1) Equivalence of NFA and DFA :- (Theorem)

Statement:- Let 'L' be a set accepted by a non deterministic finite automata then there exists a DFA which accepts 'L'.

proof :-

→ Let $M = (Q, \Sigma, \delta, q_0, F)$ be the NFA which accepts 'L'.

→ Let us define a DFA $M' = (Q', \Sigma, \delta', q_0', F')$ be an DFA which accepts 'L'.

→ The initial state of DFA is same as initial state of NFA. so,

$$q_0' = [q_0]$$
$$Q' = 2^Q$$

→ The states of the DFA are subsets of the set of states of NFA.

→ $[q_1, q_2, \ldots, q_i]$ is an individual state in DFA, where $q_1, q_2, q_3, \ldots, q_i$ are seperate states in NFA.

→ The final states of DFA are those states which Contains at least one final state in NFA.

→ The transition function δ' is defined as

$$\delta'([q_1, q_2, \ldots, q_i], a) = [P_1, P_2, \ldots, P_j]$$

. if and only if

$$\delta(\{q_1, q_2, \ldots, q_i\}, a) = \{P_1, P_2, \ldots, P_j\}.$$

→ Now Let us prove that the language accepted by the NFA and DFA are equal.

→ For this we need to prove that every string accepted by NFA is accepted by DFA.

→ Let us apply the mathematical induction on the length of string 'x', to show that there are similar type of transitions in both NFA and DFA after processing of string 'x'.

→ That is, we need to prove that

$$\delta'(q_0', x) = [q_1, q_2, \ldots, q_i]$$

iff

$$\delta(q_0, x) = \{q_1, q_2, \ldots, q_i\}$$

Basis step:- Consider a string of length 0

$$\delta(q_0, \epsilon) = q_0$$

$$\delta'(q_0', \epsilon) = q_0'$$

We know that $q_0' = [q_0]$.

→ This means that on a string of length 0, both NFA and DFA are having similar type of transitions.

Hence Basis step is proved.

Inductive Hypothesis:- Let us assume that for strings of length 'k' both NFA and DFA are having similar type of transitions.

$$\delta'(q_0', x] = [P_1, P_2, \ldots, P_j]$$

iff

$$\delta(q_0, x) = \{P_1, P_2, \ldots, P_j\}$$

Inductive step :- We need to prove that for a string of length (K+1) also, NFA and DFA will be having similar type of transitions.

i.e., we required to prove that

$$\delta'(q_0', xa) = [r_1, r_2, \ldots, r_K]$$

iff

$$\delta(q_0, xa) = \{r_1, r_2, \ldots, r_K\}.$$

→ Let us consider, $\delta'(q_0', xa) = \delta'(\delta'(q_0', x), a)$

$$= \delta'([P_1, P_2, \cdots P_j], a)$$

($\because$ From inductive hypothesis)

$$\delta'([P_1, P_2, \ldots, P_j], a) = [r_1, r_2, \ldots, r_K]$$ ($\because$ from the definition of DFA)

iff

$$\delta(\{P_1, P_2, \ldots, P_j\}, a) = \{r_1, r_2, \ldots, r_K\}$$

From the definition

$$\delta(\delta(q_0, x), a) = \{r_1, r_2, \ldots, r_K\}$$

$$\delta(q_0, xa) = \{r_1, r_2, \ldots, r_K\}$$

Hence proved.

→ Now we need to prove that every string 'x' accepted by NFA is also accepted by DFA.

→ If x is accepted by NFA.

$$\delta(q_0, x) \in F$$

i.e, one of the states $q_1, q_2, \ldots, q_i$ must be accepted by NFA.

→ From the definition, if one of the states of $\delta(q_0, x) = \{q_1, \ldots q_i\}$ is a final state then $[q_1, q_2, \ldots q_i]$ is a final state in DFA which implies that 'x' is accepted by DFA

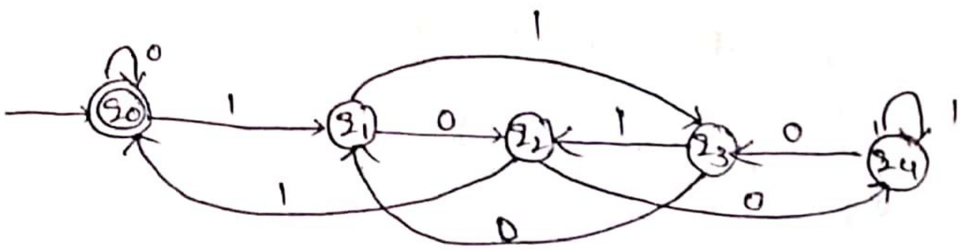
Hence language accepted by NFA and DFA are equal.

3)

Q. Let $\Sigma = \{0,1\}$. Design a finite automata for language which contain strings that are divisible by 5 when the strings are interpreted as binary numbers

Soln

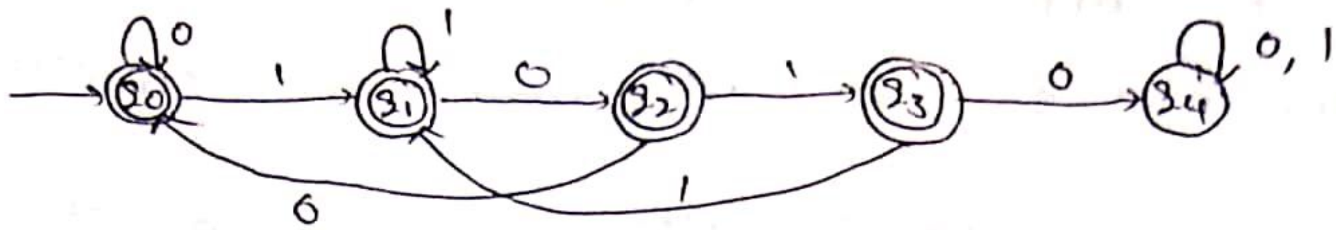
No.	Binary value	Reminder when Mod 5	state end
0	0000	0	q_0
1	0001	1	q_1
2	0010	2	q_2
3	0011	3	q_3
4	0100	4	q_4
5	0101	0	q_0
6	0110	1	q_1
7	0111	2	q_2
8	1000	3	q_3
9	1001	4	q_4
10	1010	0	q_5
11	1011	1	q_1



3)

34. Let $\Sigma = \{0,1\}$. Design a finite automata for language $L =$
$\{w \mid w$ does not contain $1010\}$

soln



4) Fo Types of Finite Automata :-

Finite Automata is of 3 forms

1. Deterministic Finite Automata (DFA)

2. Non deterministic Finite Automata (NFA)

3. ϵ - Non deterministic Finite Automata (ϵ-NFA)

1. Deterministic Finite Automata (DFA)

1. A finite automata in which every input symbol is applied on every state and is applied exactly once.

2. A deterministic Finite Automata M can be described by 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where

   i. Q is finite, non empty set of states

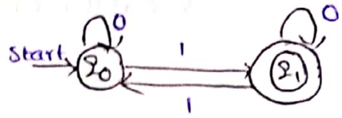
   ii. Σ is an input alphabet

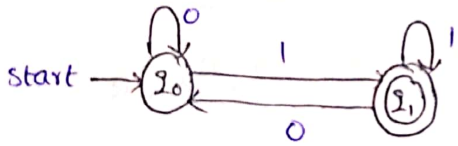
   iii. δ is transition function which maps $Q \times \Sigma \rightarrow Q$ i.e., the head reads a symbol in its present state and moves into next state.

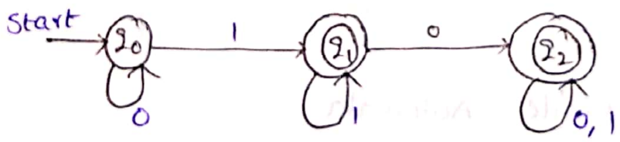
   iv. $q_0 \in Q$, known as initial state

   v. $F \subseteq Q$, known as set of final states.

3. Examples of DFA :-

1.


2.


3.


Definition of Non deterministic Finite Automata :-

A finite Automata is a 5 tuple. It is the machine

$M = (Q, \Sigma, \delta, q_0, F)$

Where,

i. $Q \longrightarrow$ Non empty set of finite ~~s~~ number of states

ii. $\Sigma \longrightarrow$ Non empty set of finite number of Symbols or Finite input alphabet.

iii. $\delta \longrightarrow$ state transition function, defined as

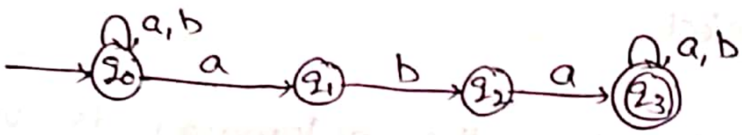
$\delta : Q \times \Sigma \longrightarrow 2^Q$ one or more states (power set of Q)

iv. $q_0 \longrightarrow$ It is initial or start state, $q_0 \in Q$.

v. $F \longrightarrow$ It is set of Final or Accepting states, $F \subseteq Q$

1. Design an NFA for the language of strings with 'aba' as substring.

Soln



Scanned with CamScanner

Epsilon NFA :-

1. The NFA which takes even epsilon (ϵ) as input and gives even transitions is called Epsilon NFA.

2. ϵ-NFA are closely related to regular expressions and useful in proving the equivalence between the classes of languages accepted by finite automata and by regular expressions.

Formal definition of ϵ-NFA :-

NFA with ϵ-moves is the machine

$M = (Q, \Sigma, \delta, q_0, F)$ where.

i. $Q \rightarrow$ Non empty set of finite number of states.

ii. $\Sigma \rightarrow$ Non empty set of finite number of symbols or Finite input alphabet.

iii. $\delta \rightarrow$ state transition function, defined as

$$\delta : Q \times \Sigma \cup \{\epsilon\} \longrightarrow 2^Q$$

iv. $q_0 \rightarrow$ It is initial or start state, $q_0 \in Q$.

v. $F \rightarrow$ It is set of Final or Accepting states, $F \subseteq Q$.

Epsilon closure :-

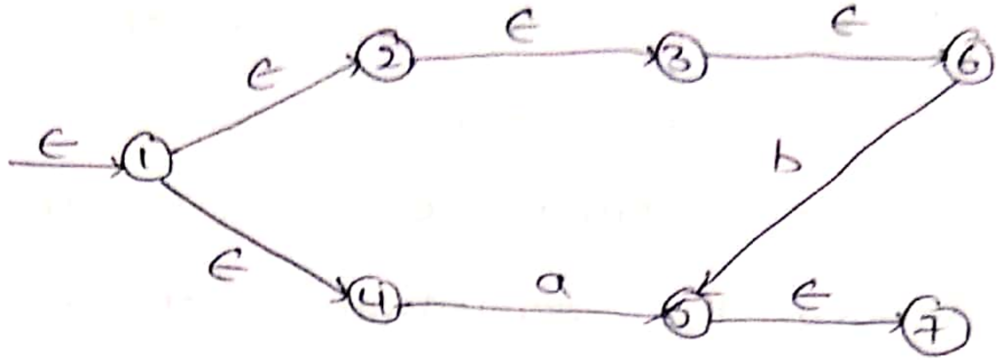
1. All the states which are reachable by just taking ϵ.

ϵ-closure $(q) = q \cup p$

where p is all the states which are reachable from q by taking 'ϵ' as input.

$$\boxed{\begin{aligned} ** \quad \hat{\delta}(q,a) &= \epsilon\text{-closure}(\delta(\hat{\delta}(q,\epsilon),a)) \\ \hat{\delta}(q,\epsilon) &= \epsilon\text{-closure}(q) \end{aligned}}$$

2. Find every state that can be reached from q along any path whose arcs are labelled ϵ.



ϵ-closure(1) = $\{1,2,3,4,6\}$

ϵ-closure(2) = $\{2,3,6\}$

ϵ-closure(3) = $\{3,6\}$

ϵ-closure(4) = $\{4\}$

ϵ-closure(5) = $\{5,7\}$

ϵ-closure(6) = $\{6\}$

ϵ-closure(7) = $\{7\}$

2) Theorem:

Let 'r' be an regular expression then there exists an NFA with ε-transitions which accepts $L(r)$.

Proof:

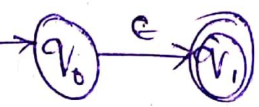
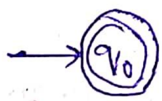
Let us apply mathematical induction on number of operators in the regular expression to show that there exists ε-NFA which accepts $L(r)$.

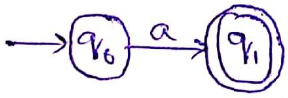
Basis step:

consider the regular expression, with zero operators which has to be one of the following forms.

(i) $\emptyset$

$\{\ \}$ 

(ii) ϵ $\{\epsilon\}$  (or) 

(iii) $a \in \Sigma$ $\{a\}$ 

there exists ε-NFA for each of the above forms.

$\{\emptyset, \epsilon, a \in \Sigma\}$

Hence the basis step is proved.

Step-II Inductive hypothesis

Let us assume that the theorem holds for a regular expression with fewer than 'i' operators. $(i \geq 1)$

Step-III Inductive step

Now let us prove that there exists an ϵ-NFA for a regular expression with i operators. Now here consider three cases.

(1) $r_1 + r_2$

(2) $r_1 \cdot r_2$

(3) r_1^* (or) r_2^*

Case (i): $r_1 + r_2$

Which have 'i' operators since r_1 & r_2 must have fewer than 'i' operators there must exists ϵ-NFA's. Let them be

$M_1 = (Q_1, \Sigma_1, \delta_1, q_1, \{f_1\})$

$M_2 = (Q_2, \Sigma_2, \delta_2, q_2, \{f_2\})$ these are the ϵ-NFA's

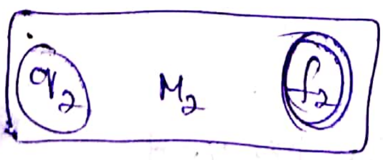
Such that $L(M_1) = L(r_1)$ and $L(M_2) = L(r_2)$

Now let us consider ϵ-NFA for M

Note:

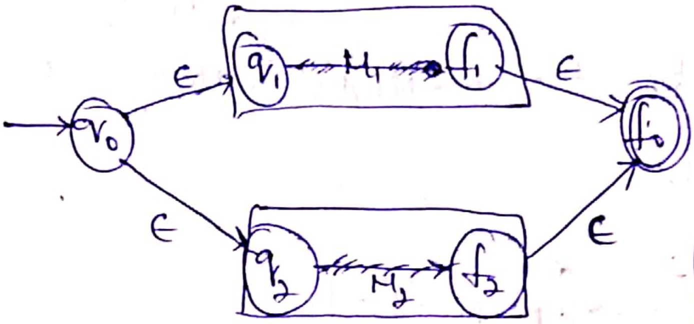
there are no transitions from final state.

M_1 

M_2 

1. $r_1 + r_2$

M:



$\boxed{M = (Q_1 \cup Q_2 \cup \{q_0, f_0\}, \Sigma_1 \cup \Sigma_2 \cup \{\epsilon\}, \delta, q_0, \{f\})}$

where δ is defined as $\boxed{\delta(q, a) = \delta_1^q(q_1, a)}$

$\delta(q, a) = \delta_2(q_2, a)$

→ if $q \in Q_1 = \{f_1\}$.

& $a \in \Sigma_1$

$\delta(q_0, \epsilon) = \{q_1, q_2\}$

→ if $q \in Q_2 - \{f_2\}$

& $a \in \Sigma_2$

→ $\delta(f_1, \epsilon) = \delta(f_2, \epsilon) = f_0$

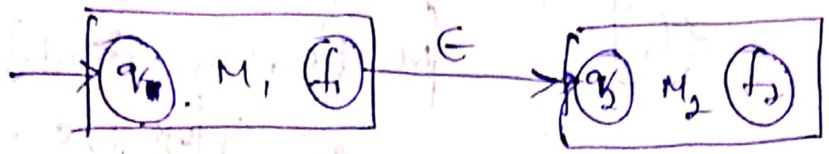
→ Any part in M from q_0 to f_0 must begin by either going from $q_0 \to q_1$ on 'ε' (or) $q_0 \to q_2$ on 'ε'. If it goes from $q_0 \to q_1$ then it follows any path 'x' from q_1 to f_1 and finally goes to 'f_0' by taking 'ε'.

→ lly it begin by going to f_2 then it follows a path 'x' and reaches f_0 by taking 'ε'. this implies that there is a part labelled 'x' in 'M' from $q_0 \to f_0$ if and only if there exists a path labelled 'x' in M_1 or M_2. a path labelled 'x' in M_2 which implies that $L(M) = L(M_1) \cup L(M_2)$

Hence verified.

② $r_1 \cdot r_2$:

Let M_1 & M_2 be the E-NFA's for r_1, r_2. Now let us define an E-NFA for M.



$$M = (Q_1 \cup Q_2, \Sigma_1 \cup \Sigma_2 \cup \{\epsilon\}, \delta, \{f_2\}, q_2)$$

$\delta(q, a) = \delta_1(q_1, a)$ if $q \in Q_1 - \{f_1\}$

$\delta(q, a) = \delta_2(q_2, a)$ if $q \in Q_2 - \{f_2\}$

$\delta(f_1, \epsilon) = q_2$

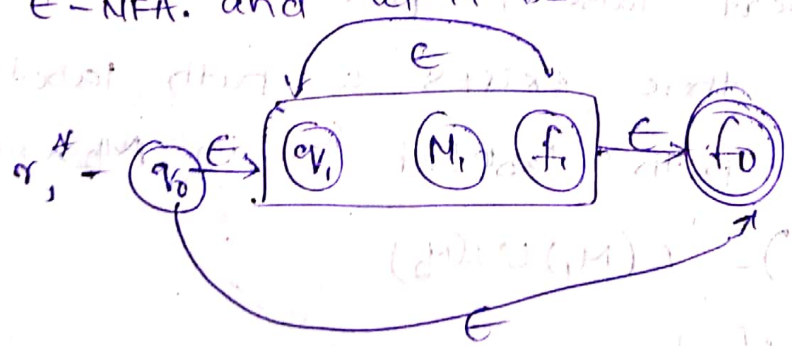
Every path in M from $q_1 \to f_2$ is of the form a path labelled x in M_1 from $q_1 \to f_1$ followed by a path labelled ϵ from $f_1 \to q_2$ followed by path labelled 'y' in M_2 from $q_2 \to f_2$ which implies $\boxed{L(M) = L(M_1) \cdot L(M_2)}$.

$$L = \{xy \mid x \in M_1 \text{ and } y \in M_2\}.$$

Hence verified.

③ r_1^* (or) r_2^*

Since r_1 contains fewer than 'i' operators there exists a E-NFA. and let it be



$M = (Q_1 \cup \{q_0, f_0\}, \Sigma_1 \cup \{\epsilon\}, \delta, q_0, \{f_0\})$

$\delta(q, a) = \delta_1(q_1, a)$, if $q \in Q_1 - \{f_1\}$.

$\delta(q_0, \epsilon) = \{q_1, f_0\}$

$\delta(f_1, \epsilon) = \{f_0, q_1\}$.

Any path in M from $q_0 \rightarrow f_0$ then either directly from $q_0 \rightarrow f_0$ on ϵ or go from $q_0 \rightarrow q_1$ on ϵ followed by x^i where $i \geq 1$ in M_1 and reach f_1 and followed a path from f_1 to f_0 on ϵ which implies that $L(M) = \epsilon + x^i$

$$L(M) = \epsilon + x^i \ (i \geq 1)$$
$$= x^*$$
$$= L(r_1^*)$$

Hence Verified.