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CS 281-3

Q1)

A	B	C	D	E	F
Ankara	x00	163	1	10	1
İzmir	x04	563	2	10	1
İstanbul	x08	267	3	20	2
İstanbul	x04	543	4	20	3
Bursa	x00	896	5	10	5
Erzurum	x08	467	6	10	6

Which of the following functional dependencies hold? Justify your answers.

I. B → C

This **doesn't hold** because of the lines:

Ankara	x00	163	1	10	1
Bursa	x00	896	5	10	5

They have the same B value but different C values.

II. AB → C

This **holds** because the A B pairs in this table are unique so there can be no case which they wouldn't hold.

III. D → BC

This **holds** because the D in this table is unique so there can be no case which it wouldn't hold.

IV. AE → F

This **doesn't hold** because of the lines:

İstanbul	x08	267	3	20	2
İstanbul	x04	543	4	20	3

They have the same AE value but their F value is different which contradicts the dependency.

V. EF → B

This **doesn't hold** because of the lines:

Ankara	x00	163	1	10	1
İzmir	x04	563	2	10	1

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Q2)

Find candidate key(s) for the following relations based on given functional dependencies.

I. R(A,B,C,D) F = {A → B, BC → AD}

$A \rightarrow A, A \rightarrow AB, AC \rightarrow ABC, AC \rightarrow ABCD$ so AC is a candidate key

$BC \rightarrow BC, BC \rightarrow BCAD$ so BC is a candidate key

AC, BC are the candidate keys.

II. R(A,B,C,D,E) F = {E → CB, D → AE, A → CB}

$D \rightarrow D, D \rightarrow DAE, D \rightarrow DAEBC$ so D is a candidate key

D is the candidate key.

III. R(X,Y,Z,T) F = {T → X, Z → YT, XY → Z}

$T \rightarrow TX, TY \rightarrow TXY, TY \rightarrow TXYZ$ So TY is a candidate key

$XY \rightarrow XY, XY \rightarrow XYZ, XY \rightarrow XYZT$ so XY is a candidate key

$Z \rightarrow Z, Z \rightarrow ZYT, Z \rightarrow ZYTX$ so Z is a candidate key

TY, XY, Z are the candidate keys.

IV. R(X,Y,Z,T) F = {T → YZ, Y → XZ, XT → Y}

$T \rightarrow T, T \rightarrow TYZ, T \rightarrow TYZX$ so T is a candidate key

$Y \rightarrow Y, Y \rightarrow YXZ, YT \rightarrow YXZT$ so YT is not a candidate key

$XT \rightarrow XT, XT \rightarrow XTY, XT \rightarrow XTYZ$ so XT is not a candidate key

T is the candidate key.

V. R(A,B,C,D,E) F = {AB → CD, D → A, BC → DE, C → DE}

$AB \rightarrow AB, AB \rightarrow ABCD, AB \rightarrow ABCDE$ so AB is a candidate key,

$D \rightarrow D, D \rightarrow DA, DB \rightarrow DAB, DB \rightarrow DABC, DB \rightarrow DABCE$ so DB is a candidate key,

$C \rightarrow C, C \rightarrow CDE, C \rightarrow CDEA, CB \rightarrow CDEAB$ so CB is a candidate key

AB, DB, CB are candidate keys.

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Q3)

Given the relation $R(X, Y, Z, U, V, T)$ and $F = \{X \rightarrow YZV, YZ \rightarrow UT, T \rightarrow XY, V \rightarrow U\}$ Is this relation in 3NF? If it is not in 3NF, decompose it into smaller relations so that it satisfies 3NF. In case you decompose it, is the decomposition lossless? Is it dependency preserving? Justify your answers.

First, we need to find the candidate keys:

$X \rightarrow X, X \rightarrow XYZV, X \rightarrow XYZVUT$ so, X is a candidate key.

$YZ \rightarrow YZ, YZ \rightarrow YZUT, YZ \rightarrow YZUTX, YZ \rightarrow YZUTXV$ so, YZ is a candidate key.

$T \rightarrow T, T \rightarrow TXY, T \rightarrow TXYZV, T \rightarrow TXYZVU$ so, T is a candidate key.

(X, YZ, T) are the candidate keys.

Then we cheek which FD's satisfy 3NF.

$X \rightarrow YZV$, X is a candidate key so this satisfies.

$YZ \rightarrow UT$, YZ is a candidate key so this satisfies.

$T \rightarrow XY$, T is a candidate key so this satisfies.

$V \rightarrow U$, V is not a candidate key and U is not part of a candidate key so this doesn't satisfy.

Now we will de compose the relation $R(X, Y, Z, U, V, T)$ using $V \rightarrow U$, we get:

$$\boldsymbol{R_1(X, Y, Z, V, T), F_1 = \{X \rightarrow YZV, YZ \rightarrow T, T \rightarrow XY\}}$$

$$\boldsymbol{R_2(V, U), F_2 = \{V \rightarrow U\}}$$

Now we cheek if R_1 , R_2 are in 3NF.

For R_1 the candidate keys are:

$X \rightarrow X, X \rightarrow XYZV, X \rightarrow XYZVT$ so, X is a candidate key.

$YZ \rightarrow YZ, YZ \rightarrow YZT, YZ \rightarrow YZTX, YZ \rightarrow YZTXV$ so, YZ is a candidate key.

$T \rightarrow T, T \rightarrow TXY, T \rightarrow TXYZV$ so, T is a candidate key.

(X, YZ, T) are the candidate keys.

Then we cheek if all FD's satisfy 3NF.

$X \rightarrow YZV$, X is a candidate key so this satisfies.

$YZ \rightarrow T$, YZ is a candidate key so this satisfies.

$T \rightarrow XY$, T is a candidate key so this satisfies.

R_1 is 3NF.

For R_2 the candidate keys are:

$V \rightarrow V, V \rightarrow VU$ so, V is a candidate key.

(V) is the only candidate key.

Then we cheek if the FD satisfy 3NF.

$V \rightarrow U$, V is a candidate key so this satisfies.

R_2 is 3NF.

They are both in 3NF, the decomposition is successful.

The **decomposition was lossless** because $R_1 \cap R_2 = V$ is the key for the relation R_2

Now to cheek if it is dependency preserving, we write the minimal cover for $R, R_1 nad\ R_2$

$$G = \{X \rightarrow Y, X \rightarrow Z, X \rightarrow V, YZ \rightarrow U, YZ \rightarrow T, T \rightarrow X, T \rightarrow Y, V \rightarrow U\}$$
$$G_1 = \{X \rightarrow Y, X \rightarrow Z, X \rightarrow V, YZ \rightarrow T, T \rightarrow X, T \rightarrow Y\}$$
$$G_2 = \{V \rightarrow U\}$$

We now cheek $(F_1 \cup F_2)^+ = F^+$:

$$G_1 \cup G_2 = \{X \rightarrow Y, X \rightarrow Z, X \rightarrow V, YZ \rightarrow T, T \rightarrow X, T \rightarrow Y, V \rightarrow U\}$$

But $YZ \rightarrow T, T \rightarrow X, X \rightarrow V, V \rightarrow U$ implies $YZ \rightarrow U$ so

$$(F_1 \cup F_2)^+ = \{X \rightarrow Y, X \rightarrow Z, X \rightarrow V, YZ \rightarrow U, YZ \rightarrow T, T \rightarrow X, T \rightarrow Y, V \rightarrow U\} = G$$

So this was a **dependency preserving** decomposition.

Q4)

Given the relation $R(A, B, C, D, E)$ and $F = \{A \rightarrow CE, C \rightarrow BD, DE \rightarrow AB\}$ Is this relation in BCNF? If it is not in BCNF, decompose it into smaller relations so that it satisfies BCNF. In case you decompose it, is the decomposition lossless? Is it dependency preserving? Justify your answers.

First, we need to find the candidate keys:

$A \rightarrow A, A \rightarrow ACE, A \rightarrow ACEBD$ so, A is a candidate key.

$C \rightarrow C, C \rightarrow CBD, C \rightarrow CBD, CE \rightarrow CBDE, CE \rightarrow CBDEA$ so, CE is a candidate key.

$DE \rightarrow DE, DE \rightarrow DEAB, DE \rightarrow DEABC$ so, DE is a candidate key.

(A, CE, DE) are the candidate keys.

Then we cheek which FD's satisfy CBNF.

$A \rightarrow CE$, A is a candidate key so this satisfies.

$C \rightarrow BD$, C is not a candidate key so this doesn't satisfy.

$DE \rightarrow AB$, DE is a candidate key so this satisfies.

Now we will de compose the relation $R(A, B, C, D, E)$ using $C \rightarrow BD$, we get:

$$R_1(A, C, E), F_1 = \{A \rightarrow CE\}$$

$$R_2(C, B, D), F_2 = \{C \rightarrow BD\}$$

Now we cheek if R_1 , R_2 are in BCNF.

For R_1 the candidate keys are:

$A \rightarrow A, A \rightarrow ACE$so, A is a candidate key.

(A) is the only candidate key.

Then we cheek if the FD satisfy BCNF.

$A \rightarrow CE$, A is a candidate key so this satisfies.

R_1 is BCNF.

For R_2 the candidate keys are:

$C \rightarrow C, C \rightarrow CBD$ so, C is a candidate key.

(C) is the only candidate key.

Then we cheek if the FD satisfy BCNF.

$C \rightarrow BD$, C is a candidate key so this satisfies.

R_2 is BCNF.

They are both in BCNF, the decomposition is successful.

The **decomposition was lossless** because $R_1 \cap R_2 = C$ is the key for the relation R_2

Now to cheek if it is dependency preserving, we write the minimal cover for $R, R_1 nad\ R_2$

$$G = \{A \rightarrow C, A \rightarrow E, C \rightarrow B, C \rightarrow D, DE \rightarrow A, DE \rightarrow B\}$$
$$G_1 = \{A \rightarrow C, A \rightarrow E\}$$
$$G_2 = \{C \rightarrow B, C \rightarrow D\}$$

We now cheek $(F_1 \cup F_2)^+ = F^+$:

$$G_1 \cup G_2 = \{A \rightarrow C, A \rightarrow E, C \rightarrow B, C \rightarrow D\}$$

This does not imply $DE \rightarrow A, DE \rightarrow B$ so we lost them:

$$(F_1 \cup F_2)^+ = \{A \rightarrow C, A \rightarrow E, C \rightarrow B, C \rightarrow D\} \neq G$$

So this was a **not dependency preserving** decomposition.

Q5)

Find the minimal cover for the following set of functional dependencies. Show your work in each step.
$R(A, B, C, D, E)\ F = \{A \rightarrow BC, B \rightarrow CE, D \rightarrow E, DE \rightarrow BC, E \rightarrow A\}$

First we put FD's in standard from,

$$F = \{A \rightarrow B, A \rightarrow C, B \rightarrow C, B \rightarrow E, D \rightarrow E, DE \rightarrow B, DE \rightarrow C, E \rightarrow A\}$$

Then we minimize the left side of each FD,

$D \rightarrow E, DE \rightarrow B$ can be combined:

$D \rightarrow D, D \rightarrow DE, D \rightarrow DEB$ this implies $D \rightarrow B$

$D \rightarrow E, DE \rightarrow C$ can be combined:

$D \rightarrow D, D \rightarrow DE, D \rightarrow DEC$ this implies $D \rightarrow C$ so we can write F as

$$F = \{A \rightarrow B, A \rightarrow C, B \rightarrow C, B \rightarrow E, D \rightarrow E, D \rightarrow B, D \rightarrow C, E \rightarrow A\}$$

Then we delete redundant FD's,

$A \rightarrow B$, $B \rightarrow C$ can be combined:

$A \rightarrow A, A \rightarrow AB, A \rightarrow ABC$ this implies $A \rightarrow C$ so we can write F as

$$F = \{A \rightarrow B, B \rightarrow C, B \rightarrow E, D \rightarrow E, D \rightarrow B, D \rightarrow C, E \rightarrow A\}$$

$D \rightarrow B, B \rightarrow C$ can be combined:

$D \rightarrow D, D \rightarrow DB, D \rightarrow DBC$ this implies D→ C so we can write F as

$$F = \{A \rightarrow B, B \rightarrow C, B \rightarrow E, D \rightarrow E, D \rightarrow B, E \rightarrow A\}$$

$D \rightarrow B, B \rightarrow E$ can be combined:

$D \rightarrow D, D \rightarrow DB, D \rightarrow DBE$ this implies D→ E so we can write F as

$$F = \{A \rightarrow B, B \rightarrow C, B \rightarrow E, D \rightarrow B, E \rightarrow A\}$$

There are no more simplifications so:

$$\boldsymbol{G} = \{\boldsymbol{A} \rightarrow \boldsymbol{B}, \boldsymbol{B} \rightarrow \boldsymbol{C}, \boldsymbol{B} \rightarrow \boldsymbol{E}, \boldsymbol{D} \rightarrow \boldsymbol{B}, \boldsymbol{E} \rightarrow \boldsymbol{A}\}$$