Mehmet Batu Özmeteler
230306

DBDM Assignment, SoSe 2021

g)

- Reactor, jacket and thermostat are perfectly mixed
- Heat is only exchanged between the jacket and the environment and between the reactor and the jacket
- The flow going to and from the thermostat $\dot{m}_T$ is constant
- When $\dot{Q}_{Heat} = 0$, the reactor and the jacket are in thermal equilibrium with the environment

input => heat source ($\dot{Q}_{Heat}$)
output -> reactor temperature (T_R)

$$\frac{dQ}{dt} = \dot{Q}_{in} - \dot{Q}_{out} , \quad Q = m c_p T$$

$$* \dot{Q}_{Heat} = \dot{Q}_{Heat} - \dot{m}_T c_{p_T} T_{Heat} + \dot{m}_T c_{p_T} T_J$$

$$* \dot{Q}_J = \dot{m}_T c_{p_T} T_{Heat} - \dot{m}_T c_{p_T} T_J - k_1 A_1 (T_J - T_e) - k_2 A_2 (T_J - T_R)$$

$$* \dot{Q}_R = - k_2 A_2 (T_R - T_J)$$

$$\dot{T}_{Heat} = \frac{\dot{Q}_{Heat}}{m_T c_{p,T}} + \frac{(T_J - T_{Heat}) \dot{m}_T}{m_T}$$

$$\dot{T}_J = \frac{(T_{Heat} - T_J) \dot{m}_T}{m_J} - \frac{k_1 A_1 (T_J - T_e)}{m_J c_{p,T}} - \frac{k_2 A_2 (T_J - T_R)}{m_J c_{p,T}}$$

$$\dot{T}_R = - \frac{k_2 A_2 (T_R - T_J)}{m_R c_{p,R}}$$

$$\begin{bmatrix} \dot{T}_R \\ \dot{T}_J \\ \dot{T}_{Heat} \end{bmatrix} = \begin{bmatrix} \frac{-k_2 A_2}{m_R Cp_{,R}} & \frac{k_2 A_2}{m_R Cp_{,R}} & 0 \\ \frac{+k_2 A_2}{m_J Cp_{,J}} & -\frac{\dot{m}_T}{m_J} - \frac{k_1 A_1}{m_J Cp_{,J}} - \frac{k_2 A_2}{m_J Cp_{,J}} & \frac{\dot{m}_T}{m_J} \\ -\frac{\dot{m}_T}{m_T} & \frac{\dot{m}_T}{m_T} & 0 \end{bmatrix} \begin{bmatrix} T_R \\ T_J \\ T_{Heat} \end{bmatrix}$$

$$+ \begin{bmatrix} 0 \\ 0 \\ \frac{1}{m_T Cp_{,T}} \end{bmatrix} \begin{bmatrix} \dot{Q}_{Heat} \end{bmatrix} + \begin{bmatrix} 0 \\ +\frac{k_1 A_1 T_e}{m_J Cp_{,J}} \\ 0 \end{bmatrix}$$

$$\dot{T}_R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{T}_R \\ \dot{T}_J \\ T_{Heat} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} \dot{Q}_{Heat} \end{bmatrix}$$

i) The estimated values of k_1 and k_2 are $k_1 = 0{,}0938$, $k_2 = 0{,}0517$

(1) $\dot{T}_{Heat} = 0{,}125\, \dot{Q}_{Heat} + 0{,}005\, (T_J - T_{Heat})$

(2) $\dot{T}_J = 0.005(T_{Heat} - T_J) - 0.0375(T_J - T_e) - 0.01\,(T_J - T_R)$

(3) $\dot{T}_R = -0.001\,(T_R - T_J)$

$T_{Heat}(0) = T_J(0) = T_R(0) = 20\ °C = 293\ K$

1 - continued) -1

$T_{Heat} = a(t)$, $T_J = b(t)$, $T_R = y(t)$, $\dot{Q}_{Heat} = u(t)$

$$\dot{a}(t) = 0.125\,u(t) + 0.005\,b(t) - 0.005\,a(t)$$

$$s\,A(s) - a(0) = 0.125\,U(s) + 0.005\,B(s) - 0.005\,A(s)$$

$$(s + 0.005)\,A(s) = 0.125\,U(s) + 0.005\,B(s) + 293 \quad (1)$$

$$\dot{b}(t) = 0.005\,a(t) - 0.005\,b(t) - 0.0375\,b(t) + 11 + 0.01\,y(t) - 0.01\,T_J$$

$$s\,B(s) - b(0) = 0.005\,A(s) - 0.0425\,B(s) + 0.01\,Y(s) + 11$$

$$(s + 0.0425)\,B(s) = 0.005\,A(s) + 0.01\,Y(s) + 304 \quad (2)$$

$$\dot{y}(t) = 0.001\,b(t) - 0.001\,y(t) \qquad B(s) = \frac{(s+0.001)\,Y(s) - 293}{0.001}$$

$$s\,Y(s) - y(0) = 0.001\,B(s) - 0.001\,Y(s)$$

$$(s + 0.001)\,Y(s) = 0.001\,B(s) + 293 \quad (3)$$

Plug (1) in (2):

$$(s + 0.0425)\,B(s) = 0.005\,\frac{0.125\,U(s) + 0.005\,B(s) + 293}{s + 0.005} + 0.01\,Y(s) + 304$$

$$(s + 0.005)(s + 0.0425)\,B(s) = 0.000625\,U(s) + 0.000025\,B(s) + (0.01\,s + 5\times10^{-6})\,Y(s) + 304\,s + 3$$

i - continued) - 2

Plug (3) in (2):

$$(s^2 + 0{,}0475s + 0{,}000187)B(s) = 0{,}000625\,U(s) + (0.01s + 0.000005)\,Y(s) + 304s + 3$$

$$(s^2 + 0.0475s + 0{,}000187)\left[\frac{(s + 0{,}001)Y(s) - 293}{0.001}\right] =$$

$$0{,}000625\,U(s) + (0{,}01s + 0{,}000005)\,Y(s) + 304s + 3$$

$$(-293000\,s^2 - 13918\,s - 54{,}79) + (1000s^3 + 48{,}5s^2 + 0{,}2345s + 0{,}000187)Y(s) - 0{,}000625\,U(s) + (0{,}01s + 0{,}00005)\,Y(s) + 304s + 3$$

$$(1000s^3 + 48{,}5s^2 + 0{,}2245s + 0{,}000137)Y(s) = 0{,}000625\,U(s) + \underbrace{(2{,}93\times10^5 + 13918s + 54{,}79)}_{\text{neglected}}$$

$$\frac{Y(s)}{U(s)} = \frac{0{,}000625}{1000s^3 + 48{,}5s^2 + 0{,}2245s + 0{,}000137}$$

$$Y(s) = \frac{25}{4\times10^7 s^3 + 1{,}94\times10^6 s^2 + 8980s + 5{,}48}$$

J) $G(s) = \frac{a_0}{b_0 + b_1 s + b_2 s^2 + b_3 s^3}$

$G(z) = (1 - z^{-1})\, Z\{H(s)\}$

$$= \frac{25}{1 \cdot 10^6 s^3 + 13 \cdot 10^4 s^2 + 825 s + 1}$$

$$H(s) = \frac{25}{(200s+1)(5000s^2 + 625s + 1)}$$

$$H(s) = \frac{r_0}{(200s+1)} + \frac{r_2 s + r_1}{(5000s^2 + 625s + 1)}$$

$$= \frac{r_0(5000s^2 + 625s + 1) + (200s+1)(r_2 s + r_1)}{(200s+1)(5000s^2 + 625s + 1)}$$

$$25 = 200s^2 r_2 + 200 s r_1 + r_2 s + r_1 + r_0 5000 s^2 + 625 s r_0 + r_0$$

From the denom root $-\frac{1}{200} \Rightarrow r_0 = -\frac{25}{2}$

Rewrite;

$$25 = s^2(200 a_2 - 62500) + s\left(200 r_1 + r_2 - \frac{15625}{2}\right) + \left(r_1 - \frac{25}{2}\right)$$

$$r_1 = \frac{75}{2}, \quad r_2 = \frac{625}{2}$$

7. continued)

$$H(s) = -\frac{25}{2(200s+1)} + \frac{625s+75}{2(5000s^2+625s+1)}$$

$$h(t) = -\frac{1}{16}e^{-\frac{t}{200}} + \frac{1}{16}e^{-\frac{t}{16}}\cosh\left(\frac{\sqrt{593}}{400}t\right) +$$

$$\frac{23\,e^{-t/16}\sinh\left(\frac{\sqrt{593}}{400}t\right)}{16\sqrt{593}}$$