EPFL
School of Computer and Communication Sciences
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github.com/epfml/OptML_course

Problem Set 7, due April 12, 2019 (Newton)

Non-convex

Solve Exercises 35, 36, 37 from the lecture notes. These exercises are carried over from last week.

Newton's Method

Solve Exercises 43, 45 from the lecture notes.

Quasi-Newton Methods

Exercise 48. *Consider a step of the secant method:*

$$x_{t+1} = x_t - f(x_t)\frac{x_t - x_{t-1}}{f(x_t) - f(x_{t-1})}, \quad t \geq 1.$$

Assuming that $x_t \neq x_{t-1}$ and $f(x_t) \neq f(x_{t-1})$, prove that the line through the two points $(x_{t-1}, f(x_{t-1}))$ and $(x_t, f(x_t))$ intersects the x-axis at the point $x = x_{t+1}$.

Solution: Let the line be $y = ax + b$. Then we have

$$\begin{aligned} f(x_t) &= ax_t + b, \\ f(x_{t-1}) &= ax_{t-1} + b. \end{aligned}$$

Subtracting the two equations yields

$$a = \frac{f(x_t) - f(x_{t-1})}{x_t - x_{t-1}}.$$

To compute the intersection with the x-axis, we need to solve

$$0 = ax + b.$$

Subtracting from this the first of the previous two equations yields

$$-f(x_t) = a(x - x_t) \quad \Leftrightarrow \quad x = x_t - f(x_t)a^{-1} = x_t - f(x_t)\frac{x_t - x_{t-1}}{f(x_t) - f(x_{t-1})}.$$

By definition of the secant method, $x = x_{t+1}$.

Fixed Point Iteration

The Jupyter notebook in `template/` contains the solution from Lab 03's exercise on fixed point iteration. Please complete the notebook and adapt the algorithm to use Newton updates.