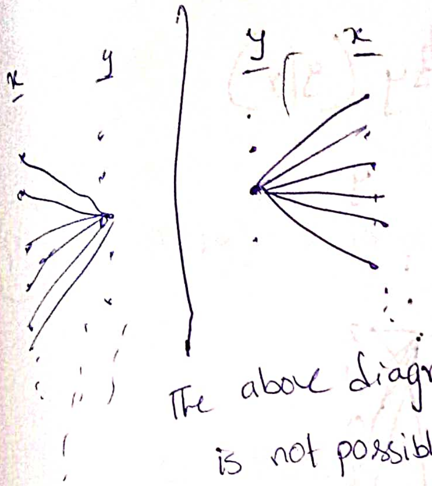
such that y is integer

∴ for any x we can define y

∴ $\forall x \exists y \, (x+y=10)$ is true

6) $\exists y \forall x$

consider $\exists y \forall x (x + y = 10)$



for some $y = y_1$

$x = 10 - y_1$

$\downarrow$

This should be satisfied for any x value ~~such~~, for fixed y

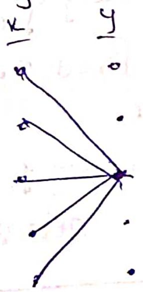
The above diagram is not possible

$\therefore \exists y \forall x (x + y = 10)$ is false

from this example we can say

$\forall x \exists y \not\rightarrow \exists y \forall x$

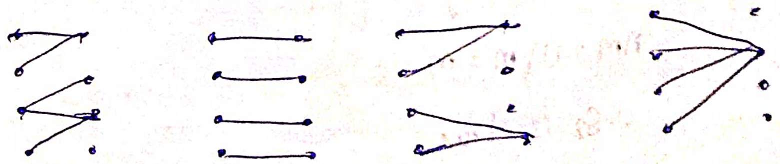
Consider $\exists y \forall x$ is true



Now the above diagram also satisfies $\forall x \exists y$

$\therefore \boxed{\exists y \forall x \rightarrow \forall x \exists y}$

→ for $\forall x \exists y$ to be true, below are possibility



etc.

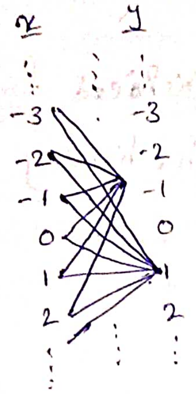
→ for $\exists y \forall x$ to be true, below are possibilities

Eg: for x & y are integers

$\exists y \forall x \, (y/x)$

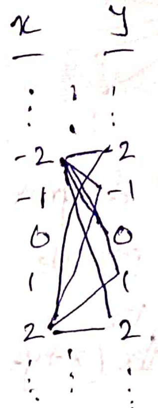
↳ y divides x

The truth value is true.



$\Longrightarrow$

$\forall x \exists y \, (y/x)$



There exist 1 that divide any integer x.

This meare every integer has a factor

→ ~~Simitl~~ Similarly

$$\boxed{\exists x \forall y \Rightarrow \forall y \exists x}$$

~~$\exists x \exists y$ mean~~

$\exists x \forall y$ means
for every y, we need to define fixed x.
$\forall y \exists x$
for every y, we need to define atleast one x

P/49 S_1: $\forall n \exists m \, (n+m=5)$

$m=5-n$

so we any integer n, we can find m such that $m=5-n$

$\therefore S_1$ is true

S_2: $\exists n \forall m \, (nm=m)$

for $n=1$ and for any m

$nm=(1)m=m$

$\therefore S_2$ is true

S_3: $\forall m \exists n \, (mn=1)$

for $m=3$, we can't define n such that $3n=1$

$\therefore S_3$ is false

S4: $\exists m \forall n (m+n=0)$

~~for a fixed m, we can't define~~

for every n, we can't have a fixed m such that

$m = -n$

$\therefore$ false

P/52

$P(x,y)$: x is a divisor of y : $x|y$

$x \in Z \quad y \in Z$

$S_1: \forall x \exists y \, P(x,y)$

for every x we ~~can~~ need to define y (multiple of x)

$\therefore$ $x|y$ but for $x=0$ its not possible

$\therefore$ false

$S_2: \forall y \exists x \, P(x,y)$

for every y we can define x (x), such that x is factor

$x=1$ & $x=y$

$\therefore$ true

Also we can check for S_2 first, if S_2 is false then we can conclude that S_3 is false too.

$\sim S_2 \to \sim S_3$

$S_3 \to S_2$

$S_3: \exists x \forall y \, P(x,y)$

for every y, we can define fixed x ($x=1$)

Such that $x|y$ i.e., $1|y$

$\therefore$ true

Also in this question check for S_3 for first. Cuz if S_3 is true, we can conclude S_2 is true as $\exists x \forall y \Rightarrow \forall y \exists x$

$S_4: \exists y \forall x \, P(x,y)$

for every x we need to define fixed y_1 such that

$x|y = x|y_1$

we need to find a number y_1 in such ~~the~~ a way that any integer can divide y_1.

This is not possible

Q6) For the following statements the universe comprises all nonzero integers. Determine the truth value of each statements

a) $\exists x \exists y \, (xy=1)$

for $x=1$ & $y=1$ above is true

$\therefore$ true

b) $\exists x \forall y \, [xy=1]$

$\underline{x}$ $\underline{y}$

we ~~the~~ need this type of graph which is not possible

$\therefore$ false.

c) $\forall x \exists y \, [xy=1]$

for $x=3$ we can't define integer y such that

$3y=1$

$\therefore$ false

d) $\exists x \exists y \, [(2x+y=5) \wedge (x-3y=-8)]$

$$2x+y=5$$
$$2x-6y=-16$$
$$7y=21 \Rightarrow y=3$$
$$\Rightarrow x=1$$

Solving above 2 eq which are integers

were values of x & y for which both eq hold

$\therefore$ true

e) $\exists x \exists y \, [(3x-y=7) \wedge (2x+4y=3)]$

~~Same as d~~

~~$\therefore$ true~~

$$12x-4y=28$$
$$2x+4y=12$$
$$14x=40$$

$x=40/14$ is not an integer

$\therefore$ false

In this type of questions always check if obtained answer belongs to the given domain or not.

Q7) For domain of integers what is truth value of

$\exists n \, (n^2=2)$

$\therefore$ false $\sqrt{2} \notin \mathbb{Z}$

English statement to Logical Expression

P/37) "For every integer n bigger than 1, there is prime strictly btw n & 2n"

$\forall$ $\quad n>1 \quad \exists \quad n<x<2n$

$$\forall n\, \exists x\, [(n>1) \rightarrow (P(x) \wedge (n<x<2n))]$$

(or)

$$\forall n\, [(n>1) \rightarrow \exists x\, (P(x) \wedge (n<x<2n))]$$

GATE 2005) which of the following is equivalen to

"Every teacher is liked by some student"

like(x,y) : x likes y

a) $\forall x\, [\text{teacher}(x) \rightarrow \exists y\, (\text{Student}(y) \rightarrow \text{Likes}(y,x))]$

b) $\forall x\, [\text{teacher}(x) \rightarrow \exists y\, (\text{Student}(y) \wedge \text{likes}(y,x))]$

c) $\exists y\, \forall x\, [\text{teacher}(x) \rightarrow (\text{student}(y) \wedge \text{likes}(y,x))]$

d) $\forall x\, [\text{teacher}(x) \wedge \exists y\, [\text{student}(y) \rightarrow \text{likes}(y,x)]$

. The gien stmt means

if a person is a teacher (the teacher(x)), then there exist ($\exists y$) some ($\wedge$) student who likes the teacher.

$\therefore$ quantifier is $\forall x$.

$\exists y \forall x$ is false because

it means every teacher is liked by same student.

$\therefore \exists y$ is placed inside.

Also not all students ~~loves~~ likes the teacher. Only some student likes. $\therefore$ '$\wedge$' is used. $\therefore$ opt (b)

GATE 2004

"Some boys in the class are taller than all the girls"

taller(x,y) : x is taller than y

a) $\exists x\,(\text{boy}(x) \rightarrow \forall y\,(\text{girl}(y) \wedge \text{taller}(x,y)))$

b) $\exists x\,(\text{boy}(x) \wedge \forall y\,(\text{girl}(y) \wedge \text{taller}(x,y)))$

c) $\exists x\,(\text{boy}(x) \rightarrow \forall y\,(\text{girl}(y) \rightarrow \text{taller}(x,y)))$

d) $\exists x\,(\text{boy}(x) \wedge \forall y\,(\text{girl}(y) \rightarrow \text{taller}(x,y)))$.

Sol:

given ~~sten~~ sentence means

$\underbrace{\text{There exist}}_{\exists x}$ a $\underbrace{\text{boy}}_{\text{boy}(x)}$ such that $\underline{\text{the boy is taller than all girls}}$

all girls are shorter than the boy
$\forall$

we use '$\wedge$'

$\therefore$ we use '$\rightarrow$'

$\therefore \exists x\,(\text{boy}(x) \wedge \forall y\,(\text{girl}(y) \rightarrow \text{taller}(x,y)))$

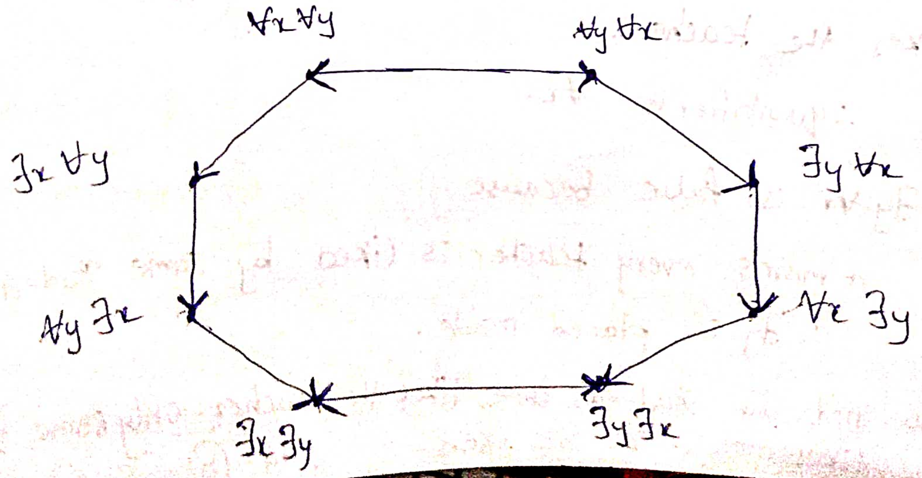
$\therefore$ opt (d)

Relation b/w different nested quantifiers (implication & equivalence)

In the below ~~bidirectional path~~. directed graph

if there is unidirectional path from A to B then $A \Rightarrow B$

if there is bidirectional path b/w A & B then $A \Leftrightarrow B$



11/05/20

Binding variables

<u>scope</u>: The part of a logical expression to which a quantifier is applied is called the scope of this quantifier.

Eg: $\exists x(P(x) \wedge Q(x)) \vee \forall x R(x)$

Here scope of $\exists x$ is $P(x) \wedge Q(x)$

$\forall x$ is $R(x)$

<u>Bound</u>: A variable which is quantified or under ~~so~~ scope of a quantifier is called bound variable

<u>free variable</u>: A variable which is not quantified is called free variable.

Eg: $\exists x(x+y=1)$

x is bound variable (it is under scope of $\exists x$)

y is free variable

and value of y is fixed and doesn't change w.r.to x.

<u>Restricted Equivalences</u>:

Let C be a well formed formula without any free occurance of x. i.e., quantification of x is ~~priv~~ provide within C.

→ $\forall x\, C \Leftrightarrow C$ & $\exists x\, C \equiv C$

→ $\forall x(A(x) \vee C) \equiv \forall x A(x) \vee C$

→ $\exists x(A(x) \vee C) \equiv \exists x A(x) \vee C$

→ $\forall x(A(x) \wedge C) \equiv \forall x A(x) \wedge C$

→ $\exists x(A(x) \wedge C) \equiv \exists x A(x) \wedge C$

* $\forall x P(x) \vee \forall x Q(x) \equiv \forall x \forall y (P(x) \vee Q(y))$

Here $Q(y)$ is free from x

* $\forall x P(x) \vee \exists x Q(x) \equiv \forall x \exists y (P(x) \vee Q(y))$

* $\forall x P(x) \wedge \exists Q(x) = \forall x \exists y (P(x) \wedge Q(y))$

→ $\forall x (C \to A(x)) \equiv C \to \forall x A(x)$

→ $\exists x (C \to A(x)) \equiv C \to \exists x A(x)$

* → $\forall x (A(x) \to C) \equiv \exists x A(x) \to C$

* → $\exists x (A(x) \to C) \equiv \forall x A(x) \to C$

Note :

~~$\forall x A(x)$~~ If domain of both x & y are same

$$\forall x A(x) \vee \forall x B(x) \equiv \underbrace{\forall x A(x) \vee \forall y B(y)}$$

As long as scope ~~don't~~ doesn't overlap, variable name can be changed

Inference Rules with quantifiers :

(i) Universal Specification (U.S):

if $\forall x P(x)$ is true then,

$P(a)$ is true where 'a' any random element from the domain

(ii) Universal Generalization (U.G)

$P(a) \longrightarrow \forall x P(x)$

a is randomly choosen from domain

This is when $P(a)$ is true for every element

Eg: $D : \{1, 2, 3\}$

$P(x) : x^2 \geq 1$

$P(1)$ $P(2)$ $P(3)$

$\therefore \forall x P(x)$

(iii) Existential Specification (ES)

if $\exists x\, P(x)$ is true, then

$P(a)$ is true for some fixed 'a'.

(iv) Existential Generalization (E.G)

if $P(a)$ is true, for a fixed a, then

$\exists x\, P(x)$ is true.

Eg: Check if below argument is valid

All students in this class will goto IITs

Some students are not solving booklet question

∴ Some students who don't solve booklet questions go to IITs

Sol:

$$\forall x\,[C(x) \to IIT(x)] \quad ①$$

$$\exists x\,[C(x) \wedge \sim B(x)] \quad ②$$

$$\overline{\exists x\,[\sim B(x) \wedge IIT(x)]}$$

The above can be written as

$$\Big[\forall x[C(x) \to IIT(x)] \wedge \exists x[C(x) \wedge \sim B(x)]\Big] \to \exists x[\sim B(x) \wedge IIT(x)]$$

from ②

$C(a) \wedge \sim B(a)$ ③ (E.S)

from ①

$C(a) \to IIT(a)$ ④ (U.S)

from ③ & ④

~~$\sim B(a)$~~ $IIT(a)$

$\Rightarrow \sim B(a) \wedge IIT(a)$

$\Rightarrow \exists x\,[\sim B(a) \wedge IIT(a)]$ (E.G)

Eg: $\exists x\,[P(x) \wedge Q(x)] \Rightarrow \exists x\, P(x) \wedge \exists x\, Q(x)$

$\exists x\,[P(x) \wedge Q(x)]$

$P(a) \wedge Q(a)$ [a is fixed] (E.S)

$P(a)$

$\exists x\, P(x)$

$Q(a)$

$\exists x\, Q(x)$

$\therefore \exists x\, P(x) \wedge \exists x\, Q(x)$

$\therefore$ valid

Now consider

$[\exists x\, P(x) \wedge \exists x\, Q(x)] \Rightarrow \exists x\,[P(x) \wedge Q(x)]$

$\exists x\, P(x)$

$P(a_1)$

$\exists x\, Q(x)$

$P(a_2)$

a_1 need not be equal to a_1

$\therefore$ above is invalid

I) $\forall x\,[D(x) \rightarrow C(x)]$

$\exists x\,[D(x) \wedge \sim G(x)]$

$\therefore \exists x\,[G(x) \wedge \sim C(x)]$

$\exists x\,[D(x) \wedge \sim G(x)]$

$D(a) \wedge \sim G(a)$

$D(a) \rightarrow C(x)$

$C(a)$

$\Rightarrow \sim G(a) \wedge C(a)$

$\therefore$ not valid

II) $\forall x[MT(x) \to \sim M(x)]$

$\exists x[M(x) \wedge P(x)]$

$\exists x[P(x) \wedge \sim MT(x)]$

$\exists x[M(x) \wedge P(x)]$

$M(a) \wedge P(a)$

$MT(a) \to \sim M(a) \equiv M(a) \to \sim MT(a)$

$\therefore \sim MT(a)$

$\therefore P(a) \wedge \sim MT(a)$

$\Rightarrow \exists x[P(x) \wedge \sim MT(x)]$

Q8) $\forall x[P(x) \vee Q(x)]$

$\forall x[(\sim P(x) \wedge Q(x)) \to R(x)]$

$\forall x[\sim R(x) \to P(x)]$

This can be seen as

$P(a) \vee Q(x)$

$[\sim P(a) \wedge Q(a)] \to R(a)$

$\sim R(a) \to P(a)$

Sol:

$P(a) \vee Q(a)$ [a is random & not fixed]

$\sim P(a) \to Q(a) \equiv \sim Q(a) \to P(a)$

$(\sim P(a) \wedge Q(a)) \to R(a)$

$\sim R(a) \to (P(a) \vee \sim Q(x))$

$[\sim R(a) \to P(a)] \vee [\sim R(a) \to \sim Q(a)]$

$[\sim R(a) \to P(a)] \vee [\sim R(a) \to P(a)]$

$\therefore [\sim R(a) \to P(a)]$

$\forall x [\sim R(a) \to P(a)]$

alternate Method

$P(a) \vee Q(a)$

$\sim(\sim P(a) \wedge Q(a)) \vee R(a)$

$P(a) \vee \sim Q(a) \vee R(a)$

$\therefore P(a) \vee R(a)$

$\therefore \sim R(a) \to P(a)$

Q9

$$\forall x\left[P(x) \rightarrow (q(x) \wedge r(x))\right]$$
$$\forall x\left[P(x) \wedge S(x)\right]$$
$$\overline{\forall x\left[r(x) \wedge S(x)\right]}$$

This can be seen as

$$p \rightarrow (q \wedge r)$$
$$p \wedge s$$
$$\overline{r \wedge s}$$

$P(a) \rightarrow (q(a) \wedge r(a))$

$P(a) \wedge S(a)$

$\therefore q(a) \wedge r(a)$

$\therefore r(a) \wedge S(a)$ —— This hold for ~~any~~ all a

$\therefore \forall x\left[r(x) \wedge S(x)\right]$ (U.G)

Q10

$$\left\{\exists x\, P(x) \wedge \exists x\left[P(x) \rightarrow Q(x)\right]\right\} \rightarrow \exists x\, Q(x)$$

$$\exists x\left[P(x) \rightarrow Q(x)\right]$$
$$\exists x\, P(x)$$
$$\overline{\exists x\, Q(x)}$$

$P(a_1) \rightarrow Q(a_1)$

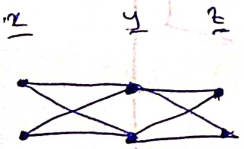
$P(a_2)$

from this we can't conclude $Q(a)$

$\therefore$ ~~false~~ not valid

Note:

* $\forall x\ \forall y\ \forall z$



$\forall x \forall y \forall z$ is like 3 for loops

Let ~~to~~ domain be $\{1,2\}$

$\forall x\ \forall y$

$1 <^{1}_{2}$

$2 <^{1}_{2}$

1-1
1-2
2-1
2-2

$\forall x\ \forall y\ \forall z$

$1-1 <^{1}_{2}$

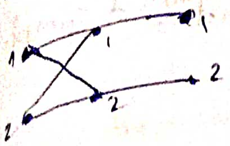
$1-2 <^{1}_{2}$

$2-1 <^{1}_{2}$

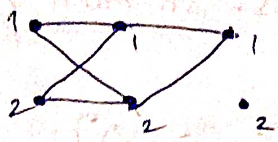
$2-2 <^{1}_{2}$

111
112
121
122
211
212
221
222

* $\forall x \forall y \exists z$



(or)



111	111
122	121
211	211
222	221

Sly $\forall x \forall y \exists z \forall w$

Q11) $\forall x [P(x) \vee q(x)]$ ①

$\exists x \sim P(x)$ ②

$\forall x [\sim q(x) \vee r(x)]$ ③

$\forall x [s(x) \to \sim r(x)]$ ④

$\therefore \exists x \sim s(x)$

Sol: ② $\exists x \sim P(x)$

$\sim P(a)$ ⑤

① $P(a) \vee q(a)$ ⑥

③ $\sim q(a) \vee r(a)$ ⑦

⑥ & ⑦ $P(a) \vee r(a)$ ⑧

⑧ & ⑤ $r(a)$ ⑨

④ $s(a) \to \sim r(a)$

$r(a) \to \sim s(a)$ ⑩

⑨ & ⑩ $\sim s(a)$

$\therefore \exists x \sim s(x)$

Q11) Let the universe for the variables in the following statements consist of all real numbers. In each case negate & simplify the given statement.

a) $\forall x \forall y [(x>y) \to ((x-y)>0)]$

b) $\forall x \forall y [(x<y) \to \exists z (x<z<y)]$

c) $\forall x \forall y [(|x|=|y|) \to (y=\pm x)]$

a) $\forall x \forall y [(x>y) \to (x-y>0)]$

Negation is

$\exists x \exists y \sim[\sim(x>y) \vee (x-y>0)]$

$\exists x \exists y [(x>y) \wedge \sim(x-y>0)]$

$\exists x \exists y [(x>y) \wedge (x-y \leq 0)]$

b) $\sim \forall x \forall y [(x<y) \to \exists z (x<z<y)]$

$\exists x \exists y \sim[\sim(x<y) \vee \exists z (x<z<y)]$

$\exists x \exists y [(x<y) \wedge \sim\exists z (x<z<y)]$

$\exists x \exists y [(x<y) \wedge \forall z \sim((x<z) \wedge (z<y))]$

$\exists x \exists y [(x<y) \wedge \forall z [(x \geq z) \vee (z \geq y)]]$

$1<2<5$

$x<z<y$

$=(x<z) \wedge (z<y)$

c) $\exists x \exists y \sim[\sim(|x|=|y|) \vee (y=\pm x)]$

$\exists x \exists y [(|x|=|y|) \wedge \sim(y=\pm x)]$

$\exists x \exists y [(|x|=|y|) \wedge (y \neq \pm x)]$

Q12 Consider the open statement:

$$P(x,y): \; y - x = y + x^2$$

where the universe for each of variables x, y comprises all integers. Determine the truth value for each of the following.

a) $P(0,0)$

$P(0,0): \; 0-0 = 0+0^2$

$0 = 0$

$\therefore$ true

b) $P(1,1)$

$1-1 = 1+1^2$

$0 = 2$

$\therefore$ false

3) $P(0,1)$

$1-0 = 1+0^2$

$1 = 1$

$\therefore$ true

4) $\forall y \; P(0,y)$

$P(0,y): \; y-0 = y+0^2$

$y = y$

$\therefore$ true

5) $\exists y, P(1,y)$

$P(1,y): \; y-1 = y+1$

we can't define such y

$\therefore$ false

6) $\forall x \exists y \; p(x,y)$:

for every x we can't define such fixed y

$\therefore$ false

for $x = 1$ no y can be defined

g) $\exists y \forall x\ P(x,y)$

for $x=1$ ~~Ga~~, we can't find appropriate y.

$\therefore$ There doesn't exist y for every x

$\therefore$ false

h) $\forall y \exists x\ P(x,y)$

~~Th~~ for $x=0$, ~~any~~ for any 'y' the expression is tre

$\therefore \exists x \forall y\ P(x,y)$ is ~~true~~

$\therefore \forall y \exists x$ is also true

(Q13) Determine whether each of the following statements is true or false. If false, provide a counter example. The universe comprises all integers.

a) $\forall x \exists y \exists z\ (x = 7y + 5z)$

b) $\forall x \exists y \exists z\ (x = 4y + 6z)$

This is explained later in book thoroughly in page: 150

Sol:

a) $\forall x \exists y \exists z\ (x = 7y + 5z)$

Let $x = x_1$ (fixed const)

~~th~~ for $x = x_1$, $\exists y \exists z\ (x_1 = 7y + 5z)$

Now $7y + 5z = x_1$

represent a line

every point on line (y_1, z_1) satisfies condition

Thus for $x = x_1$

~~then~~ there exists y & z which satisfies condition

Slly for any x, we can define y & z

b) ~~Same as (a)~~

~~this is also true.~~ (wrong)

b is false

Remaining Questions on First Order logic from ACE Material

P/34

S1: $\forall x (x^2+1<0)$

$\cancel{\forall x (x^2 = 1)}$

∴ ~~false~~

S2: $\forall x$ (x is odd)

∴ Both are open stmts & predicates

P/35

$\sim[\forall x \exists y\, P(x,y)]$

$\exists x \forall y \sim(\cancel{x}(x \vee y) \to z)$

$\exists y \forall y ((x \vee y) \wedge \sim z)$

P/36

$\phi: \forall x \exists y [x+y=17]$

1. $D_x = D_y =$ integers

   it is clearly true

2. $D_x = D_y =$ positive integers

   for ~~x~~ $x>17$

   we can't define y such that $y>0$

   ∴ false

4. ~~8.~~ $D_x =$ +ve integer $D_y =$ integers

   for any x we can define y

   ∴ true

3. $D_x =$ integer $D_y =$ +ve ints

   for $x>17$ we can't define y such that $y>0$

   ∴ false

∴ 1 & 4

P/38 "For every integer n bigger than 1, there is a prime strictly between n & 2n"

The logical ~~eq~~ equivalent is

$$\forall n\left[(n>1) \rightarrow \exists x\left(P(x) \wedge (n<x<2n)\right)\right]$$

$n<x<2n$

$\equiv (n<x) \wedge (x<2n)$

$\sim(n<x<2n)$

$\equiv (n \geq x) \vee (x \geq 2n)$

~~Negation is~~

$$\forall n\left[\sim(n>1) \vee \exists x\left(P(x) \wedge (n<x<2n)\right)\right]$$

Negation is

$$\exists n\left[(n>1) \wedge \sim\exists x\left(P(x) \wedge (n<x<2n)\right)\right]$$

$$\exists n\left[(n>1) \wedge \forall x\left(\sim P(x) \vee (n \geq x) \vee (x \geq 2n)\right)\right]$$

$$\exists n\left[(n>1) \wedge \forall x\left(P(x) \rightarrow \left((x \leq n) \vee (x \geq 2n)\right)\right)\right]$$

P/40 Direct answer

P/41 Direct answer

P/42 "There is a student who like mathematics but not history"

Some student ~~do~~ likes mathematics ~~&~~ and does not like history

$$\exists x\left[S(x) \wedge M(x) \wedge \sim H(x)\right]$$

Negation is

$$\forall x\left[\sim S(x) \vee \sim M(x) \vee H(x)\right]$$

$$\forall x\left[\sim(\sim S(x) \vee \sim M(x)) \rightarrow H(x)\right]$$

$$\forall x\left[(S(x) \wedge M(x)) \rightarrow H(x)\right]$$

P|43

Direct Answer

P|44 I. $\forall x\left[P(x)\rightarrow(Q(x)\wedge S(x))\right]$

$\forall x(P(x)\wedge R(x))$

$\forall x(R(x)\rightarrow S(x))$

This can be seen as

$$\left.\begin{matrix}P\rightarrow(Q\wedge S)\\ P\wedge R\end{matrix}\right\}\vdash Q\wedge S$$

$R\rightarrow S$

R

S

$\therefore R\rightarrow S$

$\therefore$ valid

II. All quantifier are $\forall$

$P\vee Q$ ①

$(\sim P\wedge Q)\rightarrow R\equiv\sim R\rightarrow(P\vee\sim Q)\equiv R\vee P\vee\sim Q$ ②

$\sim R\rightarrow P$

① & ② $R\vee P\equiv\sim R\rightarrow P$

$\therefore$ valid

P|45

$\forall x\left[M(x)\rightarrow I(x)\right]$ ① $M\rightarrow I$

$\forall x\left[\sim I(x)\leftrightarrow S(x)\right]$ $\sim I\rightarrow S$ ② $S\rightarrow\sim I$ ③ $\bar{S}$

$\forall x\left[G(x)\rightarrow M(x)\right]$ $G\rightarrow M$ ④

$\leftarrow G\rightarrow I$ ($\therefore$ d is true)

a) $\forall x\left[S(x)\rightarrow\sim M(x)\right]\equiv$ ~~$S\rightarrow\sim M$~~ $\equiv S\rightarrow\sim M$ ~~(not valid)~~

b) $\exists x\left[G(x)\wedge S(x)\right]$

c) $\exists x\left[G(x)\wedge I(x)\right]$

d) $\forall x\left[G(x)\rightarrow I(x)\right]$

(a)

① $M \to I \equiv \sim I \to \sim M$

③ $S \to \sim I$

$\therefore S \to \sim M$

$\therefore$ ⓐ is valid

b) $\exists x [G(x) \wedge S(x)]$

Consider its negation

$\forall x [\sim G(x) \vee \sim S(x)]$

$\forall x [S(x) \to \sim G(x)]$

$\equiv S \to \sim G$

① & ③ $S \to \sim M$

from ④ $S \to \sim G$

As negation of opt ⓑ is true

opt ⓑ is false

c) $\exists x (G(x) \wedge M(x))$

① & ④ $G \to I$

i.e., every genius is interesting

from this we can conclude

Some geniuses are interesting

P/46

$\exists x [D(x) \wedge \sim S(x)]$

Negation is

$\forall x [\sim D(x) \vee S(x)]$

$\forall x [D(x) \to S(x)]$

146

(147) S_1: $\exists x(P(x) \to Q(x)) \equiv \exists x[\sim P(x) \vee Q(x)]$

$\forall x\, P(x) \to \exists x\, Q(x) \equiv \exists x \sim P(x) \vee \exists x\, Q(x)$

$\cancel{P(a) \to Q(a)}$ ~~[fixed a]~~

now question is seen as

$\exists x(\sim P(x) \vee Q(x)) \Rightarrow \exists x \sim P(x) \vee \exists x\, Q(x)$

∴ ~~false~~ ∴ both are equivalent

∴ true

S_2: true

∴ ~~only S2~~ S_1 & S_2

(148) (a) is clearly false

b) $\exists x\, \forall y\, P(x,y) \to \forall x\, \exists y\, P(x,y)$

True

False

For above case ~~LP~~ prime is true

conclusion is false

∴ (b) is false

c) $\exists y\, \forall x\, P(y,x) \to \forall y\, \exists x\, P(x,y)$

above is same as

$\exists x\, \forall y\, P(x,y) \to \forall y\, \exists x\, P(x,y)$.

Changing variable name doesn't change meaning

∴ True

d) $\exists y\, \forall x\, P(x,y) \to \forall y\, \exists x\, P(x,y)$

T F

∴ false

148

P/50 S1:

$\exists x (A(x) \to B(x)) \Leftrightarrow \forall x\, A(x) \to \exists x\, B(x)$

$\exists x (\sim A(x) \vee B(x)) \Leftrightarrow \exists x \sim A(x) \vee \exists x\, B(x)$

clearly true

S2:

$\exists x\, A(x) \to \forall x\, B(x) \Leftrightarrow \forall x (A(x) \to B(x))$

$\forall x \sim A(x) \vee \forall x\, B(x) \Leftrightarrow \forall x (\sim A(x) \vee B(x))$

Cleary false

S3: true

S4: false

P/51 a)

$\forall x (\sim P(x) \vee Q(x)) \Rightarrow \overline{\left[\exists x \sim P(x) \vee \forall x\, Q(x)\right]}^{F}$

T	T
T	T
T	F

F v F = F

$\sim T \vee T$ | T
$\wedge$ | $\wedge$
$\sim T \vee T$ | T
$\wedge$ | $\wedge$
$\sim T \vee F$ | F

F → Valid

alternate

$\dfrac{\cancel{\forall x [P(x) \to Q(x)]}}{\cancel{\forall x\, P(x) \to \forall x\, Q(x)}} \equiv \dfrac{\cancel{\forall x (\sim P(x) \vee Q(x))}}{\cancel{\exists x \sim P(x) \vee \forall x\, Q(x)}}$

b) $[\forall x\, P(x) \to \forall x\, Q(x)] \Rightarrow \forall x (\sim P(x) \vee Q(x))$

T	→	F		$\sim T \vee F$
T		F		$\wedge$ $\sim T \vee F$
F		F		$\wedge$ $\sim T \vee F$

F → F ≡ T ———— F

∴ not valid

c) $\forall x (P(x) \vee Q(x)) \rightarrow (\forall x P(x) \vee \exists x Q(x))$

$$\frac{\forall x (P(x) \vee Q(x))}{\sim \exists x Q(x) \rightarrow \forall x P(x)}$$

This is seen as

$$\forall x (P(x) \vee Q(x))$$
$$\frac{\sim \exists x Q(x)}{\forall x P(x)}$$

$$\sim \exists x Q(x) \equiv \forall x \sim Q(x)$$

$$\therefore \; P \vee Q$$
$$\frac{\sim Q}{\therefore P}$$

This true

d) true

GATE 2008 which of following is the negation of

$$[\forall x, \alpha \rightarrow (\exists y, \beta \rightarrow (\forall u, \exists v, \gamma))]$$

a) $[\exists x, \alpha \rightarrow (\forall y, \beta \rightarrow (\exists u, \forall v, \gamma))]$

b) $[\exists x, \alpha \rightarrow (\forall y, \beta \rightarrow (\exists u, \forall v, \sim\gamma))]$

c) $[\forall x, \sim\alpha \rightarrow (\exists y, \sim\beta \rightarrow (\forall u, \exists v, \sim\gamma))]$

d) $[\exists x, \alpha \wedge (\forall y, \beta \wedge (\exists u, \forall u, \sim\gamma))]$

Sol :

$$\sim[\forall x, \sim\alpha \vee (\exists y, \sim\beta \vee (\forall u, \exists v, \gamma))]$$

$$[\exists x, \alpha \wedge (\forall y, \beta \wedge (\exists u, \forall v, \sim\gamma))]$$

$\therefore d$

☆ ☆

☆ Q13) Determine whether each of the following statements is true or false. If false, provide a counter example. The universe comprises all integers. 150

a) $\forall x \exists y \exists z \ (x = 7y + 5z)$

b) $\forall x \exists y \exists z \ (x = 4y + 6z)$

Sol:

a) $\forall x \exists y \exists z$

let $x = 1$

$1 = 7y + 5z$ should have integer solution.

for $y = -2, z = 3$

$1 = 7(-2) + 5(3)$

$1 = 1$

$\therefore$ for $x = 1$, y & z exists

Let $x = 2$

$2 = 2(7(-2) + 5(3))$

$2 = 7(-4) + 5(6)$

$\therefore$ for $x = 2$, y & z exist

Slly for ~~x~~ any value of x

we can find y & z exist

such that y, z are integers

$\therefore$ true

b) $\forall x \exists y \exists z \ (x = 4y + 6z)$

let $x = 1$

$1 = 4y + 6z$ (Also ~~mult~~ sum of multiple of 2 'even' numbers will never be odd)

$1 = 2(2y + 3z)$

$\frac{1}{2} = 2y + 3z$

for any integers y, z

$2y + 3z$ is integer which is not equal to $1/2$

$\therefore$ for $x = 1$, we can't find integers y & z

$\therefore$ false

GATE 2019

Consider the first order predicate formula ϕ:

$$\forall x\left[\left(\forall z\ z|x \Rightarrow ((z=x) \vee (z=1))\right) \Rightarrow \exists w\left((w>x) \wedge \left(\forall z\ z|w \Rightarrow ((w=z) \vee (z=1))\right)\right)\right]$$

Here $a|b$: a divides b where a, b are integers

Consider the following sets:

S1: $\{1, 2, 3, \ldots, 100\}$

S2: set of all positive integers.

S3: set of all integers.

which of the above sets satisfy ϕ?

a) S2 & S3 b) S1, S2, S3 c) S1 & S2 d) S1 & S3

sol:

observe the premise

$$\forall x\ \left(\underbrace{\forall z\ z/x}_{\text{if } z \text{ divides } x} \Rightarrow \underbrace{((z=x) \vee (z=1))}_{z=x \text{ or } z=1}\right)$$

i.e. x is prime.

The prime say for any x, if x is prime

Now observe the antecedent

$$\exists w\left(\underbrace{(w>x)}_{\text{There exists a number greater than } x} \wedge \text{ and } \underbrace{\left(\forall z\ z/w \Rightarrow ((w=z) \vee (z=1))\right)}_{w \text{ is also prime}}\right)$$

The whole logical expression say

if x is a prime, then exists another prime greater than x.

Thus the set must be infinite

So S2 & S3 clearly satisfy the condition

A few more concepts in logic:

Fallacies

(i) Fallacy of assuming the converse:

$$\begin{array}{l} P \to q \\ q \\ \hline \therefore P \end{array}$$

not valid

(ii) Fallacy of assuming the inverse

$$\begin{array}{l} P \to Q \\ \sim P \\ \hline \therefore \sim Q \end{array}$$

not valid

(iii) Non Sequitur:

$$\begin{array}{l} P \\ q \\ \hline \therefore S \end{array}$$

Quine's Method

→ This method is used to determine if compound proposition is tautology or fallacy or contingency

Eg:

$P \to (Q \wedge P)$

- P = True: $T \to (Q \wedge T) \equiv Q$
  - Q = True: True
  - Q = False: False
- P = False: True

$\therefore P \to (Q \wedge P)$ is contingency

Conditional Proof:

if $\{\{P_1, P_2 \dots P_n\} \wedge Q\} \Rightarrow R$ — ①

then

$\{P_1, P_2, \dots P_n\} \Rightarrow (Q \rightarrow R)$ — ②

whenever we see argument in form 2

we write it in form ① and prove the argument

Indirect Proof (Proof by Contradiction):

To apply this rule, first we assume that the argument is not valid.

So we take negation of conclusion as new premise.

when we combine this new premise with other premise and if we meet with any contradiction then the given argument is valid.

Inconsistency:

A set of premises $\{P_1, P_2 \dots P_n\}$ is said to inconsistent, if all the premises can't be true simultaneously.

i.e., the conjunction of all the premises is a contradiction

→ In any argument, if the premises are inconsistent then the argument is valid.

Functional Completeness:

A set of connectives is said to be functionally complete, if every propositional function can be represented using this connectives only.

Eg: $\{\vee, \sim\}$, $\{\wedge, \sim\}$, $\{\rightarrow, \sim\}$, $\{\wedge, \vee, \sim\}$

↓

This like OR gate & NOT gate

Q14 Translate below stmts into logical equivalent

a) "The sum of two positive integers is always positive"

$$\forall x \forall y \left[((x>0) \wedge (y>0)) \rightarrow ((x+y)>0) \right]$$

$x \in R$ & $y \in R$

b) "Every real number except zero has a multiplicative inverse"

This stmt is seen as

For every ~~real~~ number x, such that $x \neq 0$, x has a multiplicative inverse.

$$\forall x \left[(x \neq 0) \rightarrow \exists y \, (xy = 1) \right]$$

GATE 2016

Let p, q, r, s represent the following propositions

$p : x \in \{8, 9, 10, 11, 12\}$

q : x is a composite number

r : x is a perfect square

s : x is a prime number

The integer $x \geq 2$ which satisfies

$\sim((p \Rightarrow q) \wedge (\sim r \vee \sim s))$ is ________

Sol :

$$\sim((p \rightarrow q) \wedge (\sim r \vee \sim s)) \equiv \sim((\sim p \vee q) \wedge (\sim r \vee \sim s))$$

$$\equiv (p \wedge \sim q) \vee (r \wedge s)$$

($x \in \{8, 9, 10, 11, 12\}$ and x is not composite) or (x is perfect square and x is prime number)

This is not posible

$\therefore x = 11$

Calculating no of edges in Line graph:

In a graph, every vertex of degree k contributes $\frac{k(k-1)}{2}$ no of edges to line graph.

Now let $d_1, d_2 \ldots d_n$ be degree of graph

$\Rightarrow$ no of edge in Line graph of G, $L(G) = \frac{d_1(d_1-1)+d_2(d_2-1)+\ldots+d_n(d_n-1)}{2}$

$$= \boxed{\frac{\Sigma d_i^2 - \Sigma d_i}{2}}$$

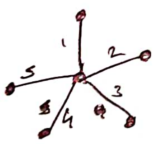
If m is no of edges in the graph

no of edges in line graph $\boxed{\frac{\Sigma d_i^2}{2} - m}$

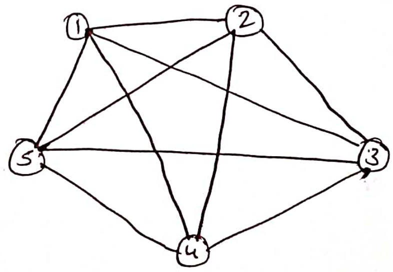
Note:

* Line graph of a star graph of n vertices is a complete graph of $(n-1)$ vertices

Eg:



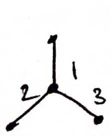
G

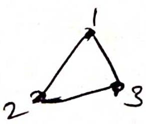


L(G)

* Line graph of a tree need not to be tree.

Eg:





* Line graph of a complete graph need not to be complete

~~reason~~: Beacause in a complete graph it is not ~~needed~~ that every edge adjacent to other edges.

* Line graph of a bipartite graph need not to be a bipartite

Reason: It is not possible to color all edges with 2 colors even if graph is bipartite.

* Max degree of Line graph can be greater that max degree of the graph

* Line graph of cycle graph is always cycle.