Energy Optimized Towing of a Kite for Seagoing Vessels

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***Abstract*— This project aims to formulate a model predictive control scheme for the energy optimal control task of a towing kite for seafaring vessels. The utilized dynamic model consists of first-order ordinary differential equations for a non-linear kite system and the economic objective is formulated to maximize the tether force. The height of the kite is considered a constraint to avoid crashing. The optimal control problem (OCP) is solved with a direct method where the problem is discretized by orthogonal collocation. Additionally, a wind model is used to implement a realistic wind trajectory. To cope with the disturbance due to varying wind speeds, a soft constraint is embedded. In this report, the design aspects are discussed with appropriate reasoning along with the numerical implementation of the OCP in CasADi.**

I. INTRODUCTION

Model predictive control (MPC) is a control scheme that uses a model to make predictions about future behaviours of a system in order to compute an optimal control input that optimizes an a priori defined performance criteria. It can handle constraints, has preview capability and can easily incorporate future reference information into the control problem to improve controller performance. Additionally, it can deal with multi-input multi-output (MIMO) systems that might have interactions between their inputs and outputs which is particularly difficult to achieve for PID controllers. However, it requires a powerful, fast processor with a large memory since it solves an online optimization problem at each time step to select the optimal control action from a sequence that drives the predicted output as close to the desired reference as possible.

Model predictive control is starting to be the industry standard for a wide range of control tasks as the cost of computational power is getting cheaper and more complex systems can be deployed in more efficient ways with the help of previously trained machine learning models [1]. MPC strategies are more robust for non-linear systems due to their ability to deal with uncertainties, the ability to impose constraints as well as being efficient in performing general control tasks. In this paper, we attempt to design an MPC controller for a non-linear towing kite system [2] to be used on seagoing vessels to reduce the fuel consumption by harnessing and optimizing high altitude wind energy [3][4].

II. SYSTEM DESCRIPTION

A. System Overview

Fig. 1 shows the schematic setup of the towing kite. The core of the propulsion system is the towing kite itself

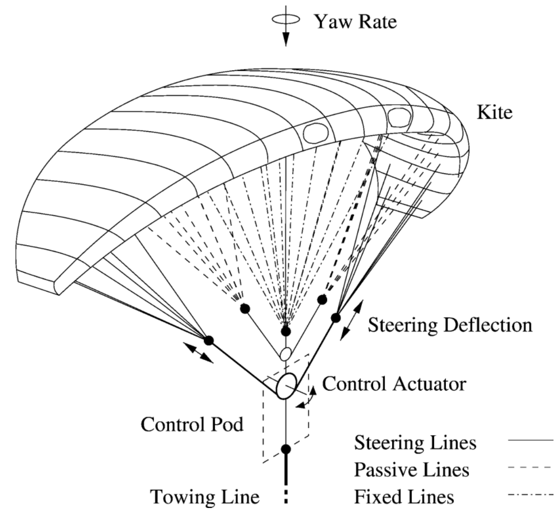

Fig. 1. Schematic Drawing of the Towing Kite

steered by a control pod located under the kite[2]. The towing force is transmitted to the ship by a tether rope. The main steering actuator in the control pod applies deflections to the kite lines leading to a curved flight. The goal of the MPC controller should be to harvest as much wind energy as possible while keeping the height above a certain limit $h_{\min}$ to avoid crashing.

B. The Kite Model

The system vectors [2] for roll, pitch and yaw is given by:

$$e_{roll} = \begin{bmatrix} -\sin\vartheta\cos\psi \\ -\cos\varphi\sin\psi + \sin\varphi\cos\vartheta\cos\psi \\ -\sin\varphi\sin\psi - \cos\varphi\cos\vartheta\cos\psi \end{bmatrix}$$

$$e_{pitch} = \begin{bmatrix} \sin\vartheta\sin\psi \\ -\cos\varphi\cos\psi - \sin\varphi\cos\vartheta\sin\psi \\ -\sin\varphi\cos\psi + \cos\varphi\cos\vartheta\sin\psi \end{bmatrix}$$

$$e_{yaw} = \begin{bmatrix} -\cos\vartheta \\ -\sin\varphi\sin\vartheta \\ \cos\varphi\sin\vartheta \end{bmatrix}$$

Neglecting gravity, the system of equations reduces to:

$$e_{roll} = \begin{bmatrix} -\sin\vartheta\cos\psi \\ -\sin\psi \\ -\cos\vartheta\cos\psi \end{bmatrix} e_{pitch} = \begin{bmatrix} \sin\vartheta\sin\psi \\ -\cos\psi \\ \cos\vartheta\sin\psi \end{bmatrix} e_{yaw} = \begin{bmatrix} -\cos\vartheta \\ 0 \\ \sin\vartheta \end{bmatrix}$$

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For the first part of the project, the airflow from wind υ_0 is assumed to be constant. The following differential equations represent the non-linear dynamic model of our system.

$$\dot{\psi} = \frac{\upsilon_a}{L}\left(\delta - \frac{\cos(\theta)}{\sin(\theta)}\sin(\psi)\right)$$
$$\dot{\theta} = \frac{\upsilon_a}{L}\left(\cos(\psi) - \frac{\tan(\theta)}{E}\right)$$
$$\dot{\phi} = -\frac{\upsilon_a}{L}\sin(\theta)\sin(\psi)$$

ψ denotes the yaw angle of the kite and θ, ϕ are the zenith and azimuth angles with respect to the anchor point in a spherical coordinate system. The kite is controlled with the steering deflection δ and influenced by E and υ_a (glide ratio and relative wind speed) [2], which are calculated as:

$$E = E_0 - c\delta^2$$
$$\upsilon_a = \upsilon_0 E\cos(\theta)$$

Lastly, two more important quantities are mentioned. The tether force T_f and the kite height h are as follows:

$$T_f = \frac{\rho v_0^2}{2} A\cos(\theta)^2(E+1)\sqrt{E^2+1}\cos(\theta)$$
$$h = L\sin(\theta)\cos(\phi)$$

Table I illustrates the parameters used for the dynamic model.

TABLE I

AN OVERVIEW OF DYNAMIC MODEL PARAMETERS

Parameter	Description	Value
L	Tether Length	400
A	Kite Surface	300
E_0	Reference Glide Ratio	6
υ_0	Ambient Wind Speed	10
c	-	0.028

C. The Wind Model

A wind model that emulates dynamic wind speeds similar to what an actual flying kite would experience in open sky is provided. In the second part of the assignment, the wind model is integrated into the system. The air flow $\vec{\upsilon}_a$ affecting the towing kite [2] is

$$\vec{v}_a = \begin{pmatrix} v_0 \\ 0 \\ 0 \end{pmatrix} - v_{\text{roll}}\,\vec{e}_{\text{roll}} - v_{\text{pitch}}\,\vec{e}_{\text{pitch}}\,.$$

The first term describes the external wind, the subsequent two terms are the flow due to the kinematic speeds v_{roll} and v_{pitch} with respect to the basis vectors $\vec{e}_{\text{roll}}$ and $\vec{e}_{\text{pitch}}$. As the wind model is added to the design, the optimal trajectories are observed to be mildly affected by the varying wind speed.

III. Numerical Implementation

A. Discretization of the OCP

Solving an OCP is basically computing an infinite dimensional object $u(\cdot)$. This challenge can be overcome by some approaches namely: the Hamilton-Jacobi-Bellman equation method, indirect methods or direct methods. When working with model predictive control, the essential step of discretization of the continuous time representation of a system allows large-scale non-linear programming (NLP) solvers to find solutions at specified intervals in a time horizon. This project solves the OCP using orthogonal collocation which is a direct method that focuses on converting the OCP into an NLP. The optimal control problem is formulated using CasADi framework in Python. The problem is first discretized and then optimized numerically.

The main goal of the control pod at the base of the kite is to maximize energy generation while preventing instability. The controller must also try to get the optimal energy harvested from the kite under the influence of the wind model and guide the kite in optimal trajectories. A discrete NMPC controller is used to accomplish these tasks as it can minimize the given cost function and satisfy constraints at the same time. The controller calculates the optimal steering deflection δ for every time instant to drive the states zenith Θ, azimuth Φ, yaw Ψ angles optimally. The primary objective is to maximize the tether force while maintaining the height of the kite above $100m$. The prediction horizon is selected to be sufficiently long to have NMPC stability and recursive feasibility. Furthermore, the discrete time-step is tuned to be small in order to increase the numerical integration accuracy of the differential equations.

B. OCP Formulation

The state and the input vector is declared as:

$$x \in \mathbb{X} \subset R^3 : [\psi, \theta, \phi]$$
$$u \in \mathbb{U} \subset R^1 : \delta$$

Optimal control trajectories are obtained by minimizing an objective functional. In order to maximize the harnessed wind energy, the economic objective includes the tether force. For a smooth control trajectory, the difference between the current and the previous input is penalized in the stage cost. The influence of the perturbations can induce a violation of the height constraint hence, it is implemented as a soft constraint which requires a slack variable ε to be adjoined to the objective to increase robustness. The state dynamics are discretized by approximating the states using orthogonal collocation on finite elements. The horizon is divided into N control stages and in each subinterval, inputs and states are discretized. The ordinary differential equations are replaced by finitely many equality constraints and collocation points become optimization variables. Moreover, the state and input bounds are included as inequality constraints in the OCP formulation.

The optimal control problem can be written as such:

$$\min_{u(k),x_k^j,\varepsilon(k)} \quad \sum_{i=0}^{N} -w_{\mathrm{F}}T_{\mathrm{F}} + w_{\mathrm{u}}\left(u-u_{\mathrm{prev}}\right)^2 + w_{\mathrm{s}}\varepsilon^2$$

subject to

$$\begin{aligned}
&\textstyle\sum_{i=0}^{n_{\mathrm{deg}}} x_k^i \frac{\partial l^i\left(t_k^j\right)}{\partial t} = f\left(x_k^j, u(k)\right) \\
&\forall k = 0,\ldots,N-1, j = 0,\ldots,n_{\mathrm{deg}} \\
&x(0) = x_0 \\
&x_{min} \le x \le x_{max} \\
&u_{min} \le u \le u_{max} \\
&\varepsilon_{min} \le \varepsilon \le \varepsilon_{max} \\
&h(x) + \varepsilon \le h_{min}
\end{aligned}$$

The parameters used in the problem formulation[1] are illustrated in table II.

TABLE II
AN OVERVIEW OF OCP FORMULATION PARAMETERS

Parameter	Values/Bounds	Units
θ	$[0, \frac{\pi}{2}]$	rad
ϕ	$[-\frac{\pi}{2}, \frac{\pi}{2}]$	rad
ψ	$[-\pi, \pi]$	rad
δ	[-10, 10]	rad
ε	[0, 5]	-
w_{F}	10^{-4}	-
w_{u}	0.5	-
w_{s}	10^3	-
ρ	1	kg m^{-3}
h_{min}	100	m
n_{deg}	2	-
Δt	0.2	s
N	80	-

IV. RESULTS

Multiple simulation scenarios were executed to indicate how the wind model and its varying wind speeds with different ambient amplitudes affect the optimal kite trajectories. The MPC solver is called once for the open-loop prediction with an initial guess of 1 for all the state variables. The obtained optimal solution from the open-loop prediction is used as the initial guess for the MPC loop (warm start) where closed-loop control steps are performed. For each iteration in the closed-loop, the most recent wind speed is used for the current prediction step. Plus, the optimal solution of the previous iteration is used as the next initial guess for the current iteration. Both the open-loop prediction and the closed-loop simulation are plotted and presented in this section. Fig. 2 shows the trajectory of the kite with and without the wind model having an ambient wind speed υ_0 of $10m/s$. One can observe that although the trajectory varies a little with the wind perturbation, the kite still follows a reasonably efficient flight path without crashing. Fig. 3 illustrates the variation of tether force T_f, kite height h and optimal control input u for a simulation time of 1 minute.
Fig. 4 shows open-loop state trajectory predictions for different constant wind speeds.

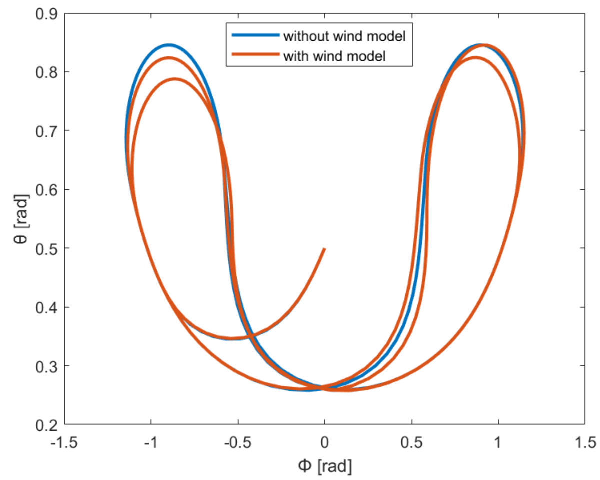

Fig. 2. Closed-Loop State Trajectories

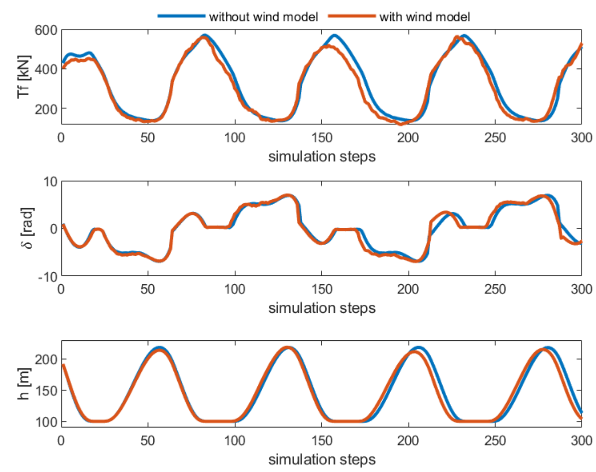

Fig. 3. Tether Force T_f, Steering Deflection δ and Kite Height h

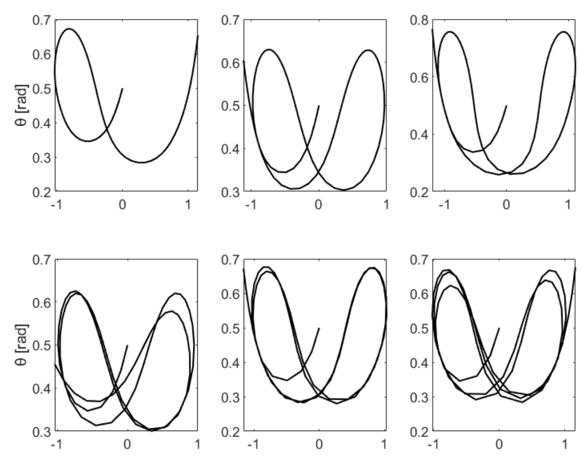

Fig. 4. Kite Trajectories with Different Constant Relative Wind Speeds υ_0 - Open Loop

Fig. 5 demonstrates how the closed-loop state trajectories fluctuate for different varying wind speeds. As the ambient wind speed increases, the trajectories are observed to be oscillating more.

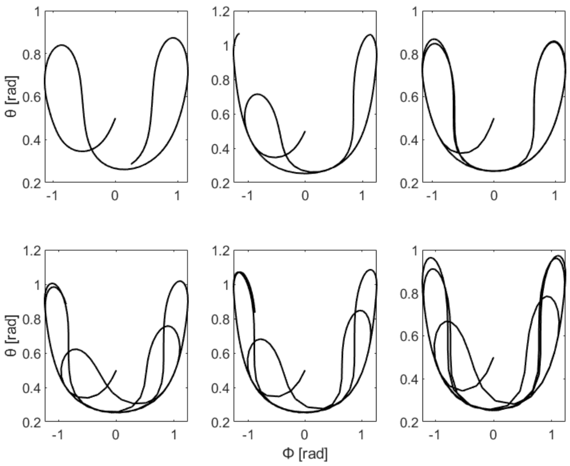

Fig. 5. Kite Trajectories with Different Varying Relative Wind Speeds υ_0 - Closed Loop

V. Conclusion

This project implemented a simple discrete NMPC scheme for the energy optimal control task of a towing kite. The numerical implementation is straight forward where the full discretization is achieved by orthogonal collocation. A wind model was included to represent external disturbances in order to render the simulation as close as possible to a real life environment. The perturbations caused by the varying wind speeds were handled with a soft constraint and a slack variable to keep the OCP feasible. Economic operation of the MPC was accomplished by maximizing the tether force while satisfying the height constraint.

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The authors worked together on the paper and have equal contribution. Nevertheless, the contributions are as follows:

- Introduction - Batu Özmeteler
- System Description - Ashwin Nedungadi
- Numerical Implementation - All Authors
- Results - Rajesh Pushparaj
- Conclusion - All Authors

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