Grammar:

A grammar is a mathematical system used to define sentences of the formal languages

A grammar G is a 4 tuple system

$$G = (V, T, P, S)$$

where

V → variables or Non-terminals

T → set of terminals

P → set of productions or re-writing rules

• Production is

S → Start symbols

Eg: Consider

$$G = (\{S, A, B\}, \{a, b\}, P, S)$$

P is given by

$S \to AB$
$S \to BA$
$A \to a$
$A \to aS$
$B \to b$
$B \to bS$

(or)

$S \to AB \mid BA$
$A \to a \mid aS$
$B \to b \mid bS$

} equal no of a's & b's

$\Rightarrow L(G) = \{ab, baba, ba \ldots\}$

Check if below strings belong to above grammar G

a) baab

$S \to BA$
$\to bA$
$\to baS$
$\to baAB$
$\to baaB$
$\to baab$

(ii) bbaa

$S \to BA$
$\to bSA$
$\to bBAA$
$\to bbAA$
$\to bbaA$
$\to bbaa$

(iii) baaabb

$S \to BA$
$\to bA$
$\to baS$
$\to baAB$
$\to baaSB$
$\to baaABB$
$\to baaaBB$
$\to baaabB$
$\to baaabb$

→ If we check we can't derive baaba using above grammar. So we can conclude the grammar is the lang producing strings with equal no of a's & b's

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Eg Consider below grammar

$S \to aB \mid bA$

$A \to a \mid aS \mid bAA$

$B \to b \mid bS \mid aBB$

check if below strings belong to the grammar

a) baaabb

$S \to bA$
$\to baS$
$\to baaB$
$\to baaaBB$
$\to baaabB$
$\to baaabb$

b) bbaaab

$S \to bA$
$\to bbAA$
$\to bbaA$
$\to bbaaS$
$\to bbaaaB$
$\to bbaaab$

c) bbbabaaa

$S \to bA$
$\to bbAA$
$\to bbbAAA$
$\to bbbaAA$
$\to bbbabAAA$
$\to bbbabaAA$
$\to bbbabaaA$
$\to bbbabaaa$

d) bbbabaaa

This string doesn't belong to the grammar

→ Thus the above grammar also represents language consisting of equal no of a's & b's.

Q: The below grammar represents

$S \to aSb \mid \epsilon$

a) $\{a^n b^n \mid n \geq 0\}$

b) $\{a^n b^n \mid n \geq 1\}$

c) $\{a^n b^m \mid m \geq 1, n \geq 0\}$

d) $\{a^n b^m \mid m \geq 0, n \geq 1\}$

Let $L = \{\epsilon, ab, aabb \ldots\}$

Ex: The below grammar represent the language

$S \to aS \mid bS \mid a \mid b$

a) $(a+b)^*$ ✓ b) $(a+b)(a+b)^*$ ✓ c) $(a+b)^*(a+b)$ d) all the above

(2005) Consider below grammar G

(N)

$S \to aA \mid bS \mid b$

$A \to bA \mid aB$

$B \to bB \mid aS \mid a$

Let $N_A(w)$ & $N_B(w)$ denote the no of a's & b's in the string w

The language $L(G) \subseteq (a+b)^*$ generated by G is _____

a) $\{w / N_a(w) > 3N_b(w)\}$

b) $\{w / N_b(w) > 3N_a(w)\}$

c) $\{w / N_a(w) = 3k, k \in \{0,1,2\cdots\}\}$

d) $\{w / N_b(w) = 3k, k \in \{0,1,2,\cdots\}\}$

Seeing options its either b or c

$L = \{b, aaa, abaa, aabba \cdots\}$

↓

Eliminate option (b)

(or)

observing grammar we can see S, A, B as ~~three~~ three states in which S is final state. A every state reading 'b' it remains in same state. Getting 'a' it moves to next state. So we can conclude that above language accepts strings contains no of a's as multiples of 3.

Classification of Grammar (Chomsky Hierarchy)

Type	Alternative name	Condition	Language	Eg	Automata
type 0	unrestricted grammar (UG)	$\alpha \to \beta$ & $\alpha \neq \epsilon$ α, β are arbitrarily long strings	Recursively Enumerable language	$S \to aBbA$ $aA \to bb$ $aB \to b$	Turing Machine (TM)
type 1	Context Sensitive (CSG) Grammar	$\alpha \to \beta$ & $\lvert\beta\rvert \geq \lvert\alpha\rvert$ α, β are arbitrarily long string	Context Sensitive Language	$S \to aBbA$ $aA \to bb$ $aB \to bA$	Linear Bounded (LBA) Automata
type 2	Context Free Grammar	$A \to \alpha$ $A \in V$ $\alpha = (V \cup T)^*$	Context free Language	$S \to AB \mid BA$ $A \to a \mid as$ $B \to b \mid bS$	Push Down Automata (PDA)
type 3	Regular Grammar	$A \to w \mid wB$ (or) $A \to w \mid Bw$ $A, B \in V$ $w \in T^*$	Regular language	$S \to aS \mid bS \mid a \mid b$	Finite (FA) Automata

★ type 3 ⊆ type 2 ⊆ type 1 ⊆ type 0 ★

→ In type 3 grammar
it is of form

$A \to w \mid wB$ (Right linear grammar)

$A \to w \mid Bw$ (Left linear grammar)

Eg: What is the type of below grammar

(N)

$S \to aA \mid bB$

$aA \to bbAb$

$Ba \to bb$

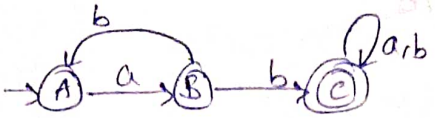
$A \to a$

a) Type 0 b) Type 1 ✓ c) Type 2 d) Type 3

→ For every automata M, we can construct an equivalent grammer G, such that $L(M) = L(G)$

FA to Reg. Grammar

Consider below FA



V = Node labels = {A, B, C}

T = edge labels = {a, b}

S = initial state = A

finding P:

$\delta(A, a) = B$

$A \rightarrow aB$

$\delta(B, b) = A \quad \delta(B, b) = C$

$B \rightarrow bA | bC | b$

$\delta(C, a) = C \quad \delta(C, b) = b$

$C \rightarrow aC | bC | a | b$

i.e., $A \rightarrow aB$

$B \rightarrow bA | bC | b$

$C \rightarrow aC | bC | a | b$

for every

$\delta(A, a) = B$

add production

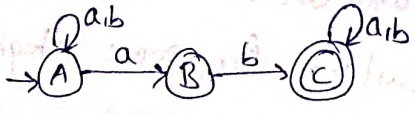
$A \rightarrow aB$, if B is non-final

else add

$A \rightarrow aB | a$

Eg: Find regular grammar corresponding to the regular Exp

$(a+b)^* ab (a+b)^*$



$S \rightarrow aA | aB | bA$

$B \rightarrow bC | b$

$C \rightarrow aC | bC | a | b$

→ The answer of above eg is a right linear grammar.

Writing it same in left linear form

$A \to Aa|Ba|b$

$B \to Cb|b$

$C \to Ca|cb|a|b$

i.e., language is reversed.

> i.e., If G is a right-linear and G' is left linear to G then G is nothing but reverse of G'

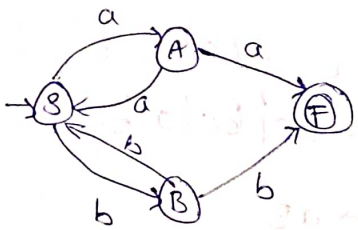
→ Also for every left-linear grammar we can construct an equivalent right linear grammar and vice versa.

Regular Grammar to FA:

Eg: $S \to aA|bB$

(2) $A \to a|aS$

$B \to b|bS$



Pumping lemma for Regular sets:

→ It is a tool used to prove certain languages are not regular

Let l be regular language and $w \in l$

with condition $|w| \geq n$ for some integer n

then $w = xyz$ such that

(i) $y \neq \epsilon$

(ii) $|xy| \leq n$

(iii) $xy^iz \in L \ \forall \ i \geq 0$

> Here n is constant and n denotes no of state in the corresponding FA

Eg: P.T $L=\{a^n b^n / n>0\}$ is not regular

Let $w = a^n b^n$

$xy = a^n = a^m a^{n-m}$, $n \neq m$

$y = a^{n-m}$

$xy^i z$ for $i=0$

$xy^0 z = a^m b^n$

since $m \neq n$

$xy^0 z \notin L$

$\therefore \{a^n b^n / n>0\}$ is not regular language

Note:

We generally apply the following principle to identify whether a given language is regular or not.

i) All finite language are regular

ii) All infinite languages, a regular language must have the following

a) A finite Automata

b) Regular expression

Note:

Consider

$L_1 = \{a^{2i} | i \geq 0\}$

$L_1 = \{\epsilon, aa, aaaa, \ldots\}$

* Here diff blw length of successive strings is const.

∴ Regular

$L_2 = \{a^{i^2} / i>0\}$

$L_2 = \{a, aaaa, aaaaaaaaa \ldots\}$

* Here diff blw length of successive strings is not const

∴ Not regular

It is always possible to write regular expressions for single symbol alphabet languages iff there exist const diff blw successive strings of language

Eg:

Identify whether below languages are regular or not

→ $\{a^{2i} \mid i \geq 0\}$ ⟶ regular

→ $\{a^{2^i} \mid i \geq 0\}$ ⟶ not regular

→ $\{a^p \mid p \text{ is prime}\}$ → not regular

→ $\{a^p \mid p \text{ is non-prime}\}$ → not regular

→ $\{a^{n!} \mid n > 0\}$ ⟶ not regular

☆ → $\{a^{i^3} \mid i < 2\}$ ⟶ Regular (∵ finite)

☆ → $\{a^m b^n \mid m, n \geq 0\}$ → Regular

☆ → $\{a^n b^n \mid n \geq 0\}$

here $\Sigma = \{a, b\}$

we have a relationship b/w $a \& b$ (i.e., $n_a(w) = n_b(w)$) which are members of Σ

∴ It is not regular

→ $\{a^i b^{2j} \mid i = j > 0\}$ → not regular

→ $\{a^i b^j \mid i \neq j\}$ ⟶ not regular

→ $\{ww^R \mid w \in \{a, b\}^*\}$ → not regular

→ $\{wcw^R \mid w \in \{a, b\}^*\}$ →

c is some terminal

· It is non-regular

→ $\{a^i b^{2j} c^{3k} \mid i, j, k > 0\}$ ⟶ regular

$a^+(bb)^+(ccc)^+ \equiv aa^* \, bb(bb)^* \, ccc(ccc)^*$

→ $L = \{a^{2i+1} b^{3j+1} c^{4k+1} d^{5m+1} \mid i, j, k, m > 0\}$ ⟶ regular

Q: Consider following languages

$S_1 = \{0^{2n} \mid n \geq 1\}$ is regular language

$S_2 = \{0^m 1^n 0^{m+1} \mid m \geq 1 \text{ and } n \geq 1\}$ is regular

which of the following statement is correct

a) only S_1 ✓

b) only S_2

c) both S_1 & S_2

d) none

Q: Find similar languages to $L = \{x^n y^n \mid n \geq 1\}$

(i) $E \to xEy \mid xy$

(ii) $xy \mid x^+xyy^+$

(iii) x^+y^+

a) I only ✓ b) I & II c) II, III d) II only

Q: which of the following is regular

(N) a) $\{ww^R \mid w \in \{0,1\}^+\}$

b) $\{ww^Rx \mid x, w \in \{0,1\}^+\} \to L = \{000, 111, 011000 \ldots\}$

c) $\{wxw^R \mid x, w \in \{0,1\}^+\} \to$ reg exp: $[0(0+1)^+0] + [1(0+1)^+1]$

d) $\{xww^R \mid x, w \in \{0,1\}^+\}$

(think) using this reg exp all possible strings of lang in opt (c) can be written

Q: which of the following are regular

$L_1 = \{ww \mid w \in \{a,b\}^*\}$

$L_2 = \{ww^R \mid w \in \{a,b\}^*\}$

$L_3 = \{0^{2^i} \mid i \text{ is integer}\}$ $L_4 = \{0^{i^2} \mid i \text{ is an integer}\}$

a) L_1, L_2 b) L_2, L_3, L_4 c) L_3, L_4 d) L_3 ✓

Right Quotient:

Right quotient $L_1/L_2 = \{x / xy \in L_1 \text{ & } y \in L_2\}$

Let $L_1 = \{rot, carrot, parrot\}$

$L_2 = \{rot\}$

$L_1/L_2 = \{\epsilon, par, car\}$

Eg: $L_1 = 010^*$ $L_2 = 0^*$ $L_1 = \{01, 010, 0100, 01000 \ldots\}$

~~$L_1/L_2 = \{01, \epsilon\}$~~ $L_2 = \{\epsilon, 0, 00, 000, \ldots\}$

$L_1/L_2 = 010^*$

Eg: $L_1 = 0^*10^*$ $L_2 = 0^*1$

$L_1 = \{1, 01, 10, 010, 0010, 00010, 000100 \ldots\}$

$L_2 = \{1, 01, 001, 0001, 00001 \ldots\ldots\}$

$L_1/L_2 = \{\epsilon, 0, 00, 000 \ldots\} = 0^*$

Eg: $L_1 = 0^*10^*$ $L_2 = 10^*1$

$L_1/L_2 = \phi = \{\}$

Because y in L_2 has two 1's but the whole xy itself in L_1 has only one 1.

Q: L_1, L_2 are two languages. Find a true statement.

a) $L_1 \cup L_2$ is regular

b) $L_1 \cap L_2$ is regular

c) $\overline{L_1}$ is regular

d) can't be determined

Q: L_1, L_2 are two regular languages. which is true

I: $L_1 - L_2$ is regular

II: $\Sigma^* - L_1$ is not regular

a) I, II b) I c) II d) both are wrong

(2014) $L_1 = \{w \mid w$ has atleast as many occurances of 110's as 011's, $w \in \{0,1\}^*\}$

(N) $L_2 = \{w \in \{0,1\}^* \mid w$ has atleast as many occurances of 000's as 111's$\}$

a) L_1 is regular, but not L_2

b) L_2 is regular, but not L_1

c) both L_1 & L_2 are regular

d) neither L_1 not L_2 are regular

Ans in Gate Overflow

Q: Which of the following are regular

I: $\{a^n b^{2m} \mid n \geq 0, m \geq 0\}$

II: $\{a^n b^m \mid n = 2m\}$

III: $\{a^n b^m \mid n \neq m\}$

IV: $\{xcy \mid x, y \in \{a,b\}^*\}$

a) I, IV b) I, III c) I only d) IV only

I $\rightarrow a^*(bb)^*$

IV $\rightarrow (a+b)^* c (a+b)^*$

Q: $X_0 \rightarrow 1X_1$

$X_1 \rightarrow 0X_1 \mid 1X_2$

$X_2 \rightarrow 0X_1 \mid \epsilon$

which of the following precisely represent the language of the grammar if X_0 is the start symbol.

a) $10(0^* + (10)^*)1$

b) $10(0^* + (10^*)^*)^* 1$

c) $1(0+10)^* 1$

d) $10(0+10)^* 1 + 110(0+10)^* 1$

solve

Q: which of the following language is/are regular

$L_1 = \{ww^R x \mid w, x \in \{a,b\}^* \text{ and } |w|, |x| > 0, w^R \text{ is reverse of } w\}$

$L_2 = \{a^n b^m \mid m \neq n \text{ & } m, n \geq 0\}$

$L_3 = \{a^p b^q c^r \mid p, q, r \geq 0\}$

a) L_1, L_3 b) L_2 c) L_2, L_3 d) L_3

Problems:

(10)

II. Infinite unions

Consider $L_1 = \{ab\}$

$L_2 = \{aabb\}$

$L_3 = \{aaabbb\}$

:

Consider $L_1 \cup L_2 \cup L_3 \cup \ldots = \{a^n b^n \mid n > 0\}$

∴ not regular

3) Any infinite intersection

Also regular language is not closed under substring operation

(26)

(i) → produces nothing i.e., empty language

(iii) → produces nothing i.e., empty language

Q: which of the following is false

a) Every subset of regular set is regular

b) Intersection of two regular sets is regular

c) Every finite subset of regular set is regular

d) None

Consider $L_1 = \{a^n b^m \mid n, m > 0\}$ ---- regular

$L_2 = \{a^n \mid b^m \mid n, m > 0 \text{ & } n = m\}$ ... non regular

$L_2 \subseteq L_1$ but L_2 is not regular ∴ option (a)

Context Free Language

→ It is language generated by context free grammar.

→ A grammar G is said to be CFG if every production is of the from

$$A \to \alpha$$

where $A \in V$

$\alpha \in (V \cup T)^*$

i.e., every production left side symbol must be single non-terminal but right side one can be combination of variables and terminals.

→ CFGs are very useful to define syntaxes of many programming language such as pascal, FORTRAN hence all these languages are called as CFLs.

→ CFLs as play important role in many fields such as natural language processing, design of programming languages, text editors etc.

→ The following are some examples of CFGs over alphabet $\{a,b\}$

(i) equal no of a's & b's

(ii) unequal no of a's & b's

(iii) palindromes

Q: Design CFG for below languages

(i) $L = \{a^n b^n \mid n>0\}$

$S \to ab \mid asb$

(ii) $L = \{a^n b^{2n} \mid n \geq 0\}$

$S \to aSbb \mid \epsilon$

(iii) $L = \{ww^R \mid w \in \{a,b\}^+$ & w^R is reverse of $w\}$

$S \to aSa \mid bSb \mid \epsilon$ (even length palindromes)

(iv) $L = \{w \mid w \in \{a,b\}^*$ and w is odd length palindrome$\}$

$S \to a \mid b \mid aSa \mid bSb$

(v) $L = \{a^n b^m \mid n > m \geq 0\}$

$S \rightarrow a \mid aSb \mid aS$

(vi) $L = \{w \mid w \in \{a,b\}^+, N_a(w) = N_b(w)\}$

$L = \{ab, ba, aabb, abab, baab, bbaa, abba \ldots\}$

$S \rightarrow ab \mid ba \mid aSb \mid bSa \mid SS$

(or)

$S \rightarrow AB \mid BA$

$A \rightarrow aS \mid a$

$B \rightarrow bS \mid b$

(or)

$S \rightarrow aB \mid bA$

$A \rightarrow a \mid aS \mid bAA$

$B \rightarrow b \mid bS \mid aBB$

(vii) $L = \{a^n b^n \mid n \neq z\}$

$L = \{\epsilon, ab, aaabbb, aaaabbbb \ldots\}$

$S \rightarrow \epsilon \mid ab \mid aAb$

$A \rightarrow aAb \mid aabb$

(or)

$S \rightarrow \epsilon \mid ab \mid aaaA bbb$

$A \rightarrow \epsilon \mid aAb$

(viii) $L = \{a^i b^j \mid i \neq j \geq 0\}$

$L = \{a, b, aa, bb, aab, abb, aaa, bbb \ldots\}$

$S \rightarrow AC \mid CB$

$A \rightarrow aA \mid a$

$B \rightarrow Bb \mid b$

$C \rightarrow acb \mid \epsilon$

Derivations:

A string 'w' can be derived from the grammar in multiple ways but in practice we use 3 methods:

(i) Left most derivation
(ii) Right most derivation
(iii) Parse trees (or) Derivation trees

Left most Derivation: (LMD)

A derivation is said to be LMD iff in each step of the derivation left most variable is replaced.

Right most Derivation (RMD)

A derivation is said to be RMD iff if a right most variable is replaced.

Parse Tree:

When a derivation is represented in the form of a tree, it is called parse tree or derivation tree.

To obtain yield of the parse tree concentrate all leaf node labels from left to right order without repetition of labels.

Eg: $S \to SaS \mid SbS \mid c$

$w = cacbc$

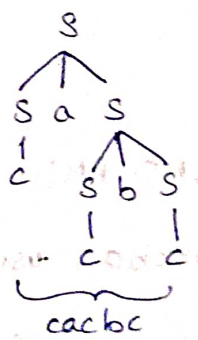
LMD:

$S \to SaS$
$\to caS$
$\to caSbS$
$\to cacbS$
$\to cacbc$

RMD:

$S \to SaS$
$\to SaSbS$
$\to SaSbc$
$\to Sacbc$
$\to cacbc$

Parse tree:



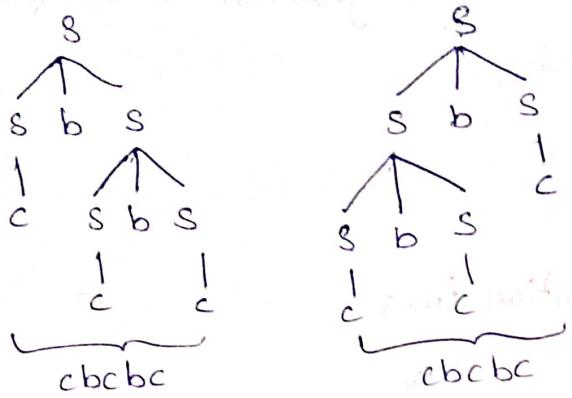
- Parse tree is same for both LMD & RMD

~~Eg: ebcbc~~

Ambiguous Grammar:

A grammar is said to be ambiguous if its language has atleast one string with more than one LMD or RMD.

Eg: Consider string cbcbc for above grammar

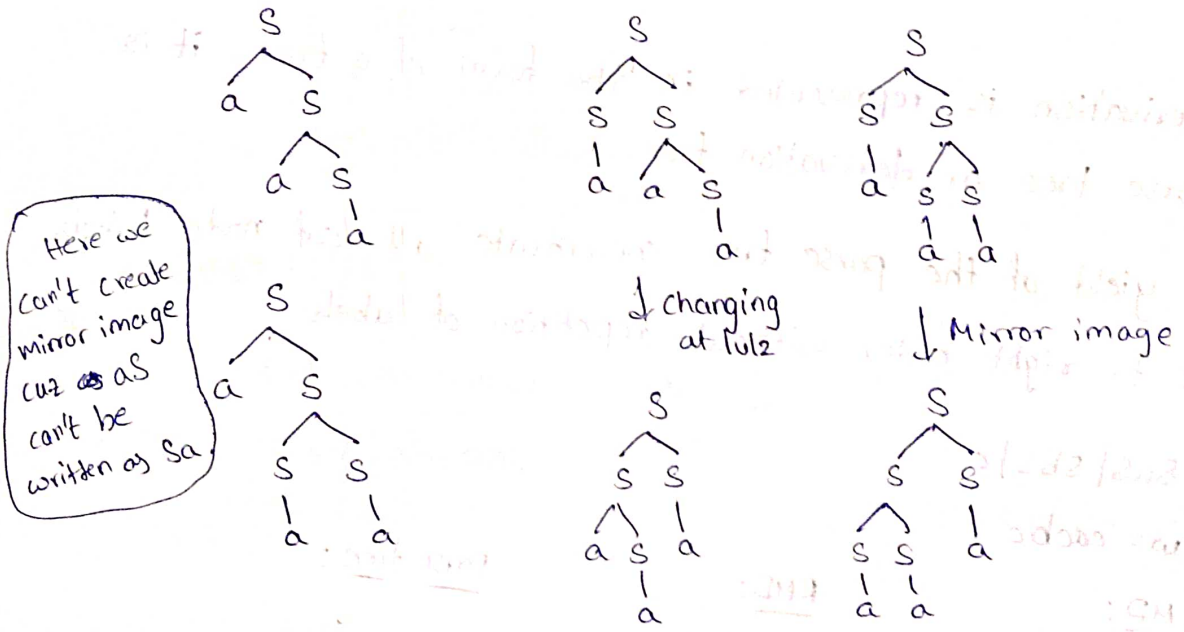


∴ It is ambiguous grammar

Q: Consider grammar

$S \rightarrow aS|a|SS$

How many parse trees can be drawn for w = aaa



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Q: Construct LMD, RMD & parse trees for the string i) w = abbbaa using grammar

$S \rightarrow aB|bA$

$A \rightarrow a|as|bAA$

$B \rightarrow b|bs|aBB$

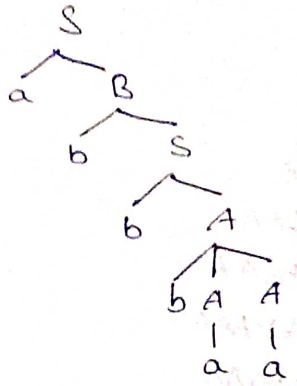
LMD

$S \to aB$
$\to abS$
$\to abbA$
$\to abbbAA$
$\to abbbaA$
$\to abbbaa$

RMD

$S \to aB$
$\to abS$
$\to abbA$
$\to abbbAA$
$\to abbbAa$
$\to abbbaa$

Parse Tree:



(ii) $w = aabbab$

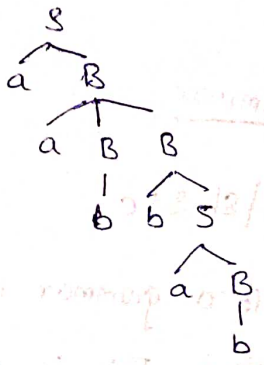
LMD

$S \to aB$
$\to aaBB$
$\to aabB$
$\to aabbS$
$\to aabbaB$
$\to aabbab$

RMD

$S \to aB$
$\to aaBB$
$\to aaBbS$
$\to aaBbaB$
$\to aaBbab$
$\to aabbab$

P.T:



Q: Construct LMD, RMD & parse tree for

$S \to AB \mid BA$
$A \to a \mid aS$
$B \to b \mid bS$

$w = abbaba$

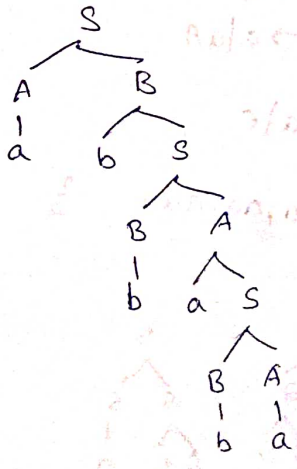
LMD:

$S \to AB$
aB
abS
$abBA$
$abbA$
$abbaS$
$abbaBA$
$abbabA$
$abbaba$

RMD:

$S \to AB$
AbS
$AbBA$
$AbBaS$
$AbBaBA$
$AbBaba$
$AbbaBa$
$Abbaba$
$abbaba$

P.T:

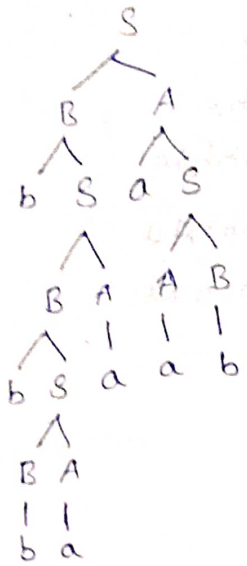


(ii) w = bbbaaaab

LMD:

S → BA
→ bSA
→ bBAA
→ bbSAA
→ bbBAAA
→ bbbaAA
→ bbbaaA
→ bbbaaaS
→ bbbaaaAB
→ bbbaaaaB
→ bbbaaaab

P.T:



Ambiguous Grammar:

Eg: S → SaS | SbS | c

Finding out if a grammar is ambiguous or not is based on trail & error method. This is undecidable problem.

Eg: Check whether below grammar is ambigous or not

(i) S → SS | a | ε

(ii) E → E+E | E-E | id

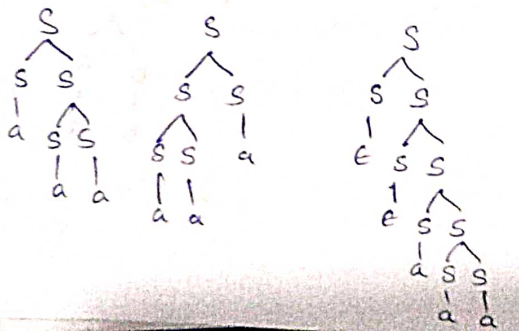
(iii) S → SaS | b

(iv) S → aaB | Ab
B → b
A → a | aA

(i) S → SS | a | ε

L = {ε, a, aa, aaa ...}

for aaa

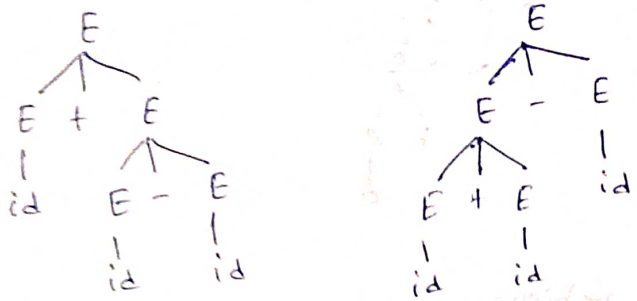


Here we can draw infinite no of parse tree for one string because S → ε is present

(ii) $E \to E+E \mid E-E \mid id$

$L = \{ id, id+id, id-id, id+id-id \cdots \}$

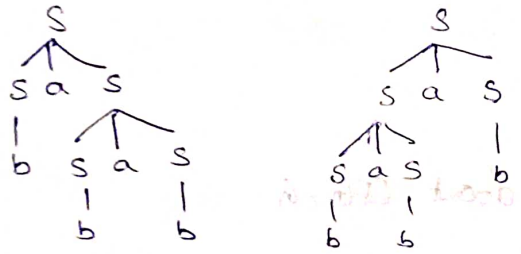
for id+id-id



(iii) $S \to SaS \mid b$

$L = \{ b, bab, babab, bababab \cdots \}$

for babab



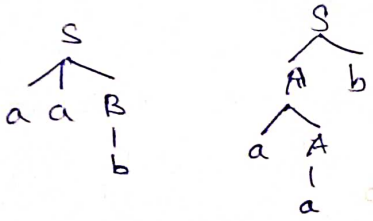
(iv) $S \to aaB \mid Ab$

$B \to b$

$A \to a \mid aA$

$L = \{ ab, aab, aaab, aaaab \cdots \}$

for aab



Q: Check if below grammar is ambiguous

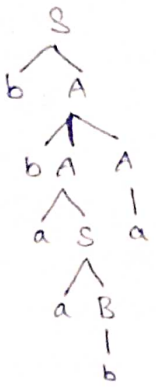
(2)

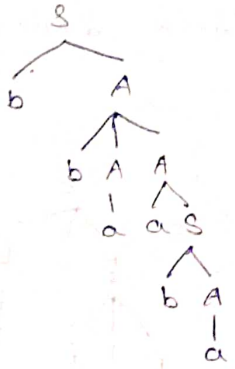
$S \to aB \mid bA$

$A \to a \mid aS \mid bAA$

$B \to b \mid bS \mid aBB$

Consider string ~~bbaaaba~~ bbaaba





∴ The grammar is ambiguous

Note:

In most of the cases (not all) if there is a production in which two non-terminals appear consecutively, then the grammar is most probably an ambiguous grammar

Eg: $T \rightarrow ABB$

Q: Consider following statements about CFG, G

$S \rightarrow SS \mid ab \mid ba \mid \epsilon$

I: G is ambiguous

II: G produces all strings with equal no of a's & b's

a) I only ✓ b) II only c) I & II d) none

Consider string ababab

$S \rightarrow SS$
$\rightarrow abS$
$\rightarrow abSS$
$\rightarrow ababab$

$S \rightarrow SS$
$\rightarrow Sab$
$\rightarrow SSab$
$\rightarrow abSab$
$\rightarrow ababab$

All strings produced by G contains equal no of a's & b's

But G does not produce all strings which contain equal no of a's & b's

i.e, aabb is not produced by G

∴ I only

Q: which of the following grammar generates

$L = \{a^i b^j \mid i \neq j\}$

a) $S \to AC \mid CB$
$A \to aA \mid \epsilon$
$B \to Bb \mid \epsilon$
$C \to aCb \mid a \mid b$

b) $S \to aS \mid Sb \mid a \mid b$

c) $S \to AC \mid CB$
$A \to aA \mid \epsilon$
$B \to Bb \mid \epsilon$
$C \to aCb \mid \epsilon$

d) $S \to AC \mid CB$
$A \to aA \mid a$
$B \to Bb \mid b$
$C \to aCb \mid \epsilon$

a) generates aabb

$S \to AC \to aaCb \to aabb$

b) generates ab

$S \to aS \to ab$

c) generates ab

$S \to AC \to \epsilon C \to acb \to ab$

d) because A & B does not produce ϵ

Q: In the correct grammar of above question what is the length of derivation (no of steps starting forom S) to generate the string $a^l b^m$ with $l \neq m$.

a) $\max(l,m)+2$

b) $l+m+2$

c) $l+m+3$

d) $\max(l,m)+3$

Consider aabbbb

$S \to CB$
$\to aCbB$
$\to aaCbbB$
$\to aaabbB$
$\to aaabbb$

5 steps i.e, $\max(2,3)+2$
$3+2=5$

$\therefore$ opt (a)

(2017) Identify the language generated by the grammar where S is the start symbol

$S \to XY$

$X \to aX \mid a$

$Y \to aYb \mid \epsilon$

a) $\{a^m b^n \mid m \geq n, n > 0\}$

b) $\{a^m b^n \mid m \geq n, n \geq 0\}$

c) $\{a^m b^n \mid m > n, n \geq 0\}$

d) $\{a^m b^n \mid m > n, n > 0\}$

XY

↑ atleast one a

$a^n b^n \quad n \geq 0$

$\Rightarrow a^m b^n$

$\Rightarrow a^m a^n b^n, \quad n \geq 0, m > 0$

$a^x b^y, \quad x > y, y \geq 0$

★

Q: Consider the following CFG

G_1: $S \to aS \mid B$

$B \to b \mid bB$

G_2: $S \to aA \mid bB$

$A \to aA \mid B \mid \epsilon$

$B \to bB \mid \epsilon$

which of the following languages generated by G_1 & G_2 respectively

a) $\{a^m b^n \mid m > 0$ (or) $n > 0\}$

$\{a^m b^n \mid m > 0$ & $n > 0\}$

b) $\{a^m b^n \mid m > 0$ & $n > 0\}$

$\{a^m b^n \mid m > 0$ (or) $n \geq 0\}$

c) $\{a^m b^n \mid m \geq 0$ (or) $n > 0\}$

$\{a^m b^n \mid m > 0$ and $n > 0\}$

d) $\{a^m b^n \mid m \geq 0$ and $n > 0\}$

$\{a^m b^n \mid m > 0$ (or) $n > 0\}$

G_1: $l = \{ab, b, aab, aabb\} \Rightarrow m \geq 0$ & $n > 0$

G_2: $l = \{a, b, ab, aab, abb \cdots\} \Rightarrow m > 0$ (or) $n > 0$

aaa, bbb

Q: Find the language of the grammar

$S \to AB$

$A \to 0A \mid \epsilon$

$B \to 1B \mid \epsilon$

a) $L = \{0^m 1^n \mid n, m \geq 0\}$

b) $L = \{0^m 1^n \mid n, m \geq 1\}$

c) $L = \{0^m 1^n \mid m > 0 \text{ and } n \geq 0\}$

d) None

Elimination of useless symbols & productions:

G: $S \to AC \mid BA$

$B \to BdD$ (useless cuz it does not produce terminating string)

$A \to a$

$C \to b$

$D \to d$ (useless since B is useless)

↓ elimination of useless symbols & productions

G: $S \to AC$

$A \to a$

$C \to b$

$L(G) = \{ab\}$ in both grammars

So in the above grammar we have 3 useless symbols

i.e., B, D, d

steps in elimination:

① Every production in P must produce terminal strings either directly or indirectly (repeated substitution). Otherwise productions are useless

Eg: $S \to BA$, $B \to BdD$ in above grammar

② Thus G is $S \to AC$

$A \to a$

$C \to b$

$D \to d$

③ Every production in the result of rule 3 must produce terminal strings by participating in atleast one sentence derivation. Otherwise they are useless

i.e., D→d in above grammar

∴ The reduced grammar is

R: S→AC
A→a
C→b

V = {S, A, C} T = {a, b}

List of useless symbols are B, D, d

★

Q: A Rank is defined as no of useless symbols in the grammar. Find the rank of the following grammar.

S→AB|BD
B→BD|BC
D→f|BD
A→a
C→c
D→DD

Step 1: Identify productions producing direct terminals

A→a
C→c
D→f

Step 2: Identify productions producing terminals ~~indirectly~~ indirectly

D→DD
A→a
C→c
D→f

Step 3: (rule 2)

B→BD|BC does not produce terminals and thus productions S→AB|BD B→BD|BC D→BD are useless production

useless symbols are S, A, B, C, D, a, c, f ⇒ rank = 8

Q: Find rank of below grammar

$S \to AC \mid BD$
$B \to BD \mid BC$
$D \to f \mid BD$
$A \to a$
$C \to c$
$D \to DD$

Step 1:

$A \to a$
$C \to c$
$D \to f$

Step2:

$A \to a$
$C \to c$
$D \to f$
$D \to DD$
$S \to AC$

Step3:

Useless productions are

$S \to BD$
$B \to BD \mid BC$
$D \to BD$
$D \to f \mid DD$

Thus useless symbols are B, D, f

reduced grammar

$S \to AC$
$A \to a$
$C \to c$

rank = 3

Q: For the grammar given below what is the equivalent CFG without useless symbols

$S \to AB \mid a$
$A \to a$

a) $S \to a$, $A \to a$
b) $S \to a$ ✓
c) $A \to a$
d) $S \to A \mid a$, $A \to a$

All the above options are equivalent to given grammar but removing useless symbols we get (b)

Elimination of Unit & Null Productions:

A production P is said to be null if it is of the form $A \to \epsilon$. For every null production grammar G we can construct an equivalent grammar G_1 without null productions such that

$$L(G_1) = L(G) - \epsilon$$

Eg: $S \to AB$
$A \to a/\epsilon$
$B \to bS/\epsilon$

$\Rightarrow L(G) = \{\epsilon, a, b, ab\}$

Removing null productions

$S \to AB/A/B$
$A \to a$
$B \to bS/b$

$L_1 = \{a, b, ab\}$

i.e., $L_1 = L - \epsilon$

Eg: $S \to AB$
$A \to a/\epsilon$
$B \to bS/\epsilon$

Step1: Identify direct null productions

$A \to \epsilon$
$B \to \epsilon$

Step2: Find productions which would become null by substitution of above production

$S \to AB$
$\to \epsilon B$
$\to \epsilon\epsilon$
$\to \epsilon\epsilon$

$\} S \to AB \to \epsilon$

$\therefore$ Final null variables are $\{S, A, B\}$

Step3 : For each production substitute null values

$S \to AB$

$\Rightarrow S \to AB|A|B$

for $A \to a$

$A \to a$

$B \to bS$

$b \to bS|b$

Find all combination and write it unless it is ϵ

final result is

$S \to AB|A|B$

$A \to a$

$B \to bS|b$

Q: Construct an equivalent grammar without null productions

(N)

$S \to ABa|BA$

$A \to aBa|b|\epsilon$

$B \to AS|b|\epsilon$

Step1 :

$A \to \epsilon$

$B \to \epsilon$

Step2 :

$S \to \epsilon$

$A \to \epsilon$

$B \to \epsilon$

Step3 :

$S \to ABa|Ba|Aa|a|A|B|ABA$

$A \to aBa|aa|b$

$B \to AS|S|A|b$

Q: Eliminate ϵ-productions from the following grammar

$S \to XY$

$X \to Zb$

$Y \to bW$

$Z \to AB$

$W \to Z$

$A \to aA|bA|\epsilon$

$B \to Ba|Bb|\epsilon$

Step1 :

$A \to \epsilon$

$B \to \epsilon$

Step2 :

$Z \to \epsilon$

$W \to \epsilon$

$A \to \epsilon$

$B \to \epsilon$

Step3 :

$S \to XY$

$X \to Zb \mid b$

$Y \to bW \mid b$

$Z \to AB \mid A \mid B$

$W \to Z$

$A \to aA \mid a \mid bA \mid b$

$B \to Ba \mid a \mid Bb \mid b$

Elimination of unit productions :

A production P is said to be unit iff production is of form

$A \to B$ where $A, B \in V$

Eg : $S \to A$, $A \to B$, $B \to C$, $C \to D$, $D \to d$ } $L(G) = \{d\}$

On removing all unit productions we get $S \to d$

Step1 : $S \to A$, $A \to B$, $B \to C$, $C \to d$

Step2 : $S \to A$, $A \to B$, $B \to d$

Step3 : $S \to A$, $A \to d$

Step4 : $S \to d$

→ For every grammar with unit productions we can produce an equivalent grammar without ~~equivalent~~ unit productions

Q: Eliminate unit & null productions from following grammar

(N)

$S \to AB$

$A \to aB \mid \epsilon$

$B \to b \mid \epsilon$

Eliminate null productions

null variables { S, A, B }

$S \to AB \mid A \mid B$

$A \to aB \mid a$

$B \to b$

> while eliminating null & unit productions first remove null production, then remove unit

Eliminating unit productions

$S \to AB \mid \underbrace{aB}_{A} \mid a \mid \underbrace{b}_{B}$

$A \to aB \mid a$

$B \to b$

→ To construct a reduced grammar for given G apply the following in sequence

(i) Eliminate null productions from G to build a new grammar G_1.

(ii) Eliminate unit productions in G_1, if any to build G_2.

(iii) Eliminate useless symbols in G_2 to build G_3.

(iv) Eliminate useless productions of given G.

Q: Consider grammar G

$S \to AB$

$A \to aAA \mid \epsilon$

$B \to bBB \mid \epsilon$

① Find null variables in the grammar

a) A b) B c) A, B, S ✓ d) A & B

② If G_1 is constructed from G after elimination of ϵ productions

G_1 is given by

$A \to \epsilon$

$B \to \epsilon$

$S \to \epsilon$

$S \to AB \mid A \mid B$

$A \to aAA \mid aA \mid a$

$B \to bBB \mid bB \mid b$

⑤ Eliminate unit productions from ②'s answer

$S \to AB \mid aAA \mid aA \mid a \mid bBB \mid bB \mid b$

$A \to aAA \mid aA \mid a$

$B \to bBB \mid bB \mid b$

☆

Q: Which of the following statements are true?

(i) Every left recursive grammar can be converted into right recursive grammar and vice versa

(ii) Every ϵ-production can be removed from any CFG by suitable transformation

(iii) The language generated by CFG $X \to w$, $X \to wY$ where w is string of terminals and Y is a non-terminal is always regular.

(iv) The derivation trees of string generated by the CFG is called as parse tree.

✓

a) i, ii, iii, iv

b) ii, iii, iv

c) i, iii, iv

d) i, ii, iv

Q: Remove all null productions from following grammar

Find no of productions

$S \to ABa \mid bz$

$z \to AB$

$w \to xz$

$X \to zB$

$Y \to az \mid bx$

$A \to aAb \mid \epsilon$

$B \to bBb \mid \epsilon$

$A \to \epsilon$
$B \to \epsilon$
$Z \to \epsilon$
$X \to \epsilon$
$W \to \epsilon$

null variables : A, B, Z, X, W

Removing ϵ-productions

$S \to ABa \mid Aa \mid Ba \mid a \mid bZ \mid b$
$Z \to AB \mid A \mid B$
$W \to XZ \mid X \mid Z$
$X \to ZB \mid Z \mid B$
$Y \to aZ \mid a \mid bX \mid b$
$A \to aAb \mid ab$
$B \to bBb \mid bb$

(23)

Instead of this long process find null variables and find no of productions directly.

Normal Forms:

Two normal forms are:

(i) Chomsky Normal Form (CNF)
(ii) Griebach Normal Form (GNF)

Chomsky Normal Form:

A Grammar G is said to be in CNF iff every production P is either of the form

$A \to BC$ (or) $A \to a$ where $A, B, C \in V$ and $a \in T$

i.e., right side of every production must have either 2 variables or single terminal

Eg: $S \to AB \mid BA$
$A \to a$
$B \to b$

Eg: Convert the below grammar into its CNF

$S \to Ab \mid Ba$
$A \to a$
$B \to b$

$S \to AX \mid BY$
$X \to b$
$Y \to a$
$A \to a$
$B \to b$

Eg: Convert the below grammar into CNF

$S \to aAB \mid BAb$
$A \to aS \mid b$
$B \to bS \mid a$

Step1: Eliminate unit and null productions if there is any

Step2: Consider the production containing terminal on RHS with variables

$S \to aAB \Rightarrow S \to XAB$, $X \to a$
$S \to BAb \Rightarrow S \to BAY$, $Y \to b$
$A \to aS \Rightarrow A \to XS$
$A \to b \Rightarrow A \to b$
$B \to bS \Rightarrow B \to YS$
$B \to a \Rightarrow B \to a$

$S \to XAB$
$X \to a$
$S \to BAY$
$Y \to b$
$A \to XS$
$A \to b$
$B \to YS$
$B \to a$

Step3: Restrict right side length

$S \to XAB \Rightarrow S \to XZ$, $Z \to AB$, $X \to a$

$S \to BAY \Rightarrow S \to BC$, $C \to AY$, $Y \to b$

$A \to XS$
$A \to b$
$B \to YS \mid a$

The final CNF form is

$S \to XZ \mid BC$
$A \to XS \mid b$
$B \to YS \mid a$
$C \to AY$
$X \to a$
$Z \to AB$

Now consider

$\omega = aaabaa \Rightarrow |\omega| = 6$

$S \to XZ$
$\to aZ$
$\to aAB$
$\to aXSB$
$\to aaSB$
$\to aaXZB$
$\to aaaZB$
$\to aaaABB$
$\to aaabBB$
$\to aaabaB$
$\to aaabaa$

Here no of steps = 11

Consider

$\omega = aba$

$|\omega| = 3$

$S \to XZ$
$\to aZ$
$\to aAB$
$\to abB$
$\to abB$
$\to aba$

Here no of steps = 5

> we can conclude that if length of string is n, and the grammar is in CNF then length of derivation is $2n-1$

> If the grammar is in CNF, then parse tree of its every string is a binary tree

Note:

full binary tree: 2 (or) 0 children

Complete binary tree: node are filled from left to right level by level.

perfect binary tree: full + complete

> The time complexity for derivation of string is in the order of n if G is in CNF

→ Convert the following grammar into CNF

$S \to aBAb \mid bb$

$A \to aS \mid b$

$B \to b$

Step1 : Remove unit & null productions

Step2 : $S \to XBAY \mid YY$

$A \to XS \mid b$

$B \to b$

$X \to a$

$Y \to b$

Step3 :

$S \to XBAY \Rightarrow S \to XM$

$M \to BN$

$N \to AY$

∴ CNF is

$S \to XM \mid YY$

$A \to XS \mid b$

$B \to b$

$M \to BN$

$N \to AY$

$X \to a$

$Y \to b$

Griebach Normal form (GNF):

A grammar G is said to be in GNF iff every production in P of G is of the form

$A \to a\alpha$

where

$a \in T$

$\alpha \in V^*$

i.e., Right side of every production must have terminal followed by any no of variables

Eg:

$S \to aAB$

$A \to aS \mid b$

$B \to bS \mid a$

Consider

$w = aba \Rightarrow |w| = 3$

$S \to aAB$
$\to abB$
$\to aba$

no of steps = 3

Consider

$w = aaabaa \Rightarrow |w| = 6$

$S \to aAB$
$\to aaSB$
$\to aaaABB$
$\to aaabBB$
$\to aaabaB$
$\to aaabaa$

no of steps = 6

i.e., If length of the string is 'n', no of derivation steps is n if grammar is in GNF
All derivations with GNF are LMD
Time complexity for derivation is of the order of n

Eg: Convert below grammar into GNF

$S \to ABC$
$A \to aBS \mid bB \mid a$
$B \to a$
$C \to d$

Substitute A in S productions

$S \to aBSBC \mid bBBC \mid aBC$
$A \to aBS \mid bB \mid a$
$B \to a$
$C \to d$

Q: Match the following List1 with List2

List-I

E: Checking that identifiers are declared before their use

F: No of formal parameters in the declaration of a function agree with no of actual parameter in use of the function

List2

P: $L = \{a^n b^m c^n d^m \mid n \geq 1;\ m \geq 1\}$

Q: $X \to Xbx \mid xcx \mid dxf \mid g$

G: Arithematic expression with matched pairs of paralysis

R: $L = \{wcw \mid w \in \{a,b\}^*\}$

S: $X \to bxb \mid cxc \mid \epsilon$

H: palindromes

	E	F	G	H
a)	P	R	Q	S
b)	R	P	S	Q
c)	R	P	Q	S
d)	P	R	S	Q

Q: $X \to \overset{E+E}{xbx} \mid \overset{E-E}{xcx} \mid \overset{(E)}{dxf} \mid \overset{id}{g}$

∴ Arithematic expression i.e., G

S: $X \to bxb \mid cxc \mid \epsilon$

palindrome i.e., H

R: $L = \{wcw \mid w \in \{a,b\}^*\}$

Here a constraint is imposed such that the language consists of exactly one 'c' and w being $\{a,b\}^*$

∴ It can be thought as a name of variable

i.e., E

P: $L = \{a^n b^m c^n d^m \mid n \geq 1; m \geq 1\}$

i.e., F

It is designed using CSG; not CFG

(2002) C language is

a) Context Free language

b) Context Sensitive Language ✓

c) Regular Language

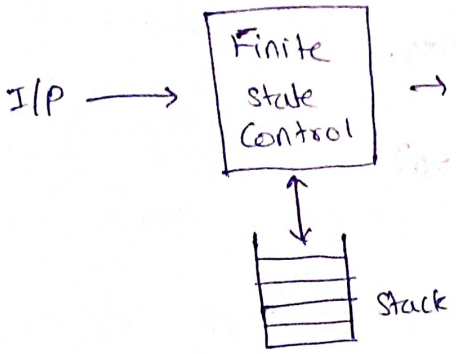
d) Recursively Enumerable Language

☆

→ CYK algo is used to ~~recogine~~ recognise tell whete language is finite or infinite

if thex is cycle → infinte, No cycle → finite, No terminal → empty

Push Down Automata



→ PDA accepts CFG.

→ The extra stack used helps recogonize CFG.

A PDA can be defined as a 7-tuple

$$P = \{Q, \Sigma, \tau, \delta, q_0, z_0, F\}$$

Q → set of state

Σ → set of i/p symbols

τ → set of stack symbols

q_0 → start state

z_0 → start symbol of stack

F → set of accepting states

δ → Transition function.

PUSH, POP operations can be performed on stack. At one time, any no of symbols can be pushed onto stack. But at one time only a single symbol can be popped

It takes a triple $\delta(q, a, X)$ as argument

$q \in Q$, $a \in \Sigma \cup \epsilon$, X is stack symbol i.e, $X \in \tau$

$$\delta: Q \times (\Sigma \cup \epsilon) \times \tau \rightarrow Q \times \tau^*$$

Instantaneous Description (ID):

→ In FA the current state defines status of FA's computation.

→ But in PDA both state and contents of stack are considered

Thus we represent configuration of PDA by a triple (q, w, r)

1. q is the current state
2. w is remaining i/p
3. r is the stack contents

This triple is called ID

For r, we generally show top of stack at left end and bottom at right end

Turnstile notation:

$\vdash$ sign is called turnstile notation and represents one move

$\vdash^*$ sign represents sequence of moves.

Eg: Define PDA for language $\{a^n b^n / n>0\}$

Sol:

$Q = \{q_0, q_1\}$

$\Sigma = \{a, b\}$

$\tau = \{A, Z\}$

$\delta(q_0, a, Z) = \{(q_0, AZ)\}$

$\delta(q_0, a, A) = \{(q_0, AA)\}$

$\delta(q_0, b, A) = \{(q_1, \epsilon)\}$

$\delta(q_1, b, A) = \{(q_1, \epsilon)\}$

$\delta(q_1, \epsilon, Z) = \{(q_1, \epsilon)\}$

Eg: Consider $w = aaabbb$

The below shows sequence of ids while computation of w

$\delta(q_0, aabb, Z)$

$\downarrow$

$\delta(q_0, abb, AZ)$

$\downarrow$

$\delta(q_0, bb, AAZ)$

$\downarrow$

$\delta(q_1, b, AZ)$

$\downarrow$

$\delta(q_1, \epsilon, Z)$

$\downarrow$

$\delta(q_1, \epsilon, \epsilon)$

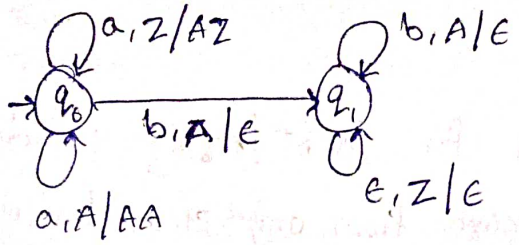
(This acceptance by empty stack)

Graphical Notation for PDA

An edge label marked "$a, X/\alpha$" from state q to P mean

$(q, a, X) \vdash (P, \alpha)$

The graphical representation for above example is



Acceptance of PDA:

A PDA can accept strings in two ways

(i) Acceptance by Final state

$$L(P) = \{ w \mid (q_0, w, z_0) \vdash^* (q_f, \epsilon, \alpha) \}$$

$$q_f \in F \quad , \quad \alpha = \Gamma^*$$

(ii) Acceptance by Empty Stack:

$$L(P) = \{ w \mid (q_0, w, z_0) \vdash^* (q, \epsilon, \epsilon) \}$$

where $q \in Q$

These two methods are equivalent i.e., A language L has a PDA that accepts it by final state if and only if L has a PDA that accepts it by empty stack.

Note:

→ if $(q, x, \alpha) \vdash_P^* (p, y, \beta)$ then

(i) $(q, xw, \alpha) \vdash_P^* (p, yw, \beta)$

(ii) $(q, x, \alpha\gamma) \vdash_P^* (p, y, \beta\gamma)$

(iii) $(q, xw, \alpha\gamma) \vdash_P^* (p, yw, \beta\gamma)$

→ if $(q, xw, \alpha) \vdash_P^* (p, yw, \beta)$ then

$(q, x, \alpha) \vdash_P^* (p, y, \beta)$

Note:

→ The example PDA designed for $\{a^n b^n / n \geq 0\}$ is deterministic PDA i.e., there is only one move from any state for every combination of i/p symbol and stack symbol.

→ The Non-deterministic PDA can have more than one move.

→ It is not always possible to convert NPDA to DPDA.

→ Thus expressive power of NPDA is greater than that of DPDA

→ DPDA can express only a subset of CFGs.

Eg: Design a PDA for

$$L = \{ww^R / w \in \{0,1\}^*\}$$

Here we design NPDA

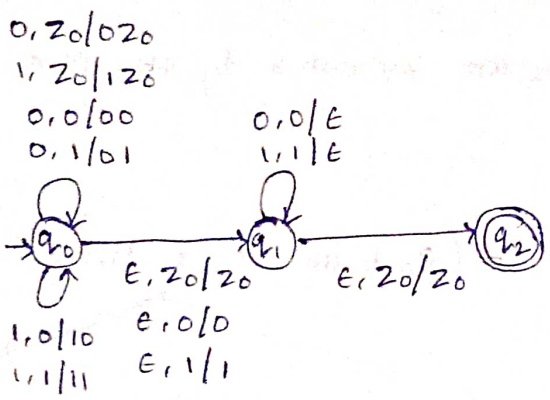
$\delta(q_0, 0, Z_0) = \{(q_0, 0Z_0)\}$
$\delta(q_0, 1, Z_0) = \{(q_0, 1Z_0)\}$
} These rules act for the first i/p read

$\delta(q_0, 0, 0) = \{(q_0, 00)\}$
$\delta(q_0, 0, 1) = \{(q_0, 01)\}$
$\delta(q_0, 1, 0) = \{(q_0, 10)\}$
$\delta(q_0, 1, 1) = \{(q_0, 11)\}$
} Here for every combination of 0,1 symbols of i/p and stack we stay at q_0 assuming we haven't reached the middle of i/p yet.

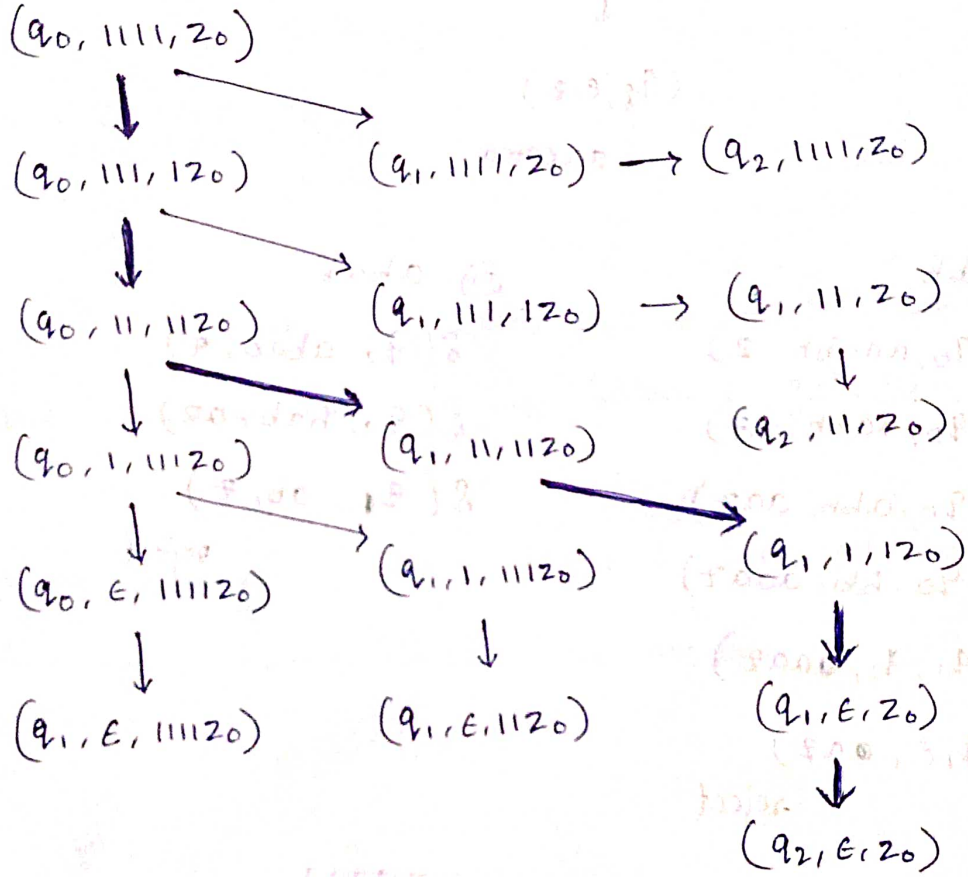
$\delta(q_0, \epsilon, Z_0) = \{(q_1, Z_0)\}$
$\delta(q_0, \epsilon, 0) = \{(q_1, 0)\}$
$\delta(q_0, \epsilon, 1) = \{(q_1, 1)\}$
} These rules allows to go for q_0 to q_1 spontaneously reading nothing. Every time we go to q_1 assuming it might the middle of the i/p.

$\delta(q_1, 0, 0) = \{(q_1, \epsilon)\}$
$\delta(q_1, 1, 1) = \{(q_1, \epsilon)\}$
} Here i/p symbols are matched to stack top and pop symbols of stack.

$\delta(q_1, \epsilon, Z_0) = \{(q_2, Z_0)\}$ Finally if we expose the bottom-of-stack(Z_0) with complete string being read. It means we found i/p of form ww^R.



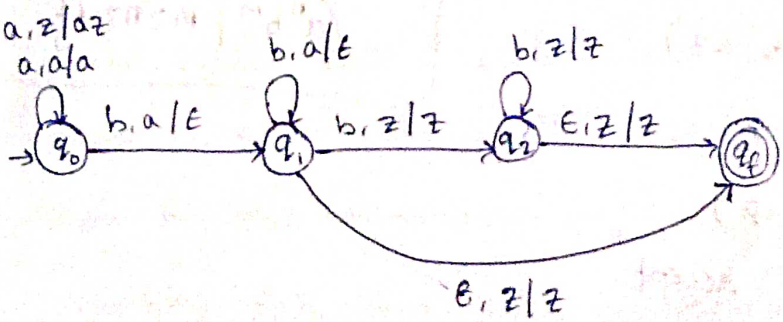
Now consider string $w = 1111$. Find all ID's of PDA that can be reached using w.



In the above thick arrows represent accepting path in which middle of string is guessed perfectly.

02/02/2020

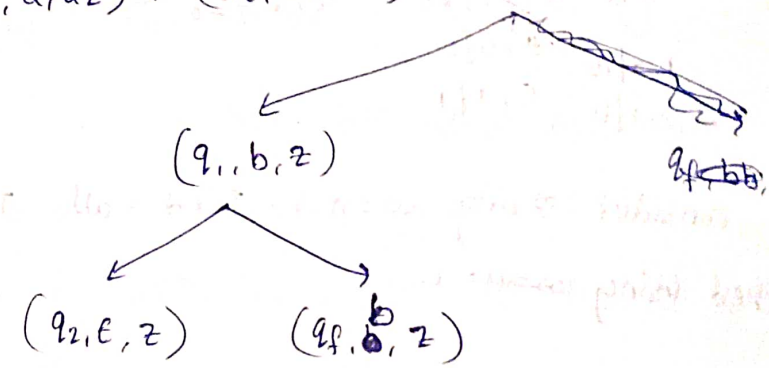
Q: Consider the following PDA



check whether following strings are accepted by the PDA

(i) aabbb

$(q_0, a, z) \vdash (q_0, a, az) \vdash (q_0, b, aaz) \vdash (q_1, b, az)$



(q_f, ϵ, z)

- accepted

(ii) aaabb

$\delta(q_0, aaabb, z)$

$\delta(q_0, aabb, az)$

$\delta(q_0, abb, aaz)$

$\delta(q_0, bb, aaaz)$

$\delta(q_1, b, aaz)$

$\delta(q_1, \epsilon, az)$

reject

(iii) abab

$\delta(q_0, abab, z)$

$\delta(q_0, bab, az)$

$\delta(q_1, ab, z)$.

reject

(iv) aaabbb

$\delta(q_0, aaabbb, z)$

$\delta(q_0, aabbb, az)$

T_*

$\delta(q_0, bbb, aaaz)$

$\delta(q_1, bb, aaz)$

$\delta(q_1, b, az)$

$\delta(q_1, \epsilon, z)$

$\delta(q_f, \epsilon, z)$

accept.

(v) bbbb

$\delta(q_0, bbbb, z)$

reject

Thus the language of PDA is

$\{a^n b^m \mid m \geq n \geq 1\}$

Q: Determine the no of string of length ≤ 3 accepted by previous question's PDA.

ab
abb ∴ 2

Design of PDA:

→ To design a PDA for regular language, we first design FA.

For every transition

$$\delta(q, a) = p \quad \text{in FA}$$

write a transition

$$\delta(q, a, z) = (p, z) \quad \text{in PDA.}$$

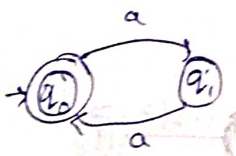
→ To design a PDA for any language we don't have a standard algorithm. But we can use the following guidelines to design a PDA

① If given language is regular design FA and don't disturb the stack

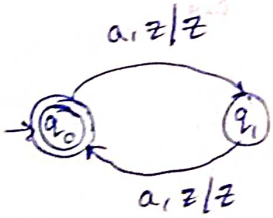
Eg: Design a PDA to recognize the language

$$L = \{a^{2n} \mid n \geq 0\}$$

FA:



PDA:



② For the non-regular but CFL languages try to design the machine with minimum no of states. This can be obtained by

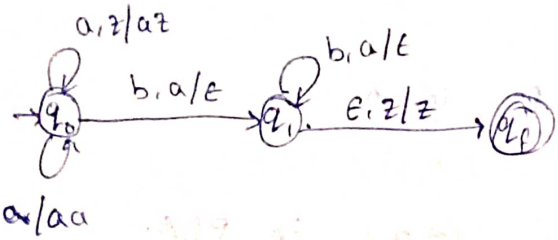
a) Change the state when there is a change in operation

i.e., when there is a change in operation from push to pop or skip to push or skip to pop or pop to skip

The main objective while designing PDA is balancing PUSH & Pop operations. In this process first part of the w, we put it on stack using push operation, the second part of the w will be in the tape for comparision.

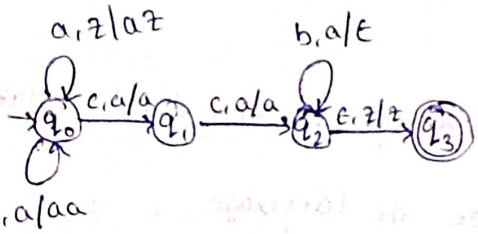
Eg: Design a PDA to recoginize the language

$$L=\{a^n b^n \mid n>0\} \Rightarrow L=\{ab, aabb, aaabbb\ldots\}$$

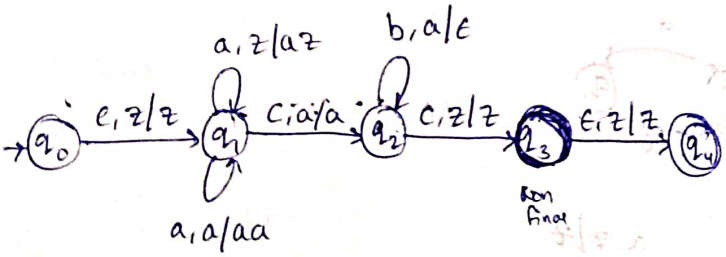


Eg: $L=\{a^n cc b^n \mid n>0\}$

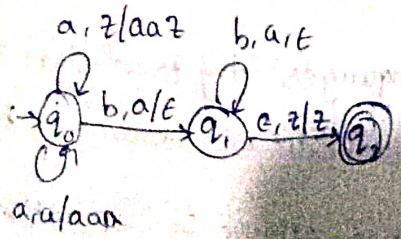
$L=\{accb, aaccbb, aaaccbbb\ldots\}$



Eg: $L=\{ca^n c b^n c \mid n>0\}$

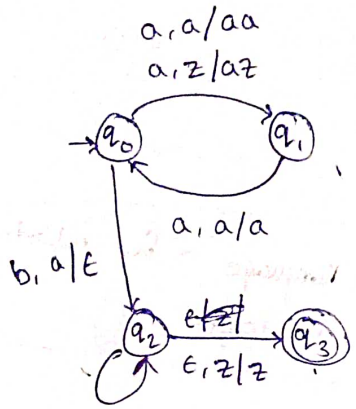


Eg: $L=\{a^n b^{2n} \mid n>0\}$



Eg: $L = \{a^{2n} b^n \mid n > 0\}$

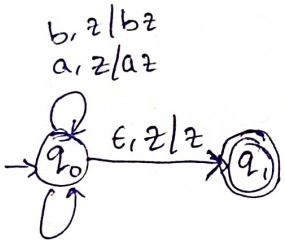
(N)



Eg: $L = \{w \mid w \in \{a,b\}^* \; N_a(w) = N_b(w)\}$

(N)

$L = \{\epsilon, ab, ba, aabb, baab \ldots\}$



a, b/ε
b, a/ε
a, a/aa
b, b/bb

for ~~abbb~~ abba

$\delta(q_0, abba, z)$

$\delta(q_0, bba, az)$

$\delta(q_0, ba, \bullet z)$

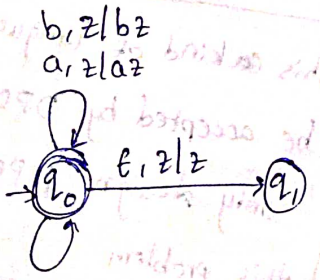
$\delta(q_0, a, bz)$

$\delta(q_0, \epsilon, z)$

$\delta(q_1, \epsilon, z)$

Eg: $L = \{w \mid w \in \{a,b\}^* \; N_a(w) \neq N_b(w)\}$,

(N)



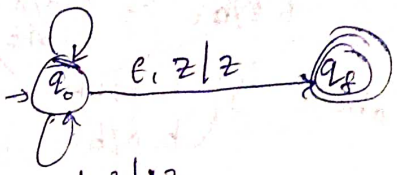
a, b/ε
b, a/ε
a, a/aa
b, b/bb

This technique of making final as non-final and non-final to final does not work for all the problems. It works only for certain problems

Eg: $L = \{w \mid w \in \{a,b\}^* \; N_a(w) = 2N_b(w)\}$

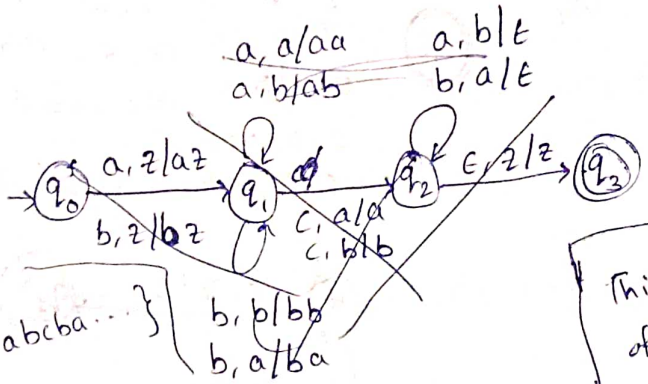
i.e., No of b's is twice the no of a's

(N)



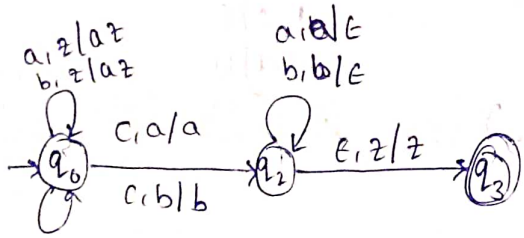
1, z/1z
1, 0/ε
1, 1/11

Eg: $L = \{wcw^R \mid w \in \{a,b\}^+$ and c is terminal$\}$



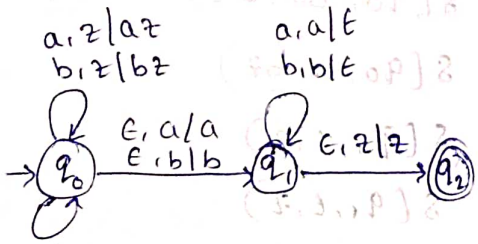
$L = \{aca, bcb, abcba \ldots\}$

This language is for palindrome of odd length



a, a/aa
b, b/bb
a, b/ab
b, a/ba

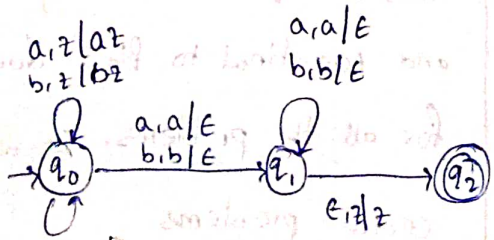
Eg: $L = \{ww^R \mid w \in \{a,b\}^+$ ~~and~~ $\}$



a, a/aa
b, b/bb
a, b/ab
b, a/ba

$L = \{aa, bb, abba, aaaa, baab \ldots\}$

(or)



a, a/aa
b, b/bb
a, b/ab
b, a/ba

This a kind of language can't be accepted by DPDA. we can only design NPDA for this problem

The transitions of above PDA are

$\delta(q_0, a, z) = \{(q_0, az)\}$

$\delta(q_0, b, z) = \{(q_0, bz)\}$

$\delta(q_0, a, a) = \{(q_0, aa), (q_1, \epsilon)\}$

$\delta(q_0, b, b) = \{(q_0, bb), (q_1, \epsilon)\}$

$\delta(q_0, a, b) = \{(q_0, ab)\}$

$\delta(q_0, b, a) = \{(q_0, ba)\}$

$\delta(q_1, a, a) = \{(q_1, \epsilon)\}$

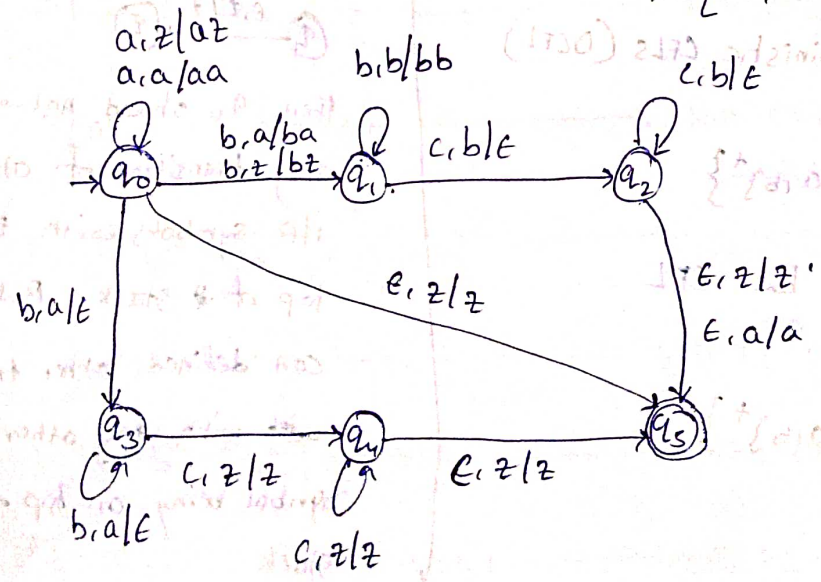
$\delta(q_1, b, b) = \{(q_1, \epsilon)\}$

$\delta(q_1, \epsilon, z) = \{(q_2, z)\}$

→ Languages accepted by deterministic PDA are called DCFL

Eg: $L = \{a^n b^m c^k \mid n=m \text{ (or) } m=k\}$

$L = \{ab, bc, aabb, abc, aabbc, abbbcc, \ldots\}$



Types of PDA:

A PDA can be classified into two types

(i) Deterministic PDA (DPDA)

(ii) Non-Deterministic PDA (NPDA) (or) (NDPDA)

Deterministic PDA:

A PDA $M = (Q, \Sigma, \tau, \delta, q_0, z_0, F)$ is deterministic if

(i) for no q in Q, z in τ and a in Σ or ϵ does $\delta(q, a, z)$ contains more than one element

(ii) for each q in Q, z in τ whenever $\delta(q, \epsilon, z)$ is non-empty then $\delta(q, a, z)$ is should be empty $\forall a \in \Sigma$

explanation

→ The languages accepted by DPDA are called deterministic CFLs (DCFL)

Eg: $L = \{ww^R \mid w \in \{a,b\}^+\}$

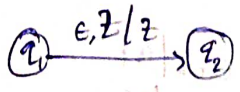
is not DCFL but CFL

$L = \{wcw^R \mid w \in \{a,b\}^+\}$

is DCFL

explation of (ii)

Consider below situation



then q_1 should not defined any transition of other i/p symbols with z being top of stack. But q_1 can defined other transition with any other stack symbol being on top of the stack

Q: Check whether following are DCFL or not

(N)

1. $L=\{a^n b^n c^m \mid n,m\geq 0\}$ ⟶ DCFL
2. $L=\{a^m b^n c^n \mid n,m\geq 0\}$ ⟶ DCFL
3. $L=\{a^n b^m c^{n+m} \mid n,m\geq 0\}$ ⟶ DCFL
4. $L=\{a^n b^{n+2} c^k \mid n,k\geq 0\}$ ⟶ DCFL
5. $L=\{a^{2n+2} b^n \mid n\geq 0\}$ ⟶ DCFL
6. $L=\{a^n b^m c^k \mid n=m \text{ (or) } m=k\}$ ⟶ CFL but not DCFL
7. $L=\{a^i b^j c^k \mid i\neq j \text{ (or) } j\neq k\}$ ⟶ CFL but not DCFL
8. $L=\{w \mid w\in\{a,b\}^+ \& N_a(w)=N_b(w)\}$ ⟶ DCFL
9. $L=\{a^i b^j \mid i=j^2\}$ ⟶ Not CFL
10. $L=\{a^n b^n c^n \mid n>0\}$ ⟶ Not CFL
11. $L=\{ww \mid w\in\{a,b\}^+\}$ ⟶ Not CFL
12. $L=\{a^n b^n c^{2n} \mid n>0\}$ ⟶ Not CFL
13. $L=\{a^n b^m c^m d^n \mid n,m>0\}$ ⟶ DCFL
14. $L=\{a^n b^n c^m d^m \mid n,m>0\}$ ⟶ DCFL
15. $L=\{a^n b^m c^n d^m \mid n,m>0\}$ ⟶ Not CFL
16. $L=\{a^i b^j c^{i+j} d^m \mid i,j,m>0\}$ ⟶ DCFL

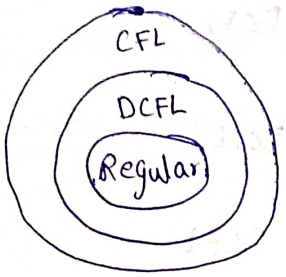
17. The language defined on single symbol alphabet is CFL iff it is regular.

Eg: $L=\{a^{2i} \mid i>0\}$ is regular & DCFL

$L=\{a^{i^2} \mid i>0\}$ is not regular & not CFL

18. $L=\{a^p / p \text{ is prime}\}$ ⟶ Not CFL (∵ not regular)

Note:



CFG to PDA:

→ For every context free grammar G we can construct an equivalent PDA M that accepts every string of G by null stack

Eg: Convert the following CFG into PDA

$S \to aSb$

$S \to ab$

Let $M=(Q, \Sigma, \tau, \delta, q_0, z_0, F)$

$G=(V, T, P, S)$

$V=\{S\}$

$T=\{a,b\}$

$S=\{S\}$

Now

$Q=\{q\}$

$\Sigma=\{T\}=\{a,b\}$

$\tau = VUT=\{S,a,b\}$

$q_0=q$

$z_0=S$

$F=\phi$

Finding δ:

for every

$A \to \alpha \Rightarrow 1.\ \delta(q,\epsilon,A)=(q,\alpha)$

~~2. for every a in Σ~~ 2. for every a in Σ

$\delta(q,a,a)=(q,\epsilon)$

Now

① $S \to aSb$

$S \to ab$

$\delta(q, \epsilon, S) = (q, aSb)$

$\delta(q, \epsilon, S) = (q, ab)$

② $\delta(q, a, a) = (q, \epsilon)$

$\delta(q, b, b) = (q, \epsilon)$

Now for aabb

$\delta(q, aabb, S)$

$\vdash_{\epsilon} \delta(q, aabb, aSb)$

$\vdash_{\epsilon} \delta(q, abb, aSbb)$

$\vdash (q_0, abb, abb)$

$\vdash (q_0, bb, bb)$

$\vdash (q_0, b, b)$

$\vdash (q_0, \epsilon, \epsilon)$

Q: Design PDA for

$L = \{ ww^R \mid w \in \{a,b\}^+ \}$ from grammar

$S \to aSa \mid bSb \mid aa \mid bb$

① $S \to aSa \Rightarrow \delta(q, \epsilon, S) = (q, aSa)$

$S \to bSb \Rightarrow \delta(q, \epsilon, S) = (q, bSb)$

$S \to aa \Rightarrow \delta(q, \epsilon, S) = (q, aa)$

$S \to bb \Rightarrow \delta(q, \epsilon, S) = (q, bb)$

② $\delta(q, a, a) = (q, \epsilon)$

$\delta(q, b, b) = (q, \epsilon)$

for us: abba

$\delta(q, abba, \$)$

$\vdash \delta(q, abba, a\$a)$

$\vdash \delta(q, bba, \$a)$

$\vdash \delta(q, bba, bba)$

$\vdash \delta(q, ba, ba)$

$\vdash \delta(q, a, a)$

$\vdash (q, \epsilon, \epsilon)$

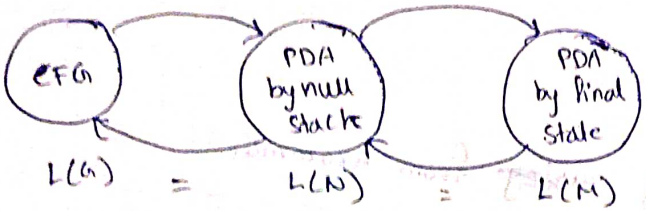
accepted

PDA to CFG:

The procedure for conversion of PDA to CFG is complicated but possible.

Note:

The following inter-conversions are possible



* → The is no conversion procedure from NPDA to DPDA

$L(NPDA) \neq L(DPDA)$

$L(DPDA) \subseteq L(NPDA)$

Closure Properties of CFL:

→ CFLs are closed under union

→ " " " " Concatenation

→ " " " " kleen closure

→ Reversal

→ positive closure

→ Homomorphism

→ Inverse homomorphism

→ Substitution

* CFLs are not closed under

(i) intersection

i.e., if l_1 & l_2 are two CFLs then $l_1 \cap l_2$ need not to be a CFL

(ii) Complementation

i.e., if l_1 is a CFL then $\bar{l_1}$ need not to be a CFL.

Some CFLs are designed only with NPDA. Here complementation technique doesn't work. Thus CFL is not close under Complementation. But DCFL is closed under complemention.

→ when CFLs are intersected with regular sets the result is CFL

if l is CFL & r is regular

$l \cap r$ is CFL

Eg: $l_1 = \{a^n b^m \mid n, m > 0\}$ → regular

$l_2 = \{a^n b^n \mid n > 0\}$ → CFL

$l_1 \cap l_2 \rightarrow l_2$ i.e., CFL

DCFL Properties

→ DCFL's are closed under complementation, but not CFLs.

→ DCFL's are not closed under intersection.

→ when DCFL is intersected with regular set, the result is DCFL.

→ Every language accepted by DPDA is unambigous, but every unambiguous grammar need not to be accepted by DPDA

Eg: $S \rightarrow 0S0 | 1S1 | \epsilon$ is unambiguous and accepted by NPDA only

Q: Consider below languages

$L_1 = \{0^p 1^q 0^r \mid p,q,r \geq 0\}$

$L_2 = \{0^p 1^q 0^r \mid p,q,r \geq 0, p \neq r\}$

which of the below statements is false

a) L_2 is CF

b) $L_1 \cap L_2$ is CF

c) L_1 is CF

d) Complement of L_1 is context free but not regular

$L_1 \rightarrow$ regular

$L_2 \rightarrow$ CFG

$L_1 \cap L_2$ i.e., regular $\cap$ CFG = CFG

L_1 is CF cuz it is regular

d) $L_1 \rightarrow$ regular $\quad \bar{L_1} \rightarrow$ regular

$\therefore$ d

(2010)

Q: Consider the languages

$L_1 = \{0^i 1^j \mid i \neq j\}$

$L_2 = \{0^i 1^j \mid i = j\}$

$L_3 = \{0^i 1^j \mid i = 2j+1\}$

$L_4 = \{0^i 1^j \mid i \neq 2j\}$

which one of the following stmt is true?

a) only L_2 is CF

b) only L_2, L_3 are CF

c) only L_1 & L_3 are CF

d) All are CF

(2014) Consider the following language over the alphabet $\Sigma = \{0, 1, c\}$

(N)

$$L_1 = \{0^n 1^n \mid n > 0\}$$

$$L_2 = \{w c w^R \mid w \in \{0,1\}^*\}$$

$$L_3 = \{w w^R \mid w \in \{0,1\}^*\}$$

Here w^R is the reverse of w

Which of these languages are deterministic CFLs

a) none b) L_1 c) L_1, L_2 d) All

(2015) which of the following languages are CFLs?

(N)

$$L_1 = \{a^m b^n a^n b^m \mid m, n \geq 1\}$$

$$L_2 = \{a^m b^n a^m b^n \mid m, n \geq 1\}$$

$$L_3 = \{a^m b^n \mid m = 2n+1\}$$

a) L_1, L_2 b) L_1, L_3 c) L_2, L_3 d) L_3

(2016) Consider the following languages

$$L_1 = \{a^n b^m c^{n+m} \mid m, n \geq 1\}$$

$$L_2 = \{a^n b^n c^{2n} \mid n \geq 1\}$$

which is true?

a) L_1 & L_2 are CF

b) L_1 is CF, but L_2 is not

c) L_2 is CF, but L_1 is not

d) none are CF

set-I

(2017) Consider the following languages over the alphabet $\Sigma = \{a, b, c\}$

(N)

$$L_1 = \{a^n b^n c^m \mid n, m \geq 0\}$$

$$L_2 = \{a^m b^n c^n \mid n, m \geq 0\}$$

Which of the following are CFs

(i) $L_1 \cup L_2$ (ii) $L_1 \cap L_2$

a) i b) ii c) i, ii d) none

$L_1 \cap L_2 = \{a^n b^n c^n / n \geq 0\}$ i.e., not CFG but CSG

set-II

(2017) Consider the languages given below

$$L_1 = \{a^p / p \text{ is a prime}\}$$

$$L_2 = \{a^n b^m c^{2m} \mid n, m \geq 0\}$$

$$L_3 = \{a^n b^n c^{2n} \mid n \geq 0\}$$

$$L_4 = \{a^n b^n \mid n \geq 1\}$$

Identify correct statement

(i) L_1 is CF but not regular

(ii) L_2 is not CF

(iii) L_3 is not CF but recursive

(iv) L_4 is a deterministic CFL

a) i, ii, iv b) ii, iii c) i, iv d) iii, iv

(2019)

(N)

(2018) Consider the following languages

(N)

I. $\{a^m b^n c^p d^q \mid m+p = n+q, m, n, p, q \geq 0\}$

II. $\{a^m b^n c^p d^q \mid m=n \text{ and } p=q \text{ & } m, n, p, q \geq 0\}$

III. $\{a^m b^n c^p d^q \mid n=n=p \ \& \ p \neq q, \ m,n,p,q \geq 0\}$

IV. $\{a^m b^n c^p d^q \mid mn = p+q\}$

which of the languages above are context free

a) I, IV b) I, II c) II, III d) II, IV

for I. perform m pushes for a^m

now pop for b^n

if $m > n$

m-n no of a's will be left on stack

else if $m < n$

perform pop for m times and push 'b' for n-m times

now push a's for p times

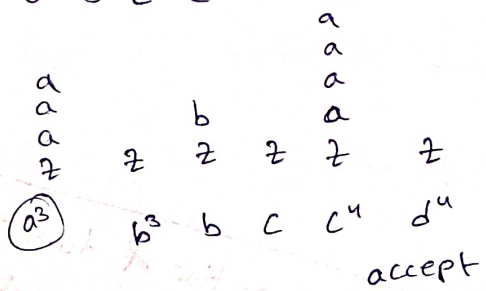
now pop the stack for d^q

pop n-m times for (n-m) in c^p

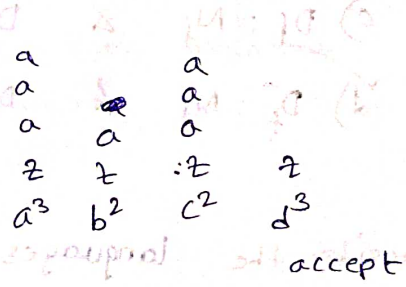
and push p-(n-m) times for c^p

now pop out for d^q

Eg: $a^3 b^4 c^5 d^4$



Eg 2: $a^3 b^2 c^2 d^3$



(2019) which of the following languages over {a,b} is not CF

a) $\{ww^R \mid w \in \{a,b\}^*\}$ b) $\{a^n b^i \mid i \in \{n, 3n, 5n\}, n \geq 0\}$

c) $\{wa^n b^n w^R \mid w \in \{a,b\}^*, n \geq 0\}$ d) $\{wa^n w^R b^n \mid w \in \{a,b\}^*, n \geq 0\}$

a => CFL

b => $\{a^n b^n\} \cup \{a^n b^{3n}\} \cup \{a^n b^{5n}\}$ => CFL .not DCFL

c => $w a^n b^n w^R$ $\left(a^n b^m c^m d^n\right)$

Push Push pop pop

d => $w a^n w^R b^n$

It is kind of in form

$a^n b^m c^n d^n$

(2005) Let N_f & N_p denote the classes of languages accepted by NFA & NPDA respectively. Let D_f & D_p denote the classes of languages accepted by deterministic FA and DPDA respectively. which one of the following is true

a) $D_f \subset N_f$ & $D_p \subset N_p$

b) $D_f \subset N_f$ & $D_p = N_p$

c) $D_f = N_f$ & $D_p = N_p$

d) $D_f = N_f$ & $D_p \subset N_p$

Q: Consider the languages

(N)

$l_1 = \{a^n b^n c^m \mid n, m > 0\}$

$l_2 = \{a^n b^m c^m \mid n, m > 0\}$

here intersection of two DCFLs need not to be CFL. So check it based on the context

which one of the following statements is false?

a) $l_1 \cap l_2$ is CF

b) $l_1 \cup l_2$ is CF

c) l_1, l_2 are CF

d) $l_1 \cap l_2$ is context sensitive language

* $l_1 \cap l_2 = \{a^n b^m c^n \mid n > 0\}$ is CSG

Q: The language

$L: \{0^i 2 1^i / i \geq 0\}$ over the alphabet $\{0,1,2\}$ are ____

a) not recursive

b) is regular language

c) is recursive and deterministic CFL

d) is not a DCFL, but CFL

(2009) Let $L = L_1 \cap L_2$ where L_1 & L_2 are the languages defined as

$$L_1 = \{a^m b^m c a^n b^n \mid m,n \geq 0\}$$

$$L_2 = \{a^i b^j c^k / i,j,k \geq 0\}$$

then L is ____ ?

a) not recursive

b) regular

c) CFL but not regular

d) recursively enumerable but not CFL

Q: Consider following statements about context free grammar

(N)

$$G = \{S \to SS, S \to ab, S \to ba, S \to \epsilon\}$$

(i) G is ambiguous

(ii) G produces all strings with equal no of a's & b's

(iii) G can be accepted by DPDA

Which combination below expresses all the true statements about G.

a) i b) i, iii c) ii & iii d) i, ii, iii

$S \to abS / baS / \epsilon$

(i) → Since $S \to \epsilon$ is present so it is ambiguous

(ii) → aabb is not produced. So (ii) is false

(iii) → DPDA can be designed (Check previous example) (we can convert G into unambiguous)

Q: Consider the languages

$L_1 = \{ww^R \mid w \in \{0,1\}^*\}$

$L_2 = \{w\#w^R \mid w \in \{0,1\}^*, \# \text{ is special symbol}\}$

$L_3 = \{ww \mid w \in \{0,1\}^*\}$

which one of the following is true

a) L_1 is DCFL b) L_2 is DCFL ✓

c) L_3 is CFL, but not DCFL d) L_3 is DCFL

★★★

Q: Let $L_1 = \{0^{n+m}1^n0^m \mid n,m \geq 0\}$

★★

(2) $L_2 = \{0^{n+m}1^{n+m}0^m \mid n,m \geq 0\}$

$L_3 = \{0^{n+m}1^{n+m}0^{n+m} \mid n,m \geq 0\}$

which of the languages are not context-free

a) L_1 only b) L_3 only ✓ c) L_1 & L_2 d) L_2 & L_3

$L_2 \rightarrow 0^{n+m}1^{n+m}0^m$

for 0's we push n+m times

for 1's we pop n+m times

we can't skip 0's because no of zeroes must be less than no of 1's because $m \leq n+m$

$L_3 \rightarrow 0^{n+m}1^{n+m}0^{n+n} \cong 0^n1^n0^n$

set-II

(2017) (2) Let L_1 & L_2 be any two context free languages and R be any regular language. Then which one of the following is correct

I. $L_1 \cup L_2$ is CF

II. $\bar{L}_1$ is CF

III. $L_1 - R$ is CF

IV. $L_1 \cap L_2$ is CF

a) I, II, IV b) I, III ✓ c) II, IV d) I only

I. $L_1 \cup L_2$ is CF ✓

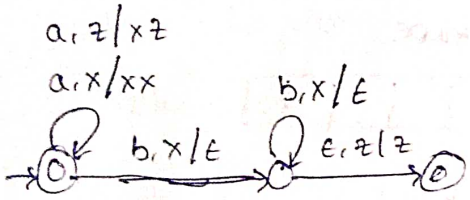
II. $\bar{L}_1$ need not to be CF

III. $L_1 - R = L_1 \cap \bar{R}$ ⇒ CF ✓
(L_1: CF, $\bar{R}$: regular)

IV $L_1 \cap L_2$ need not to be CF

8: Consider the following PDA

(N)



which one of the following is true?

a) $L = \{a^n b^n / n \geq 0\}$ and is not accepted by any DPDA

b) $L = \{a^n / n \geq 0\} \cup \{a^n b^n / n \geq 0\}$ and is not accepted by any DPDA

c) $L = \{a^n / n \geq 0\} \cup \{a^n b^n / n \geq 0\}$ and is DCFL ✓

d) none of the above

a) ⇒ $L = \{a^n b^n / n \geq 0\}$ is ~~correct~~ wrong and also it can be accepted by DPDA

~~$L = \{a^n b^n / n \geq 0\}$ is correct representation and is accepted by some DPDA~~

c) ⇒ $\{a^n / n \geq 0\} \cup \{a^n b^n / n \geq 0\}$ is correct and it is DCFL

Problems:

Pumping Lemma for CFL's

Let L be any infinite CFL. Then there is a constant 'n' depending on L such that if z is in L and $|z| \geq n$ then we may write

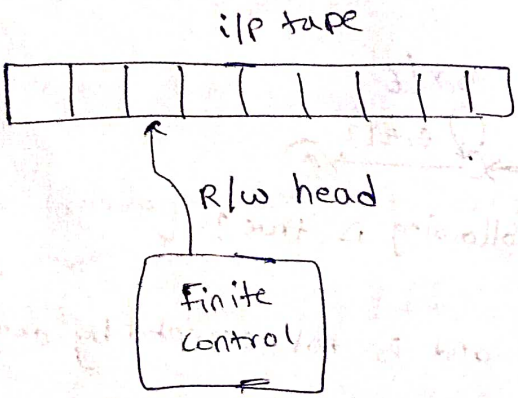
$z = uvwxy$ such that

(i) $|vx| \geq 1$

(ii) $|vwx| \leq n$ and

(iii) for $\forall i \geq 0$, uv^iwx^iy is in L

Turing Machines



→ A turing machine (T.M) is a mathematical model of digital computer. A T.M is used to recogonize Recursively Enumerable Languages.

→ According to Turing if any problem solvable by digital computer can also be solved by T.M

→ The T.M block diagram as shown in the figure consists of i/p tape with infinite length. The head can move in two directions (left or right) with read / write capabilities.

→ A TM is seven tuple system

$M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$ where

Q = finite set of states

Σ = i/p alphabet

Γ = tape alphabet

δ = transition function mapping

$$Q \times \Sigma \to Q \times \Gamma \times \{L, R\}$$

q_0 = initial state

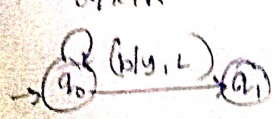
B = blank symbol

F = set of final states

A TM can be represented in multiple ways.

1. State diagram
2. State table

1. State diagram



a/x, R

a is i/p symbol

x is replacement symbol

R is direct of movement of read/write head

2. State table

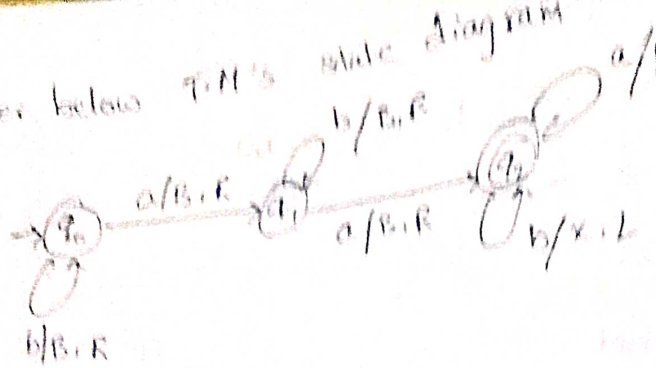
	a
$\to q_0$	x, q_0, R

→ Transition function is written as

$\delta(q_0, a) = (q_0, x, R)$

$\delta(q_0, b) = (q_1, y, L)$

5) Consider below T.M's state diagram



$Q = \{q_0, q_1, q_2\}$

$\Sigma = \{a, b\}$

$\Gamma = \{x, B\}$

$q_0 = q_0$

$F = q_2$

$\delta(q_0, a) = (q_1, B, R)$

$\delta(q_0, b) = (q_0, B, R)$

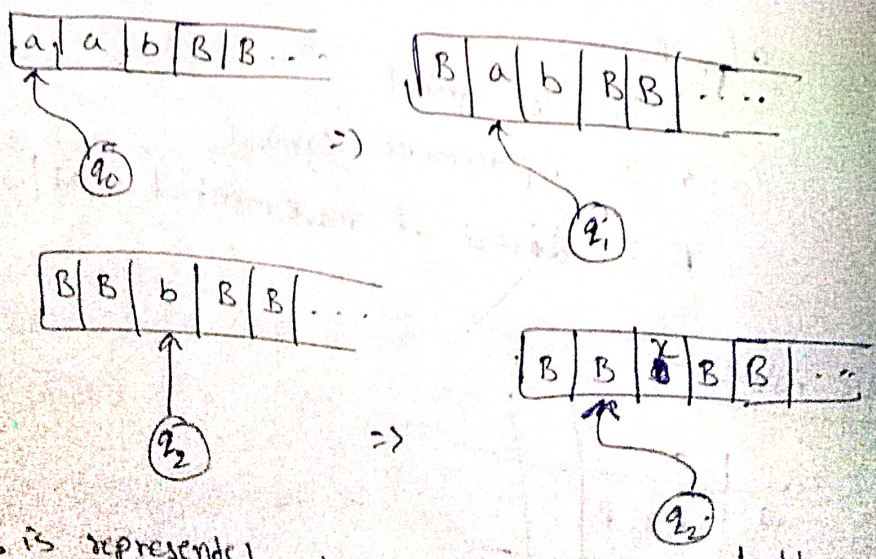
$\delta(q_1, a) = (q_2, B, R)$

$\delta(q_1, b) = (q_1, b, R)$

$\delta(q_2, a) = (q_2, B, R)$

$\delta(q_2, b) = (q_2, bx, L)$

Now consider $w = aab$



- This is represented as

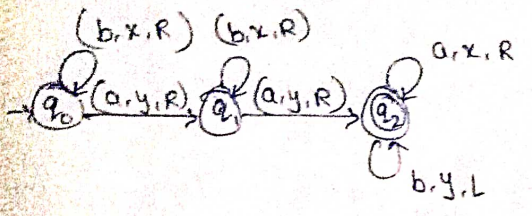
$\vdash q_0 aab$

$\vdash Bq_1 ab$

$\vdash BBq_2 b$ ~~$\vdash BBxq_1 B$ ∴ halt~~ $\vdash Bq_2 Bx$

e9/02/20

Consider the following Turing Machine



Here $Q = \{q_0, q_1, q_2\}$

$\Sigma = \{a, b\}$

$T = \{x, y, B\}$

$q_0 = q_0$

$F = \{q_2\}$

$B = B$

δ: $\delta(q_0, a) = (q_1, y, R)$

$\delta(q_0, b) = (q_0, x, R)$

$\vdots$

a) $w = abab$

$\vdash q_0abab$

$\vdash yq_1bab$

$\vdash yxq_1ab$

$\vdash yxyq_2b$

~~$\vdash yxyyB$~~

$\vdash yxq_2yy$

Halt

accept

b) $w = baab$

$\vdash q_0baab$

$\vdash xq_0aab$

$\vdash xyq_1ab$

$\vdash xyyq_2b$

$\vdash xyq_2yy$

Halt

accept

c) $w = bbbaa$

q_0bbbaa

$\vdash xq_0bbaa$

$\vdash xxq_0baa$

$\vdash xxxq_0aa$

$\vdash xxxyq_1a$

$\vdash xxxyyq_2B$

Halt

accept

Acceptance of a string 'w' by Turing Machine

A string w is said to be accepted by TM iff $q_0w \vdash^* \alpha_1 q_f \alpha_2$ for some $q_f \in F$ and $\alpha_1, \alpha_2 \in T^*$ (i.e., w is completely processed)

* Even if current state is final state and string is not completly processed then it is not accepted yet

Eg: for previous TM check acceptance of baabb

q_0baabb

$\vdash xq_0aabb$

$\vdash xyq_1abb$

$\vdash xyyq_2bb$

$\vdash xyq_2yyb$

halt & reject (because string is not completely processed)

w = aaaab

q_0aaaab

xq_1aaab

yyq_2aab

$yyxq_2ab$

$yyxxq_2b$

$yyxq_2xy$

halt & accept

Eg: Consider the following TM

	0	1	B
$\rightarrow q_0$	$(q_1, 1, R)$	$(q_1, 1, R)$	Halt
q_1^*	$(q_1, 1, R)$	$(q_0, 1, L)$	(q_0, B, L)

w = 001

q_0001

$\vdash 1q_101$

$\vdash 11q_11$

$\vdash 1q_011$

$\vdash 11q_111$

⋮

doesn't Halt and hence it is considered rejected

A string w is generally rejected by TM iff

(i) TM halts at non-final state

(ii) TM halts at final state without procession w completly

(iii) TM does not halt

b) w = 10

$q_0 10$

$1 q_1 0 B$

$11 q_2 B$

$1 q_0 1 B$

$11 q_1 B$

$1 q_0 1 B$

:

doesn't halt

and hence rejected

c) w = 0

$q_0 0$

$\vdash 1 q_1 B$

$\vdash q_0 1 B$

$\vdash 1 q_1 B$

:

doesn't halt

and hence rejected

Design of Turing Machine:

A TM must have min no of state. This can be obtained by following guidelines

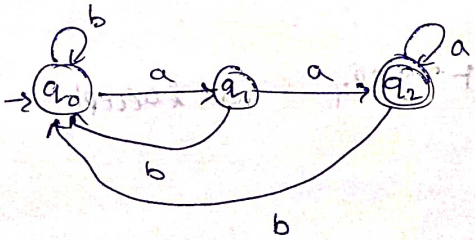
(i) Change the state if there is a change in replacement symbol.

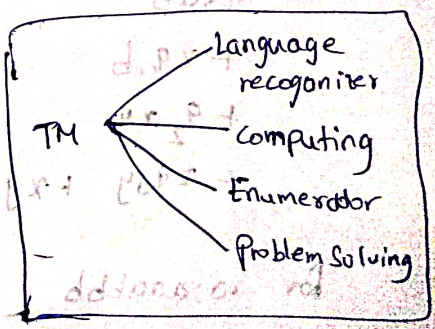
(ii) Change the state if there is a change in direction of the head

(iii) A TM for any particular problem can have n number of solutions but try achieve it with minimum no of states

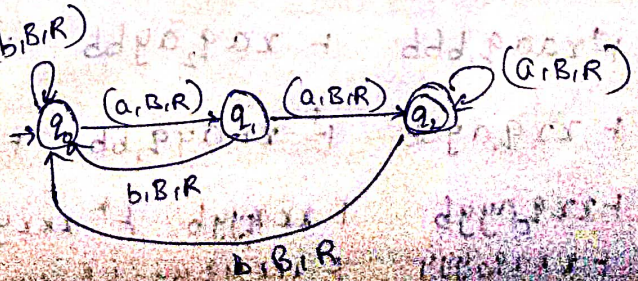
Eg: Design a TM to recogonize the language $L = (a+b)^* aa$

DFA:





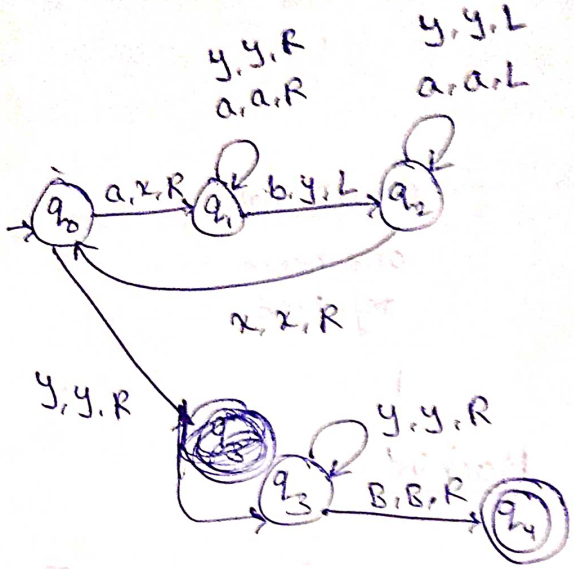
TM:



Eg: Design TM for $L = \{a^n b^n / n > 0\}$

$L = \{ab, aabb, aaabbb \cdots\}$

let $w = aabb$



$q_0 aabb$

$\vdash x q_1 abb$

~~$\vdash x x q_1$~~

$\vdash x a q_1 bb$

$\vdash x q_2 a y b$

$\vdash q_2 x a y b$

$\vdash x q_0 a y b$

$\vdash x x q_1 y b$

$\vdash x x y q_1 b$

$\vdash x x q_2 y y$

$\vdash x ~~q_2~~ x y y$

$\vdash x x q_0 y y$

$\vdash x x y q_3 y \vdash x x y y q_3 B \vdash x x y y B q_4$

halt & accept

Here we can write q_3 as final state also

for $w = ab$

$q_0 ab$

$\vdash x q_1 b$

$\vdash q_2 x y$

$\vdash x q_0 y \vdash x y q_3 B \vdash x y B q_4$ halt & accept

for $w = aaabbb$

$q_0 aaabbb$

$\vdash x q_1 aabbb \vdash^* x a a q_1 bbb \vdash x a q_2 a y bb \vdash^* q_2 x a a y bb$

$\vdash x q_0 a a y bb \vdash x x q_1 a y bb \vdash^* x x a y q_1 bb \vdash x x a q_2 y y b$

$\vdash^* x q_2 x a y y b \vdash x x q_0 a y y b \vdash x x x q_1 y y b \vdash^* x x x y y q_1 b \vdash x x x y y B q_4$

$\vdash^* x x q_2 x y y y \vdash x x x q_0 y y y \vdash x x x y q_3 y y \vdash^* x x y y y q_3 B \vdash x x y y y B q_4$

Eg: Design TM for $L=\{a^n b^{n+1} / n>0\}$ [Think how to design TM]

Eg: Design TM for $L=\{a^n b^n c^n / n>0\}$

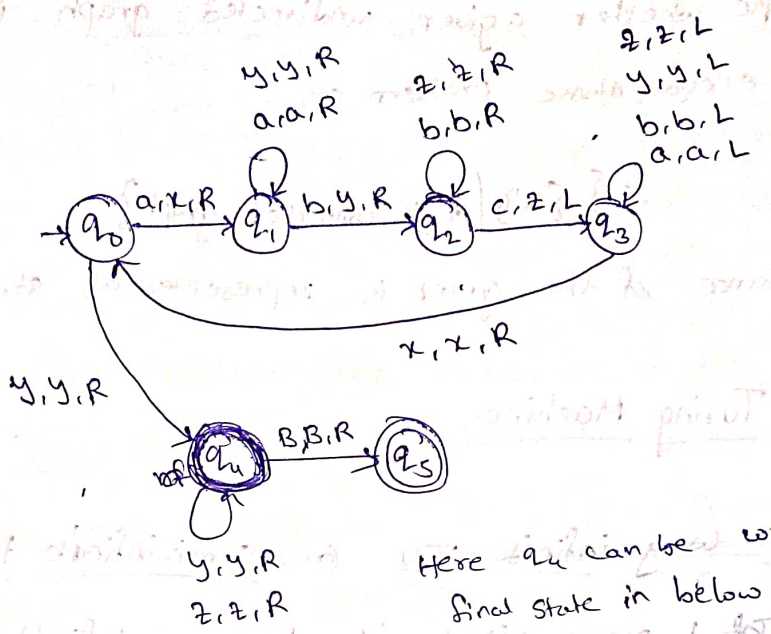
$L=\{abc, aabbcc, aaabbbccc \ldots\}$

$w=aabbcc$

$q_0 aabbcc$

$\vdash^*$

xxq_0yyzz



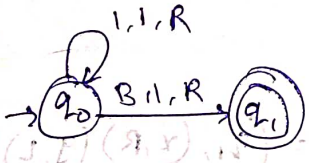
Here q_4 can be written as final state in below way

TM as Computing Machine:

Eg: Design a TM to compute

$f(n)=n+1$. where n is represented in unary form

i/p	o/p
4	5
1111	11111



$q_0 1111$

$\vdash 1q_0 111$

$\vdash 11q_0 11$

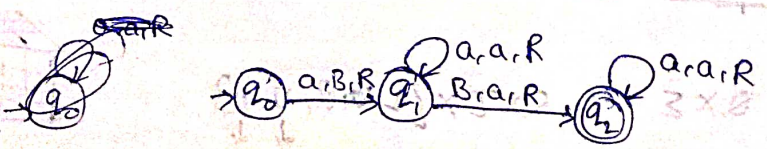
$\vdash 111q_0 1$

$\vdash 1111q_0 B$

$\vdash 11111q_1$

TM as Enumerator

i/p: aaaBaaBB... o/p: aaaaaBB...



$q_0 aaaBaaBB$

$\vdash Bq_1 aaBaaBB$

$\vdash Baaq_1 BaaBB \vdash Baaaq_2 aaB \vdash^* Baaaaaq_2 B$

TM as Problem Solver

Any arbitrary problem can be expressed as language. Any instance of the problem is encoded into string.

For example whether a given undirected graph is connected or not we can encode above problem as

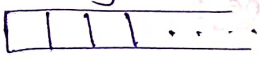
$$L = \{\{G\} / G \text{ is connected graph}\}$$

Every instance of the graph is represented as string

Types of Turing Machines

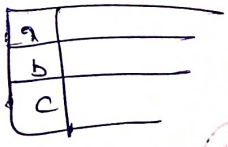
1. ~~Two way infinite TM~~ (or) Semi-infinite tape TM

L recogonited with two way infinite TM $\Leftrightarrow$ L is recogonited with one way infinite tape



2. Multitape Turing Machine

L is accepted by Multitape TM $\Leftrightarrow$ L is accepted by some single tape TM



$$\delta(q_0, a, b, c) = (q_1, (x, R)(y, L)(z, R))$$

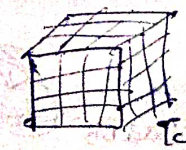
3. Non-deterministic TM:

L is accepted by NDTM $\Leftrightarrow$ L is accepted by DTM

4. Multidimensional Tape TM:

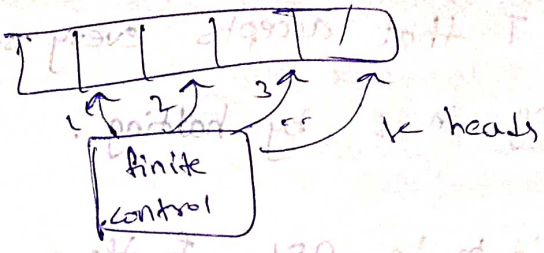
$$\delta: Q \times \Sigma \rightarrow Q \times \Sigma \times \{L, R, U, D\}$$

U: up, D: down



5. Multihead TM:

L is accepted by k head TM <=> L is accepted by one head TM.



6. Universal TM (UTM):

UTM can that can simulate the behaviour of the digital computer i.e., The UTM is a TM that reads description of some other TM along with its i/p. Then it validates whether the other TM can recogonize the given i/p or not.

Turing Theses:

→ Any that can be done on any digital computer can also be done by Turing Machine.

→ None one has an algorithm that can't be solved by a TM

→ Alternative models have been proposed but none of them is more powerful or less powerful than actual TM.

→ The set of all turing machines is although infinite, but it is countable. Similarly set of all regular sets, set of CFGs are also although infinite they are countable. i.e., Infinitely countable.

Recursive & Recursively Enumerable Languages (REL)

→ A language L is said to be Recursive language if there exist as Turing machine T that accepts every string in the L and rejects every string in 'l' by halting.

→ A language L is said to be REL if there exists a TM 'T' that accepts every string the L and rejects every string in L' by halting either by halting or entering into a loop.

i.e., TM halts for acceptance
TM may or may not halt for rejection

Recursive:

Accept(T) = l
~~Accept~~
Reject(T) = l'
loop(T) = ϕ

Recursively Enumerable:

Accept(T) = l
Reject(T) ⊂ l'
loop(T) ⊂ l'
& Reject(T) ∪ loop(T) = l'

→ For any string $w \in \Sigma^+$, the Turing Machine always halts if w is in recursive language

→ Every Recursive language is Recursively Enumerable but reverse need not to be ~~the~~ true.

Closure Properties of Recursive Languages / Recursively Enumerable

Recursive sets are closed under

(i) union
(ii) Concatenation
(iii) Kleen Closure
(iv) Complementation

Recursive languages are not closed under
(i) Homomorphism
(ii) Subset
(iii) Substitution

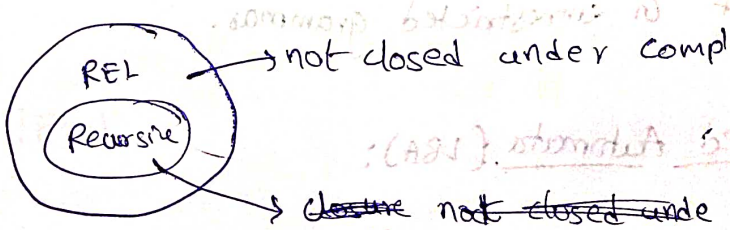
(iv) Positive closure
(vi) ~~Homomorphism~~
(vii) Intersection
(viii) Reversal
(ix) Inverse Homomorphism
(x) Quotient with Regular set

→ Recursively enumerable sets are closed under:

(i) Concatenation
(ii) Union
(iii) Intersection
(iv) Kleen closure
(v) Homomorphism
(vi) Substitution
(vii) Inverse homomorphism
(viii) Reversal
(ix) Quotient with regular set

→ All recursive languages are closed under complementation
i.e., if L is recursive then L' is also recursive

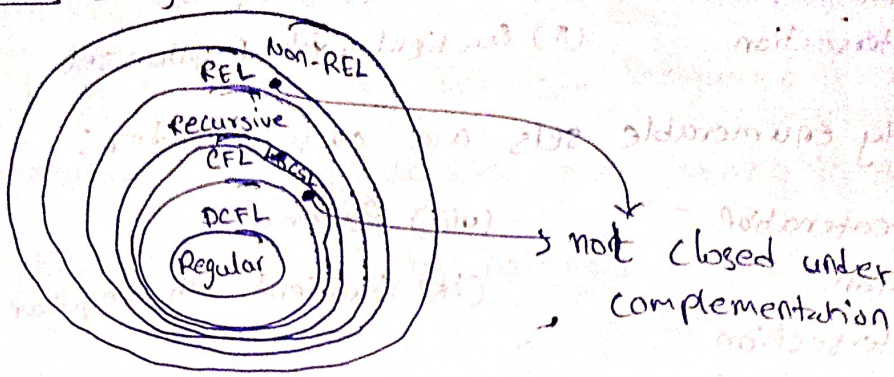
→ Recursively Enumerable languages are not closed under complementation
i.e., If L is recursively Enumerable then L' may or may not be a REL
i.e., If it is given that both L & L' are recursively Enumerable then it is possible only where L is Recursive



It means REL which are recursive are only closed under complementation

→ REL is not closed under difference

Relationship among languages:



* → If L & L' both are REL then L & L' are necessarily Recursive.
* → Complementation of 'CFL but not DCFL' is recursive.
* → The grammar of Turing Machine is Unrestricted Grammar (UG)

Every TM 'T' can be converted in UG 'G' and vice versa such that $L(T) = L(G)$

Note:

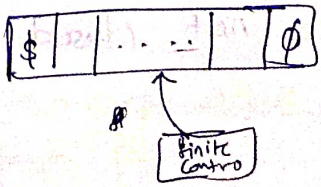
→ If given grammar 'G' is unrestricted grammar there is no specific algorithm to decide whether $L(G)$ is finite (or) infinite (or) empty

→ There is no specific algorithm to decide whether w belongs to $L(G)$ if G unrestricted grammar.

Linear Bounded Automata (LBA):

It is a Restricted model of Turing machine that contains two special symbols \$ (at leftmost of tape) and ¢ (at rightmost of tape) such that

$$\delta(q, ¢) = (q_i, ¢, L)$$
$$\delta(q, \$) = (q_i, \$, R)$$
$$\forall\, q \in Q$$



i.e., the head of finite control can't move any right of ¢ and any left of \$.

The language accepted by LBA are called CSL or RE or REL

Eg: below is CSG for $L=\{a^n b^n c^n / n>0\}$

$S \to abc \mid aAbc$

$Ab \to bA$

$Ac \to Bbcc$

$aB \to aa \mid aaA$

$bB \to Bb$

for $\omega = aabbcc$

$S \to aAbc$

$\to abAc$

$\to abBbcc$

$\to aBbbcc$

$\to aabbcc$

Eg: Design CSG for $L=\{a^n b^m c^n d^m / n,m>0\}$

$L=\{abcd, aabbccdd, \ldots\}$

$S \to aAcD \mid aBcD$

$A \to aAc \mid aBc$

$Bc \to cB$

$Bb \to bB$

$BD \to Ed \mid FDd$

$cE \to Ec$

$bE \to Eb$

$aE \to ab$

$cF \to Fc$

$bF \to Fb$

$aF \to abB$

→ Below languages are few examples of ~~recursive languages~~ CSLs

- $L = \{a^p \mid p \text{ is prime}\}$

$L = \{a^i \mid i > 0\}$

$L = \{a^{i^2} \mid i > 0\}$

$L = \{a^{\frac{n(n+1)}{2}} \mid n > 0\}$

$L = \{0^{n!} \mid n > 0\}$

Set-I

(2015) For any two languages L_1 & L_2 such that L_1 is CFG and L_2 is recursively enumerable but not recursive. Which of the following is/are necessarily true?

I. $\overline{L_1}$ is recursive

II. $\overline{L_2}$ is recursive

III. $\overline{L_1}$ is CF

IV. $\overline{L_1} \cup L_2$ recursively enumerable

a) I only b) III only c) III, IV d) I, IV ✓

$\overline{L_1}$ is recursive $\Rightarrow$ $\overline{L_1}$ is recursively enumerable too

L_2 is REL

$\therefore \overline{L_1} \cup L_2$ is REL ~~cuz~~ REL are closed under union

(2016) Consider

L_1: Regular

L_2: CF

L_3: Recursive

L_4: REL

which of the following is/are true

I. $\overline{L_3} \cup L_4$ is REL

II. $\overline{L_2} \cup L_3$ is Recursive

III. $\overline{L_1} \cap L_2$ is CF

IV $L_1 \cup \overline{L_2}$ is CF

a) I b) I & IV c) I, II, III d) I, IV

I. $\overline{L_3}$ → recursive → REL

$\overline{L_3} \cup L_4$ → REL

II. $\overline{L_2}$ is Recursive

~~to~~ $\overline{L_2} \cup L_3$ is recursive

III. $\overline{L_1}$ → Regular

$\overline{L_1} \cap L_2$ is CF

IV L_1 → regular

$\overline{L_2}$ → recursive

$L_1 \cup \overline{L_2}$ → recursive

Q: Define the languages L_0 & L_1 as follows

(N)

$L_0 = \{ \langle M, w, 0 \rangle \mid M \text{ halts on } w \}$

$L_1 = \{ \langle M, w, 1 \rangle \mid M$ ~~halts~~ doesn't halt on $w \}$,

Here $\langle M, w, i \rangle$ is a triple whose first component M is encoding of Turing machine

w is string

i is a bit

Let $L = L_0 \cup L_1$

which of the following is true

a) L is REL but $\overline{L}$ is not

b) $\overline{L}$ is REL but L is not

c) both L & $\overline{L}$ are recursive

d) neither L, nor $\overline{L}$ is recursively Enumerable

Sol:

$L_0 \rightarrow$ recursive $\rightarrow$ REL

$L_1 \rightarrow$ REL

$L = L_0 \cup L_1$

i.e., $L =$ REL

REL is not closed under complementation

Thus $\bar{L}$ is not REL

Undecidability:

A problem P is said to be decidable if there exist an alogorithm (TM) for infinite instances of P then P is called decidable problem

Post Correspondance Problem:

It is an undecidable problem

	List I	List II
1	01	0
2	111	111
3	01	101

In PCP problem the answer is stating stating sequence of number such that they produce same string from List-I & List-II

Eg: 123

List 1 : 01 111 01 = 0111101

List 2 : 0 111 101 = 0111101

Silly 123123, 2, 22 ....

Decidability overview on Regular Languages:

→ Every question about FA or regular language is decidable

where

Eg: whether given NFA and DFA accept same language or not decidable.

Eg: deciding whether union of two regular languages is CFL or not.

It CFL & decidable

Decidability Overview on CFL & PDA:

Some of the questions on CFLs are decidable, but some are not.

→ Union of two CFLs is CFLs — decidable

→ Intersection of two CFL is CFL.

Intersection of two CFL may or may not be CFL

∴ undecidable

→ Let G be a CFG.

whether $L(G) =$ finite — decidable

$L(G) =$ infinite — decidable

$L(G) = \phi$ — decidable

→ Let G is CFG and w is string

membership: whether $w \in L(G)$ — decidable

→ where G is ambiguous — ~~decidable~~ undecidable

→ whether two PDAs accepts same language — undecidable

*It undecidable for arbitrary CFG G_1 & G_2 whether

→ $L(G_1) \cap L(G_2)$ - is empty

→ $L(G) = \Sigma^*$ (it similar to checking if G is equal to some other G)

→ G is ambiguous

→ G_1 & G_2 accept same language i.e., $L(G_1) = L(G_2)$

→ $L(G_2) \subseteq L(G_1)$

→ $L(G_1) = R$

→ ~~$R \subseteq L(G_1)$~~ $L(G_1) \subseteq R$ (whether given CFG is regular)

→ $\overline{L(G_1)}$ is CFL

→ $L(G_1) \cap L(G_2)$ is CFL

* Let G be a arbitrary CFG it undecidable to say whether $L(G)$ is regular or not.

Decidability overview on Turing Machines and RELs:

→ Halting problem of TM is undecidable

i.e., finding out whether TM halts for every string or not

→ All recursive languages are decidable

→ ~~It is undecidable~~ Let G be a UG, then the problem of determining whether or not

$L(G) = \phi$ is undecidable

$L(G)$ = finite is undecidable

$L(G)$ = infinite is undecidable

→ Some language accepted by TM is regular or not — undecidable

→ whether complementation of REL is REL is undecidable

Reducibility:

Reducibility is concept used to solve the harder problems using easier problem with reduction concept.

i.e., when a harder problem doesn't have direct solution, then it is reduced to an easier problem and solutions found.

→ If reduced problem has a solution then actual problem also has a solution

→ If reduced problem has no solution then actual problem will not have a solution.

→ If there is a reduction from problem P_1 to P_2 then

I. If P_1 is undecidable then so is P_2

II. If P_1 is not REL then so is P_2

(2008) which of the following are decidable

I. whether intersection two regular languages is infinite

II. whether a given CFG is regular

III. whether two PDAs accept same language

IV. whether given grammar is CFG

a) I, II b) I, IV c) II, III d) II, IV

Q:- which of the following is/are undecidable

I. G is CFG. is $L(G) = \phi$?

II G is CFG. is $L(G) = \Sigma^*$

III. M is TM. is $L(M)$ = regular

IV. A is a DFA & N is an NFA. Is $L(A) = L(N)$?

a) III b) III, IV c) I, II, III d) II, IV

Note :

A language accepted by a TM is regular or not — undecidable

A language accepted by a TM is CFG or not — undecidable

A string w, G is unrestricted Grammar, whether $w \in L(G)$ — undecidable

membership of w for TM — undecidable

Q: (2)

Find whether the following are decidable :

1. does a given program ever produce an o/p ?
2. If L is a CF, then is $\bar{L}$ a CF ?
3. If L is regular, then is $\bar{L}$ a regular ?
4. If L is recursive, then is $\bar{L}$ a recursive ?

a) 1, 2, 3, 4 b) 1, 2 c) 2, 3, 4 d) 3, 4

2014

Q: Let <M> be the encoding of the TM over the alphabet $\Sigma = \{0, 1\}$

(2) Let

$L = \{ \langle M \rangle \mid M$ is a turing machine that accepts all strings of length 2020$\}$ then L is ____

a) decidable & REL

b) undecidable & REL

c) undecidable & but not REL

d) decidable & but not REL

It is given that M accepts all strings of length 2020
if length is not 2020 we don't know if it halts or not
∴ it is REL
thus it is undecidable

FA & Reg

2 Q

Imp: min no of states
regular expressions

CFG & PDA

1 or 2 Q

TM & UG & CSL & CFG

1 Q

Imp: undecidability
relationship blw languages

16/02/20

Numerical Ability

Average (3-times in gate):

$$\text{Avg} = \frac{\text{Sum of obs}}{\text{no of obs}}$$

type 1: The avg of 10 observations is 40, the avg of another 5 observations is 10. Find avg of total observations.

$$\frac{\text{Sum}_{10}}{10} = 40 \qquad \text{Sum}_5 = 10$$

$$\text{Sum}_{10} = 400 \qquad \text{Sum}_5 = 50$$

$$\text{Avg} = \frac{\text{Sum}_{10} + \text{Sum}_5}{15} = \frac{450}{15} = 30$$

> Avg of m observations is p and n observations is q then total average is $\frac{mp+nq}{m+n}$

obs	Avg
m	p
n	q
r	s
⋮	⋮

$$\Rightarrow \text{Avg} = \frac{mp+nq+rs+\cdots}{m+n+r+\cdots}$$

civil 2020

The ratio of the "The sum of odd positive integers from 1 to 100" to "the sum of the even positive integers from 150 to 200".

$$S_{odd} = 1+3+5+\cdots+99$$
$$= (0+2+4+\cdots 98)+50 = 2(1+2+\cdots+49)+50$$
$$= 2\times\frac{49(50)}{2}+50 = 2500$$

$$S_{even} = 150+152+\cdots+200 = \frac{150+200}{2}\times 13 = 350\times 13$$

$$\frac{S_{odd}}{S_{even}} = \frac{2500}{350\times 13} = \frac{50}{7\times 13} = \frac{50}{91}$$

Given A.P → no of terms

$S_n = \frac{n}{2}[a+l]$ ← last term

first term

Civil 2020 Insert 7 numbers in blw 2 and 84 such that the resulting sequence including 2 and 84 is in Arithematic progression Find sum of these inserted 7 numbers.

Sol:

$$S_n = \frac{n}{2}[a+l] = \frac{9}{2}[36] = 9 \times 18 = 162$$

Sum of 7 numbers = 162 − 2 − 84 = 126

(or)

Average of numbers in sequence = $\frac{34+2}{2} = 18$

Sum of 7 numbers = 18 × 7 = 126

Type 2: The avg weight of 10 students is 16. If one teacher is added to the group then averag is increased by 3. find weight of the teacher.

Sol:

$Sum_{10} = 10 \times 16 = 160$

$Sum_{11} = 11 \times 19 = 209$

$\therefore 209 - 160 = 49$

(or)

[(New strength)(change in Avg)] + [(old Avg)(no of members added)]

~~11(3) + 16 = 49~~

$11(3) + 16(1) = 49$

Type 3:

Given question is in A.P:-

6, 10, 14, 18, 22

$\text{Avg} = \frac{70}{5} = 14$

$\text{Avg} = \frac{\text{first term} + \text{last term}}{2} = \frac{28}{2} = 14$

(or)

$\text{Avg} = \frac{2^{nd} \text{ term from left} + 2^{nd} \text{ term from right}}{2} = \frac{10+18}{2} = 14$

$\text{Avg} = \frac{n^{th} \text{ term from left} + n^{th} \text{ term from right}}{2}$

→ In case no of terms is odd then we have exactly one middle number which is the avg of the series.

→ If avg is k → the middle number if k if no of terms is odd

→ the middle numbers are $k - d/2$, $k + d/2$ if no of terms is even.

where d is common difference.

Note:

For consecutive even numbers, for consecutive odd numbers the common difference is 2. Thus the middle two numbers are "k-1, k+1".

Eg: The avg of 7 consecutive even numbers is 20. Find smalled and biggest in series.

Sol:

$\frac{n + n+12}{2} = 20 \Rightarrow 2n = 28 \Rightarrow n = 14$, $n+12 = 26$

~~$2n = 26 \Rightarrow n = 13$~~

(or)

Avg = middle (∵ no of terms is odd)

14 16 18 20 22 24 26

Eg :- The avg of 20 consecutive odd numbers is 156. Find middle two numbers.

Sol :-

$$156 - \frac{2}{2}, \quad 156 + \frac{2}{2}$$

$$155, 157$$

(2014) In a sequence of 12 consecutive odd numbers, The sum of first 5 odd numbers is 425. what is sum of last 5 numbers in sequence.

$$3^{rd} = \frac{425}{5} = 85$$

$$\vdots$$

$$8^{th} = 85 + 10 = 95$$

$$12^{th} = 95 + 8 = 103$$

$$\text{last five sum} = \frac{5}{2}[95 + 103] = \frac{198 \times 5}{2} = 495$$

$$= \frac{208 \times 5}{2} = 104 \times 5 = 520$$

(2014) The sum of eight consecutive odd numbers is 656 the avg of four consecutive even numbers is 87 find sum of smallest odd and 2nd largest even.

Odd → 649, 651, 653, 655, 657 (4th, 5th)

$$\frac{656}{8} = 82$$

even → 84, 86 (2nd), 88, 90

Odd → 75, 77, 79, 81, 83

649 + 86 = 735

88 → 2nd largest

75 + 88 = 163

Blood - Relations !

[4 - times]

Notations

M = +

F = -

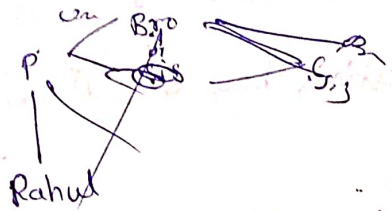
couple = ^

Parent, child => P / child

B-S, S-B, S-B, B-B } => "——" for brother & sister of same parents

Q: Pointing in a photograph Rahul said "He is the brother of my uncle's daughter". How is the man related to Rahul?

Sol:



P —. bro

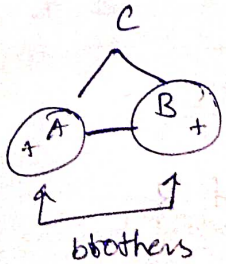
|

Rahul

daughter — brother

Cousins

Q: A and B are children of C. A is son of C but B not a daughter of C. How is A and B related.

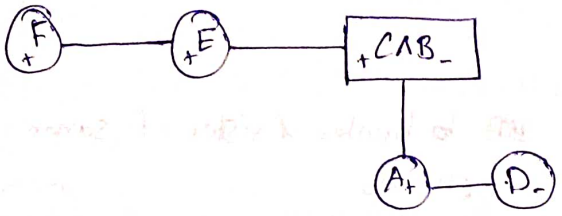


Q:- In a family A, B, C, D, E, F are travelling together. A is son of C. B is wife of C. D is daughter of B. E is brother of C, F is brother of E.

(i) How many male members in family?

(ii) Relation blw F & B?

(iii) How is C related to D?



F & B → brother-in-law

C to D → ~~daughter~~ ~~parent~~ father

Q:-

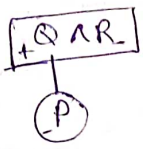
(a) A+B means A is daughter of B.

A-B means A is husband of B

A*B mean A is brother of B

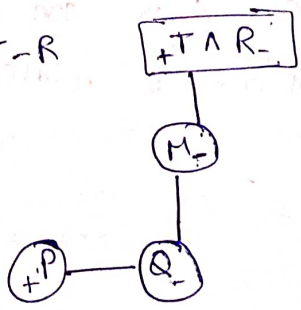
In the following questions how is "P" related to "R":

(i) P+Q-R



∴ P is daughter of R

(ii) P*Q+M+T-R



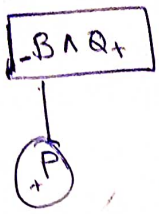
∴ P is Grandson of R

Material Questions:

(13)

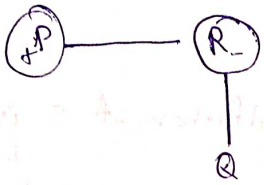
$A + B$	A is son of B
$A - B$	A is wife of B
$A \times B$	A is brother of B
$A \div B$	A is mother of B
$A = B$	A is sister of B

(i) $P + R - Q$



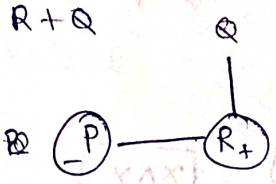
Q is father of P

(ii) $P \times R \div Q$



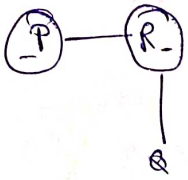
P is uncle of Q

(iii) $P = R + Q$



P is daughter of Q

(iv) $P = R \div Q$



~~Q is niece of~~ P

P is aunt of Q

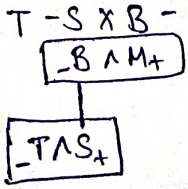
(14)

$A + B$	A is daughter of B
$A \times B$	A is son of B
$A - B$	A is wife of B

(i) $P \times Q - S$

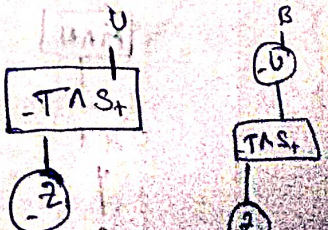
~~father~~ S is ~~wife~~ of Q

(ii) $T - S \times B - M$



∴ S is not daughter of B

(iii) $Z + T - S \times U + P$

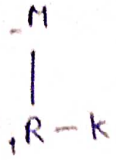


grandmother

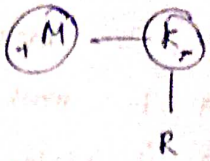
(15) ~~R+Q~~

a) M+k+R

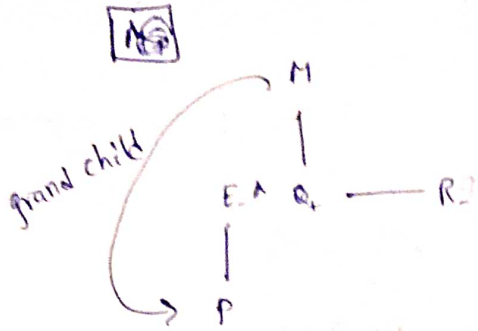
b) M-R+k



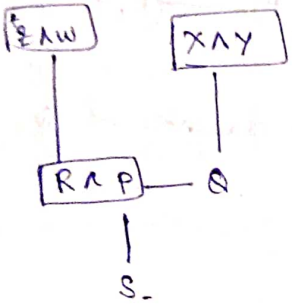
c) M+k-R



(9)



(10)



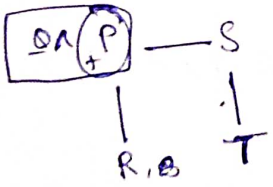
given grandfather of S is Z

but X & Y are paren of P.

So P must be married to get another grandfather

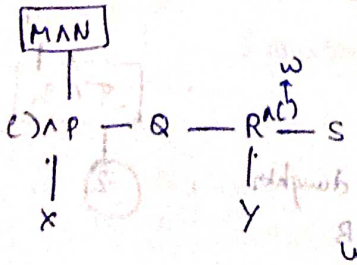
∴ Q can't be married to R is false

(11)



S is aunt of T

(12)



W can't be wife of P

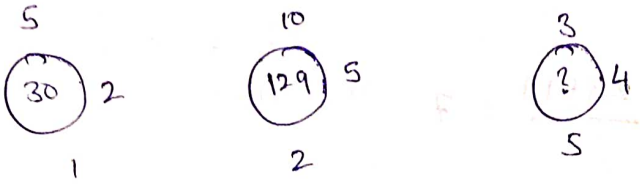
Missing No:

①

x	5	6	2
y	4	2	3
$(x+y)^2$	81	64	?

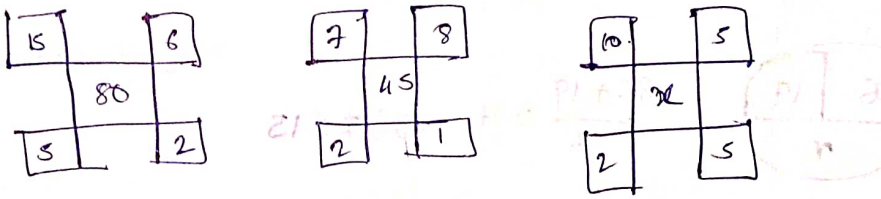
$x+1$

②



$a^2+b^2+c^2 = 3^2+4^2+5^2 = 30$

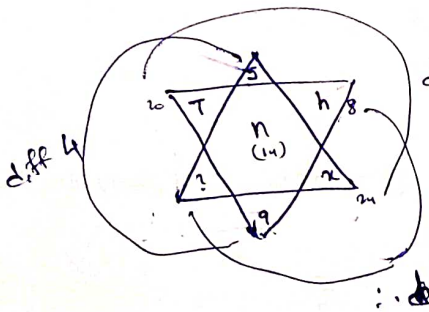
③



$(15-5)(6+2)=80$ $(7-2)(8+1) = 45$ $(10-2)(5+5)=80$

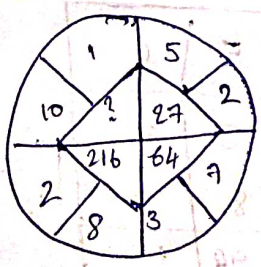
h-8
n-14
r-20
x-24

④



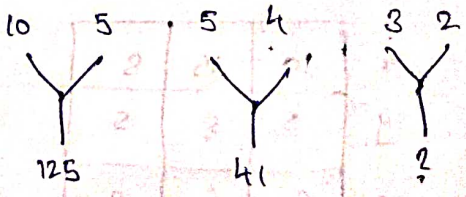
$\therefore 8-4=4 \cong d$

⑤



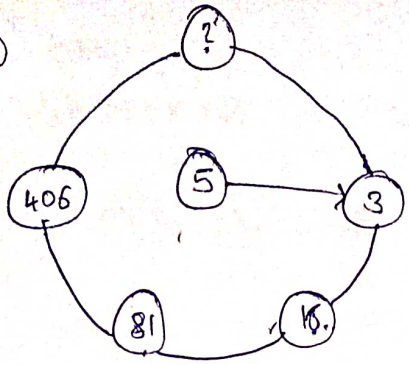
$(8-2)^3 = 216$ $(5-2)^3 = 27$
$(7-3)^3 = 64$ $(10-1)^2 = 729$

⑥



$10^2+5^2 = 125$
$5^2+4^2 = 41$
$3^2+2^2 = 13$

(7)



$5x+1$

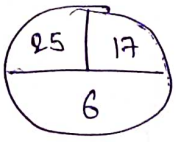
$5 \times 3 + 1 = 16$

$5 \times 16 + 1 = 81$

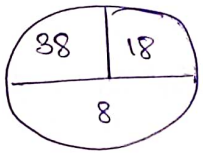
$5 \times 81 + 1 = 406$

$5 \times 406 + 1 = 2031$

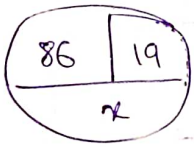
(8)



$$\frac{25+17}{6} = 7$$

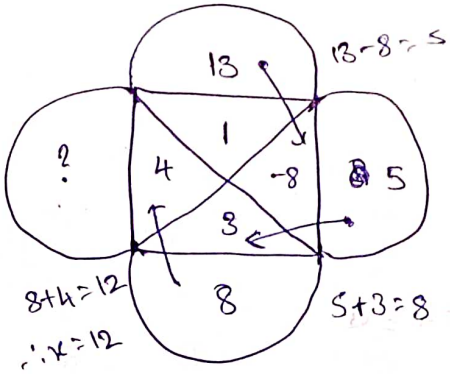


$$\frac{38+18}{8} = 7$$



$$\frac{86+19}{x} = 7 \Rightarrow x = 15$$

(9)



(10)

x	10	15	8
y	5	8	5
z	2	6	2
$x+y-z$	13	17	?

$8+5-2=11$

x	4	5	6
y	2	3	7
z	1	8	3
$x^2+y^2+z^2$	21	98	?

$36+49+9=94$

Ages:

① $P:Q = 5:7$

$P \to 5x$, $Q \to 7x$

$(7x+6) - (5x+6) = 2$

$7x - (5x+6) = 2$

$2x - 6 = 2$

$\boxed{x = 4}$

$P \to 5x = 20$

$Q \to 7x = 28$

$20 + 28 = 48$

② father $\to x$ son $\to y$

4 yrs ago

$x - 4 = 3(y - 4)$

$x - 3y = -8$

after 4 years

$x + 4 + y + 4 = 64$

$x + y = 56$

$\Rightarrow 4y = 64 \Rightarrow y = 16$

$\Rightarrow x = 56 - 16 = 40$

③ Kunal $\to x$, Sagar $\to y$

$$\frac{x-6}{y-6} = \frac{6}{5} \Rightarrow 5x - 30 = 6y - 36$$

$$\Rightarrow 5x - 6y = -6$$

$$\frac{x+4}{y+4} = \frac{11}{10} \Rightarrow 10x - 11y = 4$$

② $- 2 \times$ ① $\Rightarrow -11y + 12y = 4 + 12$

$y = 16$

(4) N : S = 5 : 6

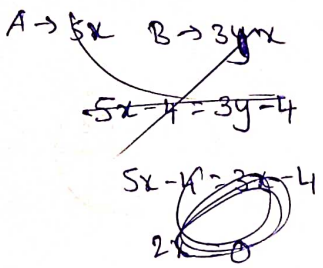
N/3 : S/2 = 5 : 9

$\frac{N}{S} = \frac{5}{6}$ $\frac{2N}{3S} = \frac{5}{9}$

$\frac{N}{S} = \frac{5}{6}$ $\frac{N}{S} = \frac{5}{6}$

∴ can't be determined

(5)



(6) A : B = 5 : 3

5x 3x

5x - 4 = 3x + 4

2x = 8 ⇒ x = 4

$\frac{5x+4}{3x-4} = \frac{24}{8} = 3:1$

Arrangement:

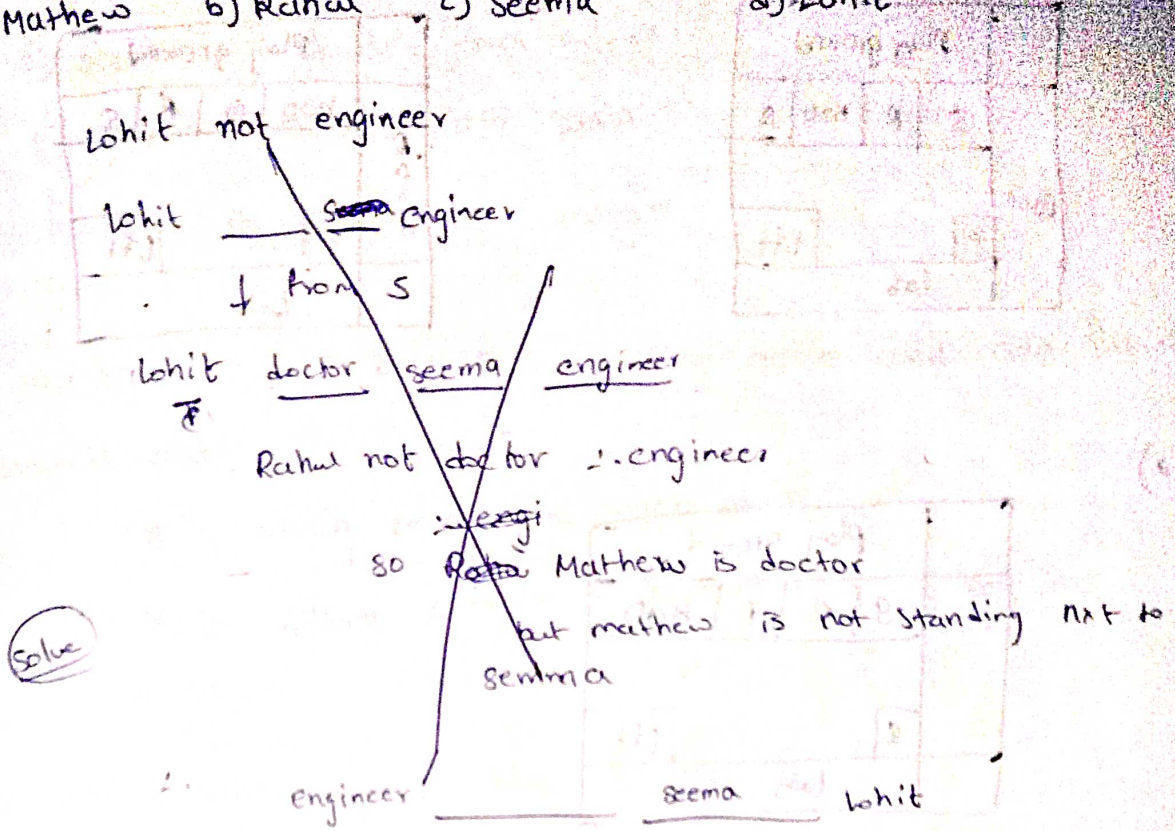
[12-times]

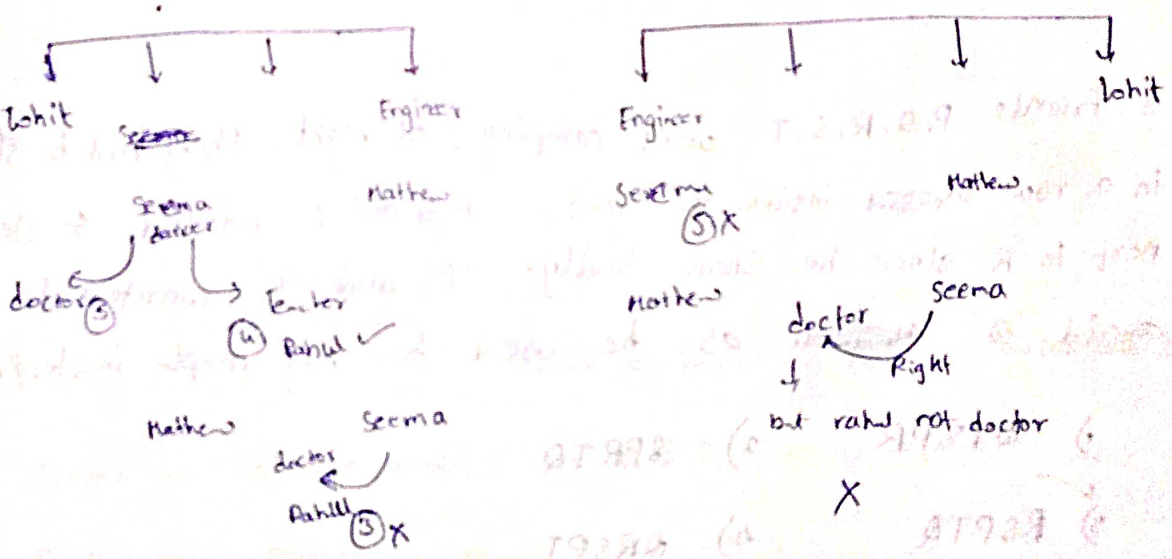
(2019) 4 people are standing in a line facing you. They are Rahul, Mathew, Seema, and Lohit. One is an engineer, one is a doctor, one a teacher and another a dancer. You are told that:

1. Mathew is not standing next to Seema
2. There are two people standing blw Lohit and the engineer.
3. Rahul is not a doctor
4. The teacher and the dancer are standing next to each other.
5. Seema is turning to her right to speak to the doctor standing next to her.

who among them is engineer?

a) Mathew b) Rahul c) Seema d) Lohit





∴ Mathew is the Engineer.

(2020) After Inauguration of the new building the "HOD" collated faculty preference for office space.

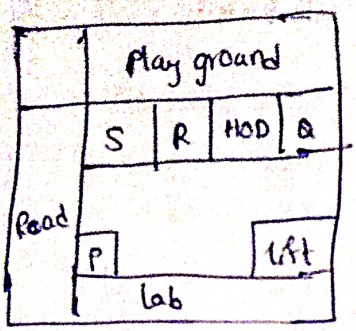
P - wanted a room Adjacent to the lab

Q - wanted to be close to the lift

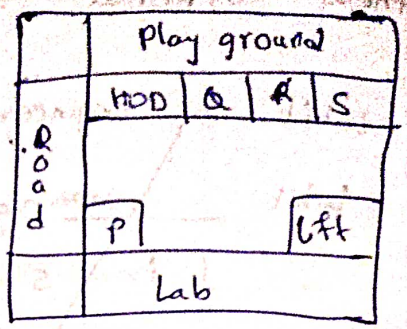
R - wanted a view of the play ground

S - wanted corner office.

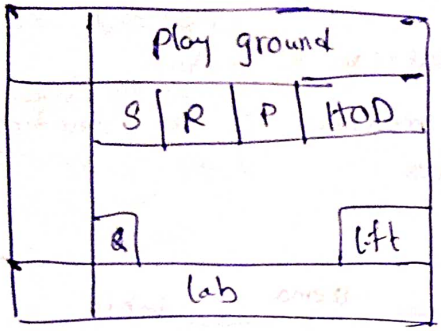
a)



b)



c)



Q: 5 Friends P, Q, R, S, T went camping. At night, they had to sleep in a row ~~in~~ inside the Tent. P, Q and T refused to sleep next to R since he snores loudly. 'P' and 'S' wanted to avoid 'Q' ~~as~~ as he used to hug people in sleep.

1) QTSPR
2) SPRTQ
3) RSPTQ ✓
4) QRSPT

(2019) Consider 5 people Mitra, Ranga, Rekha, Lakshmi and Sana.

(i) Ranga is taller than both Rekha & Lakshmi
(ii) Lakshmi is taller than Sana
(iii) Mitra is taller than Ranga

Which of the following conclusions are true

a) Lakshmi is taller than Rekha

b) Rekha is shorter than mitra

c) Rekha is taller than sana

d) Sana is shorter than ganga

9: 7 members A, B, C, D, E, F, G sitting on a wall and facing towards east

(i) C is sitting immediate right of D

(ii) B is sitting extreme end and also E as his neighbour

(iii) G is sitting in blw E and F

(iv) D is sitting 3rd from the south end

B can't be near cuz he has neighbour E

① who are extreme ends? → A & B

② How many persons are blw A & G → 3

10/7 Rahul, Murali, Srinivas, Arul → Around a square table

Rahul is sitting to the left of Murali

Srinivas is sitting to the right of Arul.

Which of the following pairs are seated opposite to each other?

1) Rahul, Murali 2) Srinu, Arul 3) Srinu, Murali 4) Srinu, Rahul.

Shortcuts for Number System:

① Every prime number has exactly 2 factors

Every square of prime number has exactly 3 factors

(2018) How many 3-factor numbers ~~are~~ present from 1 to 100.

No of factors = 3 i.e., square of prime number

$2^2, 3^2, 5^2, 7^2$

∴ 4 numbers

② Divisible by 3

Eg: 65y

$$\frac{6+5+y}{3} = \frac{11+y}{3}$$

y = 1, 4, 7

③ Divisible by 9:

Eg: 687y

$$\frac{6+8+7+y}{9}$$

$$\frac{21+y}{9} \Rightarrow y = 6$$

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y = 1, 4, 7

③ Divisible by 9:

Eg: 687y

$$\frac{6+8+7+y}{9}$$

$$\frac{21+y}{9} \Rightarrow y = 6$$

(4) Divisible by 11 :

Difference of sums of alternate digits is divisible by 11.

Eg: 764y3

$(7+4+3)-(6+y)$

$= 8-y$

$8-y=0 \Rightarrow y=8$

→ Every two digit number when followed by its reverse is divisible by 11, i.e., numbers of form xyyx

Eg: 79 ⇒ 7997

divisible by 11

→ Every three digit number when followed by itself is divisible by 11. i.e., numbers of form xyzxyz

Eg: 123 ⇒ 123123

divisible by 11

(2016) $z+\frac{1}{z}=10$, $z^2+\frac{1}{z^2}=?$

$\left(z+\frac{1}{z}\right)^2 = 10^2$

$z^2+\frac{1}{z^2}+2=100 \Rightarrow z^2+\frac{1}{z^2}=98$

(2017) $x-\frac{1}{x}=5$ $x^2+\frac{1}{x^2}=?$

$\left(x-\frac{1}{x}\right)^2 = 25 \Rightarrow x^2+\frac{1}{x^2} = 25+2 = 27$

(2019) $x+y=26$, $xy=165$, $x-y=?$

$(x-y)^2 = (x+y)^2-4xy = 26^2-4(165) = 16$

$\Rightarrow x-y=4$

(2020) $f(x) = x^2$

$$\frac{f[f(f(x))]}{f(x)} = ?$$

a) $f(x^2)^3$ b) $(f(x))^3$ c) $(f(x^3))^3$ d) none

$$\frac{f[f(f(x))]}{f(x)} = \frac{f[f(x^2)]}{x^2} = \frac{f(x^4)}{x^2} = \frac{x^8}{x^2} = x^6$$

a) $f(x^2)^3 = (x^4)^3 = x^{12}$

b) $(f(x))^3 = (x^2)^3 = x^6$

c) $(f(x^3))^3 = (x^6)^3 = x^{18}$

CIVIL

(2020) The sum of two positive integers is 100. After subtracting 5 from each number the product of resulting numbers is 0. One of the original numbers is ?

a) 85 b) 90 c) 95 d) 80

(2019)

Q: How many integers are there b/w 10 and 100, all of whose digits are even?

$$\underset{\downarrow}{4} \times \underset{\downarrow}{5} = 20$$

(2019) How many integers are there b/w 100 to 1000, All of whose digits are even?

$$4 \times 5 \times 5 = 100$$

Common Remainder:

Q: A number divisible by 12, 15, 18 and having common remainder 2 Find the least number.

$180 + 2 = 182$

LCM + Common remainder

$3 \mid 12, 15, 18$

$2 \mid 4, 5, 6$

$2, 5, 3$

$36 \times 5 = 180$

~~Q: A number~~

Different Remainder:

Q: A number when divided by 12, 15, 18 gives remainders 8, 11, 14. Find the least number that satisfies the condition.

Numbers → 12, 15, 18

Rem → 8, 11, 14

diff: 4, 4, 4 (diff must be equal to apply this technique)

LCM (12, 15, 18) − 4

$180 - 4 = 176$

When fractions are given:

$$L.C.M = \frac{L.C.M \text{ of } (N)}{H.C.F \text{ of } (D)}$$

~~$\Rightarrow H.C.F =$~~ L.

$$H.C.F = \frac{H.C.F \text{ of } (N)}{L.C.M \text{ of } (D)}$$

Co-Primes :

→ Two numbers are said to be co-primes iff H.C.F of the two numbers is 1.

$$H.C.F(x,y) = 1$$

→ The product of any two number numbers is equal to their product of LCM and HCF

→ Given two numbers = x, y

$$xy = LCM(x,y) \times HCF(x,y)$$

Hint:

Traffic signals, school bells, church bells, Circular based problems etc. This type of questions belongs to L.C.M

Note:

$x+\frac{1}{x}$ is given

$$x^2+\frac{1}{x^2} = \left(x+\frac{1}{x}\right)^2 - 2$$

$$x^3+\frac{1}{x^3} = \left(x+\frac{1}{x}\right)^3 - 3\left(x+\frac{1}{x}\right)$$

$x-\frac{1}{x}$ is given

$$\left(x^2+\frac{1}{x^2}\right) = \left(x+\frac{1}{x}\right)^2 + 2$$

$$x^3-\frac{1}{x^3} = \left(x-\frac{1}{x}\right)^3 + 3\left(x-\frac{1}{x}\right)$$

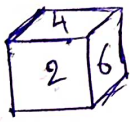
Dice:

→ Dice is a cube structure. with 6 surface.

→ If a dice is rolled, the visible face are only 3 faces at a time.

Standard Dice

(i) Sum of any two opposite sides is seven



4-oppo-3

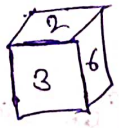
2-oppo-5

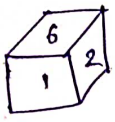
6-oppo-1

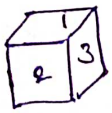
General Dice

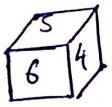
(ii) One of the adjacent surface's sum view is 7

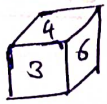
Eg:





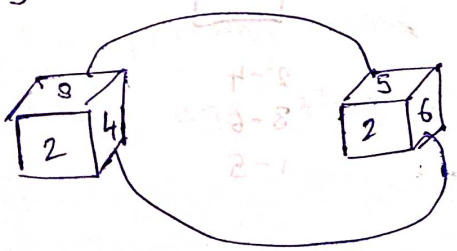






To identify the opposite face of General dice:

1. To identify opposite face we need atleast 2 views with only one number being common.



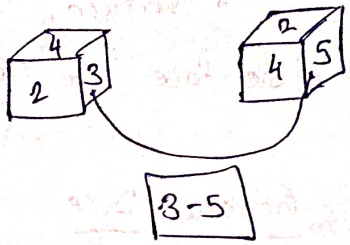
3-5
4-6
2-1

Corresponding sides are opposite

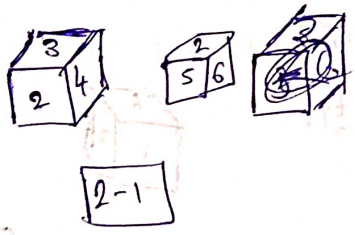
when we have one common surface then corresponding sides are opposite faces.

II. Two sides common. Then we can predict only one pair of opposite faces.

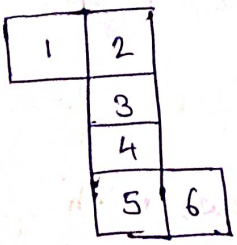
~~too~~ ~~one~~ ~~common number but different surface~~



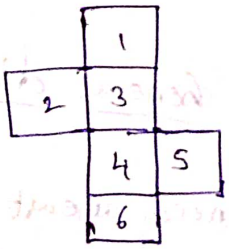
III. only one number common but different surfaces



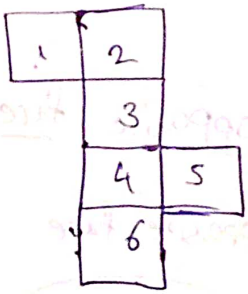
Unfolded dice given (we want to fold the DICE):



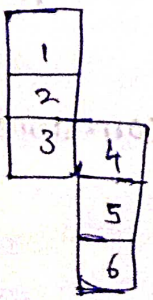
2-4
3-5
1-6



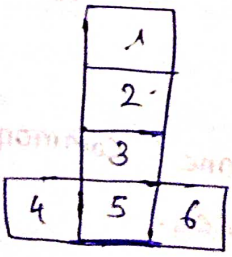
1-4
3-6
2-5



2-4
3-6
1-5



1-3
4-6
2-5



1-3
2-5
4-6

Time & work [15-times]

T/w [11] Pipes [4]

→ $T \propto \frac{1}{w}$

(or)

$w \propto \frac{1}{T}$

→ A can do a piece of work in 20 days then A's one day work is 1/20

→ one day work of A is 1/x then total work is completed in x days.

Chain Rule: [6 times]

efficiency

group 1 → $M_1, D_1, H_1, x\%$ → w_1

group 2 → $M_2, D_2, H_2, y\%$ → w_2

$$\frac{M_1 D_1 H_1 x\%}{w_1} = \frac{M_2 D_2 H_2 y\%}{w_2}$$

Q: 40 members can do the piece of work in 60 days and they are working 10 hrs per day. Find no of days to complete the work by 25 member and they are working 15 hrs per day

Sol:

$M_1 D_1 H_1 = M_2 D_2 H_2$

$40 \times 60 \times 10 = 25 \times D_2 \times 15$

$D_2 = 64$ days

(2020) It was estimated that 52 men can complete the strip in a newly constructed highway connecting the cities P and Q in 10 days. Due to an emergency 12 men were sent to another project. How many no of days more than original estimated will be required?

$M_1D_1 = M_2D_2$

$52 \times 10 = 40 \times D_2$

$D_2 = \frac{52 \times 10}{40}$

$D_2 = 13$ days

$\therefore$ 3 days more.

(2019)

Pg:51
13

$$\frac{(2 \times 8 \times 12)}{9000} = \frac{(3 \times 6 \times h)}{12000}$$

$$h = \frac{24 \times 12}{3 \times 6} = 16 \text{ hrs}$$

Pg:51
12

$$\frac{30(8)(104)}{8} = \frac{26(x)\,9}{3}$$

$$x = \frac{36 \times 16 \times 3}{9} = 160$$

Additional men = 56

Pg:51
14

$$\frac{39(7)(125)}{5} = \frac{(13)(8)(x)}{2} \Rightarrow x = 6.25$$

i.e., 7 robots

Basic formulas:

I. A → x days
B → y days
A & B → ____ days

A & B per one day → $\frac{1}{x} + \frac{1}{y}$

no of day for total work (d)

$$\left(\frac{1}{x} + \frac{1}{y}\right) d = 1$$

$$\boxed{d = \frac{xy}{x+y} \text{ days}}$$

II. A → x days
A & B → y days

A's one day work = $\frac{1}{x}$

A & B's one day work = $\frac{1}{y}$

B's one day work = $\frac{1}{y} - \frac{1}{x}$

no of days for B (d)

$$\left(\frac{1}{y} - \frac{1}{x}\right) d = 1$$

$$\boxed{d = \frac{xy}{x-y}}$$

III. A → x
B → y
C → z
A & B & C → ?

$$\boxed{d = \frac{xyz}{xy+yz+zx}}$$

IV. A & B → x days
B & C → y days
A & C → z days
A & B & C → ?

$$\boxed{d = \left(\frac{1}{2}\right) \frac{xyz}{xy+yz+zx}}$$

Eg: A → 20 days
B → 30 days
Find no of days to complete the work by A & B together.

$$\frac{20\times30}{50} = \frac{600}{50} = 12 \text{ days}$$

pg: 50 / 1

$$\frac{9\times6}{9+6} = \frac{54}{15} = \frac{18}{5} = 3\frac{3}{5} \text{ min}$$

pg: 50 / 3

P → 12 × 8
Q → 8 × 10

$$P \& Q \rightarrow \frac{12\times8\times8\times10}{8(12\times8)+(8\times10)} = \frac{12\times8\times8\times10}{8(22)} = \frac{480}{11} \text{ hrs}$$

$$\frac{480}{11}\times\frac{1}{8} \text{ days}$$

$$\frac{60}{11} = 5\frac{5}{11} \text{ days}$$

$$= 24\left(\frac{480}{11}\right) \text{ hrs}$$

$$= \frac{20}{11} \text{ days}$$

Wages Concept

(Amount wise)

A → x days
B → y days

Now work completed by A & B together. At the end of the work they recieved z rupees. Then their amounts shares is equal to $\frac{1}{x}:\frac{1}{y}$ (or) $y:x$

→ Sly if A → x days B → y days C → z days

wages ratio = $\frac{1}{x}:\frac{1}{y}:\frac{1}{z} = yz:zx:xy$

→ Silly for A, B, C, D

$$\frac{1}{w} : \frac{1}{x} : \frac{1}{y} : \frac{1}{z} = xyz : wyz : wxz : wxy$$

Pg: 50 / 8

A → 10 (x)

B → 15 (y)

$$y : x = 3 : 2$$

$$5x = 225$$

$$x = 45$$

$$A \rightarrow 3x = 135$$

Efficiency Concept:

→ A is working twice as fast as B then

A → x days

B → 2x days

→ A is working thrice as fast as B then

A → x

B → 3x

Pg: 50 / 7

A → x → y

B → x + 80 → 3y ⇒ 3y − y = 80 ⇒ y = 40

$$\frac{3y^2}{4y} = \frac{3y}{4} \text{ days}$$

$$= \frac{3}{4} \times 40 = 30 \text{ days.}$$

Work done by group:

→ 2 men (or) 6 women can do the piece of work in 60 days

Find no of days to complete the work by 4 men & 3 women

Sol:

2M → 60 ⇒ M = 120

6F → 60 ⇒ F = 360

4M → 30

3M → 120

$$\frac{30 \times 120}{30 + 120} = \frac{\cancel{30}^{6} \times \cancel{120}^{4}}{\cancel{150}_{5}} = 24 \text{ days}$$

Pg:51 / 9

5 Sk —— 20 => 2 Sk —— 50 $\left(\frac{20\times5}{2}\right)$

8 Ss —— 25 => 6 Ss —— $\frac{100}{3}$ $\left(\frac{8\times25}{6}\right)$

10 Ws —— 30 => 5 Ws —— 60 $\left(\frac{10\times30}{5}\right)$

$$\frac{50\times\frac{100}{3}\times60}{\cancel{50+60}}\ 20$$

$$50\times\frac{100}{3}+50\times60+\frac{100}{3}\times60$$

$$\frac{10\times10^4}{10^3\left(\frac{5}{3}+8+\frac{6}{3}\right)} = \frac{100\times3}{5+9+6} = \frac{\cancel{100}^{5}\times3}{\cancel{20}} = 15 \text{ days}$$

Based on remaining work:

→ A → 20 days

B → 30 days

A had started work and after 5 day B joined A and Completed remaining work. Find No of days to complete remaining work?

$$5\left(\frac{1}{20}\right)+x\left(\frac{1}{20}+\frac{1}{30}\right)=1$$

$$\frac{1}{4}+x\left(\frac{1}{12}\right)=1$$

$$3+x=12$$

$$x=9 \text{ days}$$

total days = 9+5 = 14 days.

Pg:50 / 5

$$2\left(\frac{1}{8}+\frac{1}{10}+\frac{1}{12}\right)+x\left(\frac{1}{10}+\frac{1}{12}\right)=1$$

$$2\left(\frac{30+24+20}{\cancel{240}_{120}}\right)+x\,\frac{22}{\cancel{120}}=1 \Rightarrow 74+22x=120$$

$$22x=46$$

$$x=\frac{46}{22}=\frac{23}{11}=2\frac{1}{11}$$

23/02/20

Cyclicity [or unit digit]:

Number	Unit Digits
1	1
2	2, 4, 8, 6
3	3, 9, 7, 1
4	4, 6
5	5
6	6
7	7, 9, 3, 1
8	8, 4, 2, 6
9	9, 1

Eg:

Find last digit of following calculations

→ $11^{11} \times 22^{22} \times 33^{33} \times 44^{44}$

$1 \times 4 \times 3 \times 6$

$7(2)$

→ $18^8 + 9^5 - 7^1$

$8 + 9 - 7$

$\Rightarrow 8$

→ $17^8 \times 12^6 \times 25^9 \times 9$

$\Rightarrow 0$

→ $3 \times 5 \times 7 \times 9 \times 4$

$\Rightarrow 0$

→ $11 \times 13 \times 15 \times 17 \times 19$

no even

$\therefore 5$

(2022) Find last digit of

26510^{11056}

$\therefore 1$

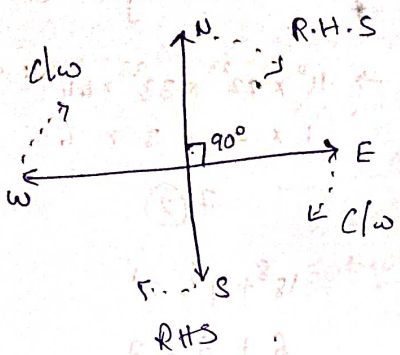
If there is a 5 and an even number in the product then it always ends with zero.

If there is a 5 and no even number in the product then it always ends with 5

Directions:

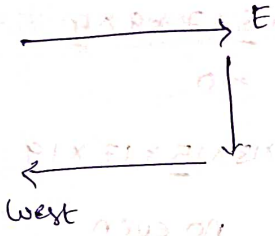
[4-times]

Type:1

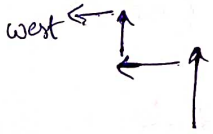


turn Right ≅ clockwise
turn left ≅ Anticlockwise

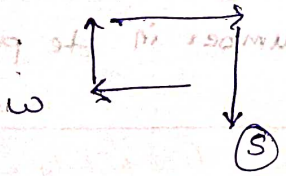
Eg: A man going to market towards east. After sometime he took right hand side. Again he takes clockwise direction. In which direction man is facing now?



Eg: A ~~man~~ man going to office towards north. After sometime he turns left. Again he takes clockwise direction. Finally he takes left hand side. Find direction he is facing



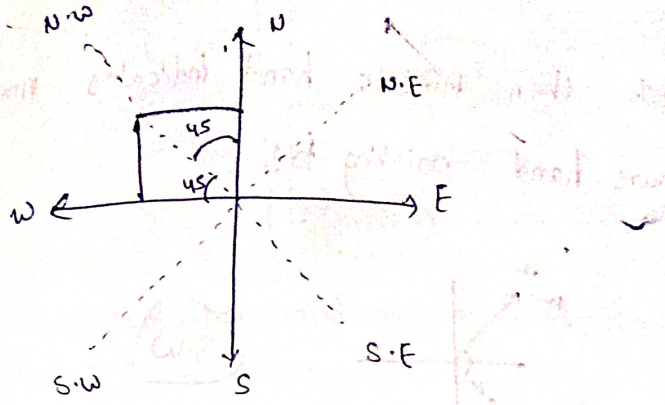
Eg: A man going home towards west. After sometime he takes clockwise direction. Again he takes right ~~clockwise~~ hand side. Finally right.



If x rights and y lefts (x,y) are given then it is equivalent to

x-y. right ≅ (x-y) % 4 rights

Type: 2



A man going to market towards east. After sometime he take 45° turn towards right, again he takes 90° towards left. After sometime he takes 135 clockwise direction. Again he takes 45° toward right. Finally he takes 90° towards left.

clw / right	left / aclw
45	90
135	90
45	
225	180

=) 45 to right i.e., Southeast.

A man going to office towards north. After sometime he takes 135° anticlockwise direction. Again he takes 90° left hand side. Finally he takes 135° clockwise direction.

Sol:

$-135 - 90 + 135 = -90$

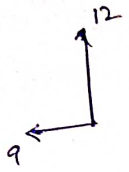
i.e., left 90°

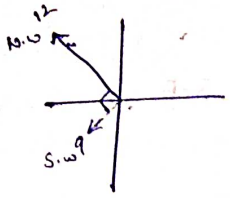
=) west.

Type:3

Eg:- If time is '9' o' clock then minute hand indicates northwest. what is direction hours hand pointing to?

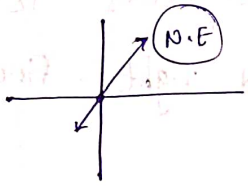
Sol:



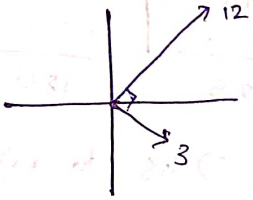


Eg: If time is 6 o' clock the hour hand indicates southwest. what is direction of minute hand.

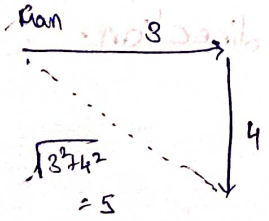
Sol:

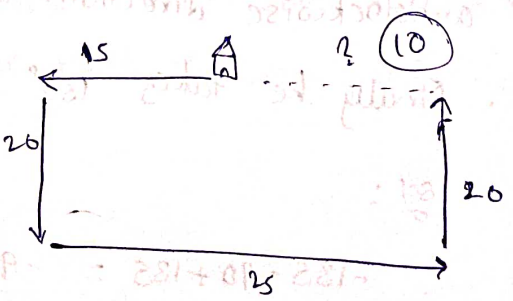


Eg: If time is 3 o' clock then hours hand indicates South East. In which direction indicates the minute hand.

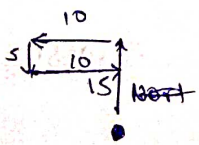


Type 4: Based on 3rd sides (or) Mathematical

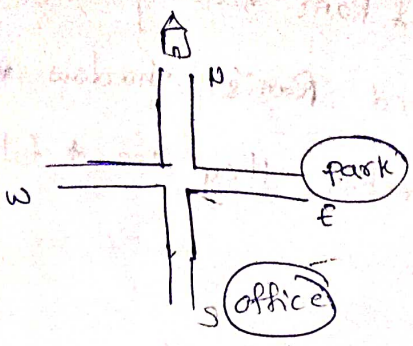




Pg:9
2

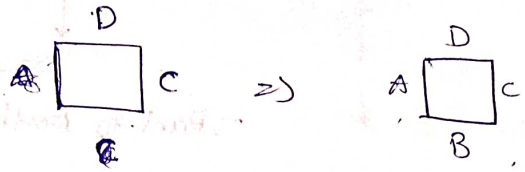


Eg: 4/6



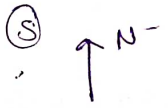
∴ Market must be in west

Eg: 4/8



∴ B to north

Eg: 4/9



2 rights + 1 left ⇒ 1 right

∴ west

Type 5: Sun Rises Concept

Shadow is back side of man ⟶ Man facing east

" " front side " " ⟶ " " west

" " right side " " ⟶ " " South

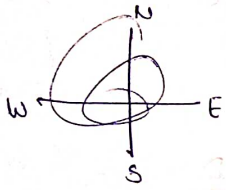
" " left side " " ⟶ " " North

Eg: One day morning man going to market. His shadow is right side of him. To what direction man is facing?

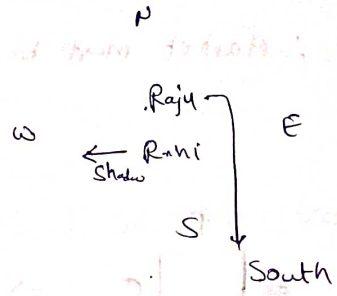
⇒ South

Eg: One day morning Raju & Rani talking to each other in a road. Suddenly Raju observed Rani's shadow is right hand side of him. In which direction they are talking. Find their individual directions.

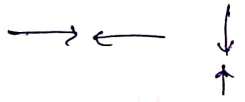
Sol:



They are talking in N-S or S-N direction



=> Rani to North

> When it is said that two persons are talking to each other it means they are talking face to face i.e., they exactly face opposite directions.
>
> 

Eg: One day morning Raju & Sunitha talking to each other. Suddenly Sunita observed Raju's shadow left hand side to side. To which direction Sunita is facing.

North

when raju's shadow is on her left then her shadow is also on her left

=> North

Note:

One day Evening a boy is going school. Suddenly he observed his shadow on his left side. To which direction he is facing.

east (sunrise) => South (sunset) (because it is evening)

Solve as in the case of sunrise then alternate the answer

Profit & Loss

[3-times]

If CP < SP => profit

If SP < CP => loss

Type: 1

$$\text{Profit \%} = \frac{P}{CP} \times 100$$

$$\text{Loss \%} = \frac{L}{CP} \times 100$$

Type: 2

$$SP = CP \times \frac{100 \pm x}{100}$$

$$CP = SP \times \frac{100}{100 \pm x}$$

L is % of profit or loss

Eg: Cost price of an item is Rs 7000. A shopkeeper got 40% profit on CP. Find selling price

$SP = CP \times \frac{100 \pm x}{100}$

$= 7000 \times$

100% → 7000

40% → 2800

SP = 140% → 9800

Eg: A shopkeeper sold an item at 14400 and he got 20% profit on CP. Find CP.

120% → 14400

100% → ?

$\frac{14400}{120} \times 100 = 12000$

Type 3:

A → B → C

10% loss

A → B

A → C

A shopkeer sold an item at 10% loss on CP. If he had sold for excess amount of 120 Rs then he would have got 20% profit on CP. Find CP.

120 — 30%

? — 100%

400

Eg: A shopkeeper sold an item at 5% loss on CP. If he had sold for an excess amount of Rs90 the he would have got 1% profit. Find CP.

6% — 90

100% — 1500

Eg: A shopkeep sold an item at 10% loss on CP. If he had sold for excess amount of Rs.60 the he would have got 5% loss on CP. Find CP & loss amount

5% — 60

100% — ? 1200

loss = 10% i.e., 120

Pg:60 / 7

108 — 22.5

? ← 12.5

$\frac{108}{22.5} \times 12.5 = \frac{108}{225} \times 125 = 60$

87.5 — x

510 — x+108

225 — 108

Type: 4

→ CP of 10 apples is equal to the SP of 8 apples. Find profit or loss percentage

Sol:

$10x = 8y$

$\frac{y-x}{x} \times 100 = \frac{10x}{8x} - x \times 100 = \frac{1}{4} \times 100 = 25$

CP[10] = SP[8]

CP → 8

SP → 10

$\Rightarrow \frac{2}{8} \times 100 = 25\%$

→ CP of 20 apples is equal to the SP of 15 apples find profit or loss percentage

Sol: $20x = 15y \Rightarrow 4x = 3y$

$\frac{y-x}{x} \times 100 = \frac{\frac{4}{3}x - x}{x} \times 100 = \frac{1}{3} \times 100 = 33.33\%$

→ CP of 5 apples = SP of 18 apples - find profit or loss %.

$5x = 6y$

$P/L\% = \frac{y-x}{x} \times 100 = \frac{5}{6} \times 100 = 83.33\%$ $\quad \frac{CP}{SP} = \frac{6}{5}$.

$\frac{-1}{6} \times 100 = -16.66\%$

i.e., Loss

Type 5:

CP of 11 apples is Rs. 10. SP of 10 apples is Rs. 11. Find P or loss %.

Sol:

CP of 11 AP = 10

1 AP = $\frac{10}{11}$

SP of 10 AP = 11

1 AP = $\frac{11}{10}$

$P\% = \frac{\frac{11}{10} - \frac{10}{11}}{\frac{10}{11}} \times 100$ $\qquad \frac{SP-CP}{CP} \times 100$

$= \frac{121-100}{110} \times 100$

= 21

CP of a items = Rs. b

SP of c items = Rs. d

$P\% = \frac{ad-bc}{bc} \times 100$

if P% is -ve then it is considered loss

Type: 6

→ A shopkeeper sold an item at Rs. 450 and he got 25% loss on CP. At what SP he would gaid 25% profit on CP

75% — 450

50% — $\frac{2}{3} \times 450 = 300$

125% — 450 + 300 = 750

→ A shopkeeper sold an item at Rs. 810 and he got 10% loss on CP. At what SP he would gain 20% profit on CP%

90% — 810

30% — 270

120% — 1080

★

→ A shopkeeper sold 36 apples for Rs. 1 and he got 10%, loss on CP. How many apples should he sell for Rs. 1 and gain 8% profit on CP

90% — 1

18% — 0.2

108% — 1.2 (At 8% profit he should give 36 apples at 1.2)

1.2 — 36

1 — ? (30)

then at 8% profit how many apples should he sell for Rs. 1

(OR)

36 AP → 1/-

1 AP — 1/36

90% — 1/36

108% — $\frac{1}{36} \times \frac{108}{90} = \frac{1}{30}$

∴ 30 apples

Type:7 : Successive Discounts (Profits)

If x, y, z ... are series of discounts or profits on a product the S.P is

$$S.P = CP \times \frac{100 \pm x}{100} \times \frac{100 \pm y}{100} \times \frac{100 \pm z}{100} \times \dots$$

Pg:60 / 8

$$(1400)\left(\frac{8}{10}\right)\left(\frac{9}{10}\right)$$

20% 10%

$14 \times 72 =$ 1008

Pg:60 / 6

$$[x+100]\left(\frac{12}{10}\right)\left(\frac{9}{10}\right)\left(\frac{11}{10}\right) = 1188$$

$x + 110 = 1000$

$x = 890/-$

Total investment of A.

Type:8

An honest dealer used 900 gms instead of 1000 gms find his profit or loss percentage

Sol:

$10x = 9y$

$\frac{CP}{SP} = \frac{9}{10}$ $\quad \frac{1}{9} \times 100 = 11.11\%$

This can be related to Type-4 problems. Here

If CP of 1000 gms = SP of 900 gms

i.e. $CP[1000] = SP[900]$

$\frac{CP}{SP} = \frac{900}{1000} = \frac{9}{10}$

A dealer used 80cm instead of 90 cm. Find profit or loss.

Sol:

$CP[90] = SP[80] \Rightarrow \frac{CP}{SP} = \frac{8}{9}$

$\Rightarrow \frac{1}{8} \times 100 = 12.5\%$ profit

→ A ~~dealer~~ dealer used 1200 gms instead of 1100 gms. Find profit or loss %?

CP[1200] = SP[1100]

$\frac{CP}{SP} = \frac{11}{12}$.

Loss = $\frac{1}{11} \times 100 = 9.99\%$ loss

Maybe wrong

Pg: 60 / 610

Inv = $150 \times 250 + 2500$

$= 160 \times 250$

SP $= 320 \times 150$

$= 160 \times 2 \times 150$

5% discount $\Rightarrow \frac{95}{100} \times 160 \times 2 \times 150$

$$\text{Profit} = \frac{\frac{95}{100} \times 320 \times 150 - 160 \times 250}{160 \times 250} \times 100$$

$$= \frac{\frac{19}{95} \times 320 \times 6 - 160 \times 10}{160}$$

$= 114 - 100 = 14\%$

Pg: 60 / 5

~~Cost of each item = 60~~

SP / item = 60

∴ CP / item = 40

Profit / item = 20

~~100~~ to make 1000 profit we need 3000 + 1000 (+ weekly expense, + profit required)

$\frac{4000}{20} = 200$ items

Coding & Decoding [5-times]

A	B	C	D	E	F	G	H	I	J
1	2	3	4	5	6	7	8	9	10
K	L	M	N	O	P	Q	R	S	T
11	12	13	14	15	16	17	18	19	20
U	V	W	X	Y	Z				
21	22	23	24	25	26				

Letter-letter coding:

① DELHI → EFMIJ

PUNE → ? QVOF

② COW → DQZ

MILK → NKOO (+1 +2 +3 +4)

③ TABLE → ATBEL

CHAIR → HCARI

④ ACE → BAH

IES → ? (JCV) (+1, -2, +3, -4 ..)

⑤ DRINK → EQJML

WATER → XZUDS (+1, -1, +1, -1 ....)

⑥ MONKEY → XDJMNL (-1)

TIGER → QDFHS

Pg:20 / 5

AMCF → EQGJ

NKUF → ROYJ

DHLP → ? (HLPT) (+4, +4 ...)

Pg:20 / 6

IMHO → JNIP

IDK → JEL

SO → TP

IDC → JED

Letter - Numbers:

① GATE = 7 - 1 - 20 - 5

IES → ? 9 - 5 - 19

② PSU → 15 - 17 - 18

BSNL → AQKH (-1, -2, -3 ...)

③ BOOK → 43

PEN → ?

2+15+15+11 = 43

P E N

16+5+14 = 35

PEN → 35

④ BOOK − PEN = 8

PEN − NIB = 1

BOOK

2 15 15 11 = 43

P E N

16+5+14 = 35

43 − 35 = 8

PEN − 35

NIB − 14+9+2

= 25

35 − 25 = 10

⑤ $\overset{1\cdot 20}{A}T = 20$

$\overset{2\,|\,20}{BA}T = 40$ (B = 2) × 20

CAT = 60 (C = 3) × 20

RAT = ?

product of numbers

R − 18

18 × 20 = 360

⑥ PEN = 105

BOOK = 172

INK = ?

P+E+N = 35 × 3 (length)

16 5 14

B+O+O+K = 43 × 4 (length)

INK

9+14+11 = 34

34 × 3 = 102

INK − 102

⑦ REASON = 5

BELIEVED = 7

GOVERNMENT = ? (9)

(length − 1)

Statement Coding:

white — black

black — Red

Red — pink

pink — green

green — Yellow

In that language what is color of blood?

i.e. pink

Pg:22
25

MANGO → 31-1-41-7-51

M – 13 – 31
A – 1 – 1
N – 14 – 41
G – 7 – 7
O – 51 – 15

JOHI → 01-12-8-9

Words Coding:

They are Fools ⟶ plane is Risky

We are wise ⟶ Train is Fast

wise never Fools ⟶ Fast always Risky

From ① & ③

Fools – Risky

From ② & ③

Fast – wise

From ③

never – always

From ① & ②

are – is

Sily They – plane

we – Train

Pg:22
24

bi nie pie — some good jokes

nie bat lik — some real stories

pie lik to — many good stories

lik – stories

good – pie

some – nie

∴ jokes – bi

pg: 8

8

```
  1 3 7
  2 7 6
-------
      5
```

7+6 → (13)

but it is given 5

So it might be octal

```
  1 3 7
  2 7 6
-------
  4 3 5
```

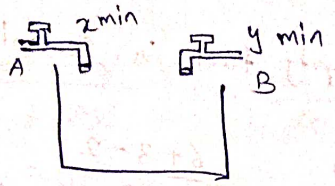
in octal

```
  7 3 1
  6 7 2
-------
1 6 3 3
    2
-------
```

Octal

Pipes & Cisterns:

Type-1:



A — x min

B — y min

A & B — $\frac{xy}{x+y}$ (same as time & work)

Eg: A = 60 min

B = 20 min

Find time taken to fill the tank by A & B together

$\frac{\cancel{60} \times \cancel{30}}{\cancel{90}} = 6\frac{2}{3}$ min

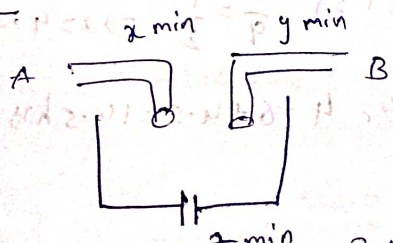
Eg: $\frac{2}{5}$th of ~~tank~~ tank filled by A in 8 min. $\frac{5}{6}$th of tank filled by B in 25 min. Find time taken to fill the tank by A & B together

A ⇒ $\frac{2}{5}$ ——— 8 ⇒ 1 ——— 20 min

B ⇒ $\frac{5}{6}$ ——— 25 ⇒ 1 ——— 30 min

A & B ⇒ $\frac{\overset{4}{\cancel{20}} \times \cancel{30}}{\cancel{50}} = 12$ min

Type-2:



Portion of tank filled per unit time

$= \frac{1}{x} + \frac{1}{y} - \frac{1}{z}$

Ex: pipes A & B can fill a tank in 10 & 20 min when opened individually. Pipe C can empty tank in 30 min.

1 min $\longrightarrow$ $\frac{1}{10}+\frac{1}{20}-\frac{1}{30} = \frac{6+3-2}{60} = \frac{7}{60}$

Time taken to fill $= \frac{60}{7} = 8\frac{4}{7}$ min

Pg:52 / 3

1 hour $\longrightarrow$ $\frac{1}{5}+\frac{1}{4}-\frac{1}{20} = \frac{4+5-1}{20} = \frac{8}{20}$

time to fill tank $\longrightarrow$ $\frac{20}{8} = 2.5$ hours

Pg:52 / 5

$4\left(\frac{1}{6}-\frac{1}{12}\right)+6\left(\frac{1}{9}-\frac{1}{12}\right)+x\left(\frac{1}{9}\right)=1$

$\frac{4}{12}+\frac{4-3}{36}+\frac{x}{9}=1$

$\frac{4}{12}+\frac{1}{6}+\frac{x}{9}=1$

$\frac{x}{9}=1-\frac{5}{6}=\frac{1}{6} \Rightarrow x=1.5$

(or) $\frac{4+2}{12}+\frac{x}{9}=1 \Rightarrow \frac{x}{9}=\frac{1}{2} \Rightarrow x=4.5$

i.e., $4+6+4.5=14.5$ hrs

(8) Pg:52 / 7

$8\left(\frac{1}{12}+\frac{1}{15}\right)+x\left(\frac{1}{15}\right)=1$

$\frac{9}{20}+\frac{x}{15}=1 \Rightarrow \frac{x}{15}=\frac{11}{20} \Rightarrow x=\frac{33}{4}=8\frac{1}{4}$ min

$= 8$ min 15 sec

opener 52

pg: 52
8

$$10\left(\frac{1}{15}+\frac{1}{20}+\frac{1}{25}\right)+x\left(\frac{1}{15}+\frac{1}{20}\right)=1$$

$$10\left(\frac{20+15+8}{300}\right)+x\,\frac{4+3}{60}=1$$

$$\frac{43}{30}+\frac{27}{30}+\frac{7x}{60}=1$$

$$2+\frac{7x}{60}=1$$

$$x\,\frac{7}{60}=\frac{3}{30}$$

$$x=\frac{6}{7}\text{ hrs}$$

pg: 52
10

$$15\left(\frac{1}{20}+\frac{1}{24}-\frac{1}{30}\right)=$$

$$15\left(\frac{1}{20}+\frac{1}{24}-\frac{1}{C}\right)=1$$

$$\frac{1}{20}+\frac{1}{24}-\frac{1}{C}=\frac{1}{15}$$

$$\frac{1}{20}+\frac{1}{24}-\frac{1}{15}=\frac{1}{C}$$

$$\frac{6+5-8}{120}$$

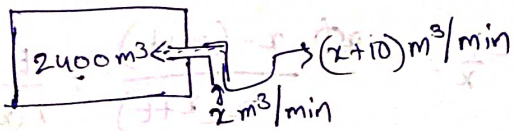
$$\frac{1}{40}=\frac{1}{C}$$

$$C=40\text{ min}$$

40×3 gallons

$=120$ gallons

pg: 52
9



filling time − emptying time = 8

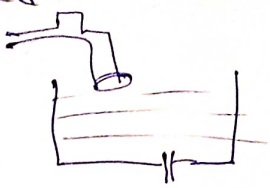
$$\frac{2400}{x}-\frac{2400}{(x+10)}=8$$

option verification

$$x=50$$

Note:

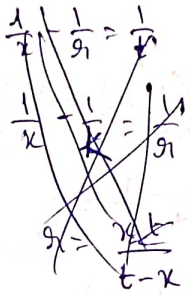
→ A pipe can fill the tank in x hours but due to the leakage it can take t hours more to fill the tank. Then time taken to empty the tank when leakage is considered.



y = time taken to empty

$$(x+t)\left(\frac{1}{x}-\frac{1}{y}\right)=1$$

t → max time



$$\frac{1}{x}-\frac{1}{y}=\frac{1}{x+t}$$

$$\frac{1}{x}-\frac{1}{x+t}=\frac{1}{y}$$

$$\frac{x+t-x}{x(x+t)}=\frac{1}{y}$$

$$\boxed{y=\frac{x(x+t)}{t}}$$

(or)

A → x hrs

A & B → $x+t$ hrs

A → $\frac{1}{x}$

A & B → $\frac{1}{x+t}$

B ⇒ $\frac{1}{x+t}-\frac{1}{x}=\frac{x-(x+t)}{x(x+t)}=\frac{-t}{x(x+t)}$

i.e., $\frac{x(x+t)}{t}$ hrs

Eg: A pipe can fill the tank in 5 hrs. But due to the leakage it can take 1 hr more to fill the tank. Find time taken to empty the tank. Sol:

Eg: A pipe can fill the tank in 10 hrs. Due to the leakage it can fill it in 12 hrs. Find time to leakage

$12 \longrightarrow 2 \times \frac{1}{10}$ leaked

? — 60 min — 1/1

Analyze shortcut

Percentages: [10-times]

-/. increase
-/. decrease
} $\frac{\text{Diff}}{\text{original}} \times 100$

Eg: A number increased from 1400 to 1500 find /. of increase

$$\frac{100}{1400} \times 100 = \frac{100}{14} = \frac{50}{7} = 7.14\ /.$$

Pg: 57 / 2

$$\frac{7}{10} \times 400$$

(294/2)

$$\frac{147}{1400} \times 100 = 10.5\ /.$$

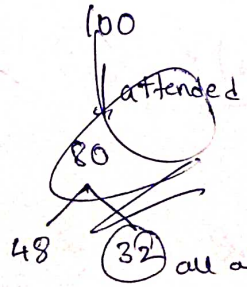
Pg: 59 / 16

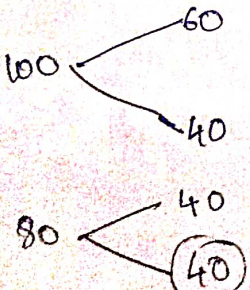
X ⟶ 300
Y ⟶ 100

taller than 6ft in X — 3
" " " " Y — 2

$$\frac{5}{400} \times 100 = 1.25$$

Pg: 59 / 17





∴ 1 : 1

2 x 25 → 1000

pg: 58 / 15

100
↓
50 infected
15 (30%) 35 not developed disease

↑ increase

pg: 59 / 19

$m + \frac{xm}{100}$, $n + \frac{yn}{100}$

$\frac{m}{n} = P$

$$P' = \frac{m + \frac{xm}{100}}{n + \frac{yn}{100}} = \frac{(100+x)m}{(100+y)n} = \frac{(100+x)P}{100+y}$$

% ↑ in P $\quad \frac{\frac{(100+x)P}{100+y} - P}{P} \times 100$

$$= \frac{(100+x-100-y)}{100+y} \times 100$$

$$= \frac{(x-y)100}{100+y}$$

OR

2001 → 2011

100 $\xrightarrow{+x\%}$ 100+x

100 $\xrightarrow{+y\%}$ 100+y

$P = \frac{x}{y} = 1$

$P' = \frac{100+x}{100+y}$

% ↑ in $P = \frac{P'-P}{P} \times 100 = \frac{100+x}{100+y} - 1 \times 100$

$= \frac{x-y}{100+y} \times 100$

Pg: 58 / 8

	GDP	Fiscal deficit
2015 =>	100	4 (4%)
2016 =>	110	5.5 (5%)

$$\frac{1.5}{4} \times 100 = \frac{150}{4} = 37.5\ \%$$

Note:

→ If a number keeps on increasing/decreasing at constant intervals at a constant percentage of r per unit time. Population P after n time intervals

$$P = P\left[1 \pm \frac{r}{100}\right]^{n}$$

Eg: In 2015, population is 10,000 which increased by 10% per ever 1 year. Find population after 3 years.

$$10000\left[1 + \frac{10}{100}\right]^3 = 10000\left[\frac{11^3}{10^3}\right]$$

$$= 10 \times 11^3 = 13310$$

Note:

→ Different timings and different rate percentage given

$$P = P\left[1 \pm \frac{r_1}{100}\right]^{t_1} * \left[1 \pm \frac{r_2}{100}\right]^{t_2} * \ldots$$

Pg: 58 / 10

$$P = 10000\left[\frac{11}{10}\right]\left(\frac{8}{10}\right)\left(\frac{13}{10}\right)$$

$$= 0.8800 \times 13$$

$$= 10400 \quad 6400$$

$$= 11440$$

88×13

264

88

1144

1144

Machine Concept:

Q: present cost of bike is ₹ 50,000/- which decreased by 20% per every year. find cost of the bike after 2 years

$$5 \times 10^4 \left(\frac{8}{10}\right)\left(\frac{8}{10}\right)$$

$$320 \times 10^2 = 32,000$$

Q: Income of mahesh is 5000. He spent 10% on rent. 20% of Remaining for food, 30% of remaining for expenditure.

Remaining are his savings?

$$5000 \times \frac{9}{10} \times \frac{8}{10} \times \frac{7}{10}$$

$$360 \times 7 = 2520$$

Pg:57 / 6

$$184 \text{ pg} \times \frac{7}{\cancel{10}} \times \frac{\cancel{16}^{8}}{\cancel{10}} \times \frac{1}{\cancel{2}}$$

$$\begin{array}{r} 184 \times 21 \\ \hline 184 \\ 368 \\ \hline 3864 \end{array}$$

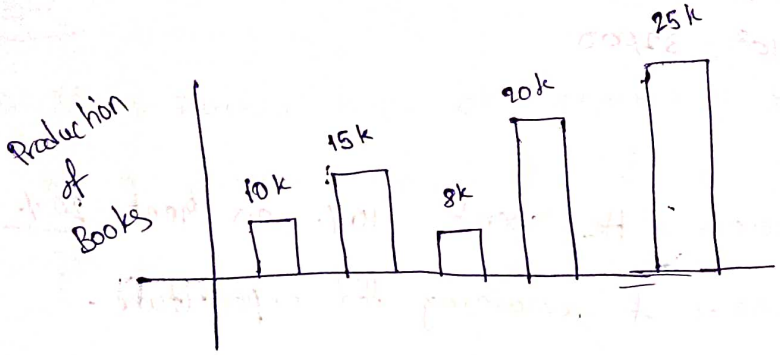
Pg:57 / 7

$$2 \frac{9}{10} \frac{\cancel{85}^{5}}{\cancel{100}} \frac{9}{10} = \frac{\cancel{153}^{51}}{\cancel{5}^{1}}$$

$$2 = \frac{\cancel{16}^{2} \times 10^3}{\cancel{8}} = 2000$$

Data Interpretation:

- Pie-Chart
- Bar-chart
- Table-chart
- Line-chart



Types of question asked

① % ↑ } $\frac{\text{diff}}{\text{original}} \times 100$
% ↓ }

② Avg

③ Ratio

④ Max, min

⑤ % of x in y = $\frac{x}{y} \times 100$

⑥ Convertion

⑦ Based on given information

Pie-Chart:

If pie-chart is divided into degrees then total circle is 360°

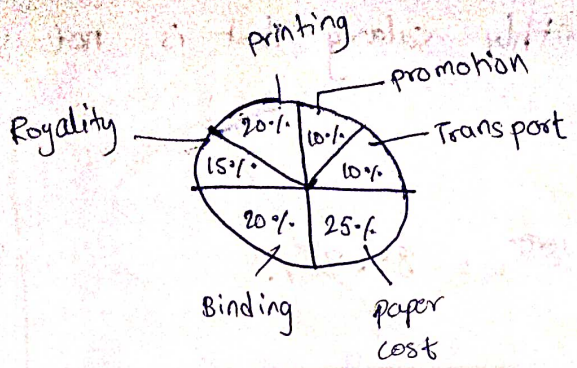
If pie-chart is divided into percentage then total circle is 100%

If " " " fraction " " " " 1

1% = 18/5°

1° = 5/18 %

Eg:



① what is the central angle corresponding to royalty?

$$15 \times \frac{18}{5} = 54^\circ$$

② The publisher has to pay 30600 as a printing cost. what is cost of royalty on those books

20% — 30600

15% — 22,950/-

③ The cost of book is marked 20% above the Cost price.

If label price is Rs. 180. Find paper cost for single copy.

$$120\% \, x = 180$$

$$\frac{6}{5} x = 180$$

$$x = 150$$

paper cost = 25% (150) = 37.5

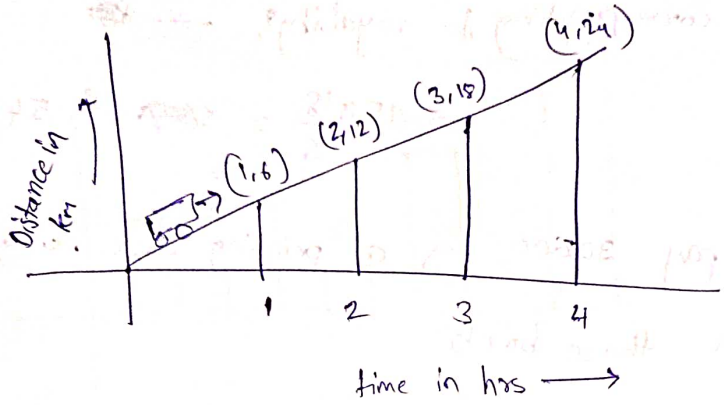
Eg:

Category	Amount
Rent	4000
Food	2000
Clothing	1800
Savings	1500
Other	1200

(i) What approximate % of monthly salary that is not spent on savings!

$$\frac{\cancel{9000}^{600}}{\cancel{10,500}_{7}} \times 100 = 85.7\%$$

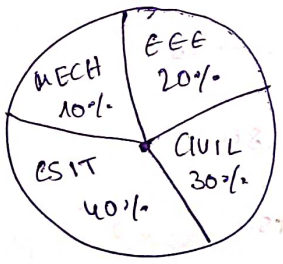
Eg:



- A car travels according to given path. Find speed of the car

(1,6) i.e., 6 km/hr

Eg:



The proportion of the male to female students in each department is 5:4. There are 40 male students in electrical department. Find diff b/w female students in civil department and female in mechanical department.

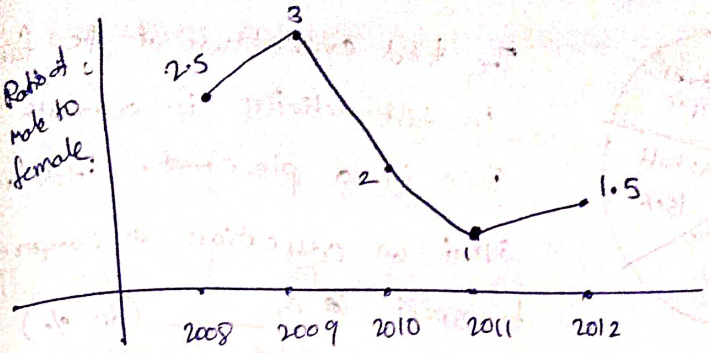
40 male — 32 female

(20%)

30% — 48

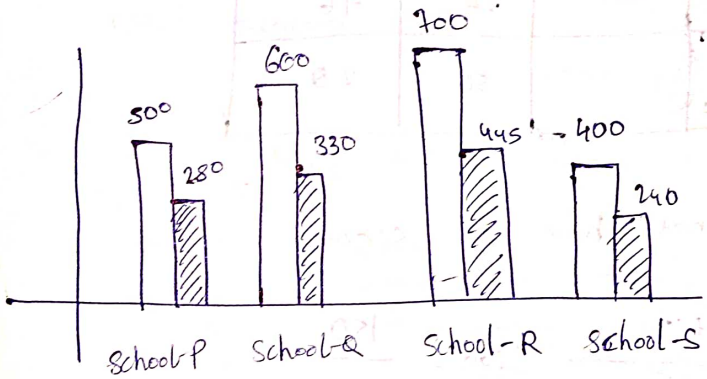
10% — 16

48 − 16 = 32



In a college the ratio of the male to female students are plotted in given line chart. If the female students 2011 and 2012 are equal, find ratio of the male students in 2012 to male students in 2012

i.e., 1.5

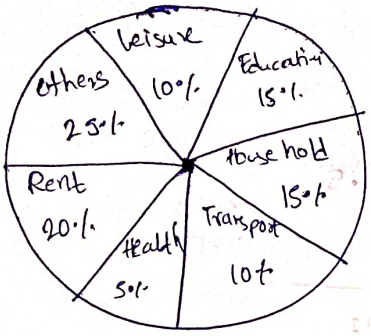


The given bar-chart shows the data of the students who are appeared and passed in an examination for four different schools P, Q, R & S. The average of sucess rates (% wise) of these 4 schools is?

56%, 55%, 63.51, 60%

$$\frac{56+55+63.5+60}{4} = 14+15+13.75+15.75 = 58.5 \approx 58.64$$

Eg: 2



The total expenditure of the family in diff activities in a month is given in a pie chart. The extra money spent on education as compared to transport is _____ (in %).

15 − 10 = 5%

Eg:

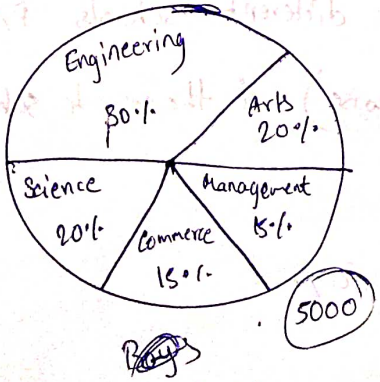
		Men	Women
Own vehicle	Car	40	~~30~~ 34
	Scooter	36	50
	Both	24	46
Does not have own vehicle		50	20

what % people does not have own scooter

$$\frac{74+76}{74+70+86+70} \times 100 = \frac{150}{}$$

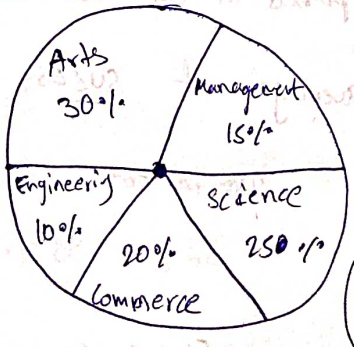
$$\frac{70+74}{74+86+70+70} \times 100 = \frac{144}{300} \times 100 = 48\%$$

Eg: (2020)



5000

Boys & Girls



1500 students

Girls

The two pie-charts given below show the data of total students and only girls registered in different ~~state~~ streams.

Total no of students = 5000

" no of girls = 1500

The ratio of boys enrolled in Arts to girls enrolled in management ~~boys~~ is :

Sol:-

girls in management

$$15\%\ (1500) = 225$$

girls in Arts

$$30\%\ (1500) = 450$$

total students in Arts

$$20\%\ (5000) = 1000$$

boys in Arts $= 1000 - 450 = 550$

$$\frac{550}{225} = \frac{22}{9} = 2.\overline{4}$$

Pg:69

Pg:70 / 81

P	Q	R	S	T
↓	↓	↓	↓	↓
20	25	25	25	5

150 (210)

600 —(+15%)→ 690

S (25) — 60%

10 — 40%

∴ S — 35%

In S

$600 \times \frac{25}{100} = 150$ —(40% (↑))→ $150 \times \frac{40}{100} = 60$ → 210 [150 + 60 (↑)]

T = 5%

$600 \times \frac{5}{100} = 30$ → 60 → 30 (old) + 30 (↑)

Units Digit for $(\Sigma n!)$:

→ no of ~~zeros~~ zeroes at the end of $n!$ result

$10! \Rightarrow 1\times2\times3\times4\times\underline{5}\times6\times7\times8\times9\times(\underline{10}\times2)$

no of 5's = 2

$\therefore$ 2 zeroes

$$n! = \left(\frac{n}{5} + \frac{n}{5^2} + \frac{n}{5^3} + \dots\right)$$

we need Quotients

$$10! = \left(\frac{10}{5} + \frac{10}{5^2} + \frac{10}{5^3} + \dots\right)$$

$$= 2$$

$$128! = \frac{128}{5} + \frac{128}{5^2} + \frac{128}{5^3} + \frac{128}{5^4} + \dots$$

$$= 25 + 5 + 1 + 0$$

$$= 31 \text{ zeroes}$$

~~1000!~~ =

(2014)

$$1000! = \frac{1000}{5} + \frac{1000}{5^2} + \frac{1000}{5^3} + \frac{1000}{5^4} + \dots$$

$$= 200 + 40 + 8 + 1$$

$$= 249$$

→ unit digit for $(\Sigma n!)$

* $1! = 1$
* $1! + 2! = 1 + 2 = 3$
* $1! + 2! + 3! = 1 + 2 + 6 = 9$
* $1! + 2! + 3! + 4! = 1 + 2 + 6 + 24 = 33$

* $\sum 5! = [1! + 2! + 3! + 4!] + 5!$

$8 + 0 = 3_{,,}$

$\sum 6! = [1! + 2! + 3! + 4!] + 5! + 6!$

$= 3 + 0 + 0$

$= 3$

$\sum 100! = 3$

01/03/20

English-3

Verbal Ability syllabus:

(i) Correct usage of parts of speech (functional grammar)

(ii) Vocabulary

(iii) Analogy

(iv) Sentence Completion

(v) Homonyms (pairs that snare)

(vi) Critical Reasoning (CR)

Functional Grammar:

Noun

Rule 1:

Eg: She has captured several ~~sceneries~~ with her new cam.

pictures of scenery

→ items of news furniture

→ persons of gentry

→ children of offsprings

→ letters of alphabet

→ pieces of advice information

→ members of faculty

→ bags of luggage.

→ items of luggage

Eg: My teacher gave me two ~~advices~~.

pieces of advice.

Rule 2:

Rule 3:

She has washed ~~a pant~~.

a pair of ~~paint~~ pants.

She has broken four ~~spectacles~~.

Pairs of sepetacles.

Rule 4:

mumphs is/are curable now.

statistics was/were a part of economics.

mumphs and measels have/has been cured.

Rule 5:

Finally the police has/have involved in the matter.

The army personnel is/are coming here.

Rule 6:

→ There are three persons/person's in charge above me in my office.

→ I have to take my person's-in-charge/person-in-charge's permission.

Rule 7:

→ A five miles journey
mile

→ A six days seminar
day

Rule: 8

→ The decision of the king is mandatory
The king's decision is mandatory
} both are right but the latter is better

→ The tower's top is very sharp

→ The cycle's bell is ringing soundly.
the bell of the cycle

the iron
g: Irons of west bengal, chattisgarh, bihar are very rich in quality

Devasena was kept in irons. ✓

air
The earth is surrounded by airs. ✗

After 6 P.M school children shold be left in airs ✓

Rule 14:

hive
she has thrown a stone at a bunch of bees

she has met the class of lectures
faculty

Rule 15:

nobody has/have helped me.

somebody was/were waiting for you.

Rule 16:

I visited kamal's and ~~the~~ vimala's house yesterday.
kamal

I visited kamal's and vimala's houses yesterday

Rule 17:

She has left four kerchiefs/kerchieves.

Vocabulary:

Identification of parts of speech:

noun: ce, ment, ness, tion, sion, ny, nary, in, ~~sg~~ sy, ist, ry, by, t, g, sident, ia, carum
thy, phy

adj: ful, less, ent, ant, eve, ic, id, ib, ble, ous, al, ar, nd

verb: se, ze, te, es, re, e, ending, fy

adverb: 'ly'

(2020) people were prohibited _____ their vehicles .....

a) from parking
b) parking
c) to park
d) to have parked

(2020) ~~Do, undo~~ ~~just like that~~ trust, _____

(i) untrust
(ii) distrust
(iii) entrust
(iv) in trust

(2020) Dear sir, please treat this as the chairman sir's personal invitation, come with family and success the party.

Chairman's
as
make the party successfull

~~gynaecologist~~
gynaecologist

Experiment

Happiness

Eradicate

Utilize

Inexplicable

amalgamate

Frequency

Circuitous

→ Mitigation

→ Ferocious

→ Judicious

→ Majestic

→ Amiable

→ antidote (verb) & (noun)

→ Reincarnation

→ Antipathy

→ panacea

→ Mollify

→ Fortitude (V & n)

rsonal invitation,

Pronoun

personal

Reciprocal

Relative

distributive

demonstrative

Reflexive

Rule 18:

Ex: J.D. Lakshmi-Narayana has ~~resigned~~ resigned to Jena Sena.
resigned himself

Ex: Pavan absented for aptitude class last week.
absented himself

Rule 19:

→ One of the dancers/dancer is/are missing

→ None of the ~~politician~~ politicians is honest.

Rule 20:

None of the doctors don't work for free. ——— Doubt

Neither of the two boys is not fit for the job.

Rule 21:

Its I/me.

I love you
- I too

08-02-20

Rule 22:

One of the boxes is/are missing.

Economics and Civics is/are not interlinked.

Each of the boys have/has given ~~bo~~ blood.

Nobody wants/want do be arrested.

~~Rahul/Ravi~~

☆ Rahul or Ravi was/were answerable.

Rule 23:

Mahesh can breake/broke records with latest movie.

His father krishna could break/broke ~~rec~~ records with ASR in those days.

Rule 24:

appoint
elect — ASX
consider

appoint - recruit as
elect - choose as
consider - treat as

Shivagami has appointed katappa (as) a chief commander.

Bijjaladev has recruited katappa as a slave. ✓

Americans have elected trump (as) the president.

Majority of Indians have choosen Modi as the Prime Minister. ✓

Sivagami has considered Bahubali (as) a commander.

Sivagami has treated Bahubali as a commander. ✓

Rule 25:

help ✓
make — to ✗
let

My friend swims to keep healthy. ✓

My friends helped me (to) bail me out.

Ravi earned to spend. ✓

Ravi made me (to) laugh by tickling.

Mr. Sharma reached airport to ~~bear~~ catch the flight. ✓

Mr. Sharma has let me (to) catch the flight.

Rule 26:

~~Ravi~~ Arundathi said to Pasupathi, "You can't anything to me"

Arundathi (told/said to) Pasupathi that he (could not/can not) to her.

Direct	Indirect
Said	said
Said to	told

Padmasree said that inspecter Ajay (has / had) used him cleverly.

Past tense is present always in indirect speech

Rule 27:

My science teacher said "The Earth is round in shape".

My science teacher said that the earth is/was round in shape.

proverbs
Monument or locations
Scientific facts
Universal truths
facts
say / says
relations
} present tense even in reported speech.

Prabhu told me that honesty is/was the best policy.

→ My grandfather said he was (is) busy then.

→ My grandfather said that the Taj is/was in Agra.

Rule 28:

She has shutted (shut) down the system.

He has hurted (hurt) her.

Some uniform verbs:

telecast, broadcast, forecast, cut, put, hurt, shut

Rule 29:

The hen has lied (laid) twice today.

The little girl has hanged (hung) the colander to the wall.

Veerappan had fallen a large number of sandal woods before he died.

My uncle looses (loses) his money during playing cards.

My cousin has raised (raged) her father by her rude behaviours.

The Britishers hung (hanged) Bhagat Singh like patriots before 1947.

The wound is soaring (coring) I can't bare (bear) it.

The girl has laid (lied) with me. several times therefore she is a liar

The naughty boy has seen sawed the beautiful girl with his sharp eye sight.

My sister has learned (learnt) kathakali.

The IT personnel have ridden (raided) several film heroines at residences and office at a time.