PERFORMANCE EVALUATION

HOMEWORK 3
DISCRETE EVENT SIMULATION OF A WEB SERVER

Jean-Yves Le Boudec
Ehsan Mohammadpour

Due date: Monday, April 1st 2019 at 11:55 PM

SUBMISSION

Submit a **single zip file** per group. This should include a pdf document containing your answers and plots, in addition to the MATLAB source code.

1 SIMULATE

A web server is modelled as a system that serves jobs First In, First Out. The web server receives requests from a large number of customers. The flow of requests is modelled as a Poisson process of intensity λ. When a request arrives, it generates a "type 1" job, which is queued at the end of the processor queue. When a type 1 job is processed, it becomes a "type 2" job, which is in turn enqueued at the end of the processor queue. When a type 2 job is processed, it leaves the system and the request is cleared.

The service times are all independent, but have different distributions (all measures are in **milliseconds**, unless otherwise specified):

type 1 Log-normal with parameters $\mu = 1.5$ and $\sigma = 0.6$. The log normal distribution is the distribution of the exponential of a normal random variable.
type 2 Uniform $(0.6, 1)$.

The goal of this homework is to implement a discrete event simulation of the system in Matlab. You should display the following variables as a function of time, for a simulation of duration such that 10^4 requests have arrived.

1. number of requests arrived and number of requests served since the beginning of the simulation (both on the same graph)
2. number of type i jobs in the processor queue ($i = 1, 2$).

Take $\lambda = 80$ requests/second. Then, compute the following averages:

1. average response time (i.e., waiting + service time) for a type i job, $i = 1, 2$
2. average number of type i jobs served per second

2 STATIONARITY

Plot the number of jobs in the processor queue versus time for various values of λ in the range $[25, 250]$ requests/second. For which values of λ is the system stationary? Determine these values analytically. What happens on the plots when the system is not stationary ?

3 REMOVE TRANSIENTS

For $\lambda = 60$ requests/s and $\lambda = 160$ requests/s:

- Compute confidence intervals for the median *and* for the mean for the average number of type i jobs in the system at level 0.95. For better results, run the simulation for $n \geq 30$ times.

Redo the computation of the confidence interval for the median and for the mean after removing an initial transient period. Explain what method you used to decide which initial period has to be removed.

4 LITTLE'S LAW

Verify Little's law on the averages you obtained, with and without transient removals.

5 PARAMETER ESTIMATION AND CONFIDENCE INTERVAL

In question 1 to 5, we assumed that a new request entering the system was of type 1. In this section, we propose a test to verify the assumption. We now assume that a new request arriving the system is of type 1 with probability $1 - \epsilon$ and of type 2 with probability ϵ and we want to assure that ϵ is zero or almost zero.

In a first experiment, we picked 10 requests at random. They were all of type 1. Give a 95% confidence interval for the parameter ϵ and a corresponding confidence interval for the stability region of the system. Do you think that 10 samples are enough?

We want to design a second experiment in order to assure that ϵ is small. What is the number of samples to be picked so that we can assure that $\epsilon < 1\%$ with a 95% of confidence (assuming that a sample is always a "type 1" request)?