HW2-236079

March 18, 2019

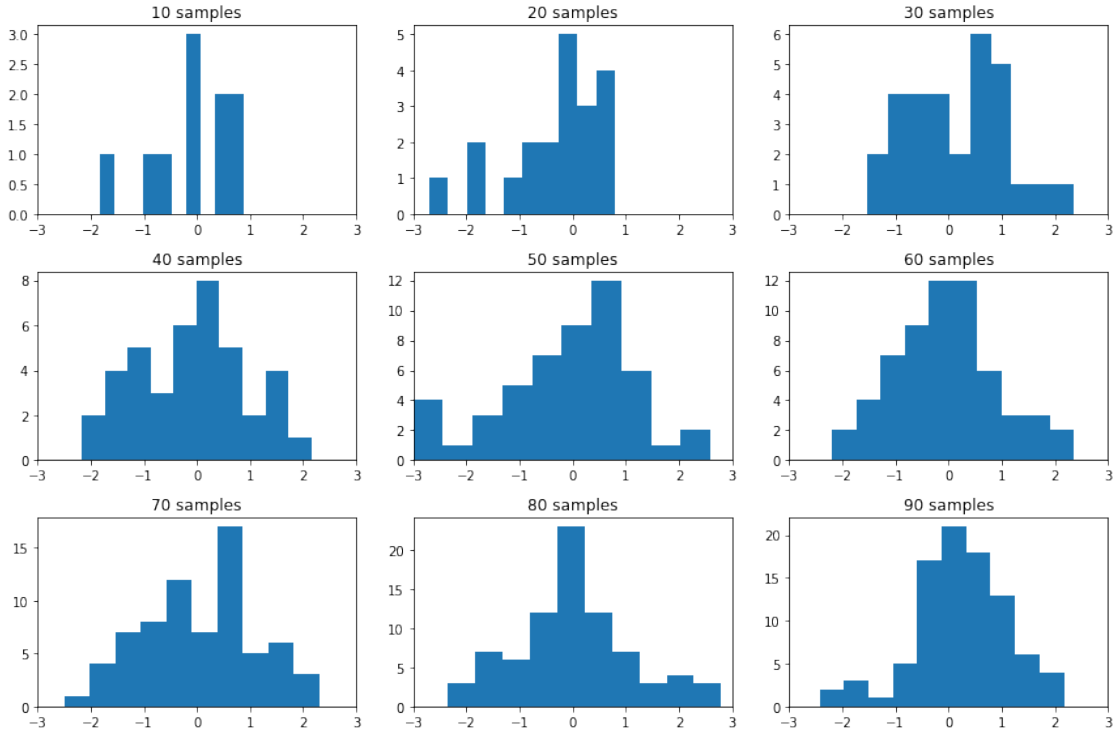
```
In [1]: import numpy as np
        import matplotlib.pyplot as plt
        import time
```

1 Question 1

1.1 Problem 1

```
In [2]: from plots import normal_plot
```

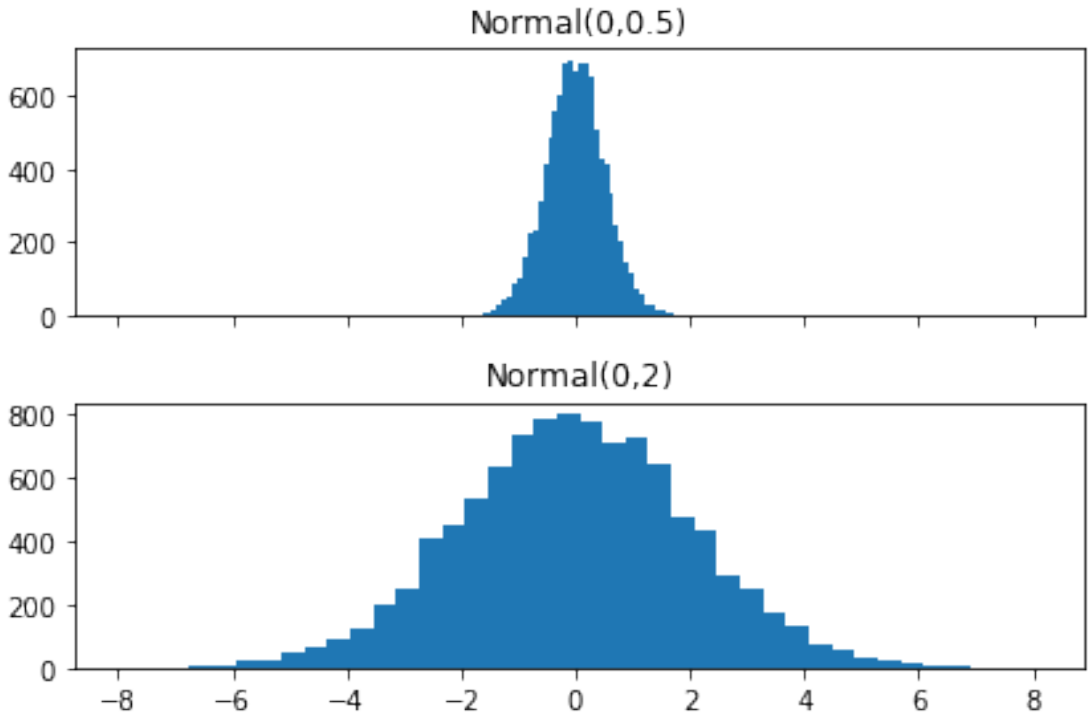
```
In [3]: fig = plt.figure(figsize=(12,8))
        for i in range(1,10):
            fig.add_subplot(3,3,i)
            normal_plot(i*10)
        plt.tight_layout()
```



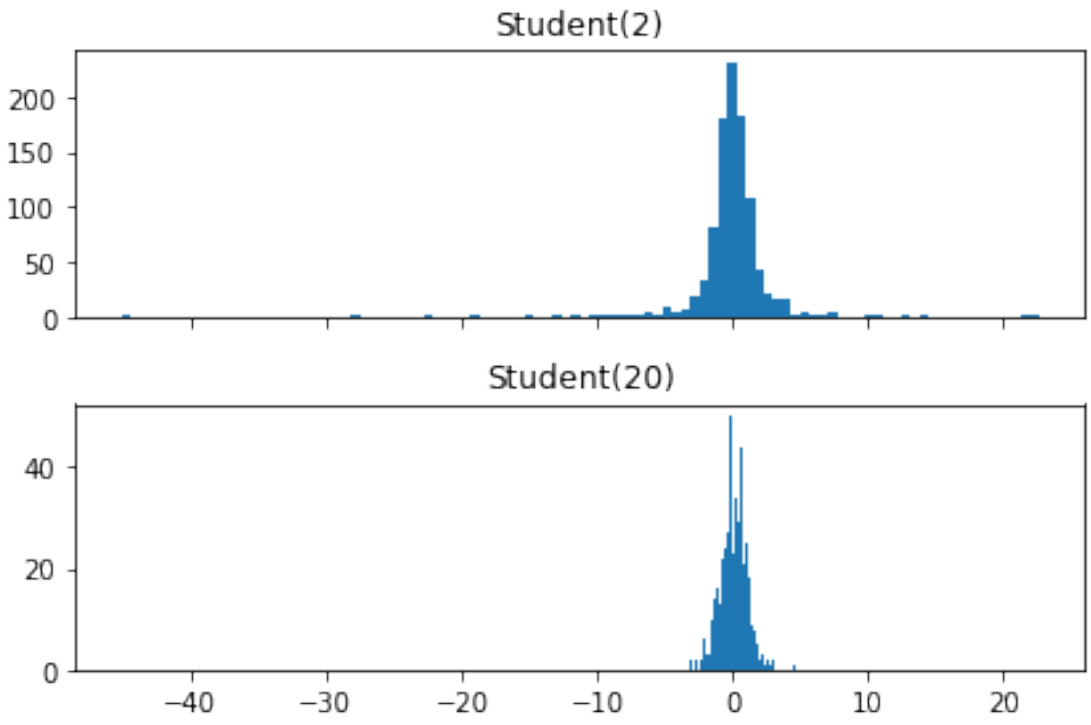
1.2 Problem 2

```
In [4]: from plots import plot_distributions
```

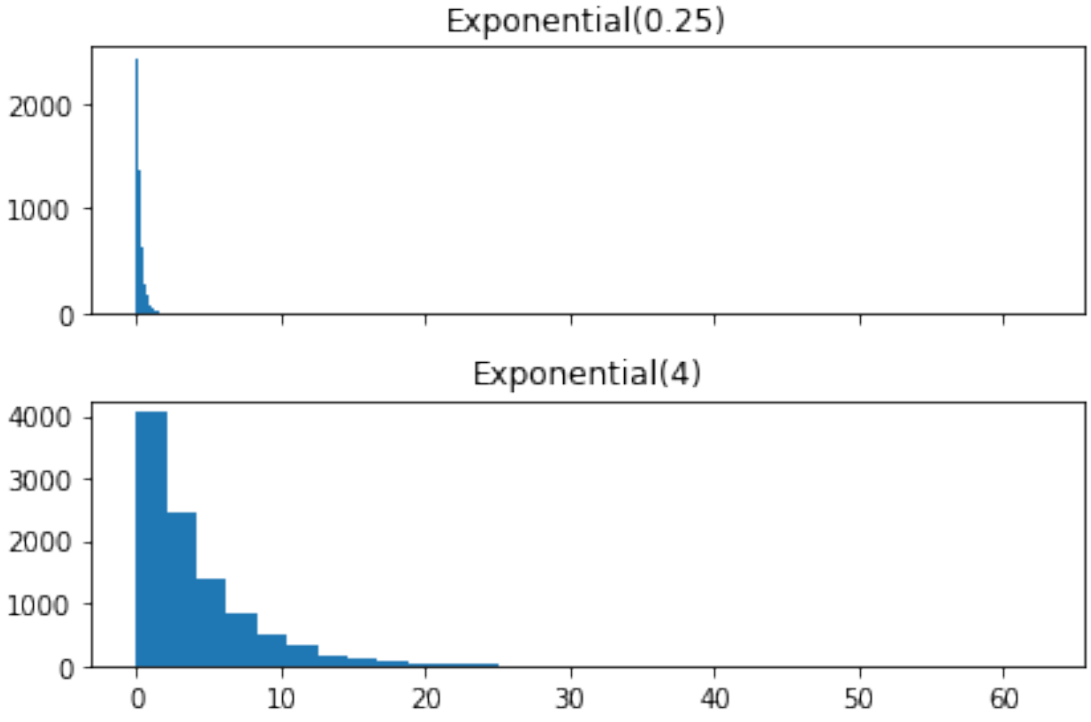
```
In [5]: plot_distributions(np.random.normal(0, 0.5, 10000),
                           np.random.normal(0, 2, 10000),
                          "Normal(0,0.5)", "Normal(0,2)", 40)
```



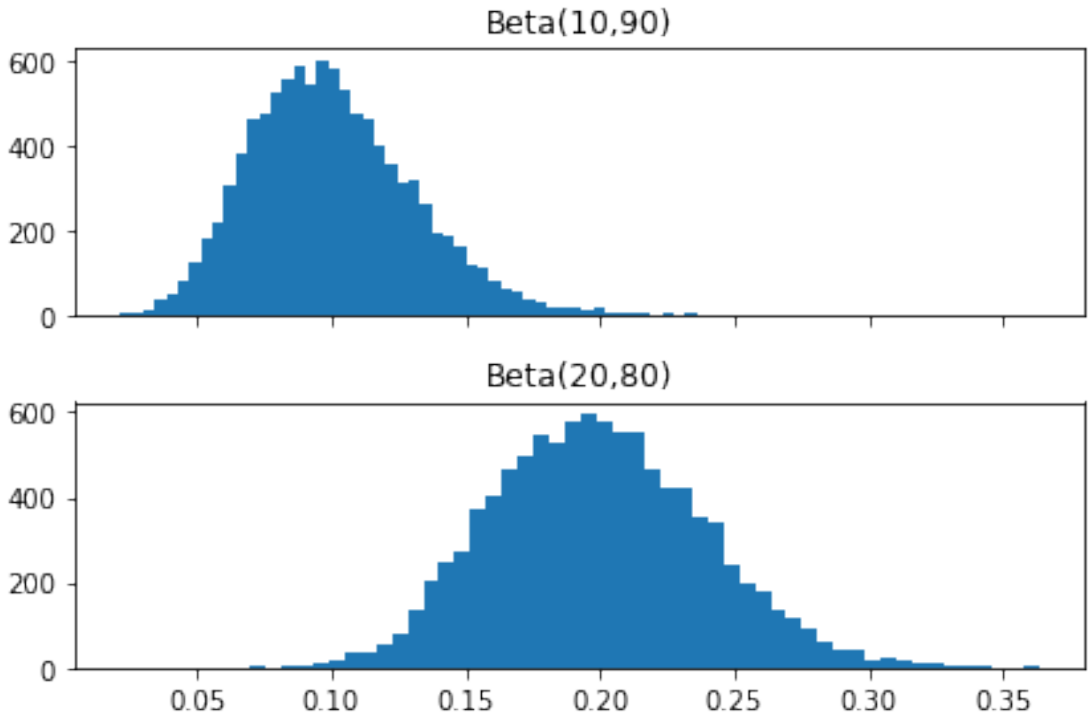

```
In [6]: plot_distributions(np.random.standard_t(2, 1000),
                           np.random.standard_t(20, 1000),
                           "Student(2)", "Student(20)", 100)
```



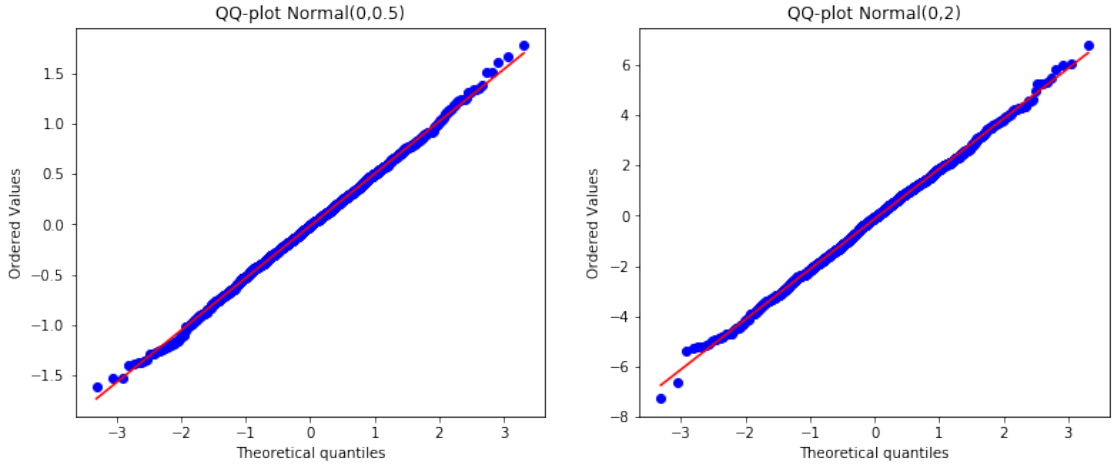
In [7]:
```
plot_distributions(np.random.exponential(0.25, 10000),
                   np.random.exponential(4, 10000),
                   "Exponential(0.25)", "Exponential(4)", 30)
```



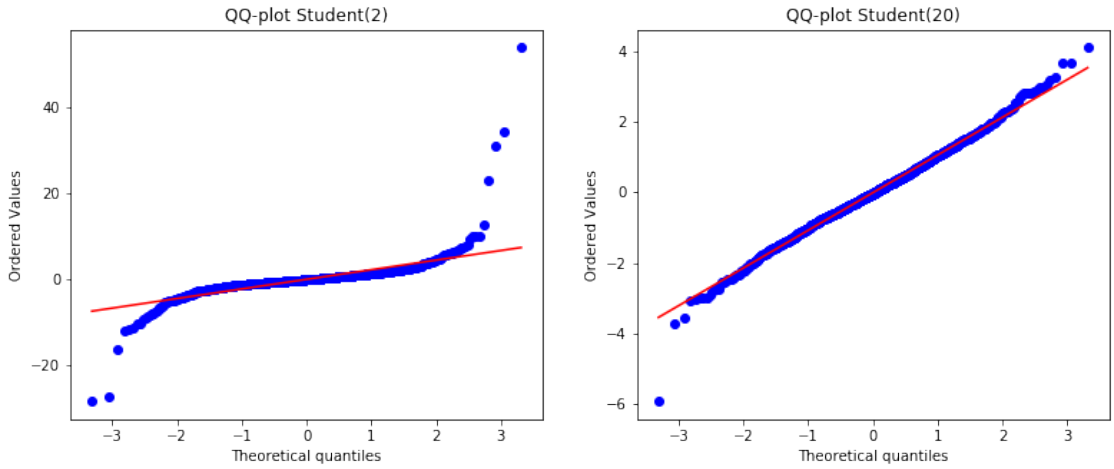
```
In [8]: plot_distributions(np.random.beta(10,90, 10000),
                           np.random.beta(20,80, 10000),
                          "Beta(10,90)", "Beta(20,80)", 50)
```



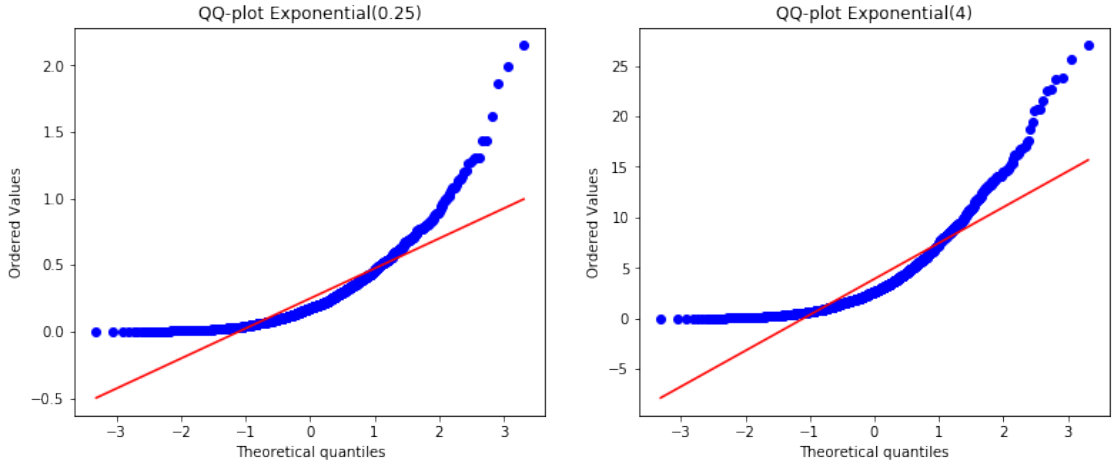

```
In [9]: from plots import qq_plots
        qq_plots(np.random.normal(0, 0.5, 1500),
                 np.random.normal(0, 2, 1500),
                 "Normal(0,0.5)", "Normal(0,2)")
```



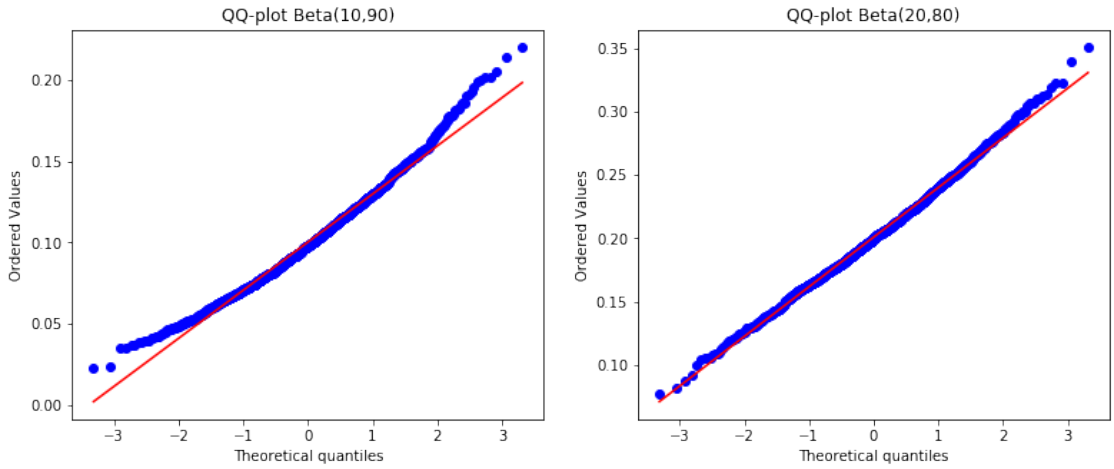
```
In [10]: qq_plots(np.random.standard_t(2, 1500),
                  np.random.standard_t(20, 1500),
                  "Student(2)", "Student(20)")
```




```
In [11]: qq_plots(np.random.exponential(0.25, 1500),
                  np.random.exponential(4, 1500),
                 "Exponential(0.25)", "Exponential(4)")
```




```
In [12]: qq_plots(np.random.beta(10,90, 1500),
                  np.random.beta(20,80, 1500),
                 "Beta(10,90)", "Beta(20,80)")
```



1.2.1 Interpretation

A QQ-Plot shows the correlation between the theoretical quantiles and the actual values. Here, the theoretical quantiles are taken from a standard normal. This means that the more the samples (blue) follow the theoretical line (red), then the distribution is normal. If they are different, here is how to interprete:

S-Shape ?

U-Shape A U-shape shows the sampeled distribution is right-skewed (and so an `n-shape` shows a left-skewed distribution). This is coherent with the QQ plot of the exponential distribution: exponential is heavily right-skewed (by definition) and thus the plot

2 Question 2

2.1 Problem 3

```
In [13]: from helpers import compute_waypoints
         # Configuration
         S = 236097
         params = {
             "N" : 100, # people
             "side_l" : 1000, # meters
             "time_limit" : 86400, # seconds, = 1day
             "v_min" :  (0.5 + 0.02*(S%21)),
             "v_max" : (3 + 0.2*(S%11))
         }

In [14]: records = compute_waypoints(params)

In [15]: %timeit compute_waypoints
```

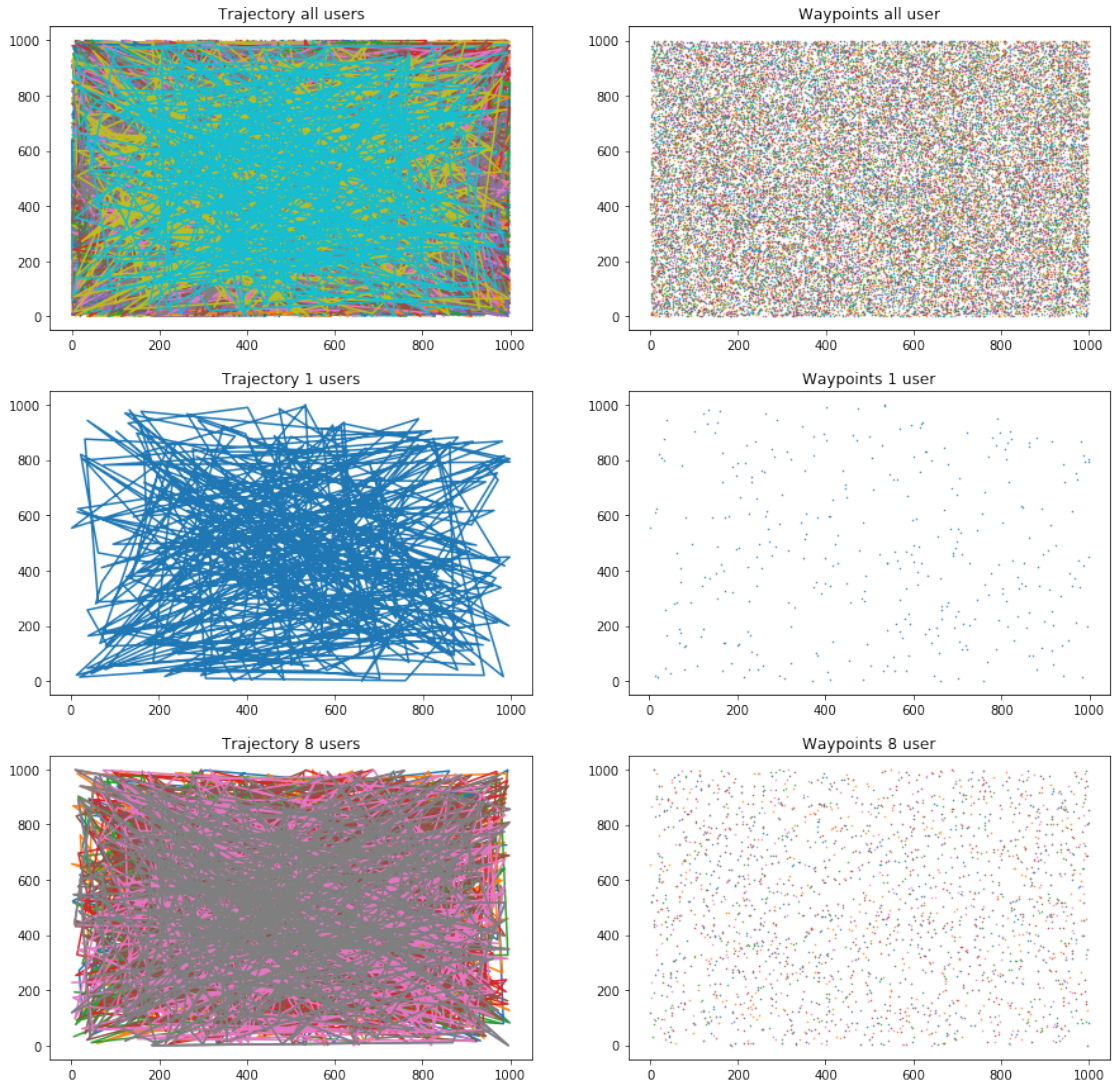
```
18.6 ns ś 0.284 ns per loop (mean ś std. dev. of 7 runs, 100000000 loops each)
```

Running the program for 1 days takes only a few nanoseconds

```
In [16]: num_waypoints_per_mobile = [len(x) for x in records]
         print("Mean:",np.mean(num_waypoints_per_mobile))
         print("Min:",np.min(num_waypoints_per_mobile))
         print("Max:",np.max(num_waypoints_per_mobile))
```

```
Mean: 321.41
Min: 294
Max: 359
```

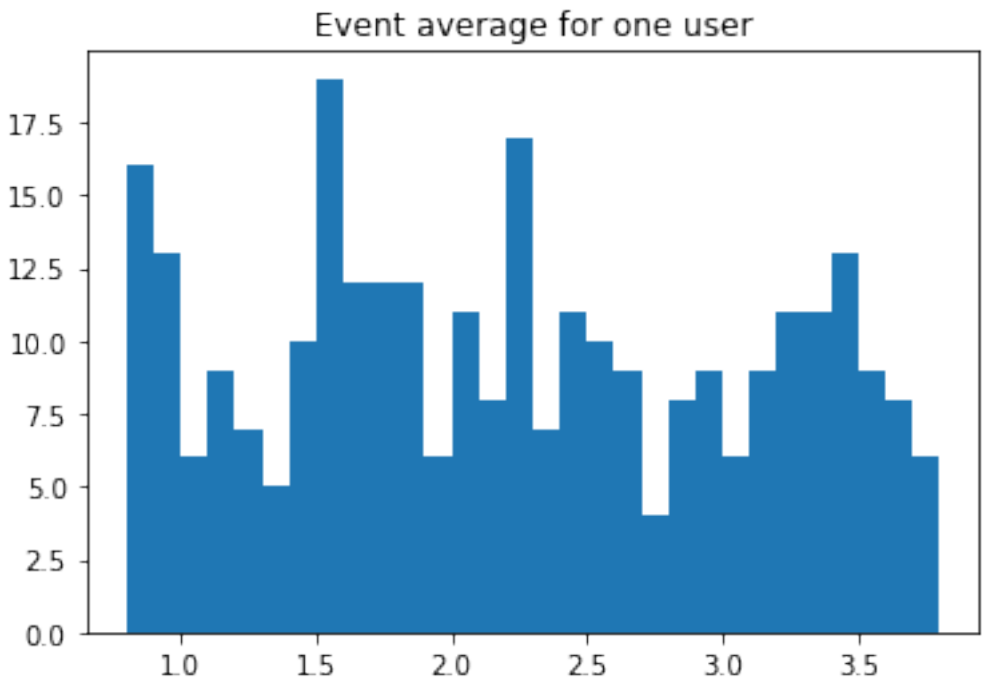
```
In [17]: f, (ax1, ax2, ax3) = plt.subplots(3,2,figsize=(15,15))

         ## plot all
         for rec in records:
             posi_x = ([x for (x,_),_,_ in rec])
             posi_y = ([y for (_,y),_,_ in rec])
             ax1[0].plot(posi_x,posi_y)
             ax1[1].scatter(posi_x, posi_y, s=0.2)
         ax1[0].set_title("Trajectory all users")
         ax1[1].set_title("Waypoints all user")

         # plot 1
         rec = np.random.choice(records)
         posi_x = ([x for (x,_),_,_ in rec])
         posi_y = ([y for (_,y),_,_ in rec])
         ax2[0].plot(posi_x,posi_y)
         ax2[1].scatter(posi_x, posi_y, s=0.2)
         ax2[0].set_title("Trajectory 1 users")
         ax2[1].set_title("Waypoints 1 user")

         # plot 8
         for rec in np.random.choice(records, 8):
             posi_x = ([x for (x,_),_,_ in rec])
             posi_y = ([y for (_,y),_,_ in rec])
             ax3[0].plot(posi_x,posi_y)
             ax3[1].scatter(posi_x, posi_y, s=0.2)
         ax3[0].set_title("Trajectory 8 users")
         ax3[1].set_title("Waypoints 8 user")



         plt.show()
```



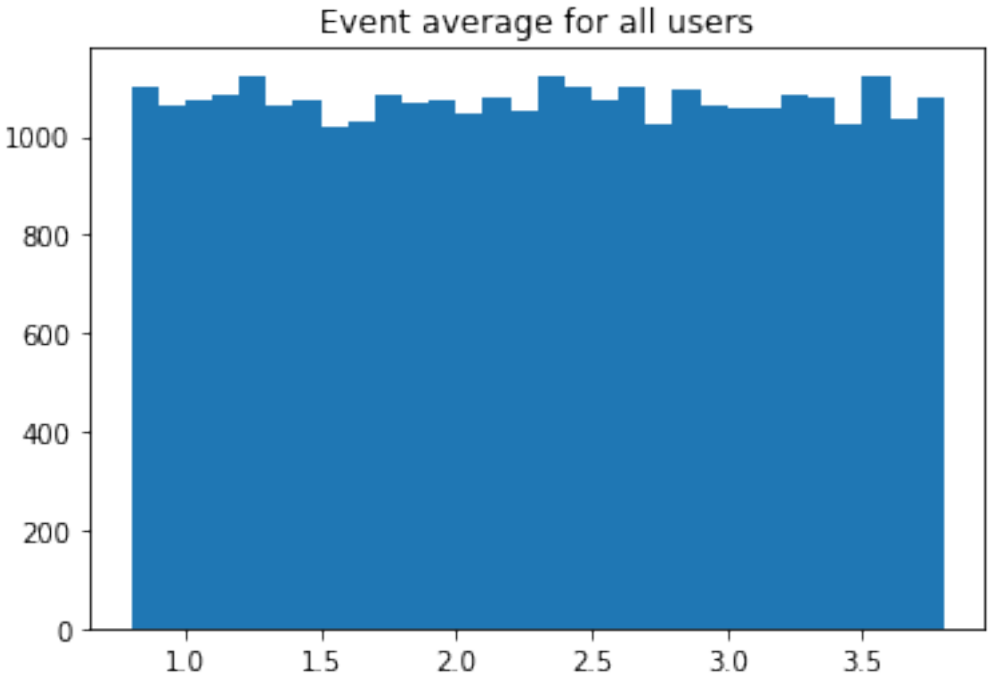
3 Question 3

3.1 Event average viewpoint

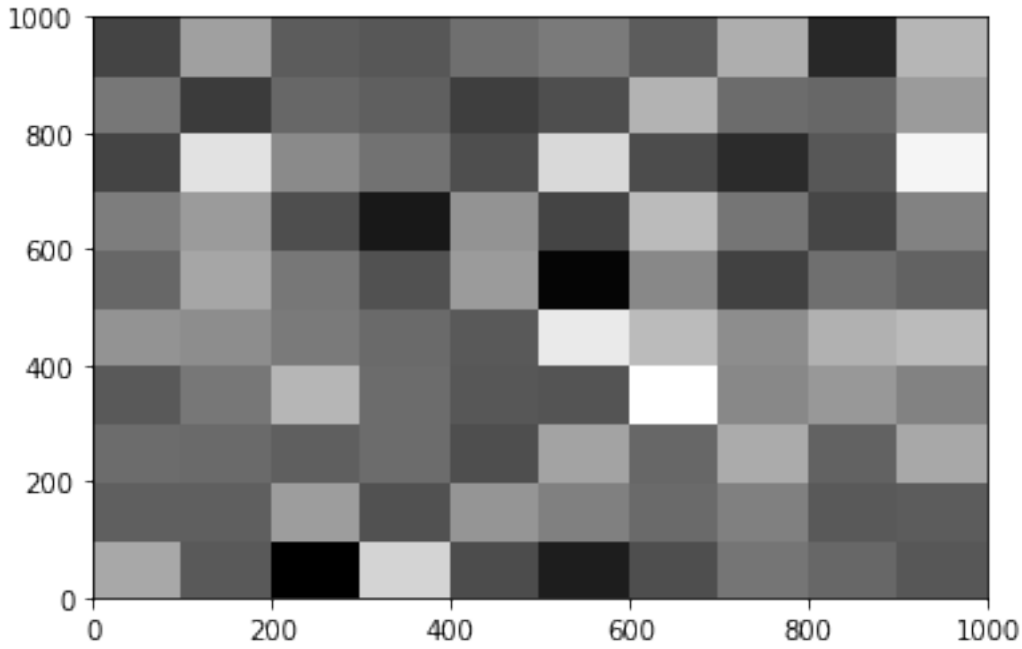
```
In [18]: rec = np.random.choice(records)
         speeds = [x[2] for x in rec]
         plt.hist(speeds, bins=30)
         plt.title("Event average for one user")

Out[18]: Text(0.5, 1.0, 'Event average for one user')
```



In [19]:
```
speeds = [x[2] for rec in records for x in rec]
plt.hist(speeds, bins=30);
plt.title("Event average for all users");
```

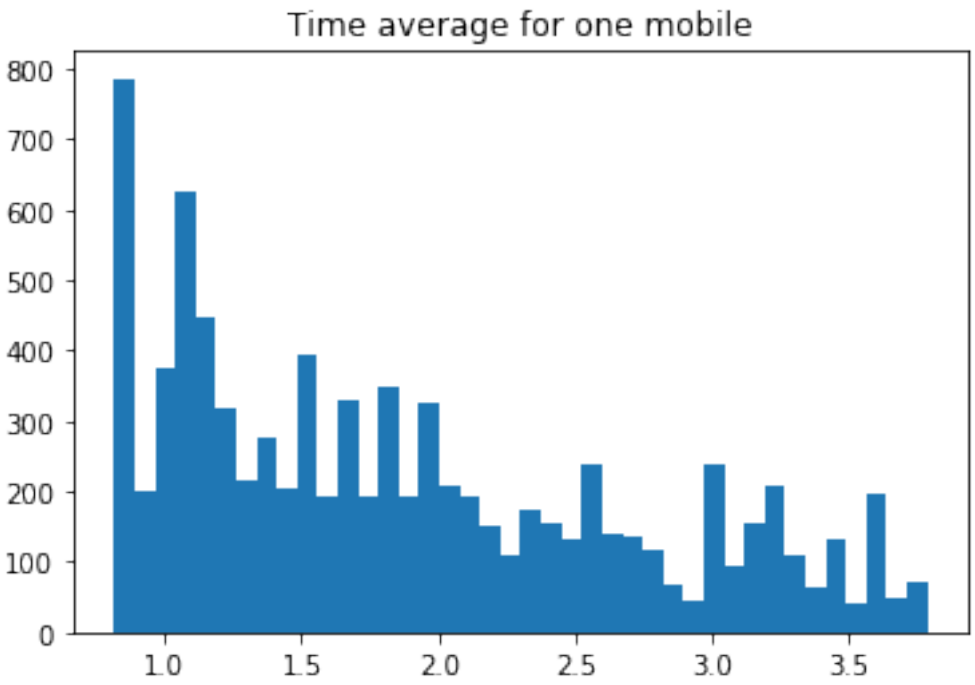


```
In [20]: x = [x[0][0] for rec in records for x in rec]
         y = [x[0][1] for rec in records for x in rec]
         plt.hist2d(x,y, cmap="gray");
```

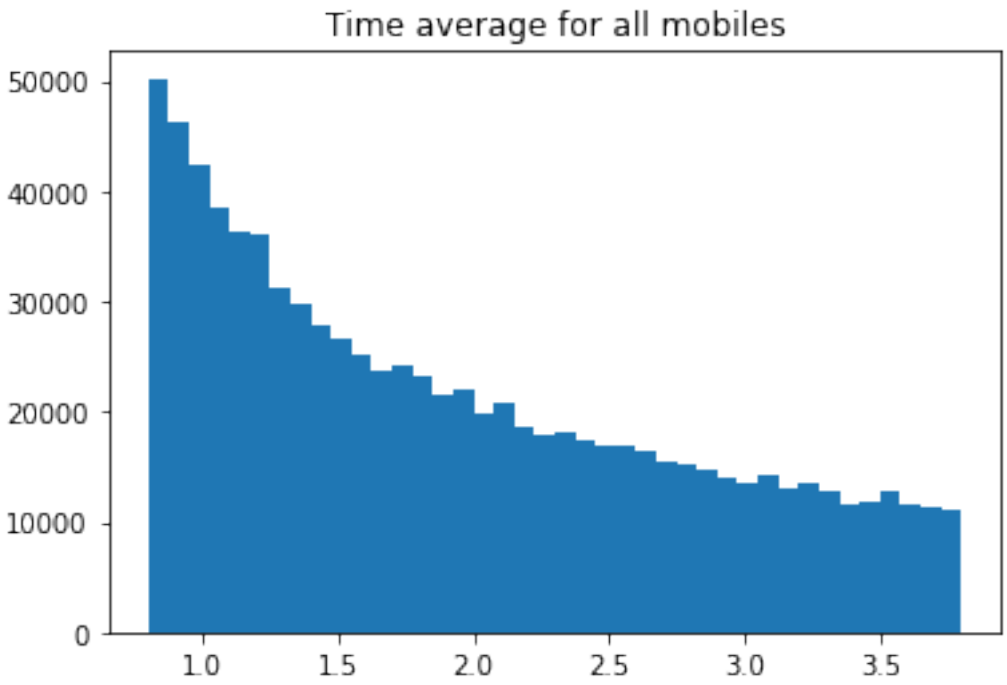


3.2 Time average viewpoint

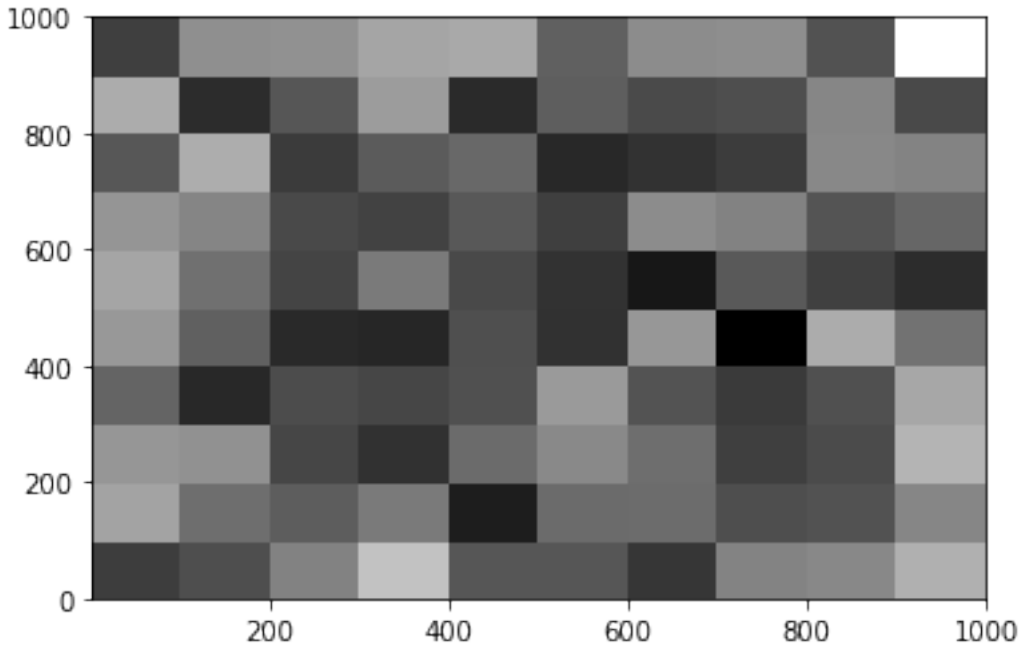
```
In [21]: from helpers import speed_at_time_t, position_at_time_t

         random_dude = np.random.choice(records)
         ten_seconds = range(0, params['time_limit'], 10) # every 10 seconds
         speeds_every_10_sec = [speed_at_time_t(t, random_dude) for t in ten_seconds]

         plt.hist(speeds_every_10_sec, bins=40);
         plt.title("Time average for one mobile")
         plt.show()
```



In [22]:
```
speeds_every_10_sec = []
ten_seconds = range(0, params['time_limit'], 10) # every 10 seconds
for person in records:
    speeds = [speed_at_time_t(t, person) for t in ten_seconds]
    speeds_every_10_sec.extend(speeds)
plt.hist(speeds_every_10_sec, bins=40);
plt.title("Time average for all mobiles")
plt.show()
```




```
In [23]: ten_seconds = range(0, params['time_limit'], 10) # every 10 seconds
         positions_evers_10_secs = []
         for person in records[:5]:
             positions = [position_at_time_t(t, person) for t in ten_seconds]
             positions_evers_10_secs.extend(positions)

         x,y = list(zip(*positions_evers_10_secs))
         plt.hist2d(x,y, cmap="gray");
```



4 Question 4

4.1 Problem 1

4.1.1 a)

```
In [24]: from plots import plot_boxplots_ci
         from helpers import get_average

In [25]: curr_N = 100
         X,Y = get_average(records, curr_N, params)
```

Finding CI for median This is found using theorem 2.1. We have 100 values. This value is too large to look up in the table. So we estimate it using the formula

$$\left[\lfloor 0.50n - 0.980\sqrt{n}\rfloor\ ,\ \lceil 0.50n + 1 + 0.980\sqrt{n}\rceil\right]$$

with $n = 100$, which yields us the results $j = \lfloor 50 - 9.8\rfloor = 40$ and $k = \lceil 50 + 9.8\rceil = 61$ So we select the 40th and 61st values as our CI for median

Finding CI for mean We use theorem 2.2.2, with 100 values:

$$\hat{\mu} \pm 0.196s \text{ with } s = \frac{1}{100}\sum(x_i - \hat{\mu})^2 \text{ and } \hat{\mu} = \frac{1}{100}\sum x_i$$

We could have used theorem 2.2.3, as the values seem to be normally distributed, but as we have a lot of values, the two theorems are similar enough to use 2.2.2 without risks. However, with $n \geq 30$, the two are roughly equal.

```
In [26]: plot_boxplots_ci(X, Y, j=40, k=61, n=curr_N)
```

```
---------------------------------------------------------------------------
ValueError                                Traceback (most recent call last)

/storage/anaconda3/envs/perfeval/lib/python3.6/site-packages/IPython/core/formatters.py
    339                 pass
    340             else:
--> 341                 return printer(obj)
    342             # Finally look for special method names
    343             method = get_real_method(obj, self.print_method)


/storage/anaconda3/envs/perfeval/lib/python3.6/site-packages/IPython/core/pylabtools.py
    242
    243     if 'png' in formats:
--> 244         png_formatter.for_type(Figure, lambda fig: print_figure(fig, 'png', **kwarg
    245     if 'retina' in formats or 'png2x' in formats:
    246         png_formatter.for_type(Figure, lambda fig: retina_figure(fig, **kwargs))


/storage/anaconda3/envs/perfeval/lib/python3.6/site-packages/IPython/core/pylabtools.py
    126
    127     bytes_io = BytesIO()
--> 128     fig.canvas.print_figure(bytes_io, **kw)
    129     data = bytes_io.getvalue()
    130     if fmt == 'svg':


/storage/anaconda3/envs/perfeval/lib/python3.6/site-packages/matplotlib/backend_bases.p
   2047                         orientation=orientation,
   2048                         dryrun=True,
-> 2049                         **kwargs)
   2050                     renderer = self.figure._cachedRenderer
   2051                     bbox_artists = kwargs.pop("bbox_extra_artists", None)


/storage/anaconda3/envs/perfeval/lib/python3.6/site-packages/matplotlib/backends/backer
    508
    509         """
--> 510         FigureCanvasAgg.draw(self)
    511         renderer = self.get_renderer()
    512


/storage/anaconda3/envs/perfeval/lib/python3.6/site-packages/matplotlib/backends/backer
```

```
    400         toolbar = self.toolbar
    401         try:
--> 402             self.figure.draw(self.renderer)
    403             # A GUI class may be need to update a window using this draw, so
    404             # don't forget to call the superclass.


    /storage/anaconda3/envs/perfeval/lib/python3.6/site-packages/matplotlib/artist.py in dr
     48                 renderer.start_filter()
     49
---> 50             return draw(artist, renderer, *args, **kwargs)
     51         finally:
     52             if artist.get_agg_filter() is not None:


    /storage/anaconda3/envs/perfeval/lib/python3.6/site-packages/matplotlib/figure.py in dr
   1647
   1648             mimage._draw_list_compositing_images(
-> 1649                 renderer, self, artists, self.suppressComposite)
   1650
   1651             renderer.close_group('figure')


    /storage/anaconda3/envs/perfeval/lib/python3.6/site-packages/matplotlib/image.py in _dr
    136     if not_composite or not has_images:
    137         for a in artists:
--> 138             a.draw(renderer)
    139     else:
    140         # Composite any adjacent images together


    /storage/anaconda3/envs/perfeval/lib/python3.6/site-packages/matplotlib/artist.py in dr
     48                 renderer.start_filter()
     49
---> 50             return draw(artist, renderer, *args, **kwargs)
     51         finally:
     52             if artist.get_agg_filter() is not None:


    /storage/anaconda3/envs/perfeval/lib/python3.6/site-packages/matplotlib/axes/_base.py i
   2626             renderer.stop_rasterizing()
   2627
-> 2628         mimage._draw_list_compositing_images(renderer, self, artists)
   2629
   2630         renderer.close_group('axes')


    /storage/anaconda3/envs/perfeval/lib/python3.6/site-packages/matplotlib/image.py in _dr
```

```
    136     if not_composite or not has_images:
    137         for a in artists:
--> 138             a.draw(renderer)
    139     else:
    140         # Composite any adjacent images together


    /storage/anaconda3/envs/perfeval/lib/python3.6/site-packages/matplotlib/artist.py in dr
     48                 renderer.start_filter()
     49
---> 50             return draw(artist, renderer, *args, **kwargs)
     51         finally:
     52             if artist.get_agg_filter() is not None:


    /storage/anaconda3/envs/perfeval/lib/python3.6/site-packages/matplotlib/text.py in draw
   2387             if self.arrow_patch.figure is None and self.figure is not None:
   2388                 self.arrow_patch.figure = self.figure
-> 2389             self.arrow_patch.draw(renderer)
   2390
   2391         # Draw text, including FancyBboxPatch, after FancyArrowPatch.


    /storage/anaconda3/envs/perfeval/lib/python3.6/site-packages/matplotlib/patches.py in d
   4307         # dpi_cor = renderer.points_to_pixels(1.)
   4308         self.set_dpi_cor(renderer.points_to_pixels(1.))
-> 4309         path, fillable = self.get_path_in_displaycoord()
   4310
   4311         if not cbook.iterable(fillable):


    /storage/anaconda3/envs/perfeval/lib/python3.6/site-packages/matplotlib/patches.py in g
   4256             self.get_mutation_scale() * dpi_cor,
   4257             self.get_linewidth() * dpi_cor,
-> 4258             self.get_mutation_aspect())
   4259
   4260         # if not fillable:


    /storage/anaconda3/envs/perfeval/lib/python3.6/site-packages/matplotlib/patches.py in _
   3216                     return path_mutated, fillable
   3217             else:
-> 3218                 return self.transmute(path, mutation_size, linewidth)
   3219
   3220     class _Curve(_Base):


    /storage/anaconda3/envs/perfeval/lib/python3.6/site-packages/matplotlib/patches.py in t
```

```
   3716                 tail_width = self.tail_width * mutation_size
   3717                 tail_left, tail_right = get_parallels(arrow_out,
-> 3718                                                       tail_width / 2.)
   3719
   3720                 patch_path = [(Path.MOVETO, tail_right[0]),


/storage/anaconda3/envs/perfeval/lib/python3.6/site-packages/matplotlib/bezier.py in ge
    379         cmx_left, cmy_left = get_intersection(c1x_left, c1y_left, cos_t1,
    380                                               sin_t1, c2x_left, c2y_left,
--> 381                                               cos_t2, sin_t2)
    382
    383         cmx_right, cmy_right = get_intersection(c1x_right, c1y_right, cos_t1,


/storage/anaconda3/envs/perfeval/lib/python3.6/site-packages/matplotlib/bezier.py in ge
     33     ad_bc = a * d - b * c
     34     if np.abs(ad_bc) < 1.0e-12:
---> 35         raise ValueError("Given lines do not intersect. Please verify that "
     36                          "the angles are not equal or differ by 180 degrees.")
     37


ValueError: Given lines do not intersect. Please verify that the angles are not equal e


<Figure size 1440x1800 with 2 Axes>
```

4.1.2 b)

Interestingly, the mean/median of the Time Average is significantly lower than for the Event Average. Also, the confidence intervals are much smaller. Because with the event average, we use "every" speed data points once, the CI is much narrower. With time average, we probably don't pick some speeds and some might be picked twice, leading to a more uncertain CI.

4.1.3 c)

```
In [27]: curr_N = 30
         X,Y = get_average(records, curr_N, params)
```

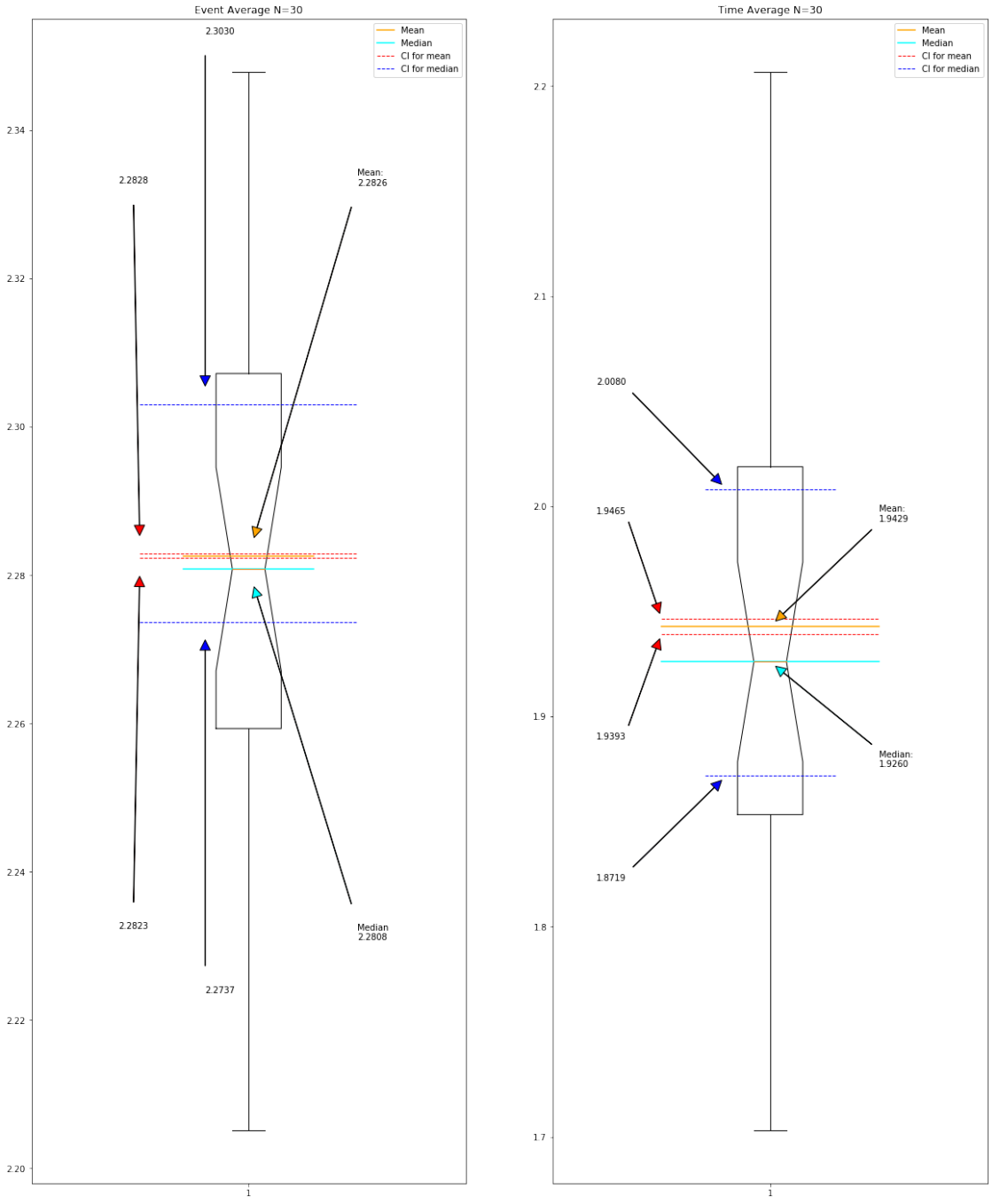
Finding CI for median This is found using theorem 2.1. We have 30 values. So we can look up in the table. We must pick the values 10 and 21 of the sorted sequence.

Finding CI for mean We use theorem 2.2.2, with 30 values:

$$\hat{\mu} \pm 0.196s \text{ with } s = \frac{1}{30}\sum(x_i - \hat{\mu})^2 \text{ and } \hat{\mu} = \frac{1}{30}\sum x_i$$

We could have used theorem 2.2.3, as the values seem to be normally distributed, but as we have a lot of values, the two theorems are similar enough to use 2.2.2 without risks. In practice however, as $n \geq 30$, the two are roughly equal.

```
In [28]: plot_boxplots_ci(X, Y, j=10, k=21, n=curr_N)
```



4.1.4 d)

We observe that the confidence intervals for the median and the mean are significantly larger with $N = 30$. As we grow to more samples, we can more effectively estimate the ''true" distribution ; indeed, with more samples we are more confident to find the true median/mean, insteand of an effect of ''bad luck" caused by some unlucky sampling.

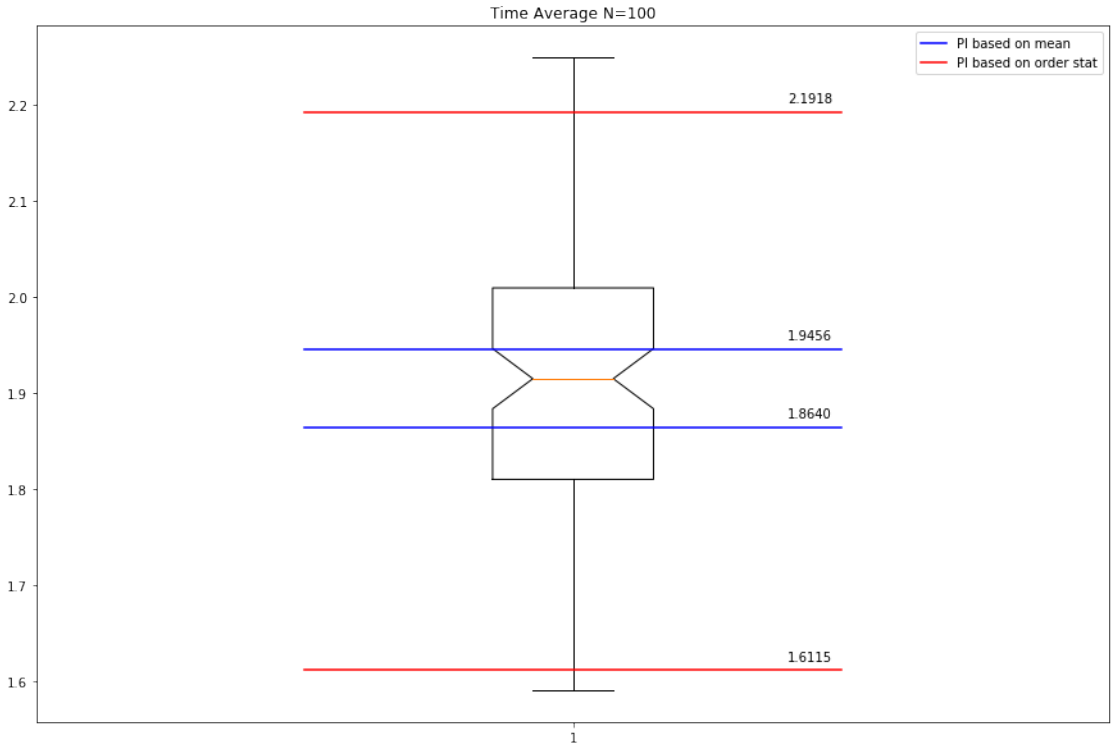
4.2 Problem 2

4.2.1 a) + b)

```
In [29]: from plots import plot_boxplots_pi
```

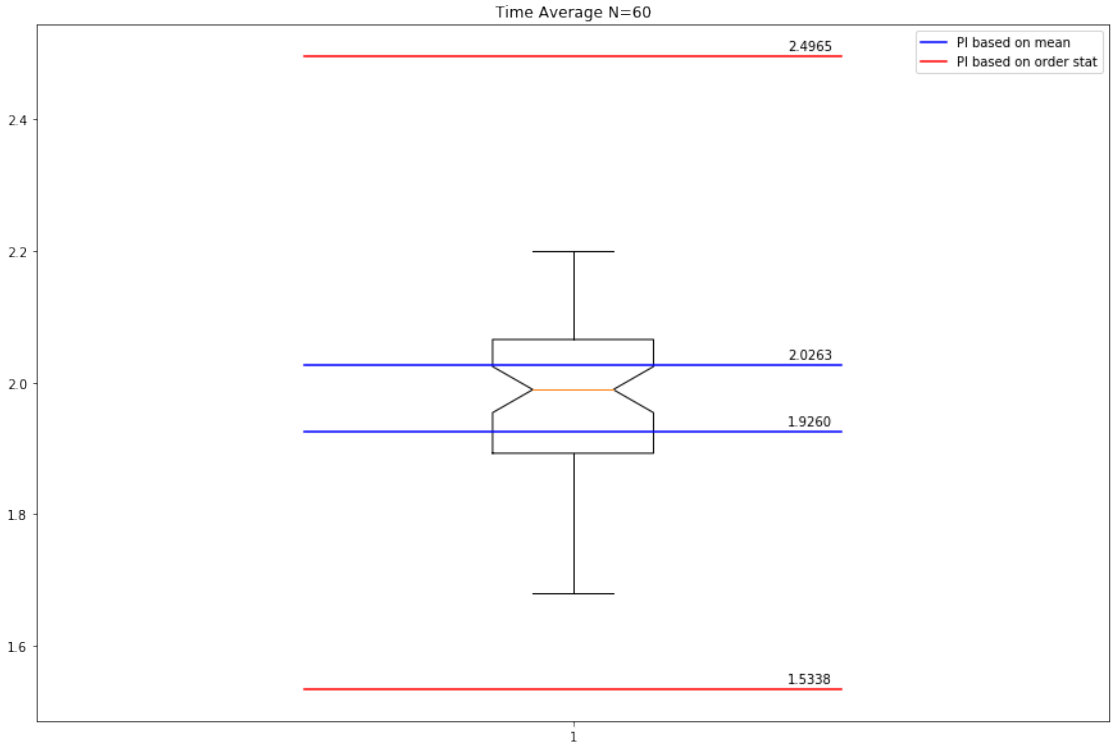
```
In [30]: curr_N = 100
         _,Y = get_average(records, curr_N, params)
         plot_boxplots_pi(Y)
```

```
1 98
```

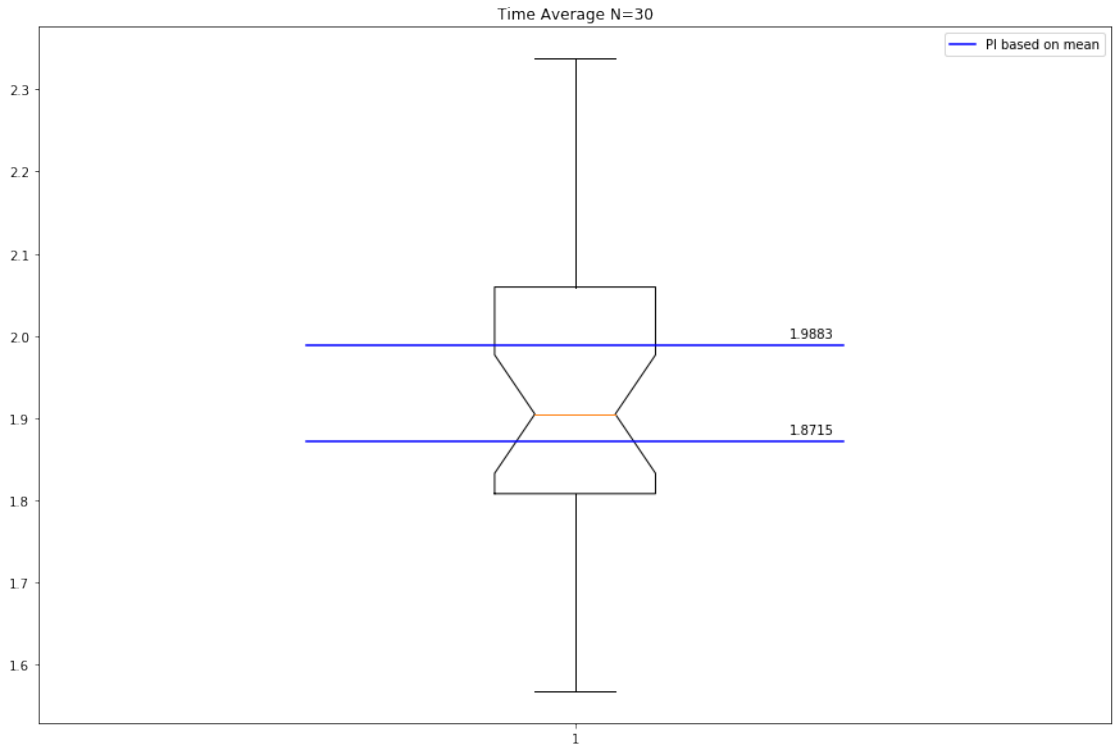



```
In [31]: curr_N = 60
         _,Y = get_average(records, curr_N, params)
         plot_boxplots_pi(Y)
```

```
0 59
```



In [32]:
```
curr_N = 30
_,Y = get_average(records, curr_N, params)
plot_boxplots_pi(Y)
```



Note that we can't plot the PI based on order statistic here, as at $n = 30 < 39$, we can't reach level 0.95.

4.2.2 c)

The Prediction Interval obtained with order statistics is extremely less precise than the one obtaines using estimates of mean and variance. Order statistics works better with a lot more data, than just 100 or 60. We see that the PI at 60 is the first and last sample, which yields absolutely no information: future points are confidently between the smallest and largest observed so far.

4.2.3 d)

PI based on order statistics is virtually useless for the two first graphs, but consistently to the extremes. Interestingly, PI based on estimates of mean/variance is not affected linearly: it shifts slightly up between N=100 and N=60 (without obvious loss of precision), and shifts down again for N=30. The interval seems to be robust to a lower amount of data, and is still usable.