Epsilon NFA

EPSILON NFA

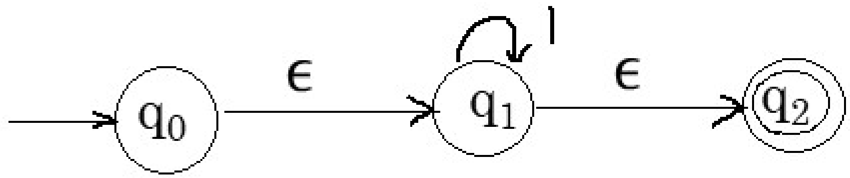
- The NFA which takes even epsilon (ϵ) as input and gives even transitions is called Epsilon NFA.
- NFA with ϵ moves is the machine
  $M = (Q, \Sigma, \delta, q_0, F)$ where

  - Q – Non empty set of finite number of states.
  - Σ – Non empty set of finite number of symbols or Finite input alphabet.
  - δ – State transition function, defined as
    $\delta: Q \times \Sigma \cup \{ \epsilon \} \rightarrow 2^Q$.

  - q_0 – It is initial or start state, $q_0 \in Q$.
  - F – $F \subseteq Q$, It is set of Final or Accepting states.

Epsilon Closure

- All the states which are reachable by just taking ϵ.
- ϵ - closure(q) = q $\cup$ p.
- where p is all the states which are reachable from q by taking ϵ as input.



ϵ-closure(q_0) = {q_0, q_1, q_2}

$$\hat{\delta}(q, a) = \epsilon - \text{closure}(\delta(\hat{\delta}(q, \epsilon), a)$$

$$\hat{\delta}(q, \epsilon) = \epsilon - \text{closure}(q)$$

Conversion of ϵ - NFA to NFA without epsilon moves

- Step1: The states of NFA are same as states of ϵ – NFA.

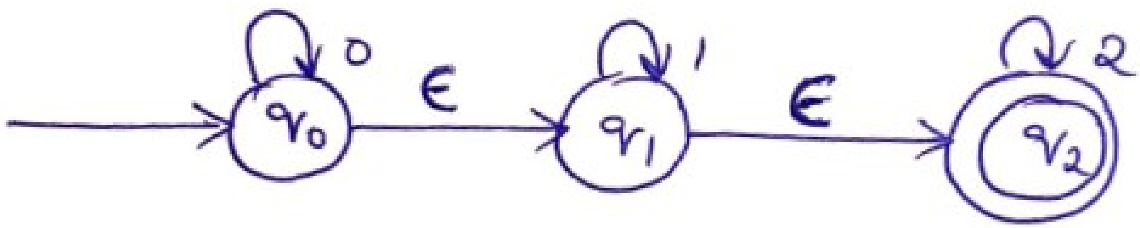
- Step2: The transitions in NFA are found by the following transitions in ϵ – NFA.

  $\delta'(q, a) = \hat{\delta}(q, a)$

- Step3: The final states of NFA are
  if ϵ – closure (q) contains a final state of ϵ – NFA then q is a final state in NFA.

CONVERSION OF ϵ - NFA TO NFA WITHOUT ϵ MOVES

- **Example 1**: Convert the following NFA with ϵ moves to NFA without ϵ moves.



- **Solution**: We find ϵ – closure at each state.
  i.e., ϵ – closure $(q_0) = \{q_0, q_1, q_2\}$
  ϵ – closure $(q_1) = \{q_1, q_2\}$
  ϵ – closure $(q_2) = \{q_2\}$

CONVERSION OF ϵ - NFA TO NFA WITHOUT ϵ MOVES

- Let δ' be transition function in NFA without ϵ moves.

- $\delta'(q_0, 0) = \epsilon - \text{closure}\ (\delta\ (\epsilon - \text{closure}\ (q_0), 0))$
  $= \epsilon - \text{closure}\ (\delta\ (\{q_0, q_1, q_2\}, 0))$
  $= \epsilon - \text{closure}\ (\delta\ (q_0, 0) \cup \delta\ (q_1, 0) \cup \delta\ (q_2, 0))$
  $= \epsilon - \text{closure}\ (q_0 \cup \emptyset \cup \emptyset)$
  $= \epsilon - \text{closure}\ (q_0)$
  $= \{q_0, q_1, q_2\}$

- $\delta'(q_0, 1) = \epsilon - \text{closure}\ (\delta\ (\epsilon - \text{closure}\ (q_0), 1))$
  $= \epsilon - \text{closure}\ (\delta\ (\{q_0, q_1, q_2\}, 1))$
  $= \epsilon - \text{closure}\ (\delta\ (q_0, 1) \cup \delta\ (q_1, 1) \cup \delta\ (q_2, 1))$
  $= \epsilon - \text{closure}\ (\emptyset \cup q_1 \cup \emptyset)$
  $= \epsilon - \text{closure}\ (q_1)$
  $= \{q_1, q_2\}$

Conversion of ϵ - NFA to NFA without ϵ moves

- $\delta'(q_0, 2) = \epsilon - \text{closure}\ (\delta\ (\epsilon - \text{closure}\ (q_0), 2))$

$$= \epsilon - \text{closure}\ (\delta\ (\{q_0, q_1, q_2\}, 2))$$
$$= \epsilon - \text{closure}\ (\delta\ (q_0, 2) \cup \delta\ (q_1, 2) \cup \delta\ (q_2, 2))$$
$$= \epsilon - \text{closure}\ (\emptyset \cup \emptyset \cup q_2)$$
$$= \epsilon - \text{closure}\ (q_2)$$
$$= \{q_2\}$$

CONVERSION OF ϵ - NFA TO NFA WITHOUT ϵ MOVES

- $\delta'(q_1, 0) = \epsilon - \text{closure}\ (\delta\ (\epsilon - \text{closure}\ (q_1), 0))$
  $= \epsilon - \text{closure}\ (\delta\ (\{q_1, q_2\}, 0))$
  $= \epsilon - \text{closure}\ (\delta\ (q_1, 0) \cup\ \delta\ (q_2, 0))$
  $= \epsilon - \text{closure}\ (\emptyset \cup\ \emptyset)$
  $= \emptyset$

- $\delta'(q_1, 1)\ = \epsilon - \text{closure}\ (\delta\ (\epsilon - \text{closure}\ (q_1), 1))$
  $= \epsilon - \text{closure}\ (\delta\ (\{q_1, q_2\}, 1))$
  $= \epsilon - \text{closure}\ (\delta\ (q_1, 1) \cup\ \delta\ (q_2, 1))$
  $= \epsilon - \text{closure}\ (q_1 \cup\ \emptyset)$
  $= \epsilon - \text{closure}\ (q_1)$
  $= \{q_1, q_2\}$

CONVERSION OF ϵ - NFA TO NFA WITHOUT ϵ MOVES

- $\delta'(q_1, 2) = \epsilon - \text{closure}\ (\delta\ (\epsilon - \text{closure}\ (q_1), 2))$

  $= \epsilon - \text{closure}\ (\delta\ (\{q_1, q_2\}, 2))$

  $= \epsilon - \text{closure}\ (\delta\ (q_1, 2) \cup\ \delta\ (q_2, 2))$

  $= \epsilon - \text{closure}\ (\emptyset \cup q_2)$

  $= \epsilon - \text{closure}\ (q_2)$

  $= \{q_2\}$

Conversion of ϵ - NFA to NFA without ϵ moves

- $\delta'(q_2, 0) = \epsilon - \text{closure}\ (\delta\ (\epsilon - \text{closure}\ (q_2), 0))$
  $= \epsilon - \text{closure}\ (\delta\ (\{q_2\}, 0))$
  $= \epsilon - \text{closure}\ (\delta\ (q_2, 0))$
  $= \epsilon - \text{closure}\ (\emptyset)$
  $= \emptyset$

- $\delta'(q_2, 1) = \epsilon - \text{closure}\ (\delta\ (\epsilon - \text{closure}\ (q_2), 1))$
  $= \epsilon - \text{closure}\ (\delta\ (\{q_2\}, 1))$
  $= \epsilon - \text{closure}\ (\delta\ (q_2, 1))$
  $= \epsilon - \text{closure}\ (\emptyset)$
  $= \emptyset$

CONVERSION OF ϵ - NFA TO NFA WITHOUT ϵ MOVES

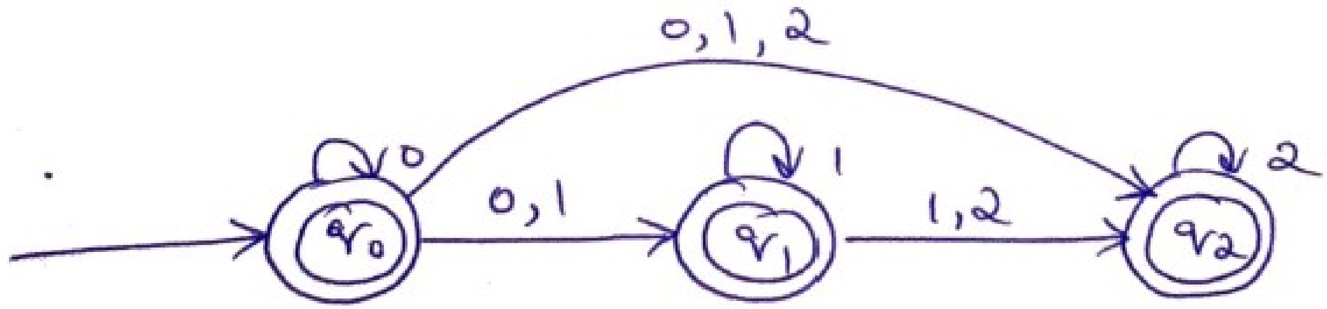
- $\delta'(q_2, 2) = \epsilon - \text{closure}\ (\delta\ (\epsilon - \text{closure}\ (q2), 2))$

  $= \epsilon - \text{closure}\ (\delta\ (\{q_2\}, 2))$

  $= \epsilon - \text{closure}\ (\delta\ (q2, 2))$

  $= \epsilon - \text{closure}\ (q_2)$

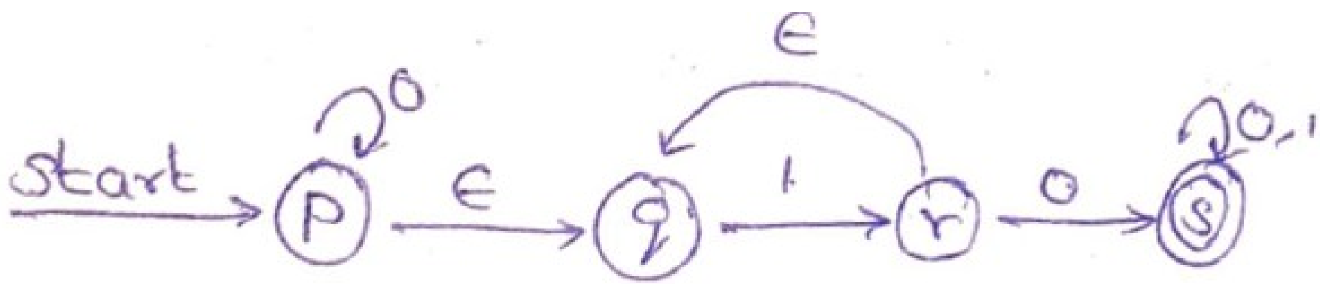
  $= \{q_2\}$



- Note: If ϵ – closure of the state consists of the final state then make that state as a final state.

CONVERSION OF ϵ - NFA TO NFA WITHOUT ϵ MOVES

- **Example 2**: Convert the following NFA with ϵ moves to NFA without ϵ moves.



- **Solution**: We find ϵ – closure at each state.
  i.e., ϵ – closure (p) = {p q}
  ϵ – closure (q) = {q}
  ϵ – closure (r) = {r, q}
  ϵ – closure (s) = {s}

Conversion of ϵ - NFA to NFA without ϵ moves

- Let δ' be transition function in NFA without ϵ moves.

- $$\begin{aligned}\delta'(p, 0) &= \epsilon - \text{closure } (\delta\ (\epsilon - \text{closure } (p), 0)) \\ &= \epsilon - \text{closure } (\delta\ (\{p, q\}, 0)) \\ &= \epsilon - \text{closure } (\delta\ (p, 0) \cup \delta\ (q, 0)) \\ &= \epsilon - \text{closure } (p \cup \emptyset) \\ &= \epsilon - \text{closure } (p) \\ &= \{p, q\}\end{aligned}$$

- $$\begin{aligned}\delta'(p, 1) &= \epsilon - \text{closure } (\delta\ (\epsilon - \text{closure } (p), 1)) \\ &= \epsilon - \text{closure } (\delta\ (\{p, q\}, 1)) \\ &= \epsilon - \text{closure } (\delta\ (p, 1) \cup \delta\ (q, 1)) \\ &= \epsilon - \text{closure } (\emptyset \cup r) \\ &= \epsilon - \text{closure } (r) \\ &= \{r, q\}\end{aligned}$$

CONVERSION OF ϵ - NFA TO NFA WITHOUT ϵ MOVES

- Let δ' be transition function in NFA without ϵ moves.

- $$\begin{aligned} \delta'(q, 0) &= \epsilon - \text{closure } (\delta\ (\epsilon - \text{closure } (q), 0)) \\ &= \epsilon - \text{closure } (\delta\ (\{q\}, 0)) \\ &= \epsilon - \text{closure } (\delta\ (q, 0)) \\ &= \epsilon - \text{closure } (\emptyset) \\ &= \emptyset \end{aligned}$$

- $$\begin{aligned} \delta'(q, 1) &= \epsilon - \text{closure } (\delta\ (\epsilon - \text{closure } (q), 1)) \\ &= \epsilon - \text{closure } (\delta\ (\{q\}, 1)) \\ &= \epsilon - \text{closure } (\delta\ (q, 1)) \\ &= \epsilon - \text{closure } (r) \\ &= \{r, q\} \end{aligned}$$

Conversion of ϵ - NFA to NFA without ϵ moves

- Let δ' be transition function in NFA without ϵ moves.

- $\delta'(r, 0) = \epsilon - \text{closure}\ (\delta\ (\epsilon - \text{closure}\ (r), 0))$
$$= \epsilon - \text{closure}\ (\delta\ (\{r, q\}, 0))$$
$$= \epsilon - \text{closure}\ (\delta\ (r, 0) \cup \delta\ (q, 0))$$
$$= \epsilon - \text{closure}\ (s \cup \varnothing)$$
$$= \epsilon - \text{closure}\ (s)$$
$$= \{s\}$$

- $\delta'(r, 1) = \epsilon - \text{closure}\ (\delta\ (\epsilon - \text{closure}\ (r), 1))$
$$= \epsilon - \text{closure}\ (\delta\ (\{r, q\}, 1))$$
$$= \epsilon - \text{closure}\ (\delta\ (r, 1) \cup \delta\ (q, 1))$$
$$= \epsilon - \text{closure}\ (\varnothing \cup r)$$
$$= \epsilon - \text{closure}\ (r)$$
$$= \{r, q\}$$

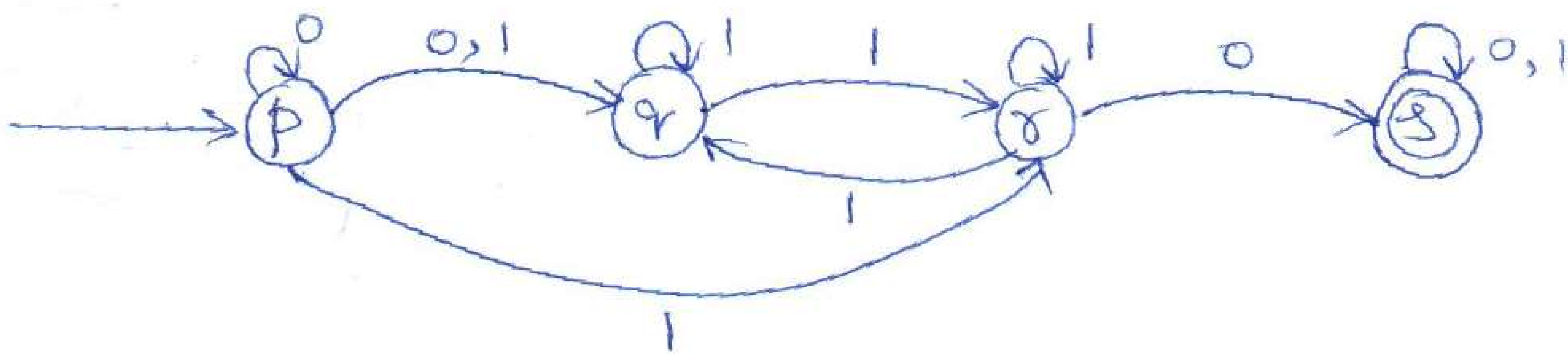
CONVERSION OF ϵ - NFA TO NFA WITHOUT ϵ MOVES

- Let δ' be transition function in NFA without ϵ moves.

- $$\begin{aligned}\delta'(s, 0) &= \epsilon - \text{closure}\ (\delta\ (\epsilon - \text{closure}\ (s), 0)) \\ &= \epsilon - \text{closure}\ (\delta\ (\{s\}, 0)) \\ &= \epsilon - \text{closure}\ (\delta\ (s, 0)) \\ &= \epsilon - \text{closure}\ (s) \\ &= \{s\}\end{aligned}$$

- $$\begin{aligned}\delta'(s, 1) &= \epsilon - \text{closure}\ (\delta\ (\epsilon - \text{closure}\ (s), 1)) \\ &= \epsilon - \text{closure}\ (\delta\ (\{s\}, 1)) \\ &= \epsilon - \text{closure}\ (\delta\ (s, 1)) \\ &= \epsilon - \text{closure}\ (s) \\ &= \{s\}\end{aligned}$$

Conversion of NFA with Epsilon moves to NFA without Epsilon moves



Applications of Finite Automata

- String Processing.
- For designing Lexical Analysis of a compiler.

- For recognizing pattern using regular expression.
- Natural Language Processing.

- Used in text editors.
- For the designing of the combination and sequential circuits using Mealy and Moore Machines.

- Video Games
- CPU Controllers
- Protocol Analysis

- Speech Recognition