

Project: Rest-to-Rest control of a 2-DoF Robot

An often occurring control task in industry is to compute optimal input trajectories for robots to reach a given end position while consuming minimal energy, or to reach the end position in minimal time. In this project we want to tackle these challenges by controlling a two-degree-of-freedom (2-DoF) robot shown in Figure 1. The dynamic model of the robot is given by a vector-valued second order differential equation

$$B(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = u \quad (1)$$

in terms of the joint angles $q = (q_1, q_2)^\top \in \mathbb{R}^2$ and the input torques $u = (u_1, u_2)^\top \in \mathbb{R}^2$. The coefficient matrices are given by

$$B(q) = \begin{pmatrix} b_1 + b_2 \cos(q_2) & b_3 + b_4 \cos(q_2) \\ b_3 + b_4 \cos(q_2) & b_5 \end{pmatrix} \in \mathbb{R}^{2 \times 2},$$
$$C(q, \dot{q}) = -c_1 \sin(q_2) \begin{pmatrix} \dot{q}_2 & \dot{q}_1 + \dot{q}_2 \\ -\dot{q}_1 & 0 \end{pmatrix} \in \mathbb{R}^{2 \times 2},$$
$$g(q) = \begin{pmatrix} g_1 \cos(q_1) + g_2 \cos(q_1 + q_2), & g_2 \cos(q_1 + q_2) \end{pmatrix}^\top \in \mathbb{R}^{1 \times 2}.$$

If you are not familiar with robot modeling take a look at [1] to recall the basic modeling steps.

Modeling

- What would be your approach to obtain model (1) from physical laws? Why is the model formulated in q_1 - q_2 -coordinates and not in x - y -coordinates? Explain briefly.
- Model (1) is given as an implicit system of second-order differential equations. Reformulate it as an explicit first-order system in state space form $\dot{x} = f(x, u)$. Choose an appropriate state vector x .

Open-loop Optimal Control

- Formulate at least two OCPs that drive the robot from the initial position $q = (-5, -4)^\top \text{ rad}$, $\dot{q} = (0, 0)^\top \text{ rad/s}$ to the upper position $q = (\frac{\pi}{2}, 0)^\top \text{ rad}$, $\dot{q} = (0, 0)^\top \text{ rad/s}$ in 3s. Thereby, the input constraints $u \in [-1000 \ 1000] \text{ Nm}$ and the state constraints $\dot{q} \in [-\frac{3}{2}\pi \ \frac{3}{2}\pi] \text{ rad/s}$ should be satisfied for all times. Motivate your problem formulation.
- Solve one of the OCPs of a) numerically. Use different numbers of sampling points and at least two different integrators. Compare the simulation results. Plot your results in the q_1 - q_2 -plane and the x - y -plane.
- Now formulate a problem that reaches the end position from a) in minimum time.
- Reformulate the OCP of c) as a fixed end-time problem.
- Solve your problem formulation from d) numerically. What is the minimum time needed? Try also a problem formulation to find a compromise between minimal time and minimal control energy.

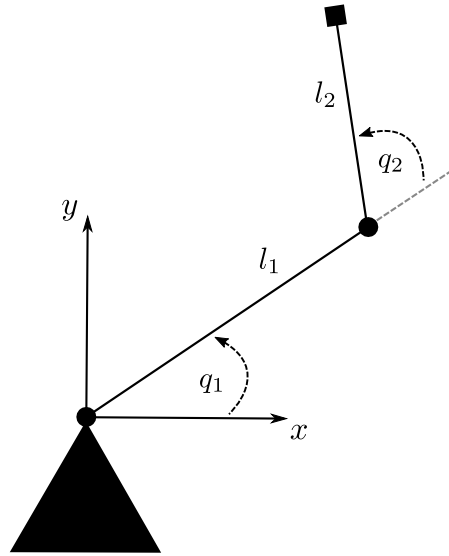


Figure 1: 2-DoF robot.

Table 1: Parameters for system (1).

b_1	200.0	[kg m ² /rad]	b_2	50.0	[kg m ² /rad]
b_3	23.5	[kg m ² /rad]	b_4	25.0	[kg m ² /rad]
b_5	122.5	[kg m ² /rad]	c_1	-25.0	[Nms ⁻²]
g_1	784.8	[Nm]	g_2	245.3	[Nm]
l_1	0.5	[m]	l_2	0.5	[m]

Model Predictive Control

- What type of uncertainties/disturbances can you imagine for the robot? Formulate them mathematically.
- Design an MPC controller for the fixed end-time problem a) from the previous exercise.
- Assume incorrect parameters in the model, such as $b_1 = 180 \text{ kg m}^2/\text{rad}$ and $b_2 = 45 \text{ kg m}^2/\text{rad}$. When/how would this happen in practice? Simulate the open loop and the closed loop system. Plot your results and describe the differences.
- Now assume an additive Gaussian measurement error; choose a meaningful value for the variance. Implement this situation and simulate your closed-loop system for the fixed end-time problem. What do you observe? Plot your results.
- Analyze the stability properties of your designed MPC controller.

References

- [1] Siciliano, B., Sciavicco, L., Villani, L., & Oriolo, G. (2010). "Robotics: Modelling, Planning and Control". Springer Science & Business Media.