TECHNISCHE UNIVERSITÄT DORTMUND
Department of Bio- and Chemical Engineering
Process Dynamics and Operations Group
Prof. Dr. Sebastian Engell

Control Theory & Applications - Assignment **WS 20/21**

- The assignment must be submitted **before 14:00 on 10.01.2021**.
- The assignment constitutes 15% of your final grade.
- No points will be given for beauty, but points will be subtracted if the solution is incomprehensible. Furthermore, you are required to prepare the solution by yourself, not in teams. If we found that several students submit the same (or a very similar) solution, we will divide the points of the solution among all students who have copied (including the ”original” version).
- It is required to provide the procedure followed to obtain your results (i.e. only writing the result is not sufficient). Furthermore, justification of your conclusions must be provided.
- Your submission must be a **single PDF file**. Use the MATLAB publish function to do this, e.g., `publish('assignment.m','pdf')`. Make sure text/code is not cut at the page borders!
- The MATLAB code shall be commented. The comments shall also include to which task does this part of the code belong.
- The name of the PDF file should be your **Matriculation Number**.
- The first page of your submission should be the filled in cover sheet for personal information that is provided on **MOODLE**.
- All plots (with appropriate labels) must be included in the PDF file.
- Upload your solution in the corresponding field for 'assignment submission' on the **MOODLE** webpage.
- All the calculations done on paper must be easily readable, scanned/photographed with a decent quality, and part of the PDF file.
- Please use the assignment consultation hours:

Friday	**18.12.2020**	*at*	**16:00**	via	**Zoom**
Monday	**04.01.2021**	*at*	**16:00**	via	**Zoom**

Please use the same zoom link as that of the tutorials for both consultation hours.

1 Mechanical System Modeling and Control (35 points)

A small pick-and-place robot of mass M affected by the gravitational acceleration g moves on an inclined plane along an axis z and is attached to a wall-mounted spring with constant k_s as illustrated in Figure 1. The inclination of the plane is θ with respect to the horizontal plane. The robot picks objects from the position $z = 0$ (as indicated in Figure 1) and the velocity of the robot is $\dot{z}$. The spring is used to prevent the robot from accidental free fall in the case of power failure and the natural length of the spring (the length for which the force generated by the spring is equal to zero) is $8m$. The effects of friction from the inclined plane are very small and are therefore neglected.

The robot has a motor which provides a linear force F in the positive z-direction (upwards). The motor can provide a maximum force value of $F_{max} = 115N$. If the force supplied by the motor exceeds F_{max}, the motor will fail. Assume that all the states are perfectly measured and that the robot is modeled as a point mass.

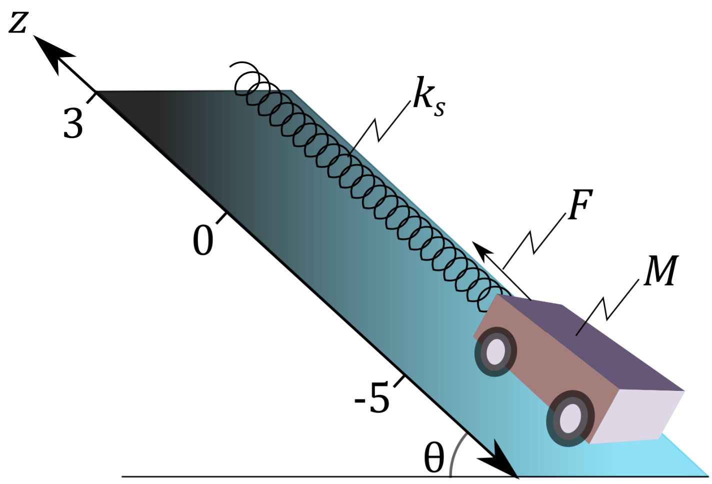

Figure 1: Mechanical System

1. What are the state variables of this system? (**2 pts.**)

2. Use the coordinates and positions indicated in Figure 1 to derive the differential equations that describe the motion of the system. Declare each step in your calculation. (**6 pts.**)

3. Put the model in state space form.
   Hint: Choose the input such that the state equation is exactly $\dot{x} = Ax + Bu$ with no extra terms, and the output equation is exactly $y = Cx + Du$. (**4 pts.**)

   From hereon, assume that $M = 15Kg$, $\theta = 30^o$, $k_s = 3\frac{N}{m}$, and assume that the gravitational acceleration is $g = 10\frac{m}{s^2}$.

4. Substitute the constants in the state space matrices with the given parameter values. (**1 pt.**)

5. Check the controllability of this system using the Kalman criterion. (**2 pts.**)

   You will now design two state feedback controllers for this system: Design (I) and Design (II). Assume that the robot is initially at rest ($\dot{z} = 0$) and is at position $z = -5m$ as shown in Figure 1.

6. Design (I): Design a controller for the system such that eigenvalues of the controlled system (closed-loop poles) are $\lambda_1 = -0.8$ and $\lambda_2 = -1$. The controller shall move the robot to the position $z = 0$ on the inclined plane and the robot shall remain there afterwards. Determine the gain matrix K and show all the steps of your calculations. (**6 pts.**)

7. Use MATLAB to plot the closed-loop response. Plot the states and the force supplied by the motor over a time period of 20 seconds. (**3 pts.**)

8. From the system description and the plots of the previous task, why is this design problematic?(**1 pts.**)

9. Design (II): Design a controller for the system such that the problem that resulted from Design(I) is mitigated. Specify the eigenvalues of the controlled system (closed-loop poles). The controller shall move the robot to the position $z = 0$ on the inclined plane and the robot shall remain there afterwards. Determine the gain matrix K and show all the steps of your calculations. (**5 pts.**)

10. Use MATLAB to plot the closed-loop response. Plot the states and the force supplied by the motor over a time period of 20 seconds. (**3 pts.**)

11. For Design(II), what is the final value (equilibrium value) of the the force supplied by the motor. Compute it by hand and compare it with the results from the MATLAB plot. (**2 pts.**)

Notes:

All the tasks should be solved by hand (without MATLAB) except for the closed-loop simulations, i.e. tasks 7 and 10.

Please do all the mathematical modeling and computations on paper (in clear handwriting) and make a clear scan and include it in the PDF.

Solution

1. The state variables are the position on the inclined plane $x_1 = z$ (**1 pt.**) and the velocity $x_2 = \dot{z}$ (**1 pt.**).

2. Position:
   $\dot{x_1} = \dot{z} = x_2$ (**1 pt.**).

   Velocity:
   Since the natural length of the spring is $8m$ and $z = 0$ corresponds to $5m$ compression: (**1 pt.**)
   $M\ddot{z} = F - Mg.sin(\theta) - k_s(z+5)$ (**2 pts.**).
   $\ddot{z} = \frac{-k_s z}{M} + \frac{F}{M} - g.sin(\theta) - \frac{3k_s}{M}$ (**1 pt.**).
   $\dot{x}_2 = \frac{-k_s x_1}{M} + \frac{F}{M} - g.sin(\theta) - \frac{3k_s}{M}$ (**1 pt.**).

3. Let $u = \frac{F}{M} - gsin(\theta) - \frac{3k_s}{M}$ (**1 pt.**)
   $A = \begin{pmatrix} 0 & 1 \\ -\frac{k_s}{M} & 0 \end{pmatrix}$ (**1.5 pts.**)

   $B = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ (**0.5 pts.**)

   $C = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ (**0.5 pts.**)

   $D = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ (**0.5 pts.**)

4. $A = \begin{pmatrix} 0 & 1 \\ -0.2 & 0 \end{pmatrix}$
   The other matrices are independent of the parameters.

   (**1 pt.**)

5. Kalman:

   $S = [B|AB]$
   $S = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
   $Rank(S) = 2$. Therefore the system is controllable.

   (**2 pts.**)

6. Desired characteristic polynomial:
   $(\lambda + 0.8)(\lambda + 1) = \lambda^2 + 1.8\lambda + 0.8$ (**1 pt.**)

   $A_K = A + BK = \begin{pmatrix} 0 & 1 \\ -0.2 & 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} K_1 & K_2 \end{pmatrix}$(**1 pt.**)

   $A_K = \begin{pmatrix} 0 & 1 \\ K_1 - 0.2 & K_2 \end{pmatrix}$(**1 pt.**)
   $det(\lambda I - A_K) = \lambda(\lambda - K_2) + (0.2 - K_1)$
   $det(\lambda I - A_K) = \lambda^2 - K_2\lambda + 0.2 - K_1$ (**1 pt.**)

Equate to the desired characteristic polynomial:
$K_2 = -1.8$ (**1 pt.**)
$K_1 = -0.6$ (**1 pt.**)

7. Design(I) plots:

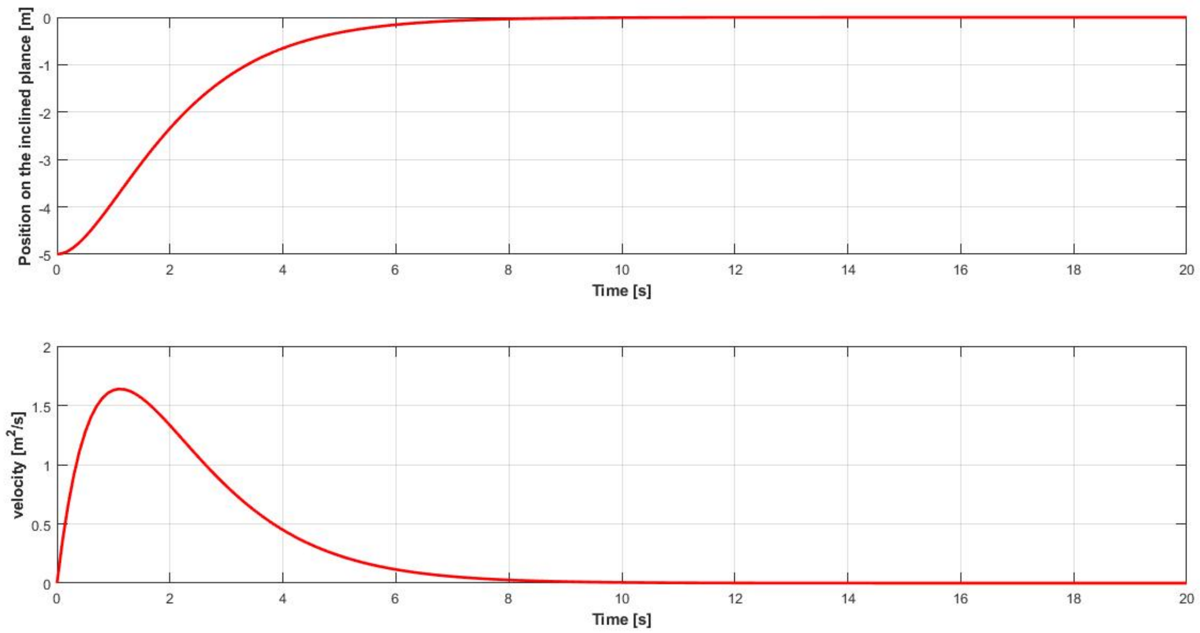

Figure 2: Design I: States

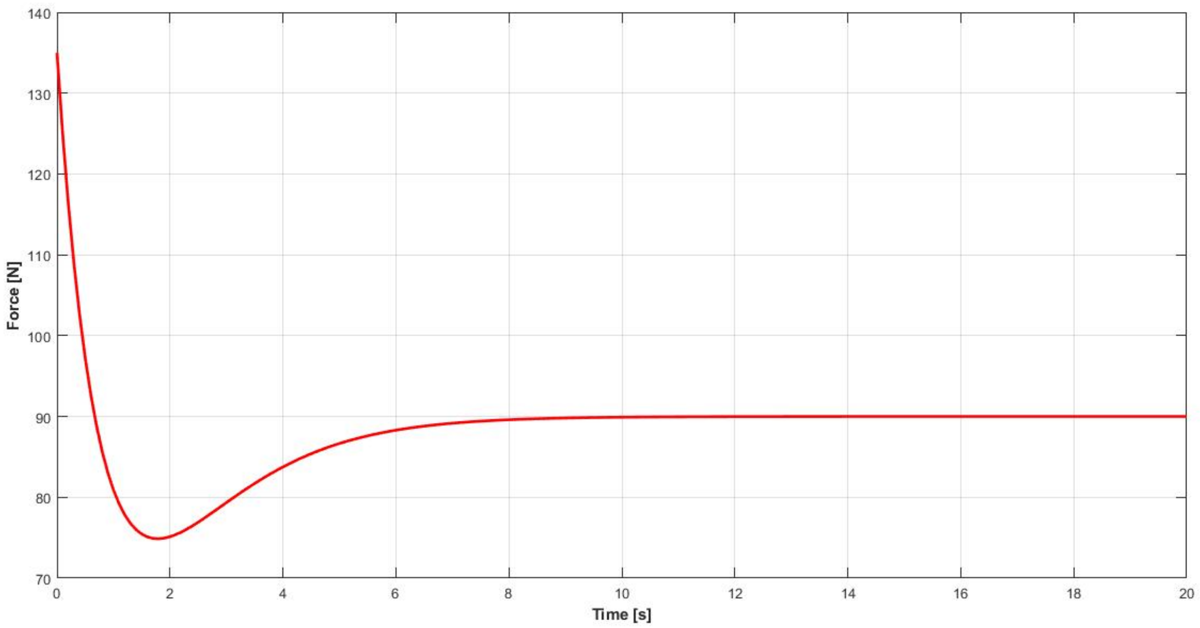

Figure 3: Design I: Force

(**3 pts.**)

8. This is because the motor force exceeds $115N$ as shown in Figure 3 and hence the motor will fail.(**1 pts.**)

9. We have to choose the eigenvalues of the closed-loop system such that the motor force does not exceed the specified limit ($115N$). By trail and observing the force plot we can (for example) choose the eigenvalues of the closed-loop to be $\lambda_1 = -0.5$ and $\lambda_2 = -1$(**1 pts.**)

   Desired characteristic polynomial:
   $(\lambda + 0.5)(\lambda + 1) = \lambda^2 + 1.5\lambda + 0.5$ (**1 pt.**)

   $$A_K = A + BK = \begin{pmatrix} 0 & 1 \\ -0.2 & 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} K_1 & K_2 \end{pmatrix}$$

   $$A_K = \begin{pmatrix} 0 & 1 \\ K_1 - 0.2 & K_2 \end{pmatrix}$$
   $det(\lambda I - A_K) = \lambda(\lambda - K_2) + (0.2 - K_1)$
   $det(\lambda I - A_K) = \lambda^2 - K_2\lambda + 0.2 - K_1$ (**1 pt.**)
   Equate to the desired characteristic polynomial:
   $K_2 = -1.5$ (**1 pt.**)
   $K_1 = -0.3$ (**1 pt.**)

   Note: Different choices of the eigenvalues will result in a different gain matrix. This has to be taken into account during the correction.

10. Design(II) plots:

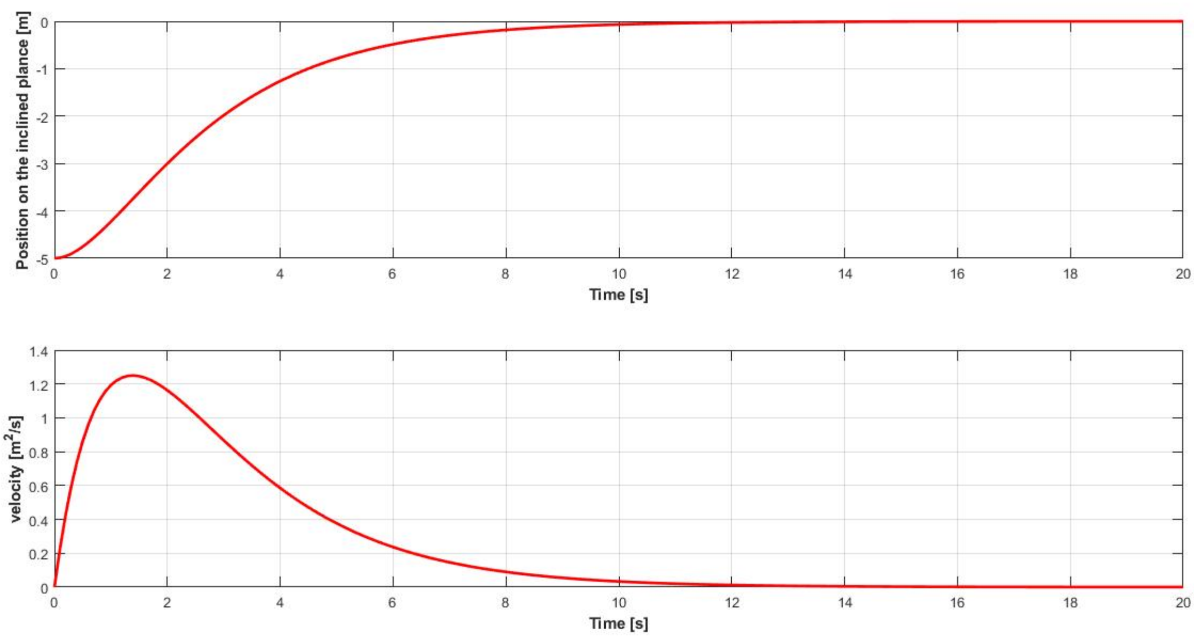

Figure 4: Design II: States

(**3 pts.**)

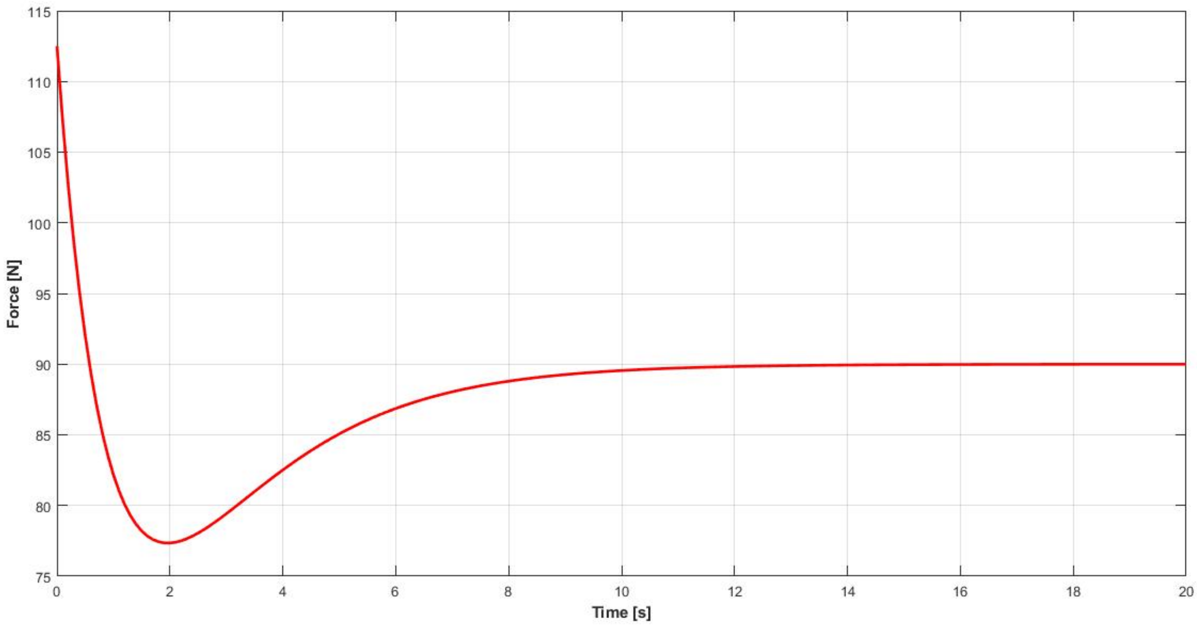

Figure 5: Design II: Force

As we can see, the motor force never exceeds the specified limit $(115N)$. **Note: Different choices of the eigenvalues will result in different plots, however the force must not exceed $115N$. This has to be taken into account during the correction**.

11. At the controlled equilibrium $z = 0$:

    $F - k_s z - Mg.sin(\theta) - 5k_s = 0.$
    $F - 75 - 15 = 0.$
    $F = 90N.$

    Same as the plot.

    (**2 pts.**)

2 Process System Modeling and Control (50 points)

Consider an isothermal continuous stirred tank reactor (CSTR), shown in Figure (6). Component A is fed with the flow rate F_r and inlet concentration $C_{A_{in}}$. In the reactor, the following irreversible exothermic first order reactions take place:

$$A \xrightarrow{k_1} B \xrightarrow{k_2} C$$

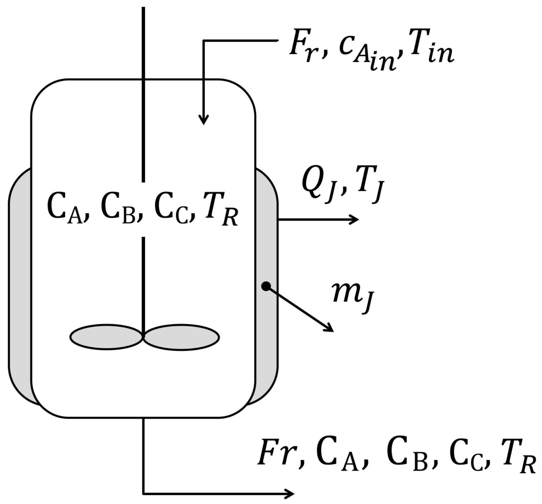

Figure 6: Reactor system

To prevent the reaction system from a thermal runaway, a cooling jacket is built around the reactor. Further assumptions are as follows:

- The feed inlet flow only contains component A.
- The volume of the reactor V is constant at all times.
- The reaction mixture is completely filled with liquid and is ideally mixed.
- The liquid is in-compressible.
- The reactions are elementary and obey the Arrhenius relation:

  $$r_i = k(T_R) \cdot C_i = k0_i \cdot exp(-E_R/T_R) \cdot C_i.$$

- The amount of heat that is removed by the jacket can be adjusted by Q_J.
- The heat transfer coefficient between the reaction medium and the jacket (kA) is constant.
- The initial concentrations in the reactor are $C_{A,0} = C_{B,0} = 0$ [kmol/m^3] and the initial reactor and jacket temperatures are $T_{R,0} = T_{J,0} = 387.05$ [K].

Variables and parameters are listed in Table 1.

Quantity	Symbol	Value	Unit
Feed volumetric flowrate	Fr	-	$\frac{\mathrm{m}^3}{\mathrm{min}}$
Component A inlet concentration	$\mathrm{C_{A_{in}}}$	5.1	$\frac{\mathrm{kmol}}{\mathrm{m}^3}$
Concentration of component i at the reactor outlet	$\mathrm{C_i}$	-	$\frac{\mathrm{kmol}}{\mathrm{m}^3}$
Reactor volume	V	0.01	m^3
Pre-exponential factor - first reaction	$\mathrm{k0_1}$	2.145e10	$\frac{1}{\mathrm{min}}$
Pre-exponential factor - second reaction	$\mathrm{k0_2}$	2.145e10	$\frac{1}{\mathrm{min}}$
Reaction activation energy - first reaction	$\mathrm{E_{R,1}}$	9758.3	K
Reaction activation energy - second reaction	$\mathrm{E_{R,2}}$	9758.3	K
Heat of reaction - first reaction	$\Delta\mathrm{H_{R,1}}$	-4200	$\frac{\mathrm{kJ}}{\mathrm{kmol}}$
Heat of reaction - second reaction	$\Delta\mathrm{H_{R,2}}$	-11000	$\frac{\mathrm{kJ}}{\mathrm{kmol}}$
Inlet temperature	$\mathrm{T_{in}}$	387.05	K
Reactor temperature	$\mathrm{T_R}$	-	K
Jacket temperature	$\mathrm{T_J}$	-	K
Liquid density	ρ	934.2	$\mathrm{kg/m^3}$
Heat capacity reaction medium	$\mathrm{c_p}$	3.01	$\frac{\mathrm{kJ}}{\mathrm{m^3 \cdot K}}$
Heat capacity jacket medium	$\mathrm{c_{p,J}}$	2.0	$\frac{\mathrm{kJ}}{\mathrm{m^3 \cdot K}}$
Coolant mass	$\mathrm{m_J}$	5.0	kg
Heat transfer coefficient	kA	14.448	$\frac{\mathrm{kJ}}{\mathrm{min \cdot K}}$
Heat removal by the jacket	$\mathrm{Q_J}$	-	$\frac{\mathrm{kJ}}{\mathrm{min}}$

Table 1: List of parameters and variables

Tasks:

Note: For tasks 1 to 4 you are asked to solve them by hand on paper. Only for part 3, you may use MATLAB for the solution of the nonlinear equations. For the rest of the tasks you can use MATLAB where needed.

1. What are the required balances that describe the dynamic behavior of the system? (**2 pts.**)

2. Set up the differential balance equations that describes the concentration profiles of component A and B as well as the reactor and jacket temperatures T_R and T_J. Is the system linear or nonlinear? Justify your answer. (**11 pts.**)

3. Calculate the equilibrium point(s) of the system for the given steady-state inputs: $Fr_{ss} = 0.002365$ [m^3/min]$, Q_{J,ss} = 18.5583$ [kJ/min]. (**2 pts.**)

4. Linearize the system around the computed equilibrium point(s) in part (2). Put the linearized system in the standard state space representation, assume that all the states are measured. Check the local stability of the computed equilibrium point(s). (**5 pts.**)

5. Check the validity of the linearization by simulating the linearized system against the original model at the equilibrium point(s) for a $\Delta u = \pm 10\%$ of the steady state input. (**4 pts.**)

6. Check the operability of the linearized system(s). What is the dimension of the steady-state subspace? What do you infer from the matrices V and U (in terms of the input and output directions)? And how to explain this physically? (**5 pts.**)

7. Assuming that all of the states are measured, design a state feedback controller to regulate the nonlinear system around the selected equilibrium point? Check the condition needed to assign the closed loop poles freely? Give a reason for your selection of the closed-loop poles. Simulate the closed-loop with the nonlinear system at the following 2 initial conditions $x_{0,1} = [C_{A,ss} * 0.9, C_{B,ss} * 0.9, T_{R,ss} - 10, T_{J,ss} - 10]$ and $x_{0,2} = [C_{A,ss} * 1.1, C_{B,ss} * 1.1, T_{R,ss} + 10, T_{J,ss} + 10]$. (**8 pts.**)

8. Now assume that the concentrations C_A and C_B are not measured, and the temperatures of the reactor and the jacket are the only available measurements. Design a Luenberger observer to estimate the unmeasured states. Show the convergence of the estimated concentrations to the true states of the nonlinear system by simulation from different initial conditions. show the results using same initial conditions as task (7) (**7 pts.**)

9. Simulate the nonlinear system with the observer-based feedback controller. Test the closed-loop with the nonlinear system at the same 2 initial conditions from task (7) and compare between the simulation results with state feedback and the with observer-based state feedback in terms of the closed-loop performance. (**6 pts.**)

Solution

1. Molar and energy balances, total mass is not required as the volume inside the reactor is constant. **2 pts.**

2. Reaction rate for the first reaction:

$$r_1 = k0_1 \cdot exp(-E_{R,1}/T_R) \cdot C_A \tag{1}$$

Reaction rate for the second reaction:

$$r_2 = k0_2 \cdot exp(-E_{R,2}/T_R) \cdot C_B \tag{2}$$

Differential balance for the concentration of A: **2 pts.: 1 pt. for correct inlet/outlet terms, 1pt. for correct sign of reaction rate**

$$\frac{dC_A}{dt} = \frac{Fr}{V}(C_{A_{in}} - C_A) - r_1 \tag{3}$$

Differential balance for the concentration of B:**2 pts.: 1 pt. for correct inlet/outlet terms, 1pt. for correct sign of reaction rates**

$$\frac{dC_B}{dt} = \frac{-Fr}{V}C_B + r_1 - r_2 \tag{4}$$

Differential balance for the reactor temperature:**4 pts.: 1 pt. for correct inlet/outlet terms, 1pt. for correct sink term, 2 pts for correct source terms (1 for each)**

$$\frac{dT_R}{dt} = \frac{Fr}{V}(T_{in} - T_R) - \frac{kA}{\rho c_p V}(T_R - T_J) + \frac{1}{\rho c_p}((\Delta H_{R,1})(-r_1) + (\Delta H_{R,2})(-r_2)) \tag{5}$$

Differential balance for the jacket temperature:**2 pts.: 1 pt. for correct heat removal sign, 1pt. for correct source term**

$$\frac{dT_J}{dt} = \frac{1}{m_J \cdot c_{p,J}}(-Q_J + kA(T_R - T_J)) \tag{6}$$

The system is nonlinear due to the multiplication of the concentration to the exponential function of temperature.**1 pt.**

3. Solving the above system of equation for , $\dot{f}(x) = 0$, the equilibrium point is calculated as: $x_{ss} = [1.6329, 1.1101, 398.6581, 397.3736]$. **2 pts.: 1pt. for correct way of calculation, e.x fsolve; 1 pt. for correct results**

4. Computing the corresponding derivatives in the jacobian matrices J_A and J_B and substitution of the calculated equilibrium states and input variables, the jacobian matrices are calculated as: **4 pts.: 2 per each correct jacobian (1 for correct derivatives and 1 for correct substitutions)**

$$J_A = \begin{pmatrix} -0.7386 & 0 & -0.0503 & 0 \\ 0.5021 & -0.7386 & 0.0161 & 0 \\ 0.75 & 1.9643 & -0.5412 & 0.5138 \\ 0 & 0 & 1.4448 & -1.4448 \end{pmatrix} \tag{7}$$

$$J_B = \begin{pmatrix} 346.7 & 0 \\ -111 & 0 \\ -1160.8 & 0 \\ 0 & -0.1 \end{pmatrix} \tag{8}$$

Then the linearized system will look like:

$$\Delta\dot{x} = J_A\Delta x + J_B\Delta u, \tag{9}$$

with the output equation,

$$y = Cx = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} x. \tag{10}$$

For the local stability of the equilibruim point, the eigenvalues of the linearized system are computed as follows, $\lambda_i = \{-0.1205, -0.4637, -0.9058, -1.9734\}$. it is noticed that they are all negative real eigenvalues which implies local asymptotic stability of the equilibrium point.**2 pts**

5. Validity of linearization around the equilibrium point with step input $\Delta u = 0.1 * u_{ss}$. **(4.0 points)** for the correct simulation results.

Figures (7 and 8) show the state evolution of the nonlinear and the linearized systems for the desired step input. From the simulation it is noticed that the linearized system captures the essential dynamical behavior of the nonlinear system in the vicinity of the equilibrium point. It is also noticed that the state C_B has slightly different steady value, however the main dynamics are captured pretty well.

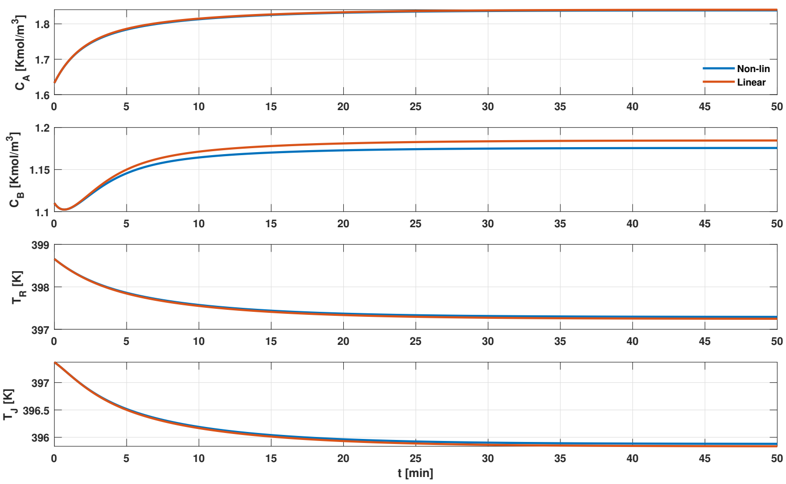

Figure 7: Linearized against nonlinear system around equilibrium point for $(\Delta u = 10\%)$

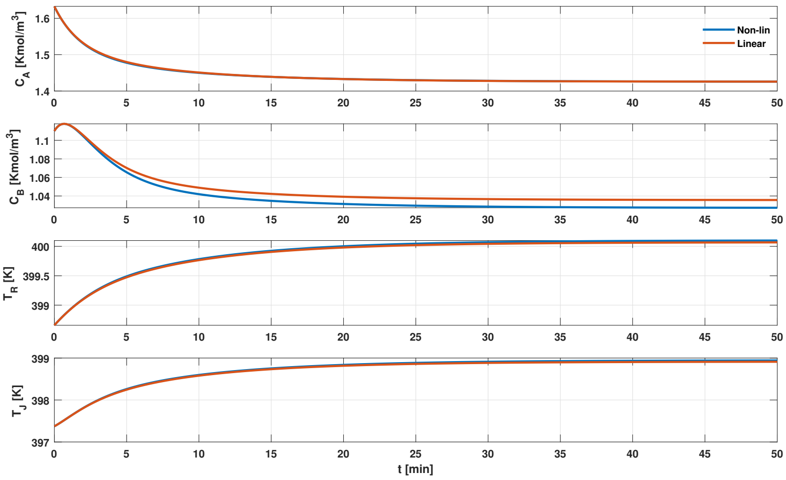

Figure 8: Linearized against nonlinear system around equilibrium point for ($\Delta u = -10\%$)

6. **(5.0 points)** for the correct answer.

$$M = -J_A^{-1} J_B, \qquad = \begin{pmatrix} 726.1 & 0.019 \\ 261.1 & 0.006 \\ -3766.6 & -0.28 \\ -3766.6 & -0.349 \end{pmatrix} \textbf{(1.0 point)} \tag{11}$$

Rank of the operability matrix is $2 < n$, therefore not fully operable.**(1.0 point)**
It can be also observed that the steady state subspace is of dimension 2 at maximum as the system has only 2 inputs.**(1.0 point)**

$$V = \begin{pmatrix} -1.0000 & -0.0001 \\ -0.0001 & 1.0000 \end{pmatrix} \tag{12}$$

$$U = \begin{pmatrix} -0.1349 & -0.6211 & 0.7363 & 0.2322 \\ -0.0485 & -0.2234 & 0.1077 & -0.9675 \\ 0.6998 & 0.4593 & 0.5411 & -0.0809 \\ 0.6998 & -0.5945 & -0.3917 & 0.0585 \end{pmatrix} \tag{13}$$

$$Sigma = \begin{pmatrix} 5382.4 & 0 \\ 0 & 0.06 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \tag{14}$$

$$\tag{15}$$

The singular values are $\sigma_{1,2} = \{5382.4, 0.06\}$ and the condition number is then:

$$\gamma = \frac{\sigma_1}{\sigma_2} = 8.19e + 04 \tag{16}$$

So according to the limit given in the lecture, the matrix is on the verge of being ill-conditioned. Therefore $v_1 = [-1; -0.0001]$ is the direction with a very large system amplification and in direction $v_2 = [-0.0001; 1]$ the system gain is very low. **(1.0 point)**

2 points for the correct interpretation and the physical explanation
If the input is applied along v_1, largest amplification occurs along vector u_1. On this output direction, it can be seen that the concentrations decrease and the temperatures both increase with same ratio. So it is very easy to increase the temperatures if the first input F_r is decreased while the heat removal Q_J is almost the same.

If the input is applied along v_2, smallest amplification occurs along vector u_2. On this output direction, it can be seen that the concentrations also decrease but the temperatures move in different directions. This make sense as it is very hard to make the reactor temperature increase while decreasing the jacket temperature or vice versa. For this you need a very fast heat removal from the jacket and same time the heat production from the reactions should be little reduced by less inflow. So F_r is slightly decreased while the heat removal Q_J is increased a lot more.

The steady state can not move in the directions of u_3 and u_4, using the given input variables.

7. **(8.0 points)** for the correct answer.

The pair (J_A, J_B) is fully controllable as the rank of the controlability matrix is $(n = 4)$. **(1.0 points)**

Therefore the closed-loop eigenvalues can be freely assigned to the desired locations. The closed-loop eigenvalues can be $\lambda_i^{cl} = \{-0.5, -1.5, -2, -5\}$ which is almost 3 times faster than the open-loop ones. This leads to a gain matrix, **(2.0 points)**

$$K = \begin{pmatrix} 0.0031 & 0.0054 & -0.0034 & -0.0009 \\ -39.4073 & 2.5278 & -33.4397 & -20.7700 \end{pmatrix}$$

The close loop now has much faster response as is shown in the simulation studies. **(1.0 points)**

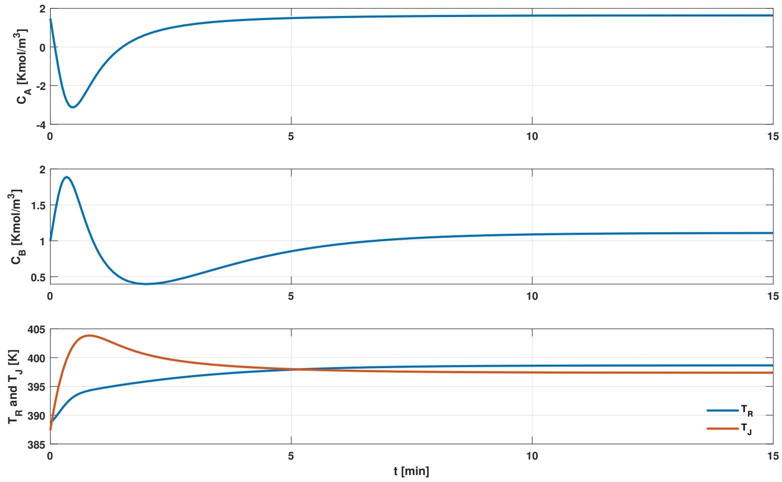

Figure 9: Closed-loop system simulation with linear state feedback controller ($x_0 = x_{0,1}$).(**(2.0 points)** for correct simulation)

8. **(7.0 points)** for the correct answer.

With the given information, the measurements matrix is: **(1.0 point)**

$$C = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{17}$$

Then the pair (J_A, C) of the linearized system is completely observable. The Kalman observability check renders the observability matrix with full rank $n = 4$. **(1.0 points)**

Therefore the observer can be built and its eigenvalues can be freely assigned. We choose $\lambda_i^o b = \{-1.5, -3, -6, -10\}$. This leads to a gain matrix, **(2.0 points)**

$$L = \begin{pmatrix} 111.6636 & 0 \\ -1.1131 & 0 \\ 16.9815 & 0.5138 \\ 1.4448 & 0.0552 \end{pmatrix}$$

From the given initial conditions, the convergence is shown in figures 11 and 12. **(3.0 points)**

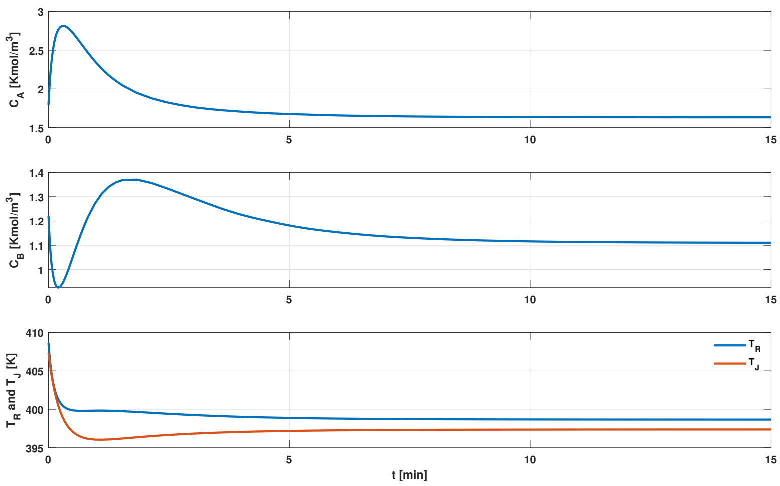

Figure 10: Closed-loop system simulation with linear state feedback controller ($x_0 = x_{0,2}$).(**(2.0 points)** for correct simulation)

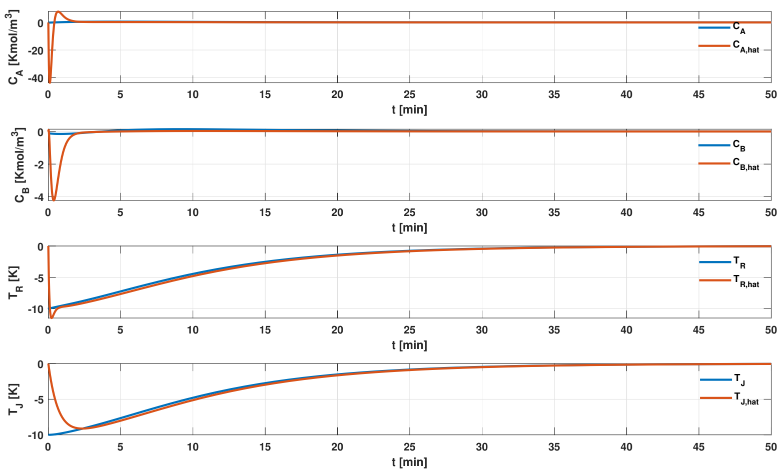

Figure 11: convergence of the observer for initial conditions $x_{0,2}$

9. **(6.0 points)** for the correct answer.

   Output feedback simulations are shown in figures (13) and (14). For the observer based feedback case, the performance degraded a bit due to wrong estimates of the C_A and C_B. Larger overshoots with more oscillations can be observed in the output feedback cases. Also the closed loop takes longer to regulate the nonlinear system to the steady state. **(2.0 points)**

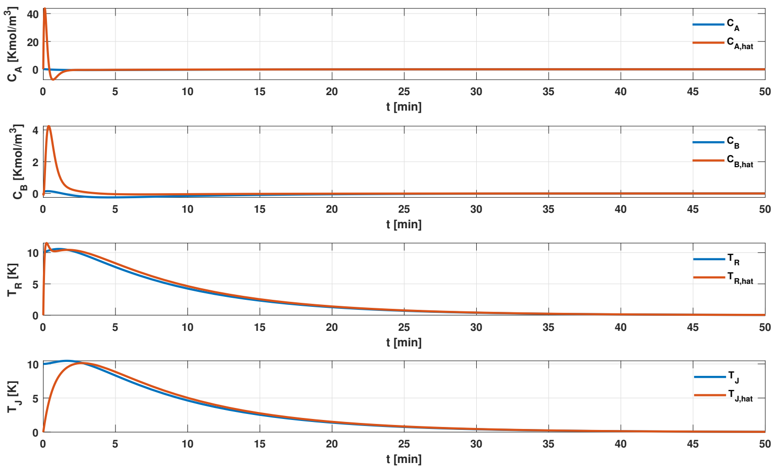

Figure 12: convergence of the observer for initial conditions $x_{0,2}$

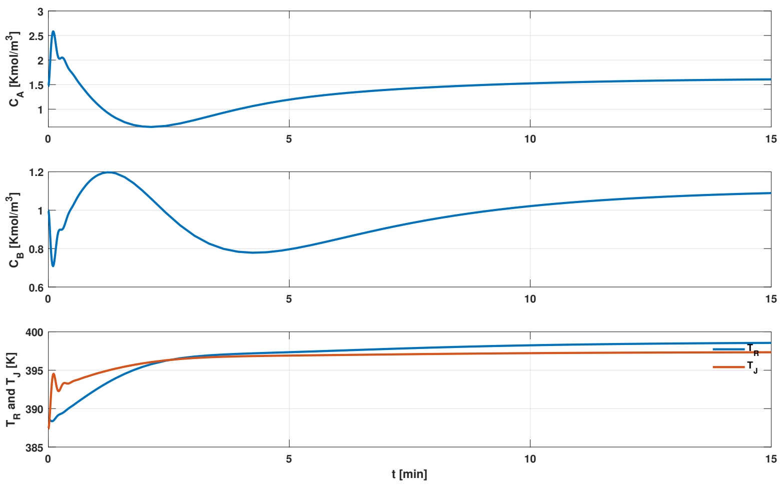

Figure 13: Closed-loop system simulation with linear state feedback controller ($x_0 = x_{0,1}$).(**(2.0 points)** for correct simulation)

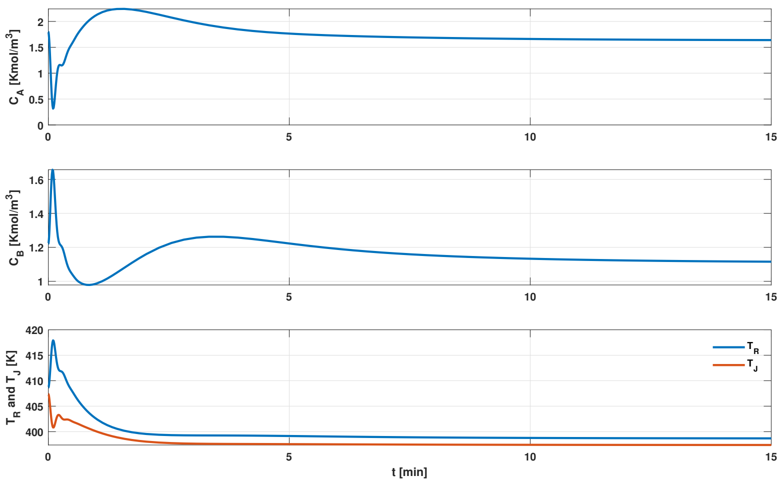

Figure 14: Closed-loop system simulation with linear state feedback controller ($x_0 = x_{0,2}$).(**(2.0 points)** for correct simulation)