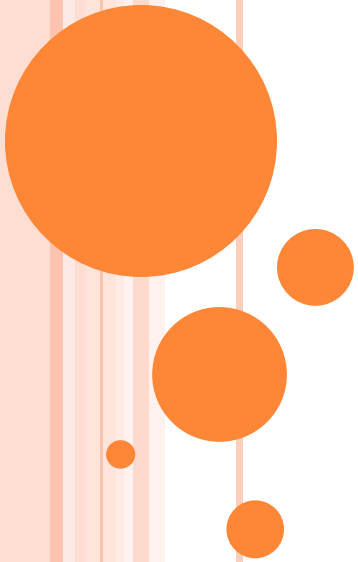


Finite Automata with output

FINITE AUTOMATA WITH OUTPUT

- The FA will take an input as well as they will keep on writing the output.
- FA with output has two categories:
  - Moore machine
  - Mealy machine
- These two machines are deterministic.

FINITE AUTOMATA WITH OUTPUT

- **Moore machine**

  $M = (Q, \Sigma, \delta, q_0, \Delta, \lambda)$ where

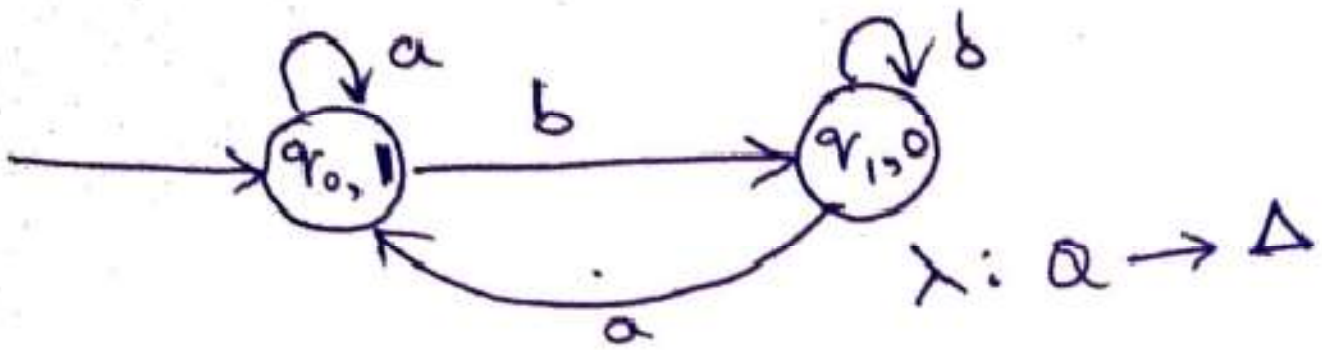
  - Q – Non empty set of finite number of states.
  - Σ – Non empty set of finite number of symbols or Finite input alphabet.
  - δ – State transition function, defined as
    $\delta: Q \times \Sigma \rightarrow Q$.

  - q_0 – It is initial or start state, $q_0 \in Q$.
  - Δ – output alphabet, the symbols which are supposed to be printed or the symbols which will be outputted by the automata.
  - λ – output function, determines what will be printed as output.
    $\lambda: Q \rightarrow \Delta$, for every state output is associated.

FINITE AUTOMATA WITH OUTPUT

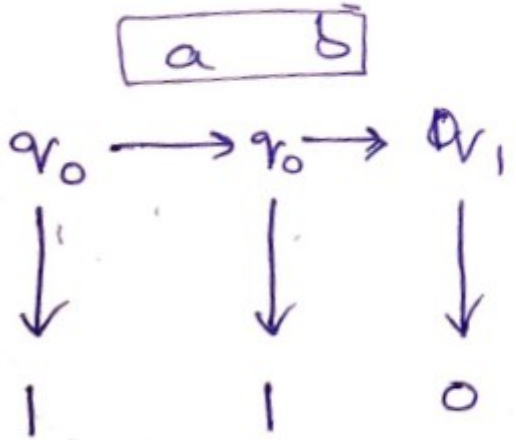
- **Moore machine Example**:

- For every state, we have one output.
- If I give the string of length as 'n' input, then the output produce will be string of length 'n+1'.



FINITE AUTOMATA WITH OUTPUT

- Input string ab.



On seeing input "ab", the moore machine has printed the output "110".

FINITE AUTOMATA WITH OUTPUT

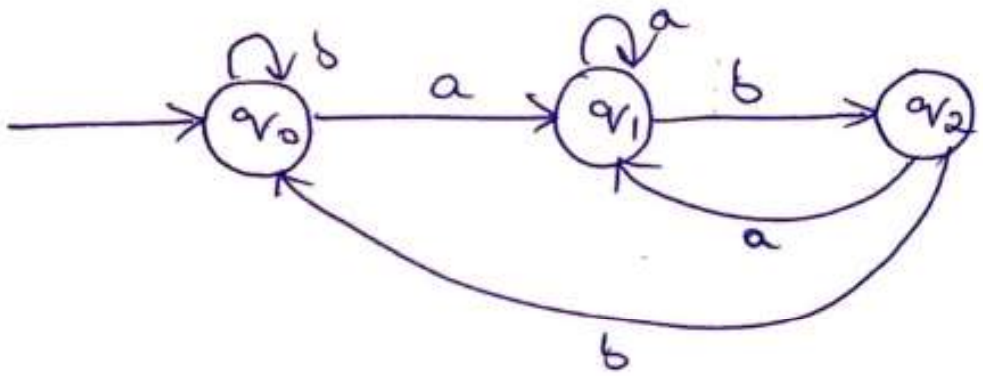
- **Example**: Construct a Moore machine that takes set of all strings over {a, b} as input and prints '1' as output for every occurrence of 'ab' as a substring.

- **Solution**:
  $\Sigma = \{a, b\}$
  $\Delta = \{0, 1\}$

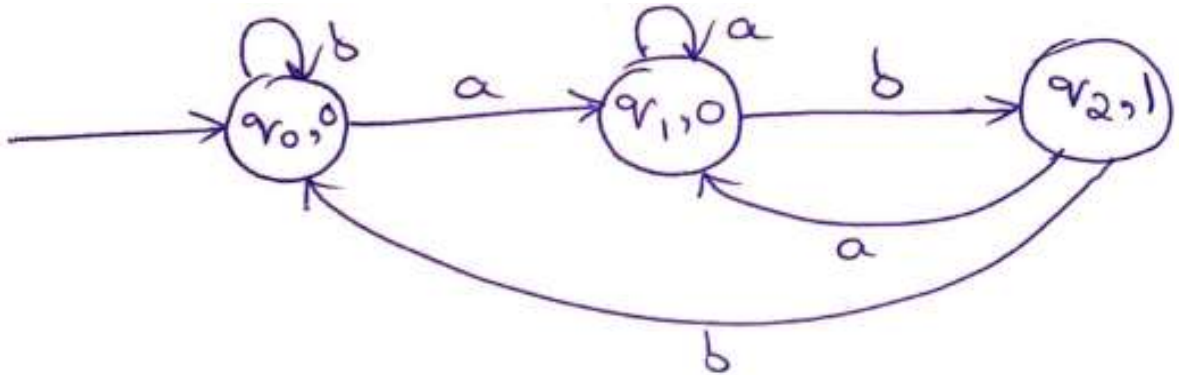
- Design the deterministic finite machine (DFA) for the language having substring 'ab'

FINITE AUTOMATA WITH OUTPUT

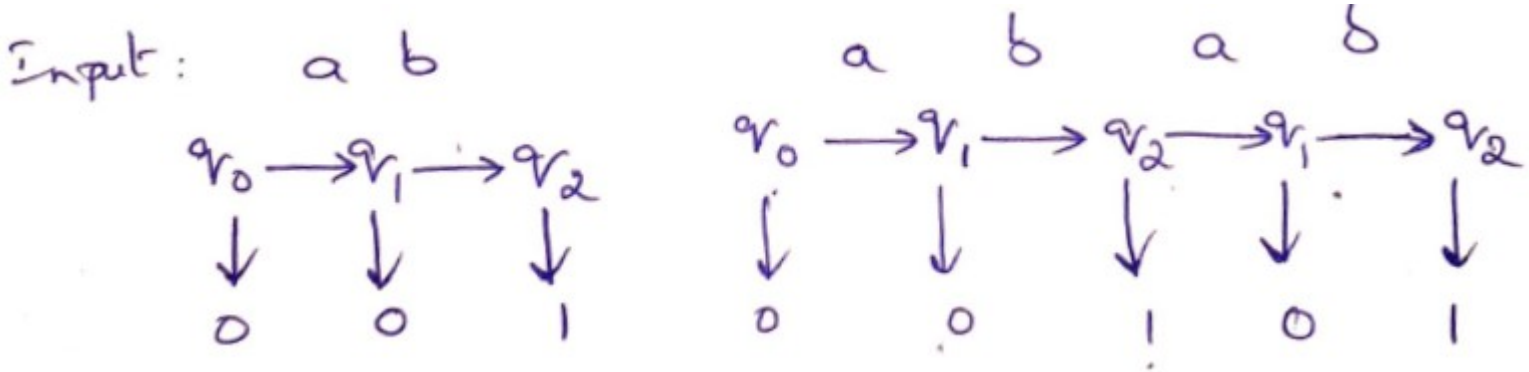
- Make the final state as non final state and make the transitions which accepts strings of any length (for **counting no. of 'ab's**).



- For every state, associate the output to make it as Moore machine.



FINITE AUTOMATA WITH OUTPUT

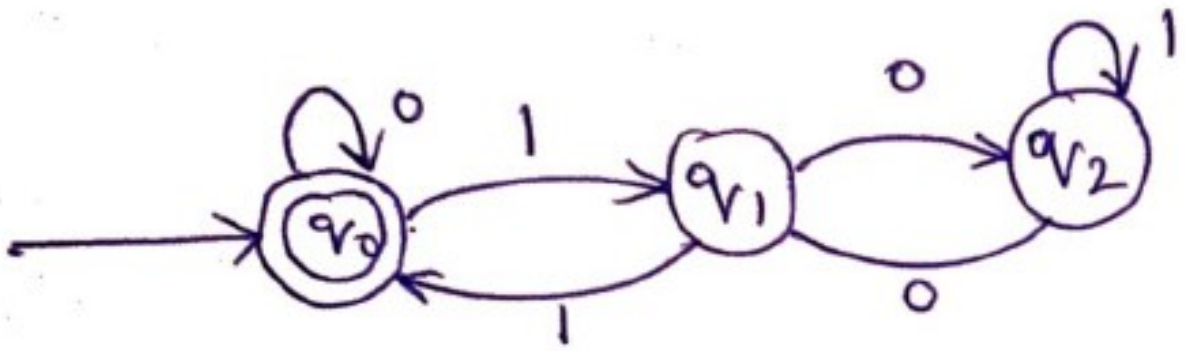


FINITE AUTOMATA WITH OUTPUT

- **Example**: Design a Moore machine for residue mod 3 of a binary string treated as a binary number.

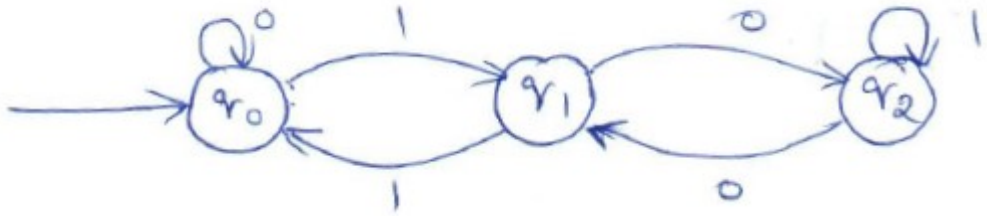
- **Solution**:
  $\Sigma = \{0, 1\}$
  $\Delta = \{0, 1, 2\}$

- Design the deterministic finite machine (DFA) for the given language.

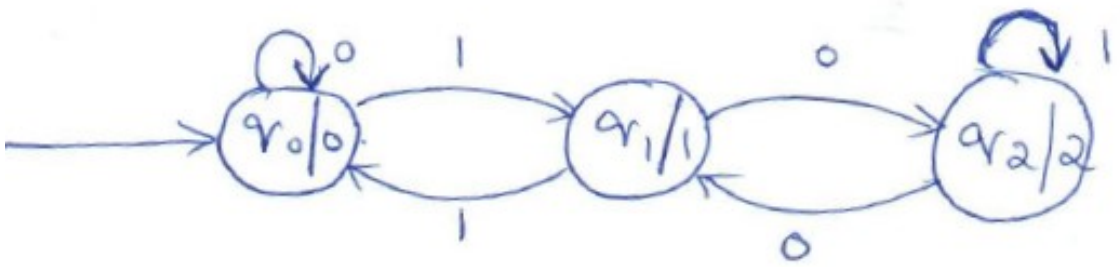


FINITE AUTOMATA WITH OUTPUT

- Make the final state as non final state and make the transitions which accepts strings of any length.



- For every state, associate the output to make it as Moore machine.



FINITE AUTOMATA WITH OUTPUT

- **Mealy machine**

  $M = (Q, \Sigma, \delta, q_0, \Delta, \lambda)$ where

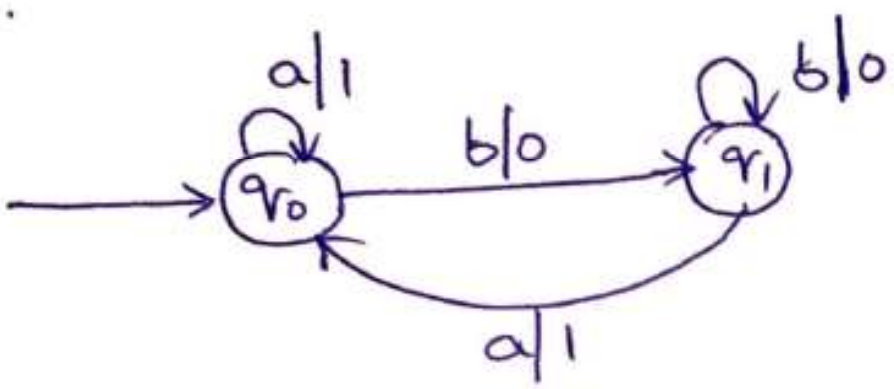
  - Q – Non empty set of finite number of states.
  - Σ – Non empty set of finite number of symbols or Finite input alphabet.
  - δ – State transition function, defined as
    $\delta: Q \times \Sigma \rightarrow Q$.

  - q_0 – It is initial or start state, $q_0 \in Q$.
  - Δ – output alphabet, the symbols which are supposed to be printed or the symbols which will be outputted by the automata.
  - λ – output function
    $\lambda: Q \times \Sigma \rightarrow \Delta$, for a state, for the given input there will be some output.

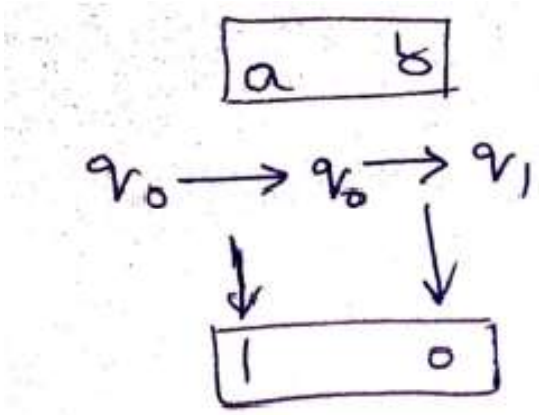
FINITE AUTOMATA WITH OUTPUT

- **Mealy machine Example**:

- For every state, every input, we have one output.
- If I give the string of length as 'n' input, then the output produce will be string of length 'n'.



$\lambda : Q \times \Sigma \rightarrow \Delta$



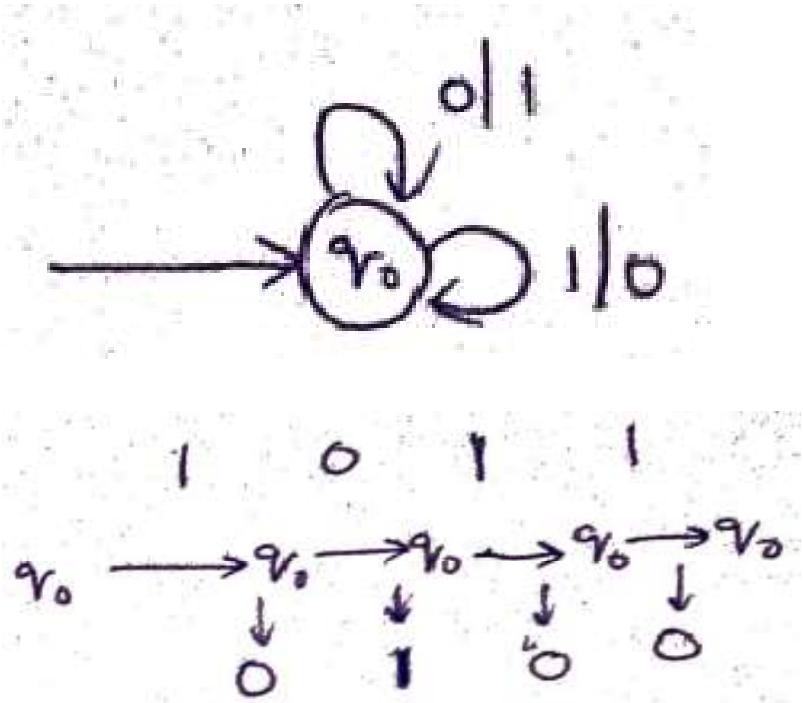
FINITE AUTOMATA WITH OUTPUT

- **Example**: Construct a Mealy machine that takes binary number as input and produces 1's complement of that number as output. Assume the string is read LSB to MSB and end carry is discarded.

- **Solution**:

$\Sigma = \{0, 1\}$

$\Delta = \{0, 1\}$



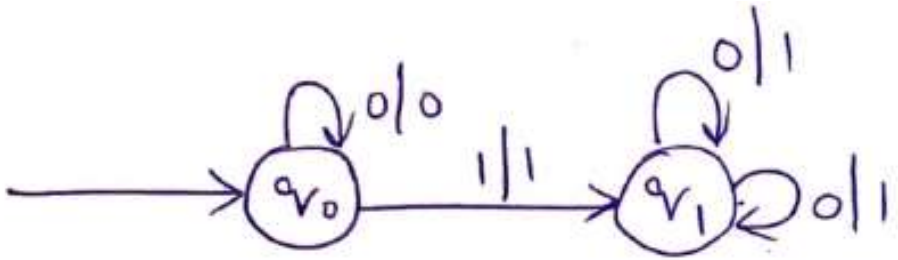
FINITE AUTOMATA WITH OUTPUT

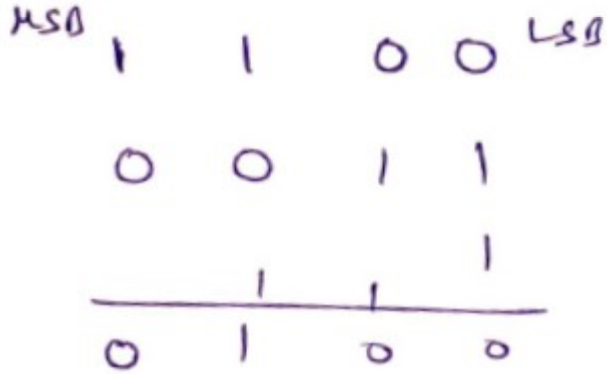
- **Example**: Construct a Mealy machine that takes binary number as input and produces 2's complement of that number as output. Assume the string is read LSB to MSB and end carry is discarded.

- **Solution**:
  $\Sigma = \{0, 1\}$
  $\Delta = \{0, 1\}$

- If I read from LSB to MSB, whenever I have 0's, leaving them as 0's and whenever I see first 1, I am leaving it as 1. And whatever is there after a first 1, I am going to apply 1's complement.

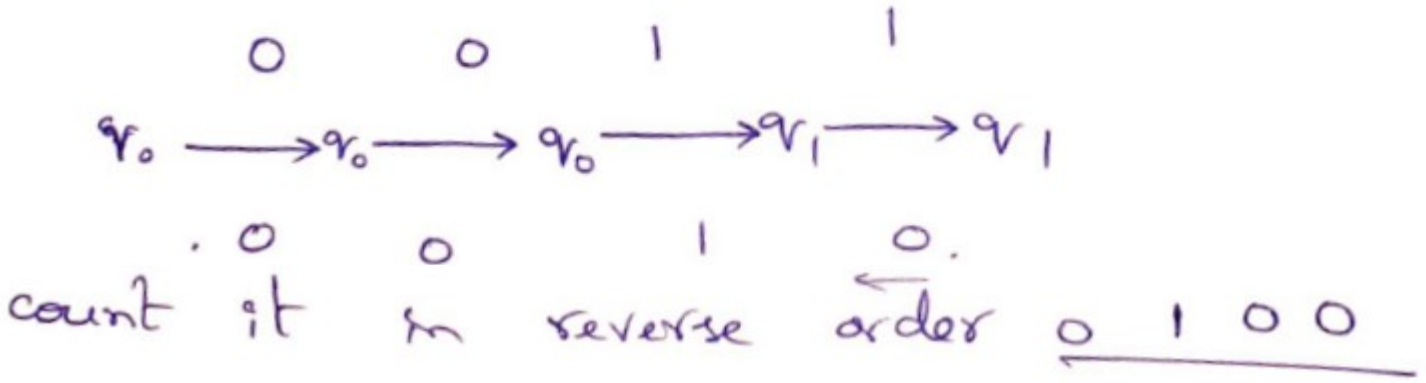
FINITE AUTOMATA WITH OUTPUT





FINITE AUTOMATA WITH OUTPUT

- Read from LSB, so I should write it as



FINITE AUTOMATA WITH OUTPUT

- **Conversion of Moore machine to Mealy**:

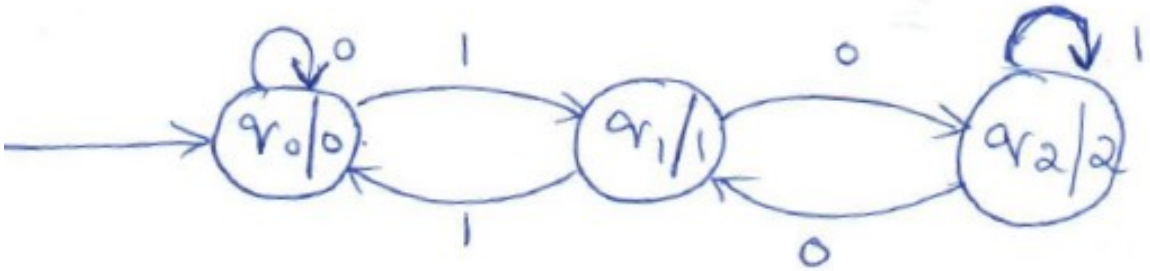
  - Let λ be output function in Moore
  - Let λ' be output function in Mealy

  - δ be transition function of Moore.
  - Now to convert a Moore to Mealy

$$\lambda'(q, a) = \lambda(\delta(q, a))$$

FINITE AUTOMATA WITH OUTPUT

- **Example**: Design a Moore machine for residue mod 3 of a binary string treated as a binary number and convert into Mealy machine.



- **Solution**:

  $\Sigma = \{0, 1\}$

  $\Delta = \{0, 1, 2\}$

- Formula is: $\lambda'(q, a) = \lambda(\delta(q, a))$

Finite Automata with output

- $\lambda'(q_0, 0) = \lambda(\delta(q_0, 0))$
  $= \lambda(q_0)$
  $= 0$

- $\lambda'(q_0, 1) = \lambda(\delta(q_0, 1))$
  $= \lambda(q_0)$
  $= 1$

- $\lambda'(q_1, 0) = \lambda(\delta(q_1, 0))$
  $= \lambda(q_2)$
  $= 2$

- $\lambda'(q_1, 1) = \lambda(\delta(q_1, 1))$
  $= \lambda(q_0)$
  $= 0$

FINITE AUTOMATA WITH OUTPUT

- $\lambda'(q_2, 0) = \lambda(\delta(q_2, 0))$
  $= \lambda(q_1)$
  $= 1$
- $\lambda'(q_2, 1) = \lambda(\delta(q_2, 1))$
  $= \lambda(q_2)$
  $= 2$

- The equivalent Mealy machine is