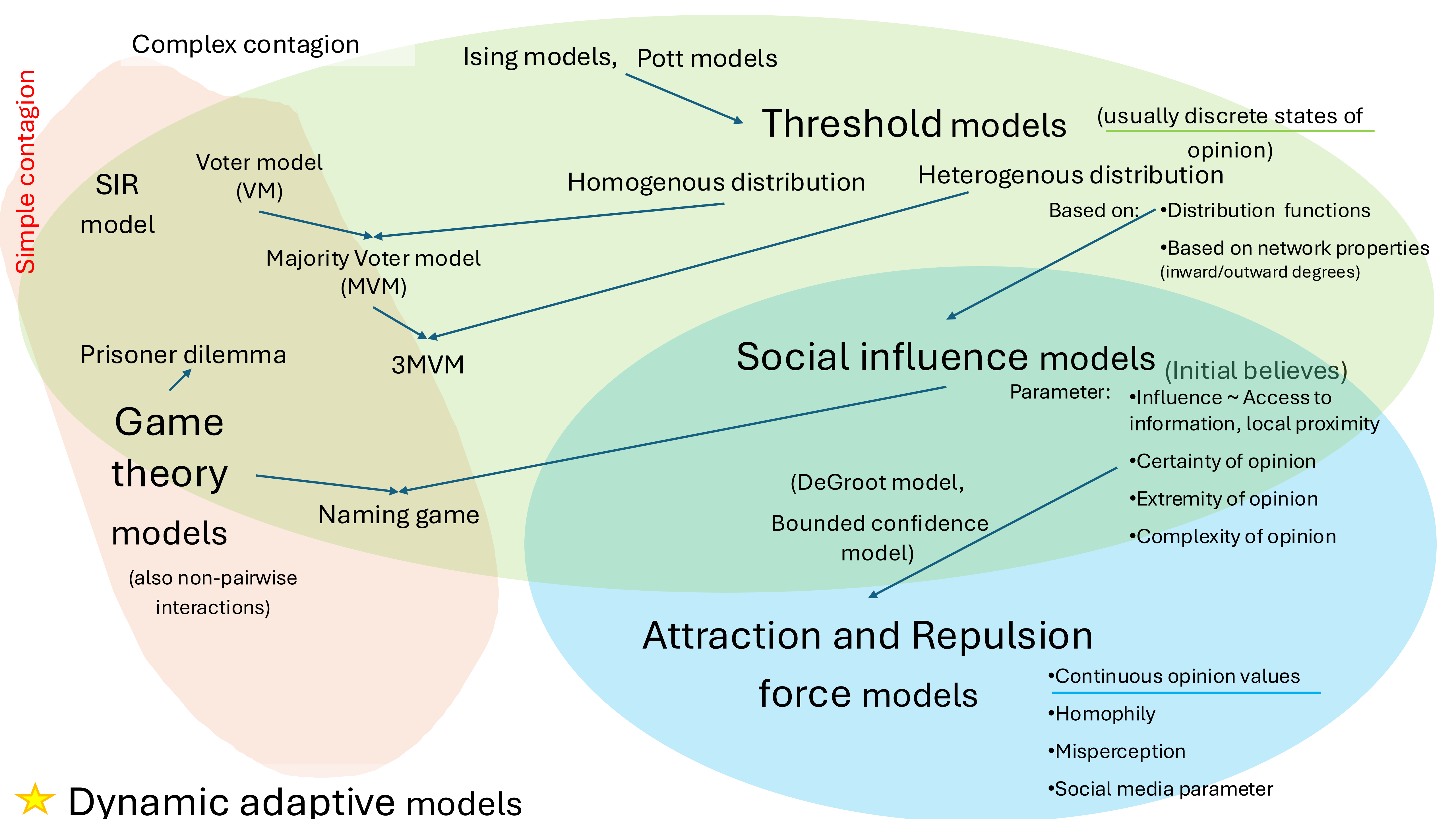
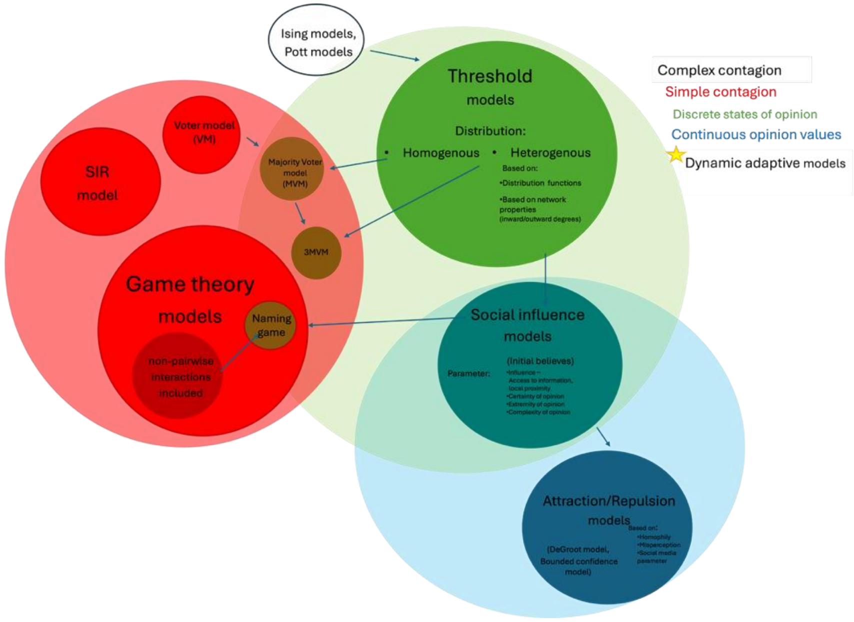
Opinion dynamics

SynoSys Deep work and Focus Days Jun 2025, Barbara Vinatzer

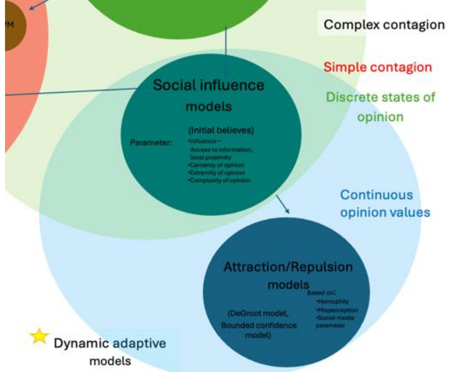

Complex contagion
Simple contagion
Ising models, Pott models
Threshold models (usually discrete states of opinion)
SIR model
Voter model (VM)
Homogenous distribution
Heterogenous distribution
Based on: •Distribution functions
•Based on network properties (inward/outward degrees)
Majority Voter model (MVM)
3MVM
Prisoner dilemma
Game theory models
(also non-pairwise interactions)
Naming game
Social influence models (Initial believes)
Parameter: •Influence ~ Access to information, local proximity
•Certainty of opinion
•Extremity of opinion
•Complexity of opinion
(DeGroot model, Bounded confidence model)
Attraction and Repulsion force models
•Continuous opinion values
•Homophily
•Misperception
•Social media parameter
Dynamic adaptive models

Base assumptions

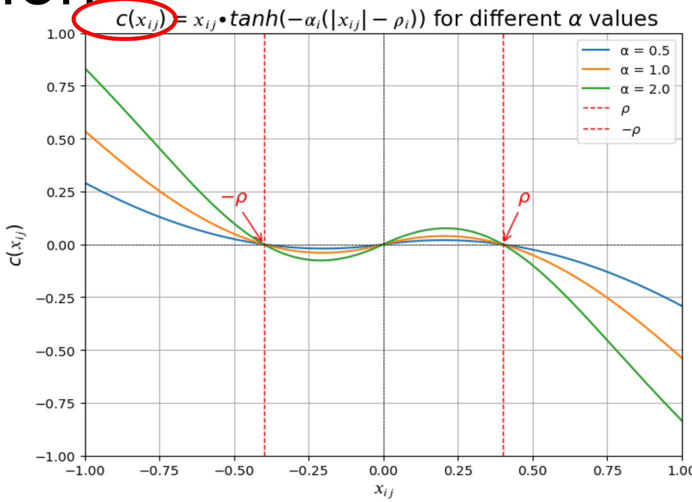


- ➢States of opinion
  - o Discrete
  - o Continuous
- ➢Thresholds
  - o Homogenous
  - o Heterogenous
- ➢Networks
  - o Static
  - o Dynamic
- ➢Social influence
- ➢Repulsion allowed?
- ➢Allowing non pairwise/bipartite interactions
- ➢Stubborn agents/leaders

$$x_i(t+1) = x_i + \frac{\beta_i}{\sum_{j\neq i} w_{ij}} \sum_j w_{ij}\, x_{ij} \cdot \tanh(-\alpha_i(|x_{ij}| - \rho_i))$$

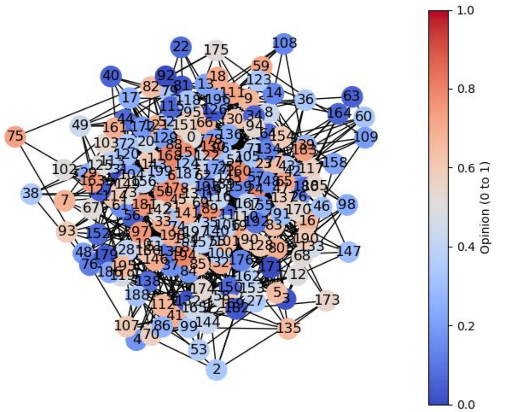


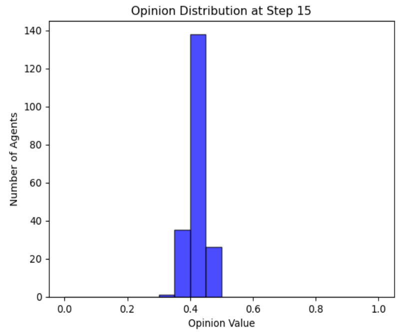
- $x_i(t), x_j(t) \in [0,1]$ → Opinions of individuum x_i and its neighbors
- $x_{ij}(t) = x_j(t) - x_i(t) \in [-1,1]$ → difference of opinion
- $w_{ij} \in [0,1]$ → social influence parameter (edge weights, unsymmetrical influence)
- $\alpha_i \in [0,+\cdots]$ → controversy of the topic (nonlinearity parameter,)
- $\rho_i \in [0,1]$ → max opinion difference before repulsion
- β_i → coupling strength (stubborn agents)

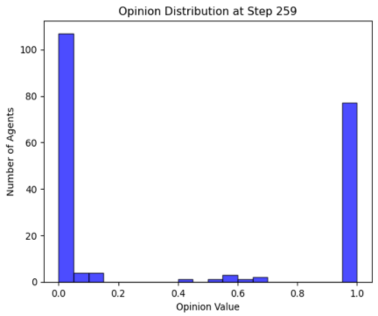


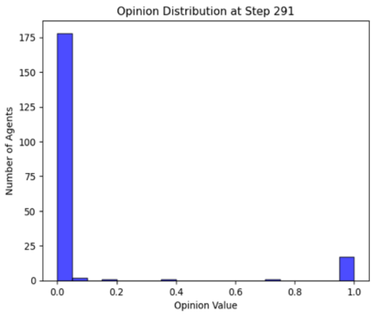
Simulations

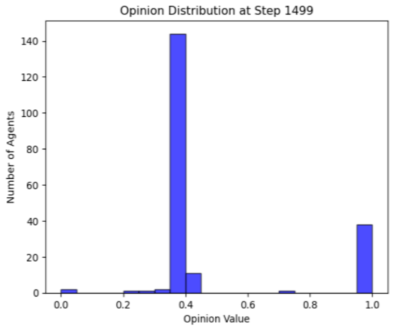
$$x_i(t+1) = x_i + \frac{\beta_i}{\sum_{j\neq i} w_{ij}} \sum_j w_{ij} \;\; x_{ij} \cdot \tanh(-\alpha_i(|x_{ij}| - \;\rho_i))$$

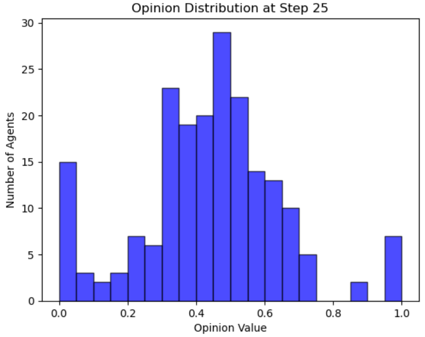












Variance: 0.000618
Bipolarity: 0.0

Variance: 0.22913
Bipolarity: 0.94

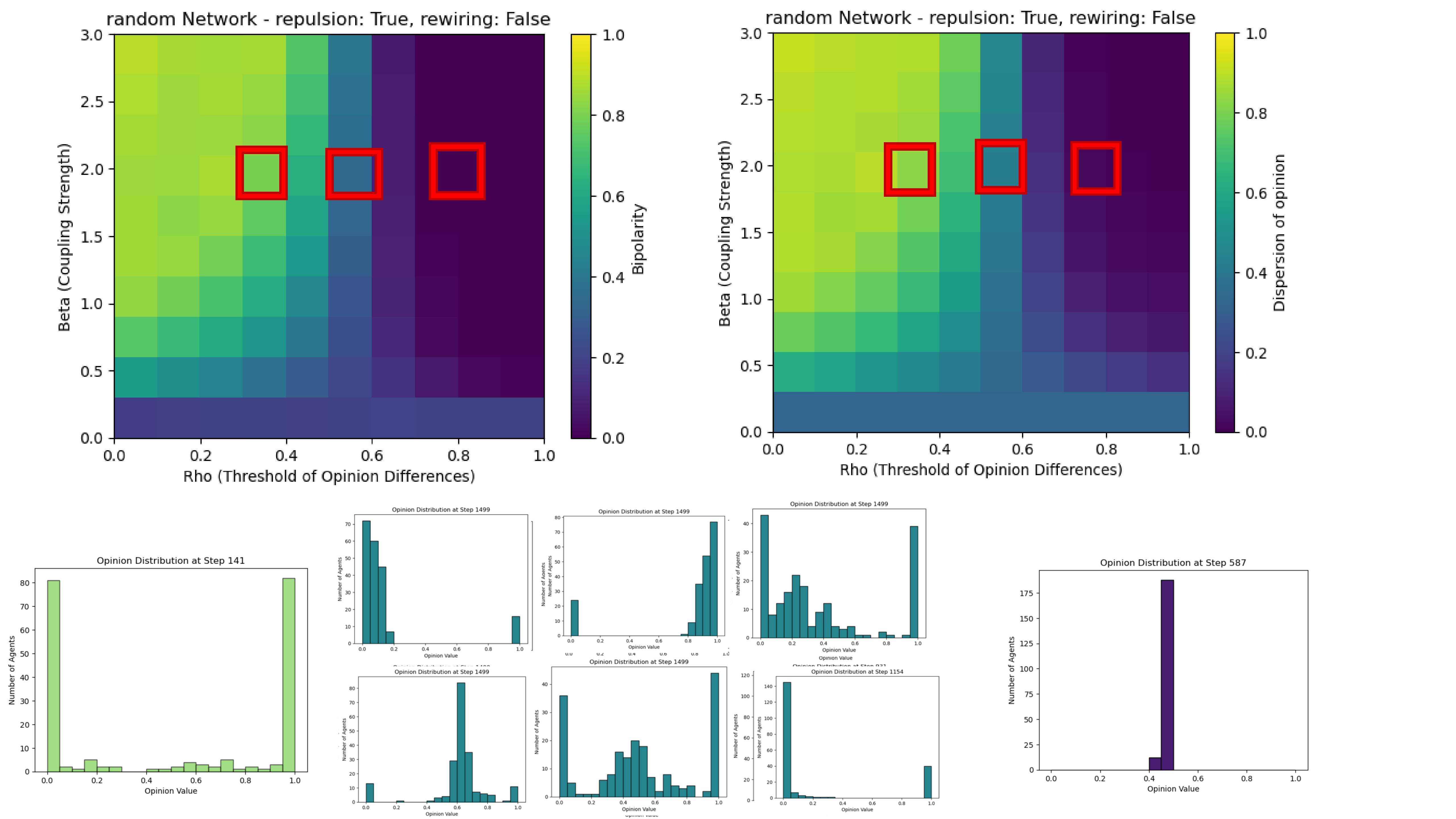
Variance: 0.07950
Bipolarity: 0.985

Variance: 0.05958
Bipolarity: 0.2

Variance: 0.04452
Bipolarity: 0.125

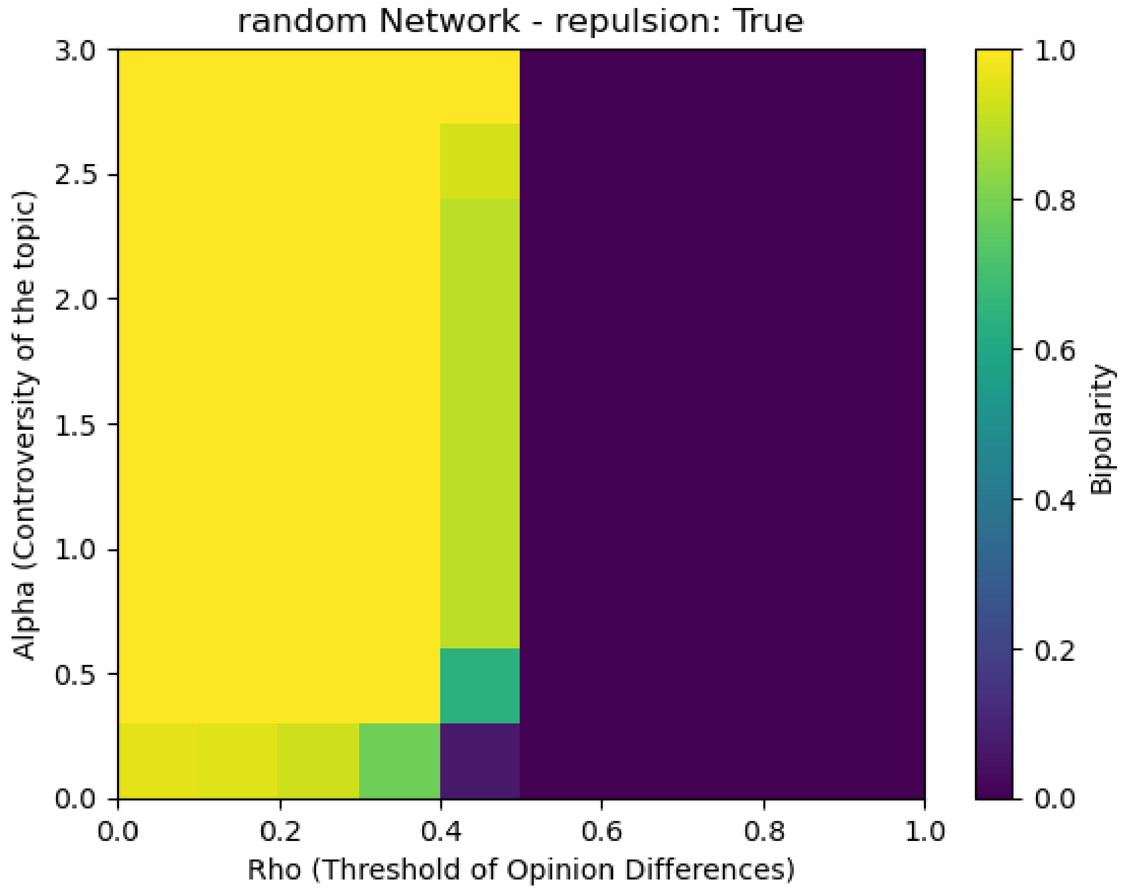
$$\text{Bipolarity} = \frac{count(|x-0| + count([x-1| < e)}{N}$$

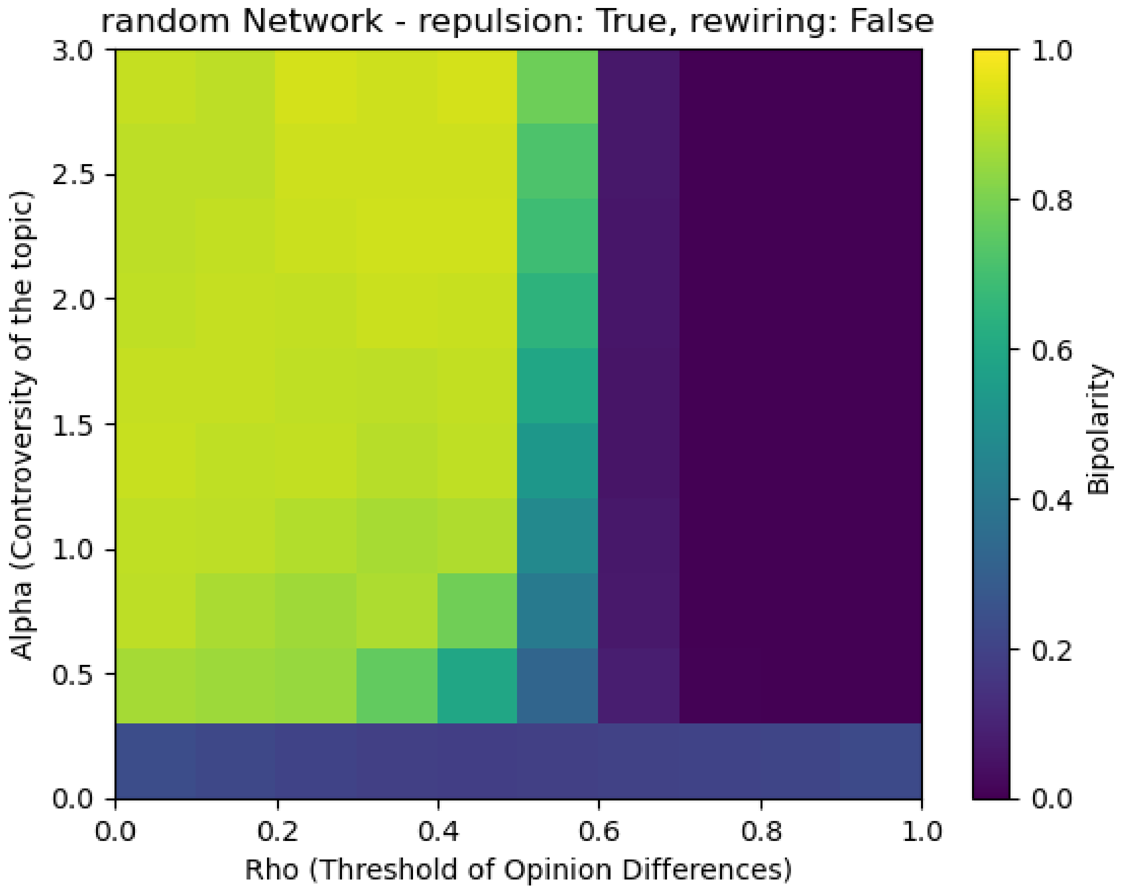
$with\ e \dashrightarrow small\ threshold;$
$custom\ made\ put\ of\ bimodularity\ literature$



random Network - repulsion: True, rewiring: False
Beta (Coupling Strength)
Rho (Threshold of Opinion Differences)
Bipolarity
random Network - repulsion: True, rewiring: False
Beta (Coupling Strength)
Rho (Threshold of Opinion Differences)
Dispersion of opinion
Opinion Distribution at Step 141
Number of Agents
Opinion Value
Opinion Distribution at Step 1499
Opinion Distribution at Step 1499
Opinion Distribution at Step 1499
Opinion Distribution at Step 1499
Opinion Distribution at Step 1499
Opinion Distribution at Step 1154
Opinion Distribution at Step 587
Number of Agents
Opinion Value

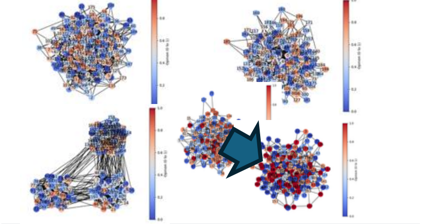
Number of agents updating per step:





1 agent updates per step:
- Very sharp phase transition

10 agents update per step:
- Sharp phase transition is blurred



With repulsion

Bipolarity

Variance of opinion

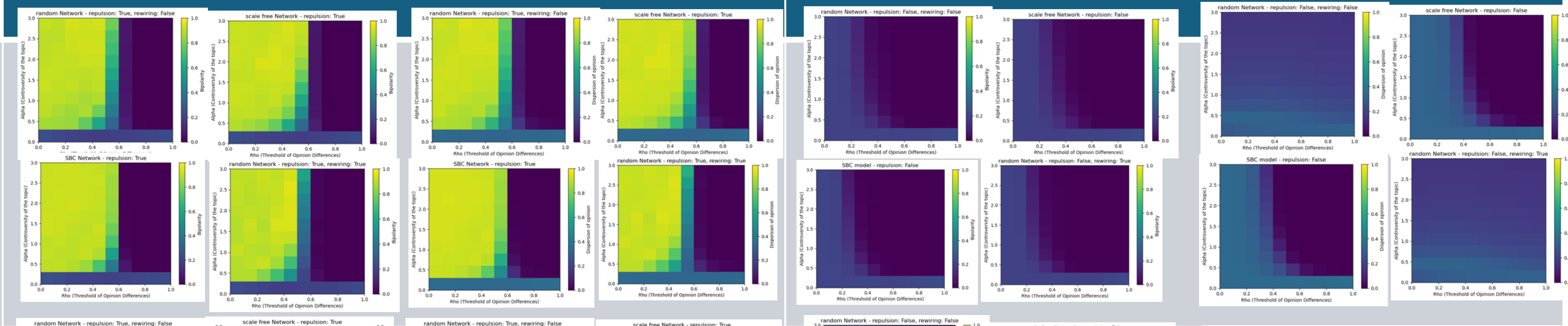
Without repulsion

Bipolarity

Variance of opinion

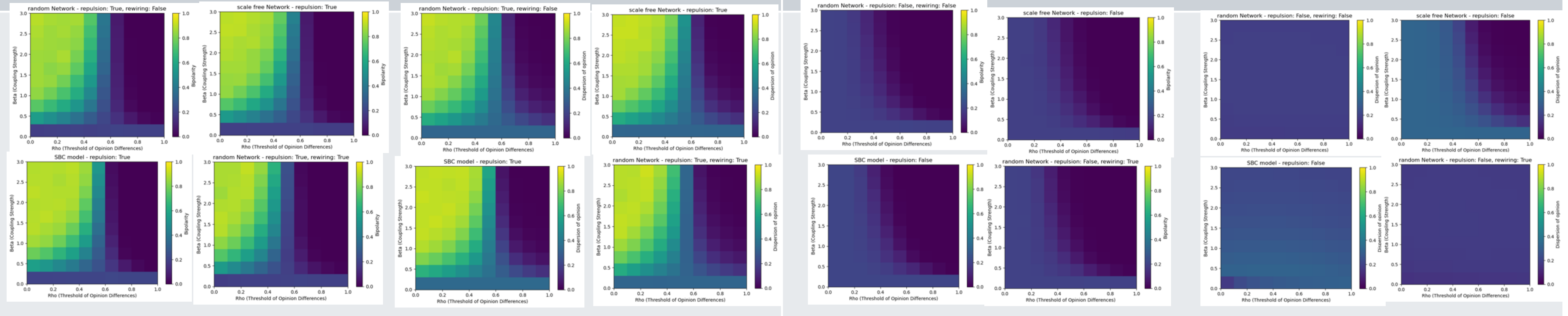
Controversy of topic (α) against threshold of opinion difference (ρ)

Coupling parameter (β=1)



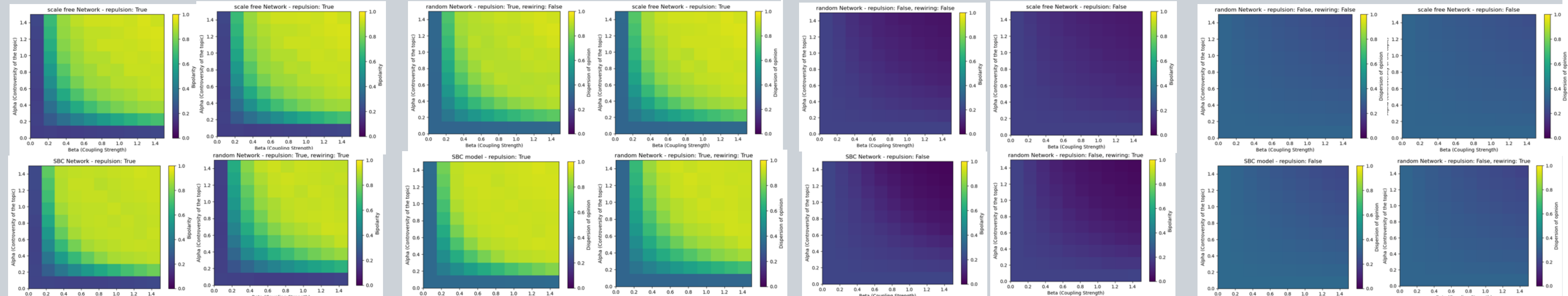
Coupling parameter (β) Against threshold of opinion difference (ρ)

Controversy of topic (α) = 0.2

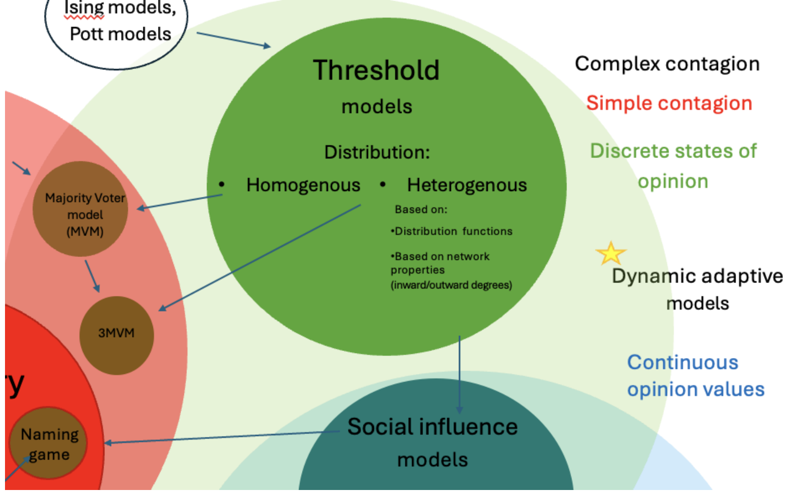


Controversy of topic (α) against Coupling parameter (β)

threshold of opinion difference (ρ)= 0.3



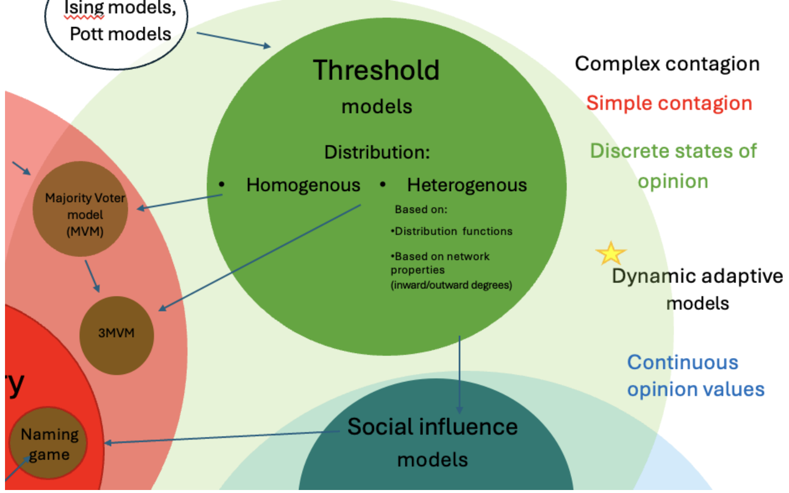
Complex contagion – Threshold model



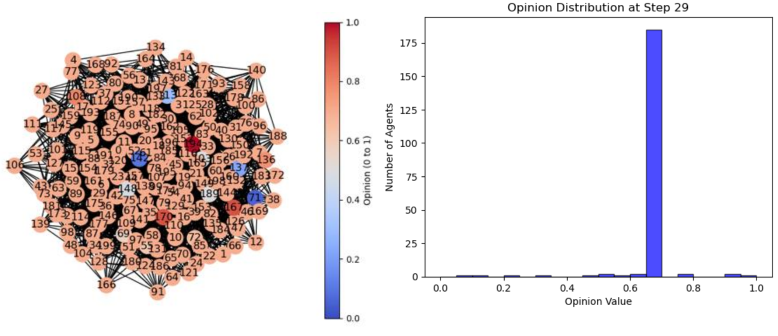
$$x_i(t+1) = \begin{cases} x_j & for \quad majority(\{w_{ij}x_j(t): j \in N(i)\}) > \beta_i \\ x_i & else. \end{cases}$$

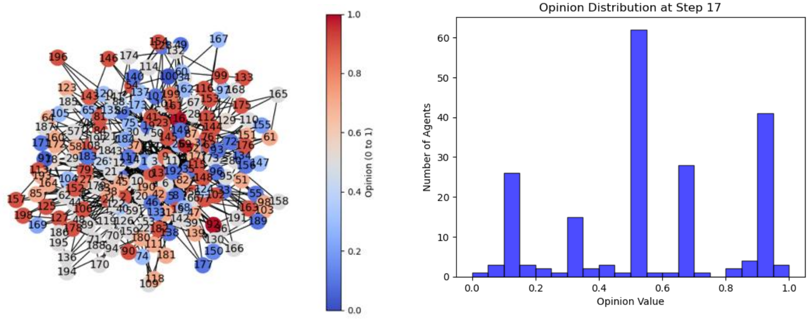
- Opinions are discretized
- Relevant for decision-based behavior

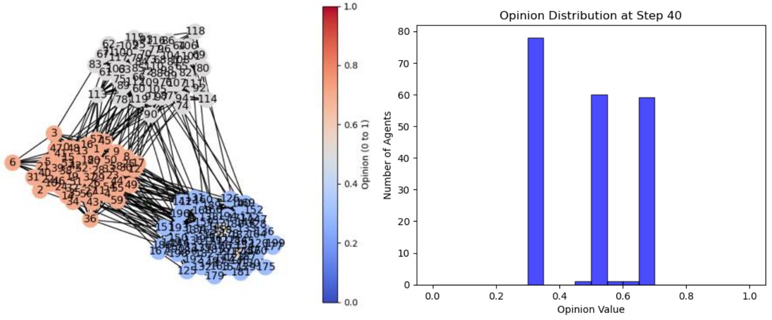
Complex contagion – Threshold model



- Highly dependency to initial condition
- Random Network with low noise: (Radical) consensus
- Scale free: consensus or coexistence of states
- Stoch block model : coexistence

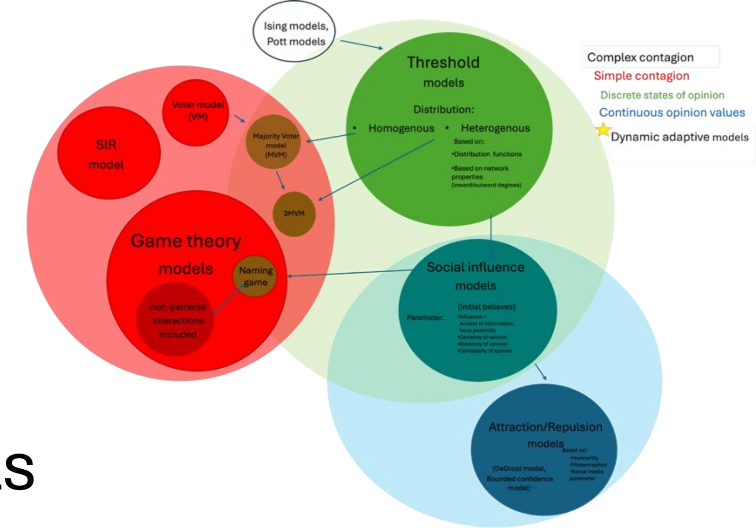






- Other equilibria: higher noise and randomized tie breaker functions

Summary



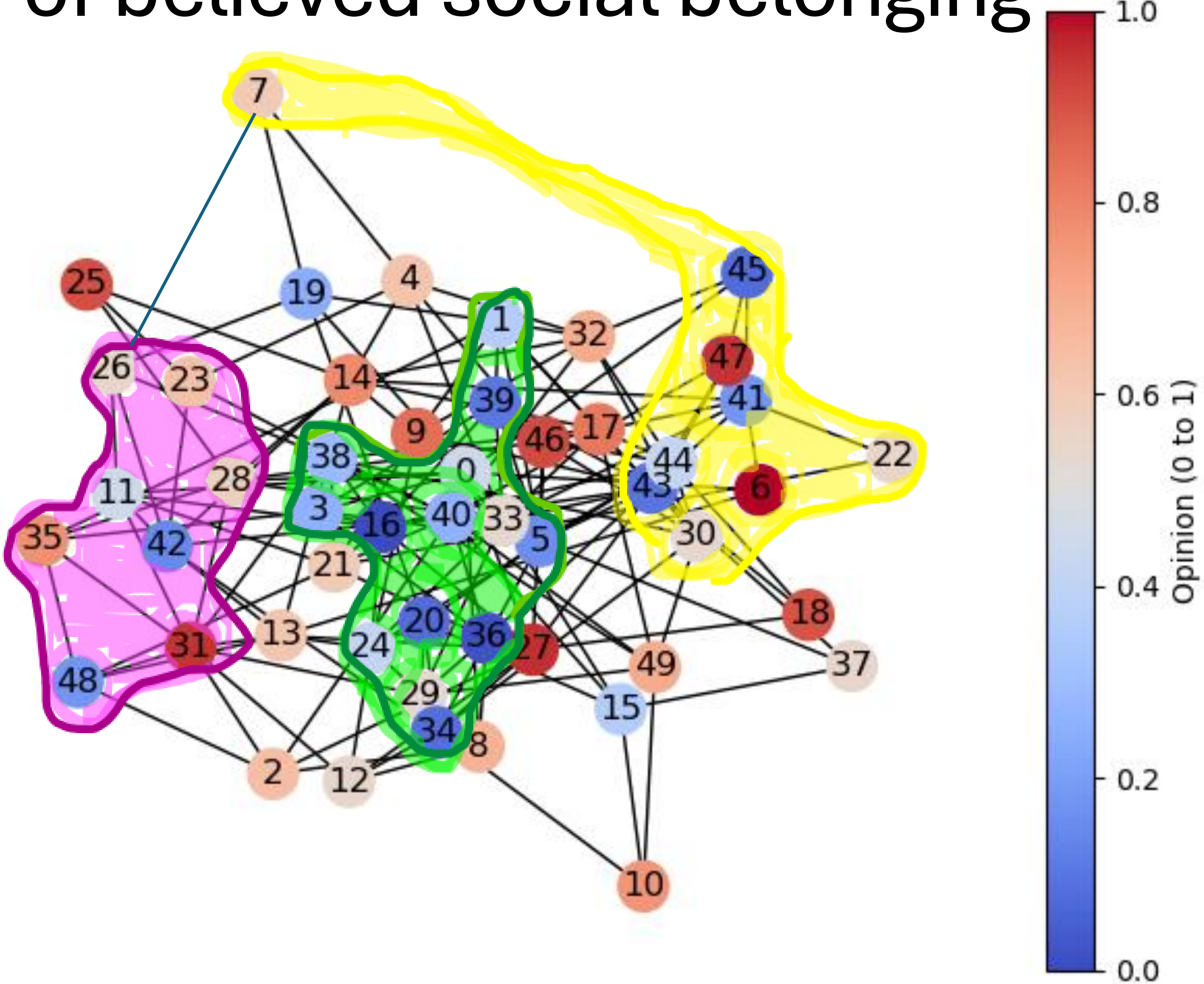
Social influence models

- Fairly structurally independent
- Sharp phase transition for Threshold of difference in opinion
- Only in transition phase very dependent on initial conditions

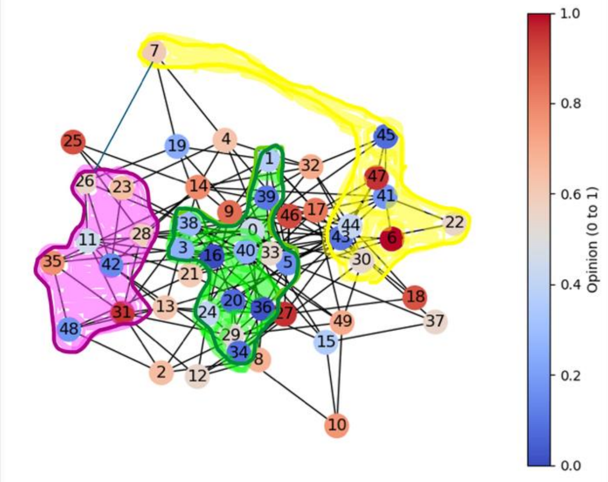
Threshold models

- High dependency on initial condition and network structure

Layer of believed social belonging



Layer of believed social belonging



$$x_i(t+1) = x_i(t) + \frac{\beta_i}{\sum_{j \neq i} w_{ij}} \sum_j w_{ij} T_{ij} \; x_{ij} \cdot \tanh(-\alpha_i(|x_{ij}| - \rho_i(t))$$

Trust matrix $T_{ij} \in [0,1]$
$T_{ij} = \max(0, B_{ij}(\mathrm{t}))$

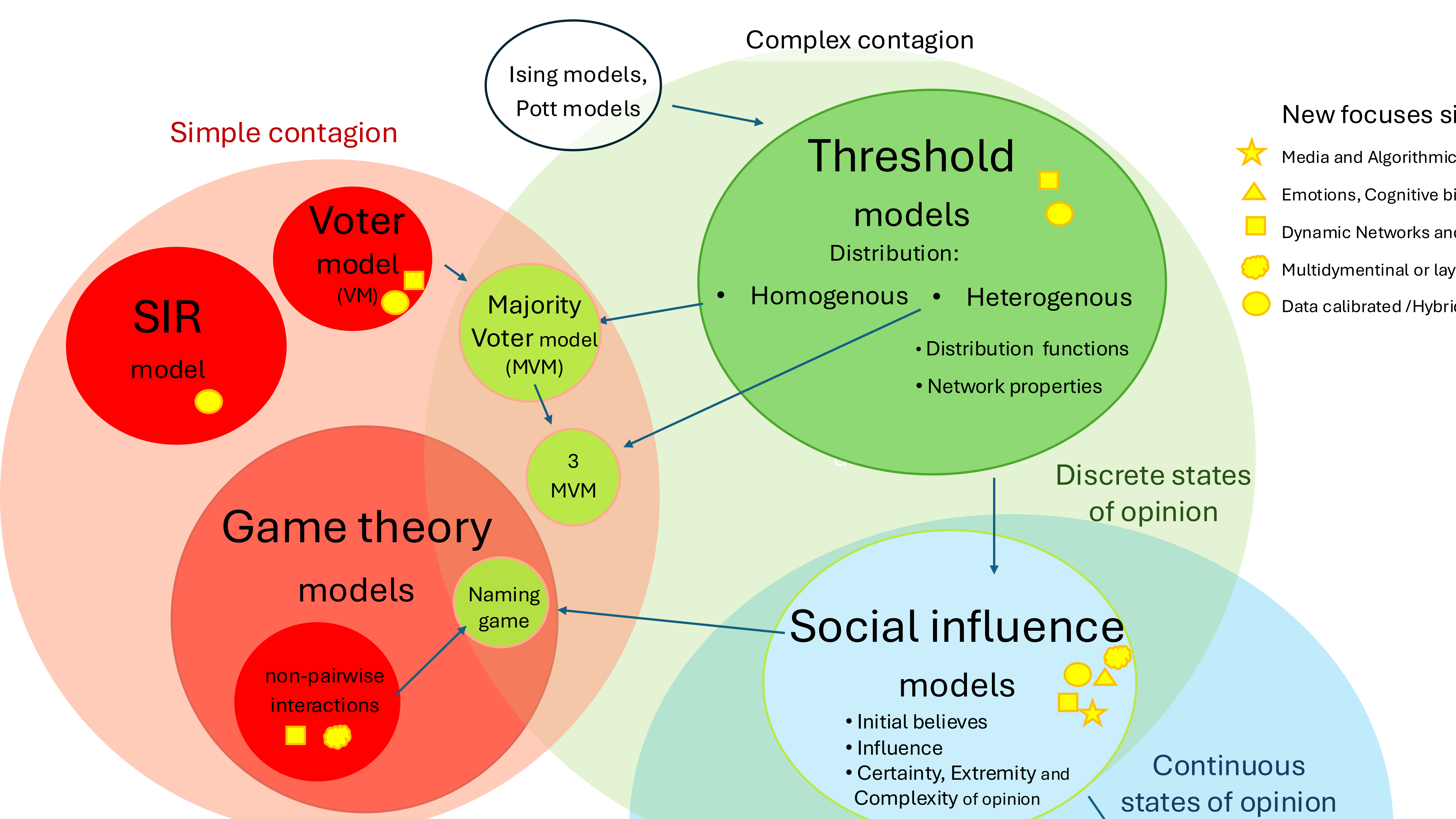
Threshold of tolerance
$\rho_i(t) \sim \mathrm{var}\,(x_{in})$

Trust modifier
$B_{ij} \in$ [-1,1]

Learning factor
$n \in$ [-1,1]

$$B_{ij}(t+1) = \begin{cases} B_{ij} + n \cdot \alpha_i \cdot \overbrace{\left(1 - |x_j(t) - x_i(t)|\right)}^{\text{Agreement}} \cdot \overbrace{\left(|x_j(t) - 0.5| + |x_i(t) - 0.5|\right)}^{\text{Distance from center}} & for\ |x_j(t) - x_i(t)| < \rho_i(t) \\ B_{ii} - n \cdot \alpha_i \cdot \left(1 - |x_j(t) - x_i(t)|\right) \cdot \underbrace{(B_{ij} + 1)}_{\substack{\text{Shifting interval:} \\ \text{betrayal weights more}}} & else. \end{cases}$$

Thank you!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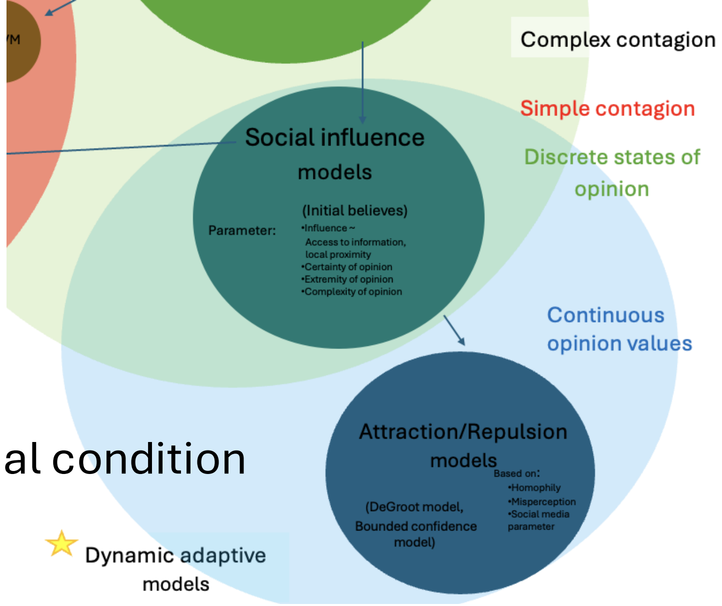
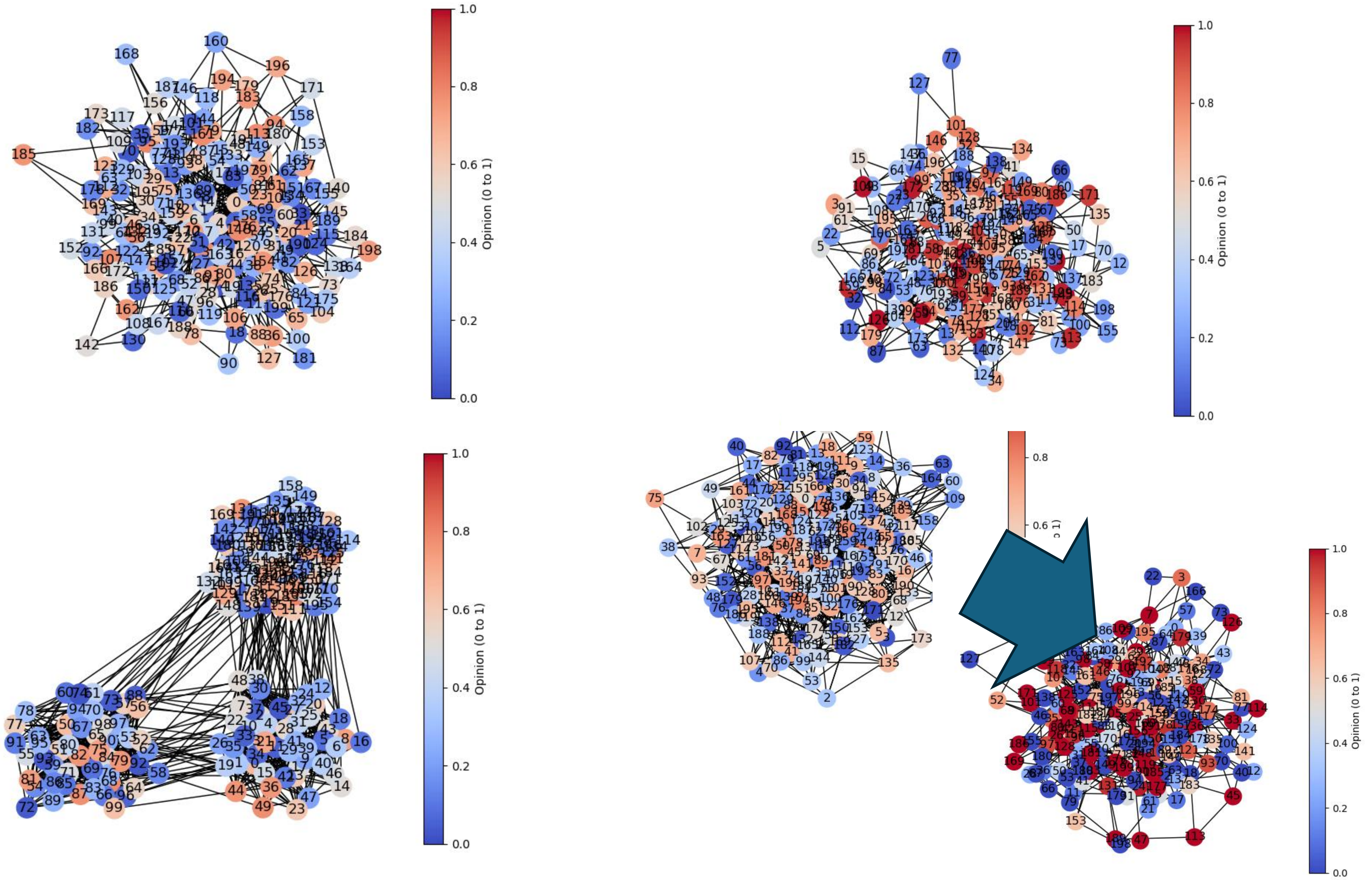
Threshold
models
Distribution:
• Homogenous
• Heterogenous
• Distribution functions
• Network properties
Voter
model
(VM)
SIR
model
Majority
Voter model
(MVM)
3
MVM
Discrete states
of opinion
Game theory
models
Naming
game
non-pairwise
interactions
Social influence
models
• Initial believes
• Influence
• Certainty, Extremity and
Complexity of opinion
Continuous
states of opinion
New focuses si
Media and Algorithmic
Emotions, Cognitive bi
Dynamic Networks anc
Multidymentinal or lay
Data calibrated /Hybric

summary



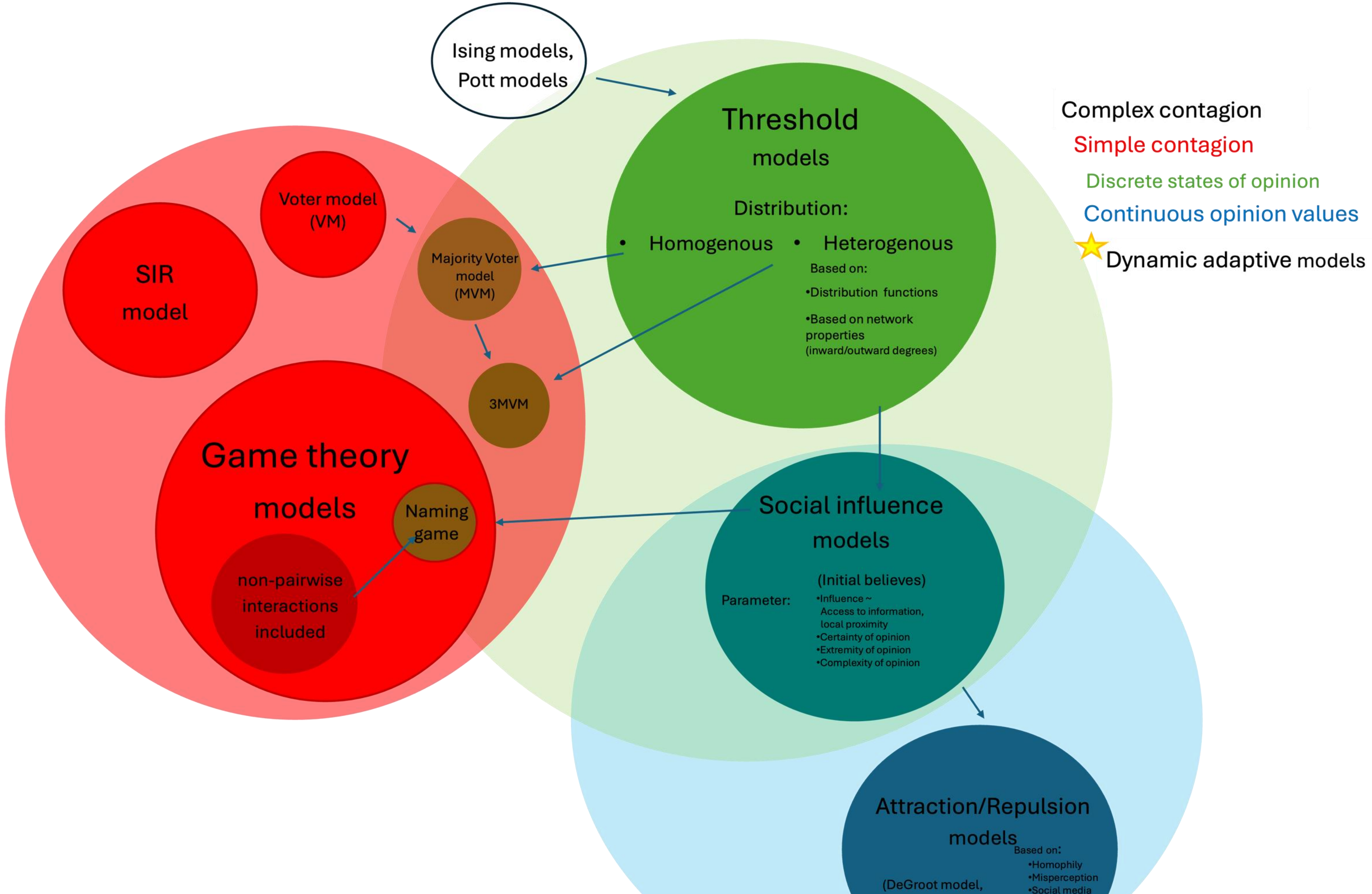
- Fairly structurally independent
- Sharp Phase transition
- Threshold of difference in opinion is most important for phase transition
- Only in transition phase very dependent on initial conditions:
- Changes in controversy of topic and the coupling strength are leading to continuous phase transitions
- higher numbers of agents updating at a timestep → blurring of transition

- High dependency to initial condition and network structure
- Random Network with low noise: (Radical) consensus likely
- Scale free: hubs can lead to consensus or coexistence of states
- Stoch block model – coexistence, the groups work as opinion hubs.
- Higher noise and randomized tie breaker functions: dynamic equilibria,



Opinion (0 to 1)
0.0
0.2
0.4
0.6
0.8
1.0

Fig. 1


Ising models,
Pott models
Threshold
models
Distribution:
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Based on:
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•Based on network properties (inward/outward degrees)
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Discrete states of opinion
Continuous opinion values
Dynamic adaptive models
Voter model (VM)
SIR model
Majority Voter model (MVM)
3MVM
Game theory models
Naming game
non-pairwise interactions included
Social influence models
(Initial believes)
Parameter:
•Influence ~ Access to information, local proximity
•Certainty of opinion
•Extremity of opinion
•Complexity of opinion
Attraction/Repulsion models
Based on:
•Homophily
•Misperception
•Social media
(DeGroot model,

Missing parts: (not simulated yet)

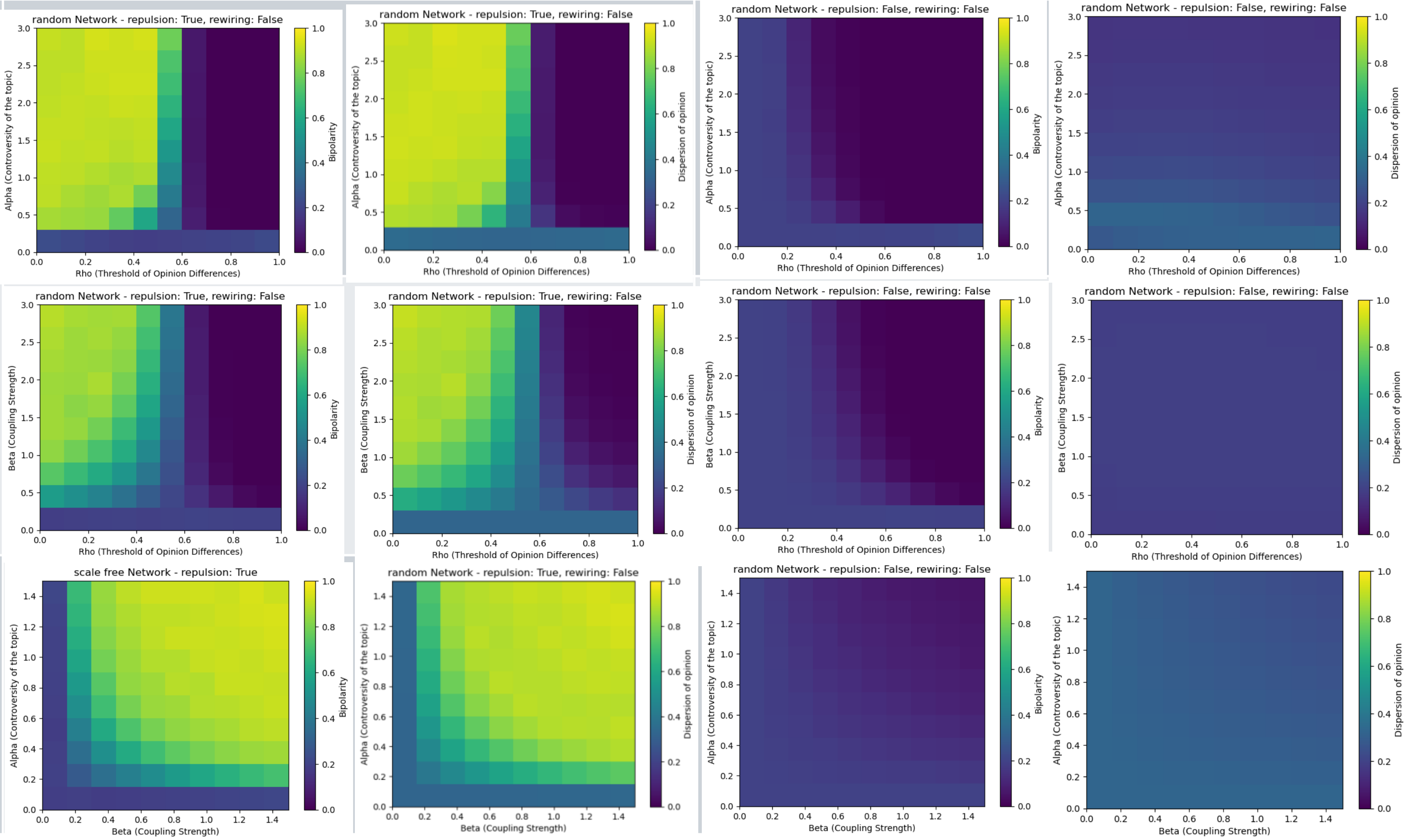
- No directed networks
- No high noise in parameters, heterogeneity is low
- No stubborn agents tested
- Grid network is missing

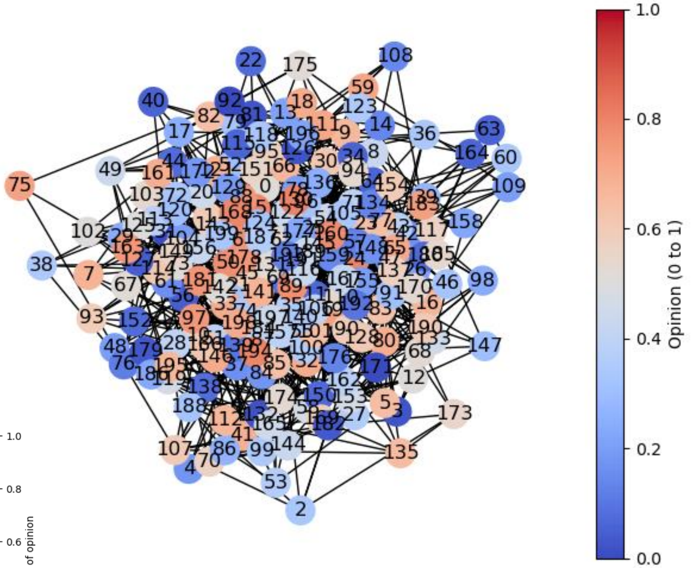
- No game theory, no

Random network

Rand network Bipolarity heatmaps

	repulsion		*no repulsion*	
	Bipolarity	Variance of op.	Bipolarity	Variance of op.
$alpha, rho$ beta = 1				
$beta, rho$ alpha = 0.2				
$alpha, beta$ Rho = 0.3				



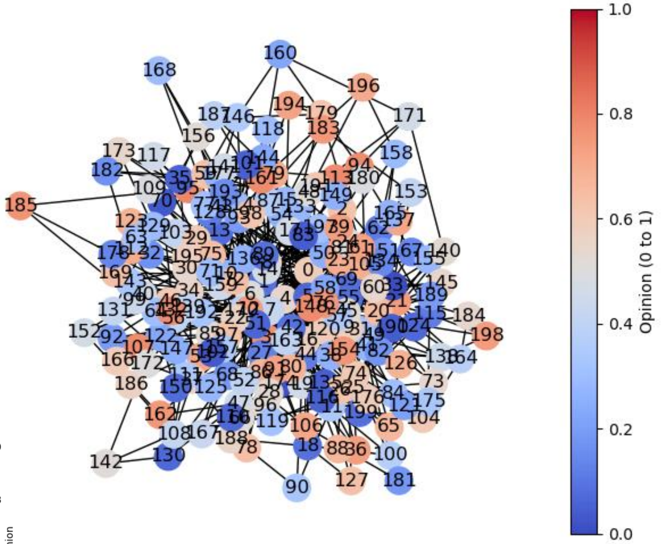


$$x_i(t+1) = x_i + \frac{\beta_i}{\sum_{j\neq i} w_{ij}} \sum_j w_{ij}\ c(x_{ij})$$

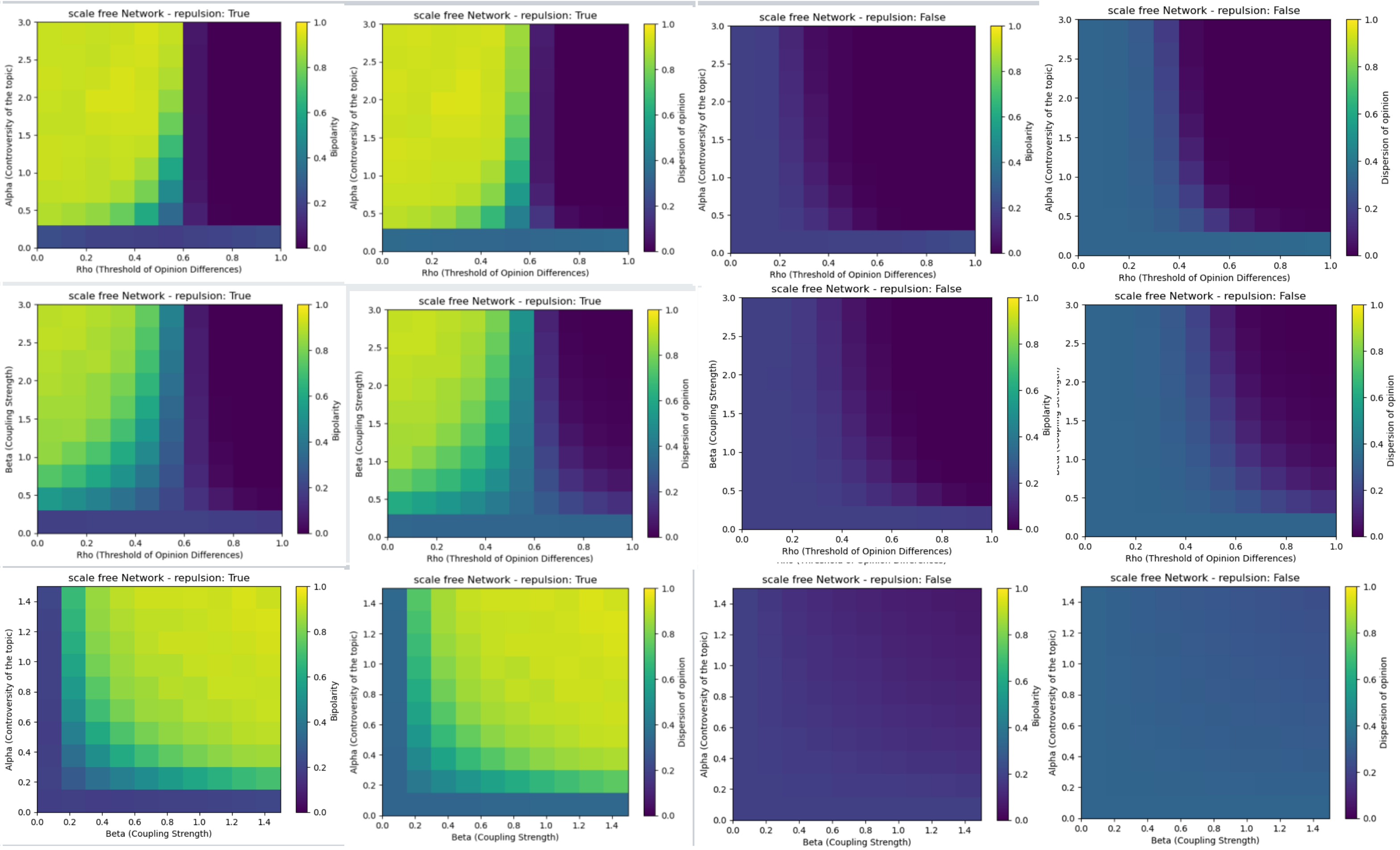
with $c(x_{ij}) = x_{ij} \cdot \tanh(-\alpha_i(|x_{ij}| - \rho_i))$

Heterogenous parameter:
Homogenous + noise

Scale free networks



Scale free network Bipolarity heatmaps	*repulsion*		*no repulsion*	
	Bipolarity	Variance of op.	Bipolarity	Variance of op.
$alpha, rho$ beta = 1				
$beta, rho$ alpha = 0.2				
$alpha, beta$ Rho = 0.3				



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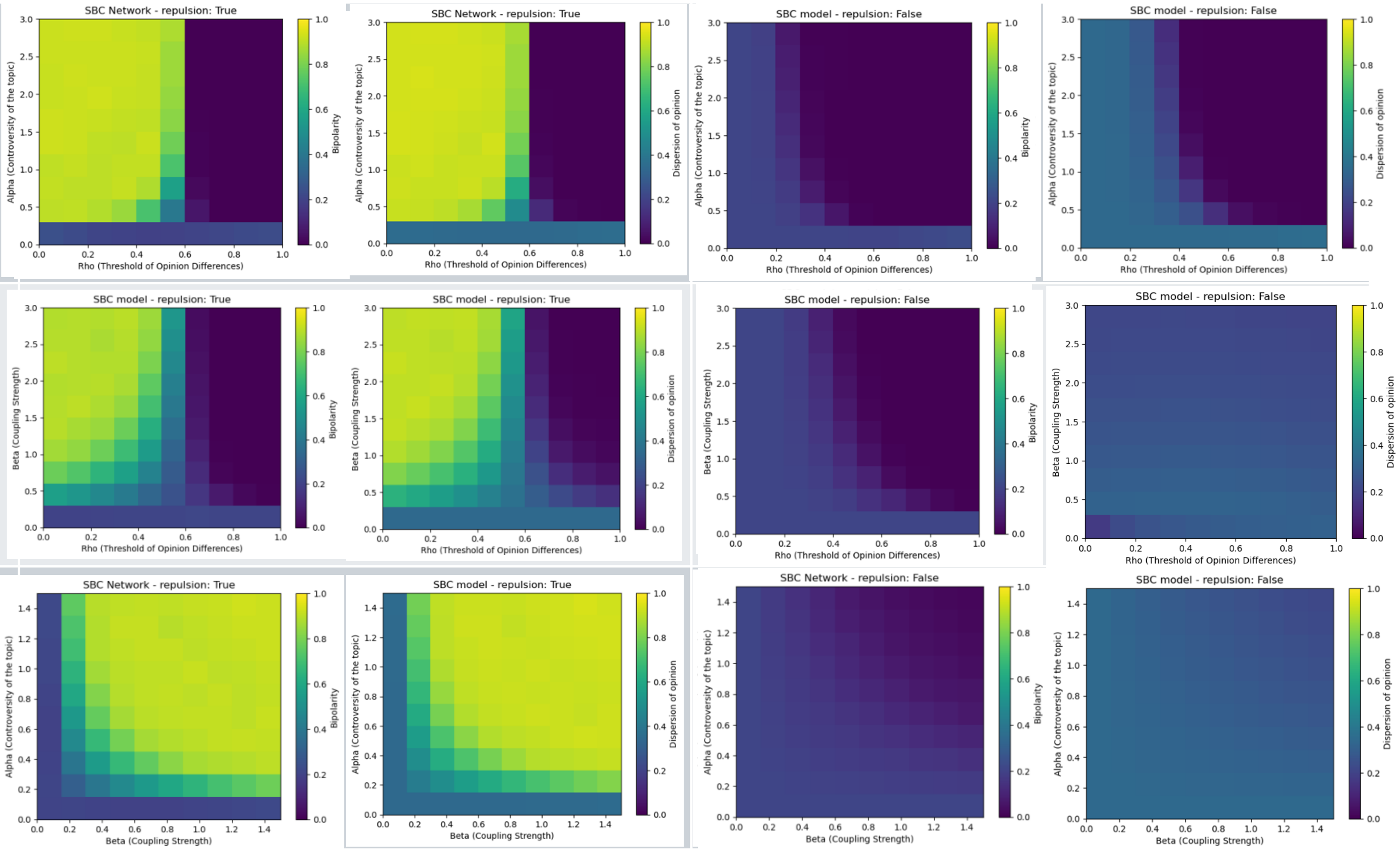
with $c(x_{ij}) = x_{ij} \cdot \tanh(-\alpha_i(|x_{ij}| - \rho_i))$

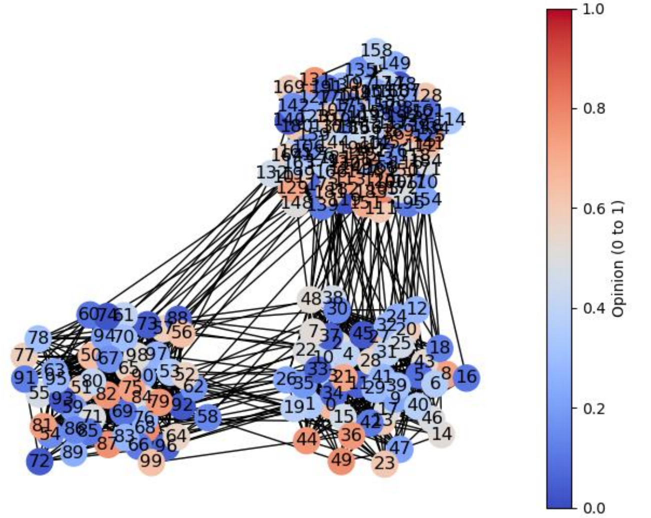
Heterogenous parameter:
Homogenous + noise

Stochastic block chain models

Stochastic block chain model Bipolarity heatmaps

	repulsion		*no repulsion*	
	Bipolarity	Variance of op.	Bipolarity	Variance of op.
alpha, rho beta = 1				
beta, rho alpha = 0.2				
alpha, beta Rho = 0.3				



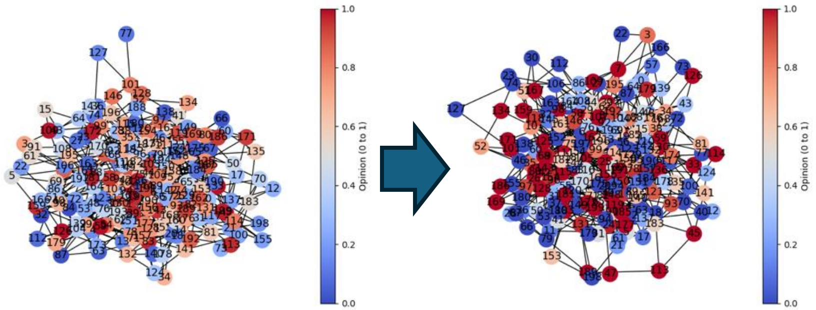


$$x_i(t+1) = x_i + \frac{\beta_i}{\sum_{j\neq i} w_{ij}} \sum_j w_{ij}\ \ c(x_{ij})$$

with $c(x_{ij}) = x_{ij} \cdot \tanh(-\alpha_i(|x_{ij}| - \rho_i))$

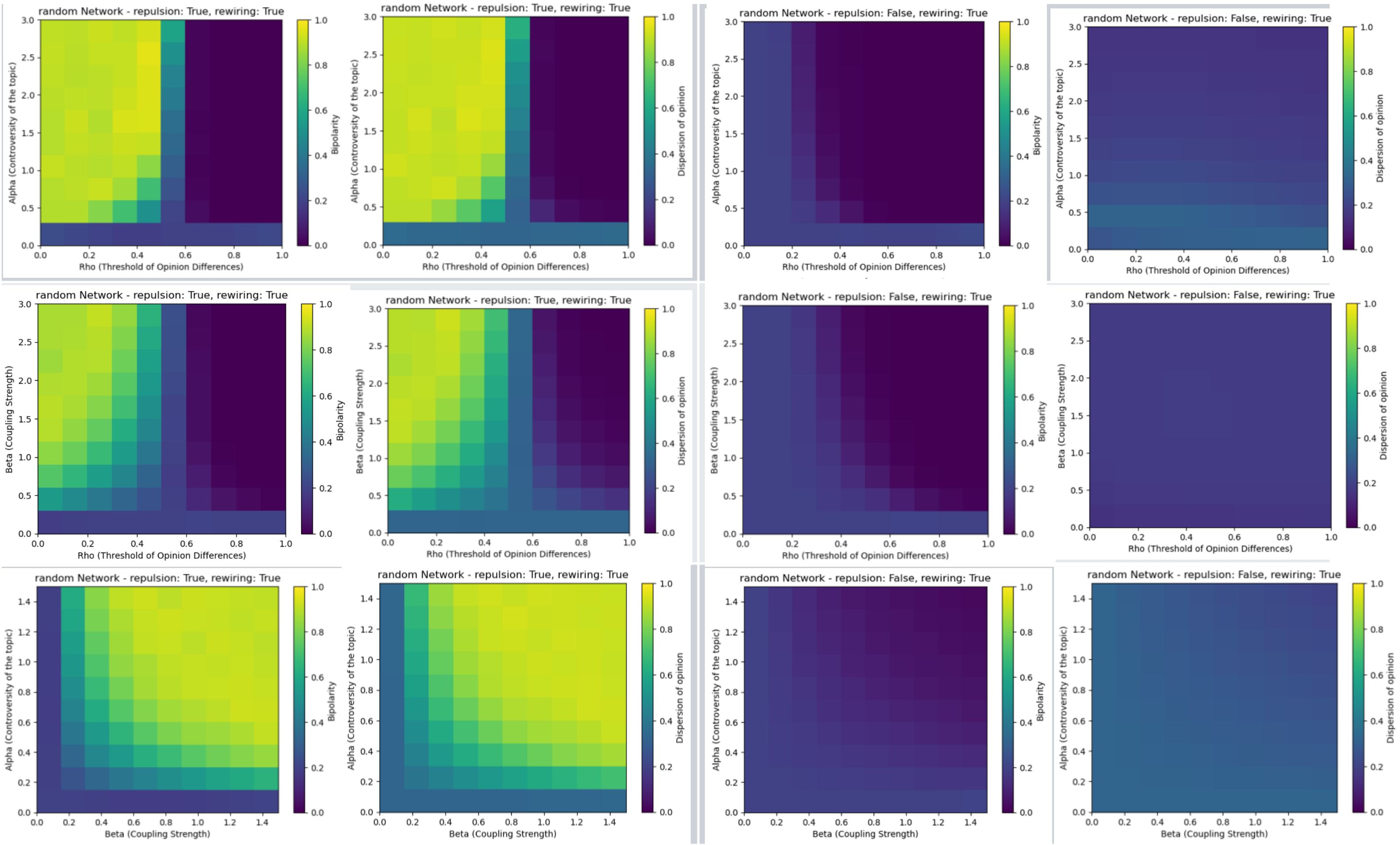
Heterogenous parameter:
Homogenous + noise

Dynamic adaptive models



Rand adaptive network Bipolarity heatmaps

	repulsion		*no repulsion*	
	Bipolarity	Variance of op.	Bipolarity	Variance of op.



Rows of heatmaps:

$alpha, rho$ — beta = 1

$beta, rho$ — alpha = 0.2

$alpha, beta$ — Rho = 0.3

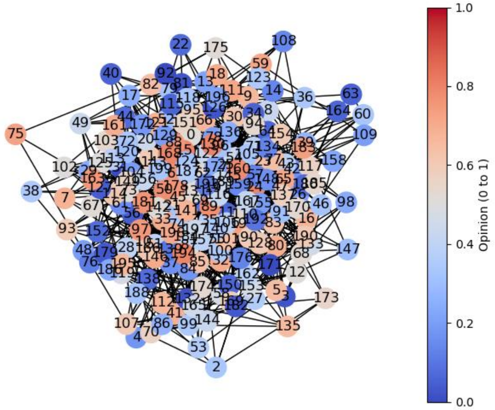
$$x_i(t+1) = x_i + \frac{\beta_i}{\sum_{j\neq i} w_{ij}} \sum_j w_{ij}\ c(x_{ij})$$

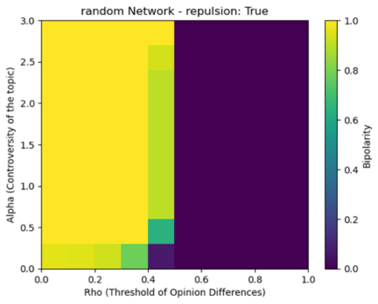
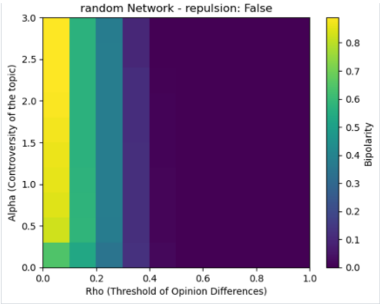
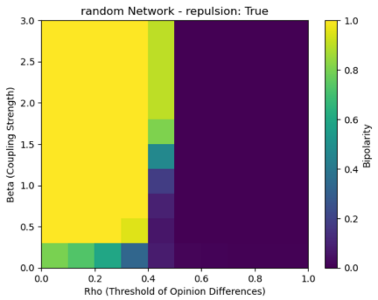
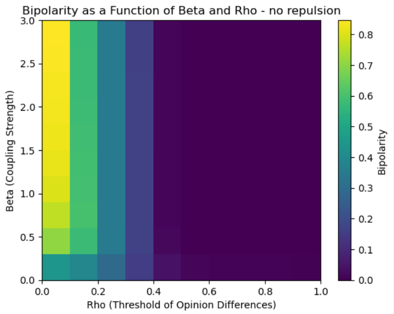
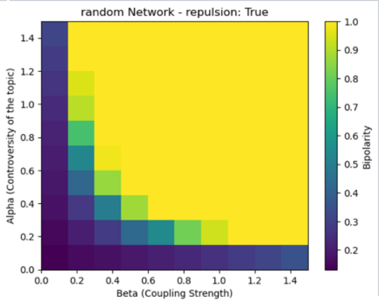
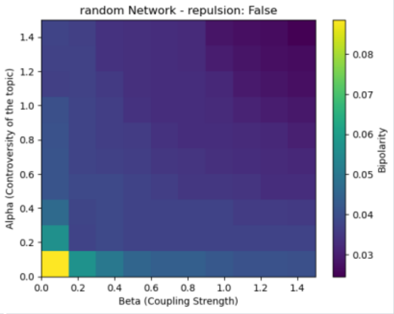
with $c(x_{ij}) = x_{ij} \cdot \tanh(-\alpha_i(|x_{ij}| - \rho_i))$

Heterogenous parameter:
Homogenous + noise

Random Networks

(update one per step)



Rand network Bipolarity heatmaps	*repulsion*	*no repulsion*
alpha, rho beta = 1		
beta, rho alpha = 0.2		
alpha, beta Rho = 0.4		

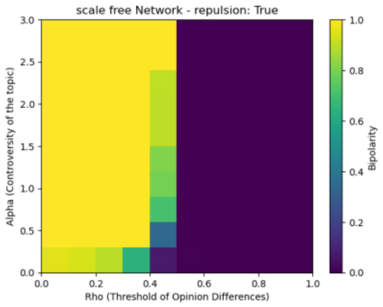
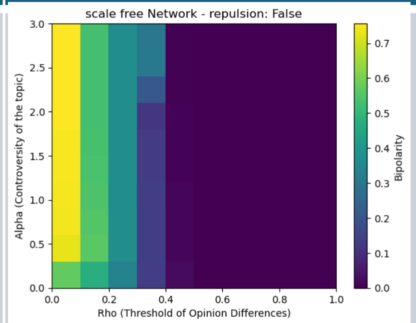
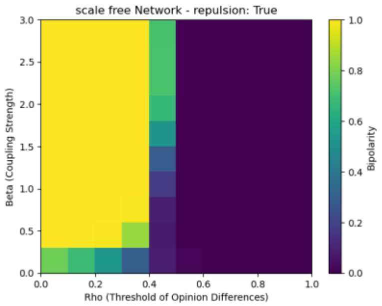
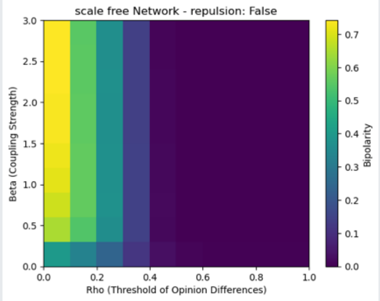
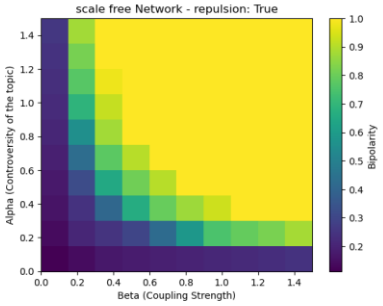
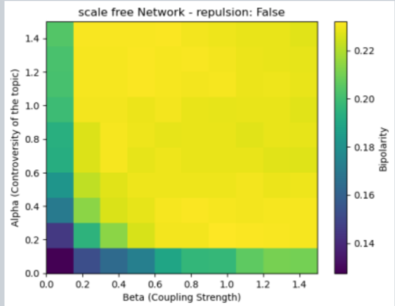
$$x_i(t+1) = x_i + \frac{\beta_i}{\sum_{j\neq i} w_{ij}} \sum_j w_{ij} \; c(x_{ij})$$

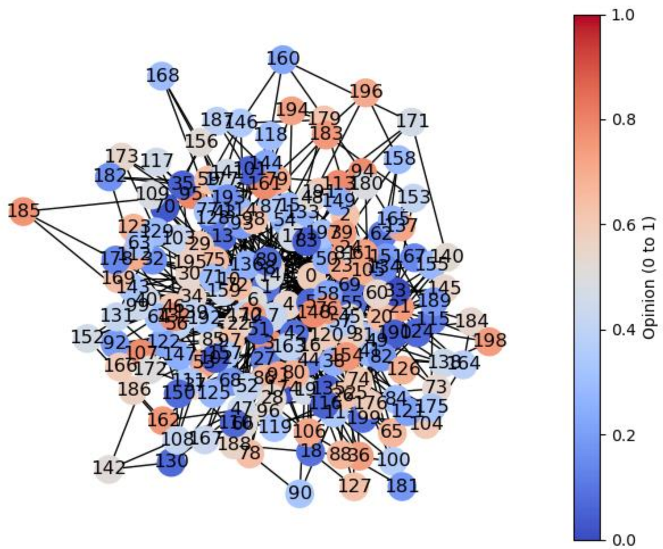
with $c(x_{ij}) = x_{ij} \cdot \tanh(-\alpha_i(|x_{ij}| - \rho_i))$

Heterogenous parameter:
Homogenous + noise
repulsion allowed

Scale free networks

(update one per step)

Scale free network Bipolarity heatmaps	*repulsion*	*no repulsion*
$alpha, rho$ beta = 1		
$beta, rho$ alpha = 0.2		
$alpha, beta$ Rho = 0.4		



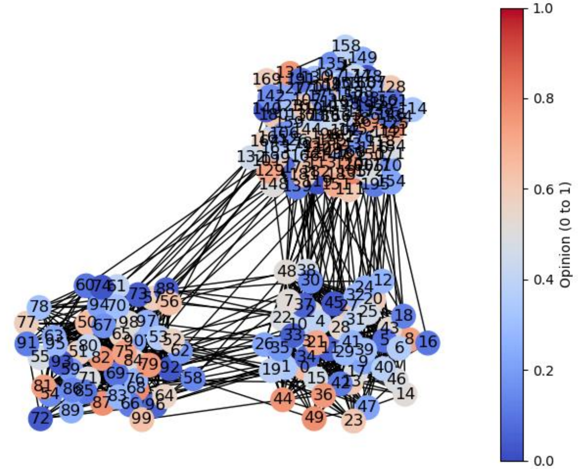
$$x_i(t+1) = x_i + \frac{\beta_i}{\sum_{j \neq i} w_{ij}} \sum_j w_{ij}\ c(x_{ij})$$

with $c(x_{ij}) = x_{ij} \cdot \tanh(-\alpha_i(|x_{ij}| - \rho_i))$

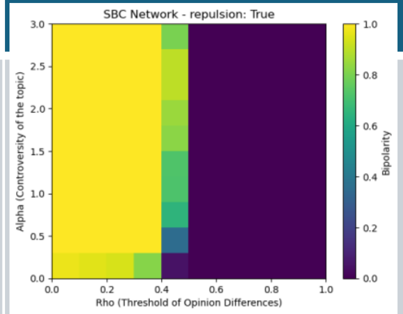
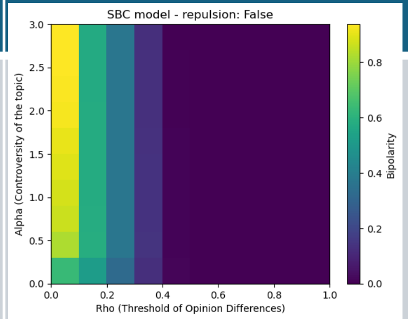
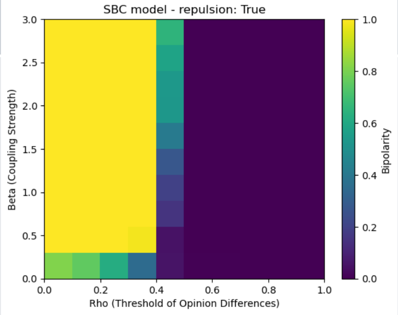
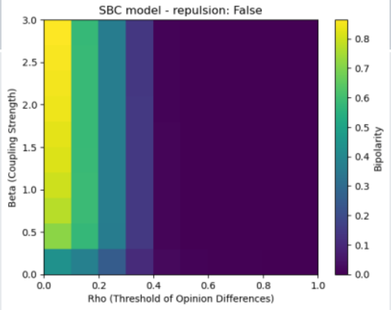
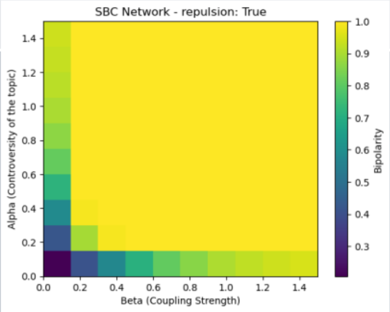
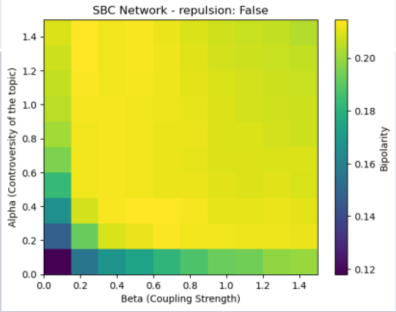
Heterogenous parameter:
Homogenous + noise
repulsion allowed

Stochastic block chain models

(update one per step)



Stochastic block chain model Bipolarity heatmaps

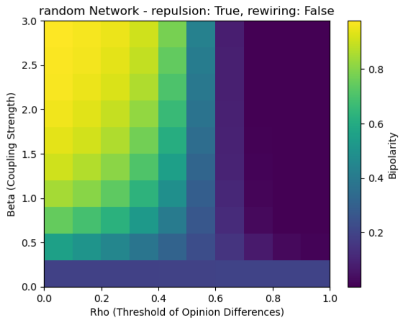
	$repulsion$	$no\ repulsion$
$alpha, rho$ beta = 1		
$beta, rho$ alpha = 0.2		
$alpha, beta$ Rho = 0.4		

$$x_i(t+1) = x_i + \frac{\beta_i}{\sum_{j \neq i} w_{ij}} \sum_j w_{ij}\ c(x_{ij})$$

with $c(x_{ij}) = x_{ij} \cdot \tanh(-\alpha_i(|x_{ij}| - \rho_i))$

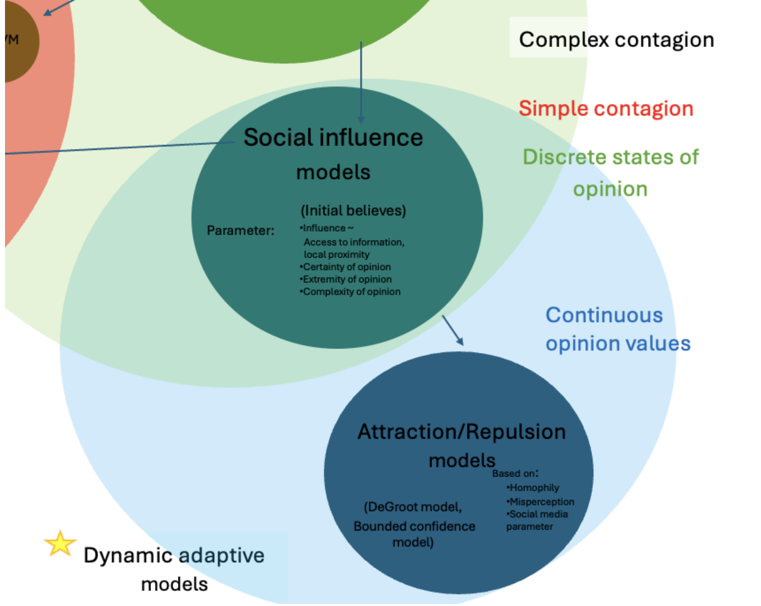
Heterogenous parameter:
Homogenous + noise
repulsion allowed

Redoing stuff with medium sized nw. (1,000 nodes)

Rand network Bipolarity heatmaps	*repulsion*	*no repulsion*
$alpha, rho$ beta = 1		
$beta, rho$ alpha = 0.2		
$alpha, beta$ Rho = 0.3		

$$x_i(t+1) = x_i + \frac{\beta_i}{\sum_{j\neq i} w_{ij}} \sum_j w_{ij}\ c(x_{ij})$$

with $c(x_{ij}) = x_{ij} \cdot \tanh(-\alpha_i(|x_{ij}| - \rho_i))$



Social Influence models

- Fairly structurally independent/ network size independent
- Sharp Phase transition (polarization to neutral consensus)
- Threshold of difference in opinion is most important for phase transition
- In transition phase very dependent on initial conditions:
  - different outcomes, from radicalization, polarization, to stages in between
- Changes in controversy of topic and the coupling strength are leading to continuous phase transitions changes
- higher numbers of agents updating at a timestep → blurring of transition

Simple contagion

Voter model

Information /opinion dynamics

Complex contagion

Ising models

DeGroot model

Pott models

Threshold models

Discrete/ continuous opinions

Bound confidence model

Homogenous/heterogenous distribution

Based on distribution functions/ network properties

Assumption/data based

Echo chambers and Hubs

Majority Voter model

Social influence models

Misinformation

Based on network properties

Mean field approaches

synchronous/asynchronous updating

Polarization

Social media influence

Homophily

Network Structural dependency

Game theory model

Attraction and Distraction force models

Prisoner dilemma, chicken game, stag hunt naming game....

Visibility of opinion

Dynamic adaptive models

Pairwise/ non-pairwise interaction

Fake news modeling

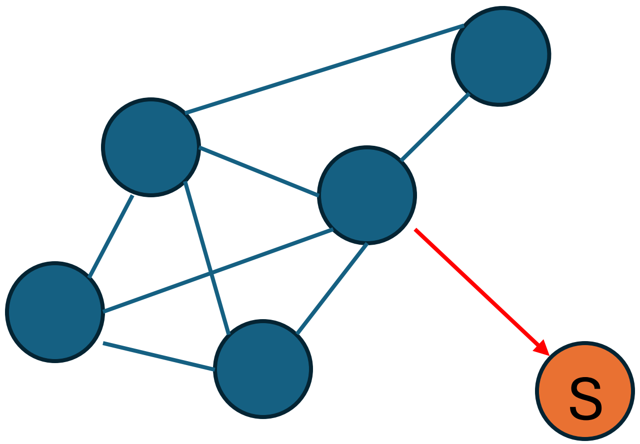
multigraphs

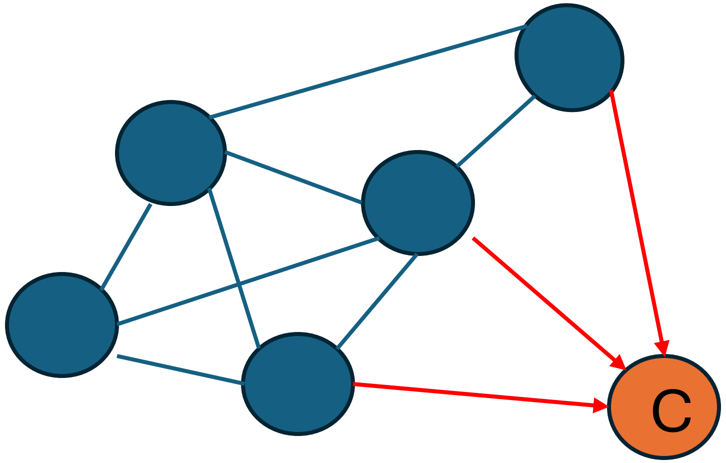
Motivation to research opinion dynamics

- Opinions growing further apart in Europe's population
- Radicalization → thread to society and democracy
- Political situation in Europe

- Huge chaotic amount of existing opinion dynamic models

Simple contagion – Complex contagion





- Information flow

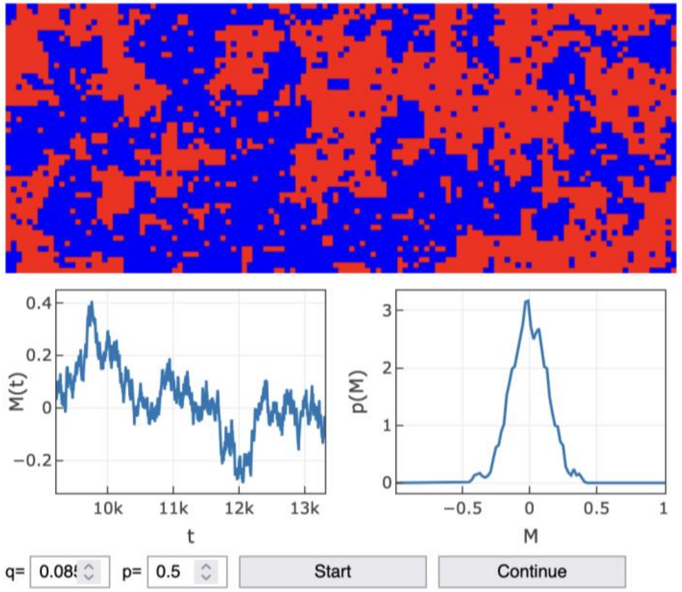
- Opinion dynamics

Threshold models

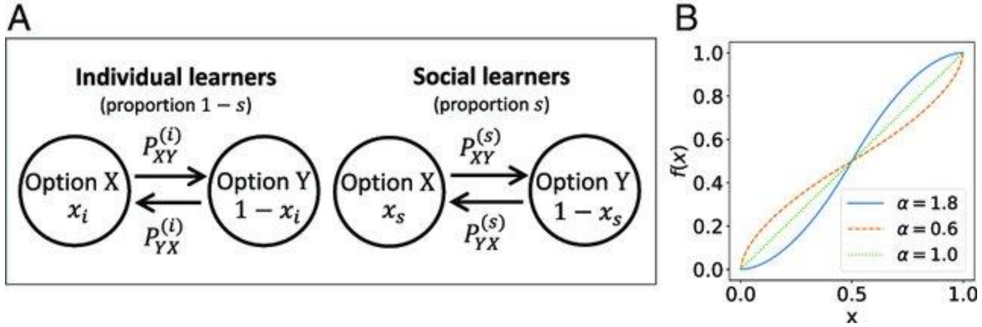
- Proportion of neighbors needed for an individual's opinion change

- Majority vote model

- Individuals can have different thresholds



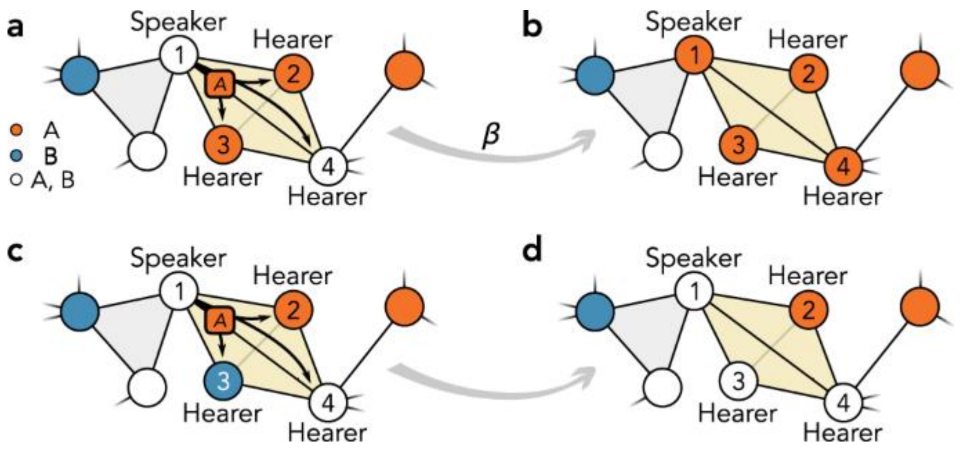
Ref: Kononovicus (2018, Dec 11). **Majority-vote model.** https://rf.mokslasplius.lt/majority-vote-model/

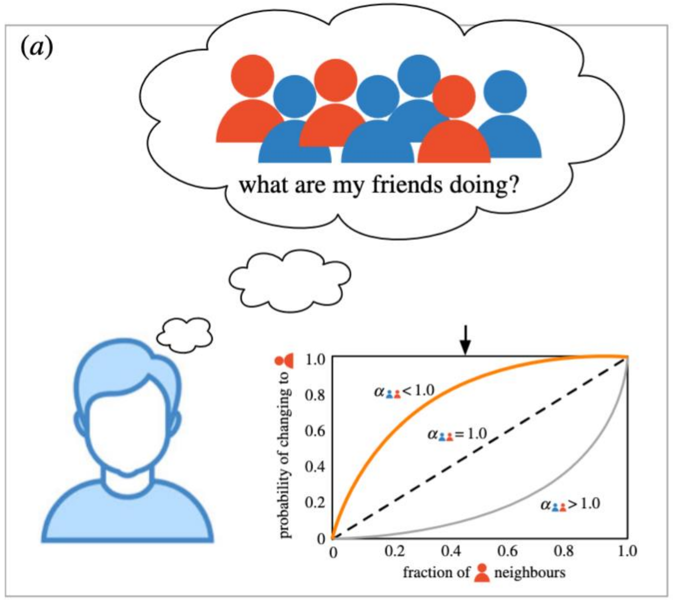


Yang, V. C., Galesic, M., McGuinness, H., & Harutyunyan, A. (2021). Dynamical system model predicts when social learners impair collective performance. PNAS, *118*(35)

Social influence models

- Include social reinforcement and initial beliefs





Iacopini, I., Petri, G., Baronchelli, A. Barrat, A. (2022) Group interactions modulate critical mass dynamics in social convention. *Commun Phys* 5, 64.

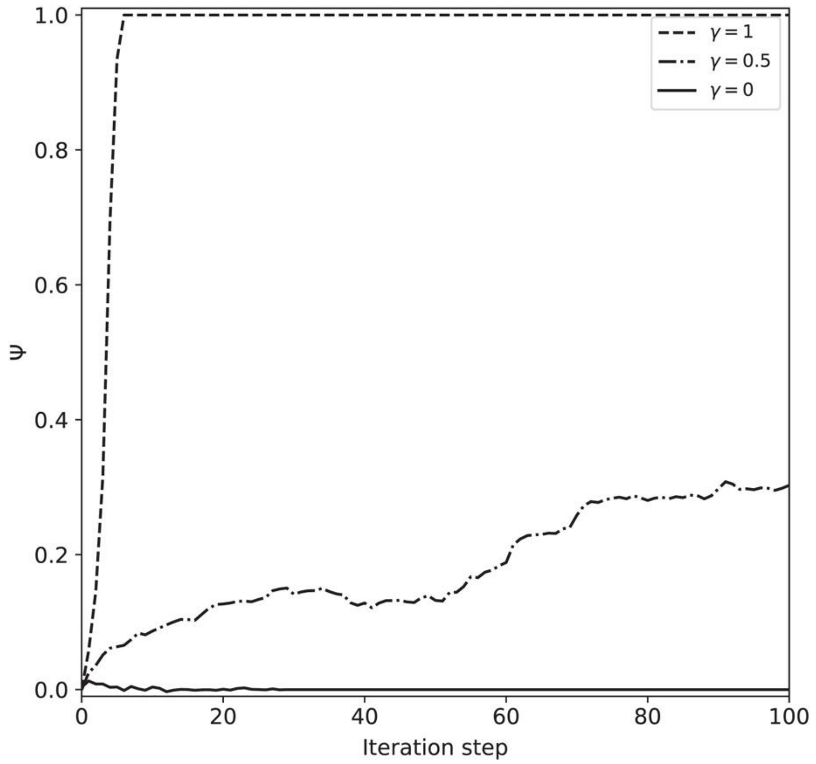
Vasconcelos, V., Levin, S. & Pinheiro, F. (2019). Consensus and polarization in competing complex contagion processes. *J. R. Soc. Interface* 16, 155.

Attraction and repulsion forces

- Homophily
- Ignorance and misperception (misleading claims)
- Social media
- Bot experiments

Parameter for social media

Sorting of groups through similarity



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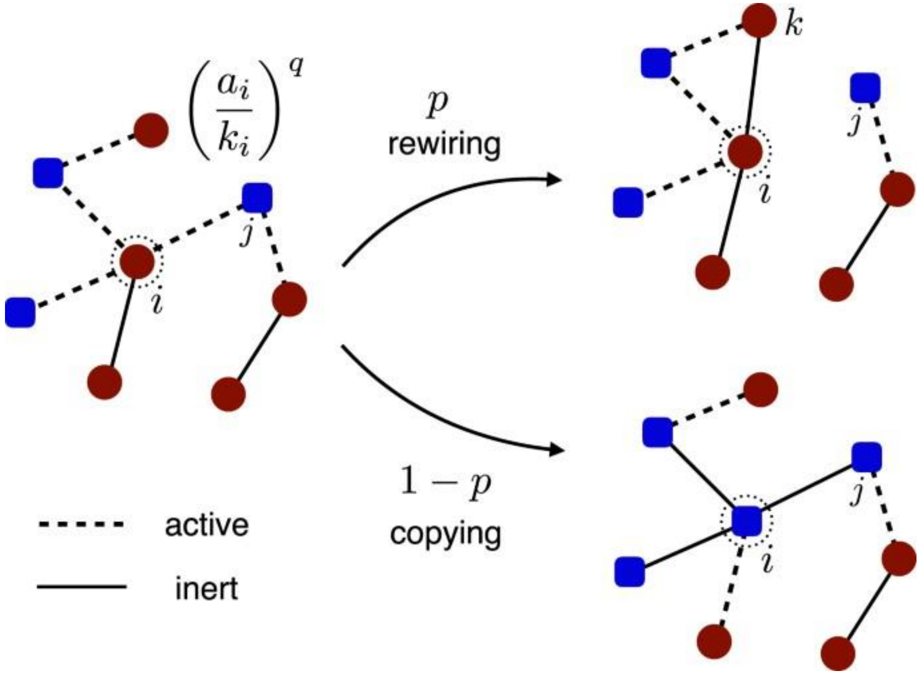
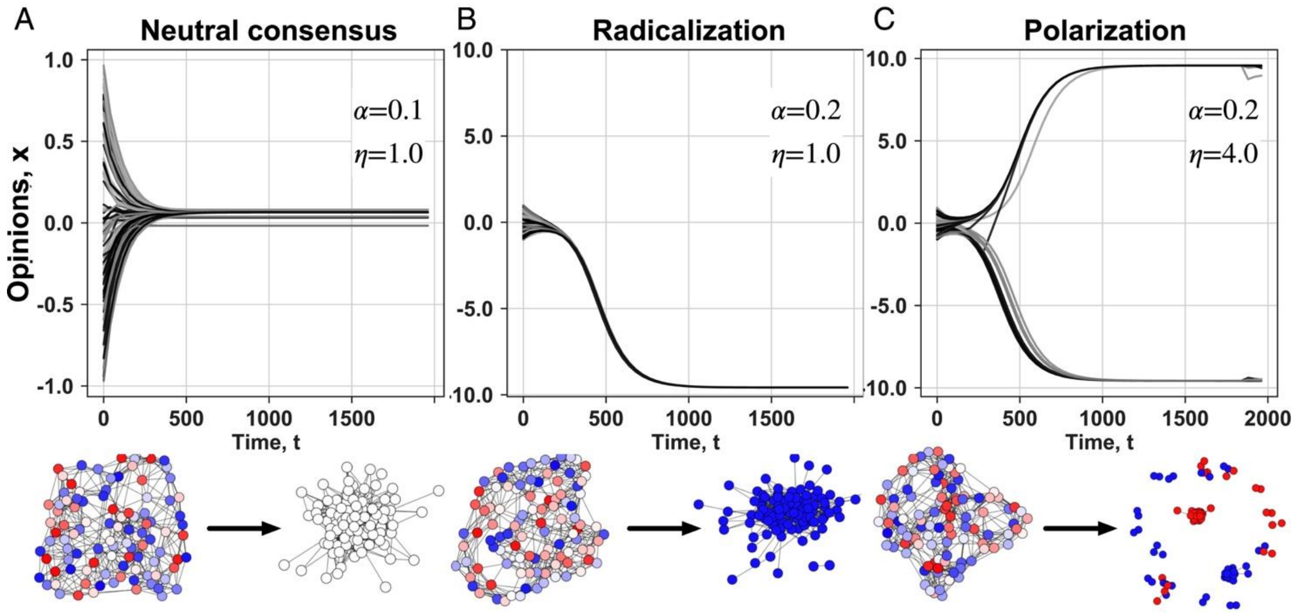
Nyhan, B., Reifler, J. (2010). When Corrections Fail: The Persistence of Political Misperceptions. *Polit Behav.* 32, 303–330

Bail, C. A., Argyle, L. P., Brown, T. W., Bumpus, J. P., Chen, H., Hunzaker, M. F., ... & Volfovsky, A. (2018). Exposure to opposing views on social media can increase political polarization. *PNAS. 115,*37

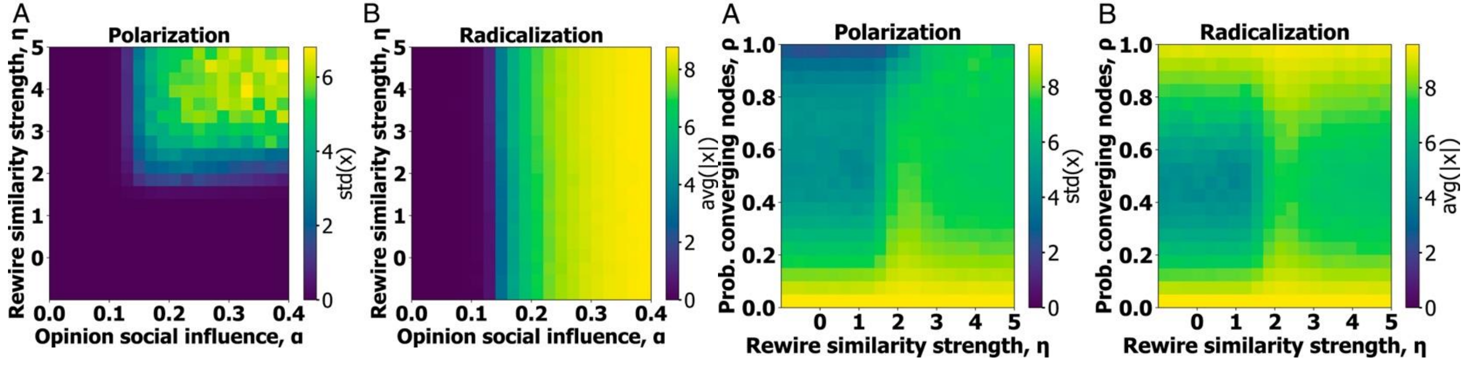
Dynamic networks



Min, B. & Miguel, M. (2017). Fragmentation transitions in a coevolving nonlinear voter model. *Sci. Rep.* 7, 12864



Santos, F. P., Lelkes, Y., & Levin, S. A. (2021). Link recommendation algorithms and dynamics of polarization in online social networks. *PNAS*. *118,* 50



Outlook

- Taxonomy of assumptions

- Make different models comparable
- Framework matrix of assumptions from the models
- Testing on real data
- Show paradoxes

- Creating an own model:
  - involving social belonging, prob. beyond pairwise interactions

One of the old tries

$$x_i(t+1) = \frac{1}{\sum_j w_{ij}} \sum_j w_{ij}(sign(x_{ij}) \cdot \frac{x_j}{2} \cdot \tanh(\alpha_i(|x_{ij}| - \rho_i)) + \frac{1}{2})$$

- Heterogentity of parameters $(allows\ stubborn\ agents..)$
- Allows single and complex contagion trough α_i
- Continuous states of opinion x_i with discrete being special case of it (if opinions will be rounded after summation)
- Not limited to pairwise interactions if x_j can also be neighborhood clusters average
- Social influence is included through w_{ij}
- Repulsion is existing: without repulsion cutting function: for $|x_{ij}| > \rho_i$

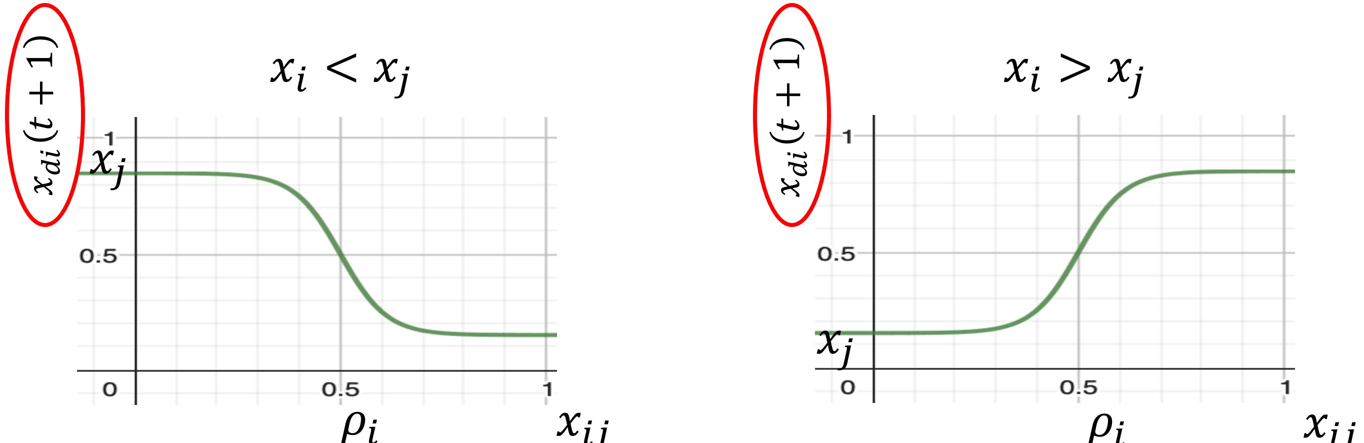
$$x_i(t+1) = \sum_j \frac{1}{w_{ij}} \sum_j w_{ij}\, x_i$$

- x_i, x_j → Opinions of an individual
- x_{ij} → difference of opinion
- w_{ij} → social influence parameter
- α_i → controversy of the topic
- ρ_i → threshold of repulsion

One of the old tries

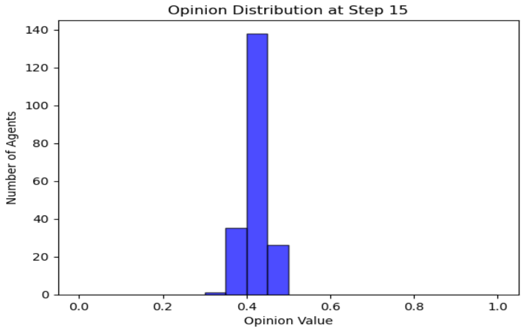
$$x_i(t+1) = \frac{1}{\sum_j w_{ij}} \sum_j w_{ij} \left(sign(x_{ij}) \cdot \frac{x_j}{2} \cdot \tanh(\alpha_i(|x_{ij}| - \rho_i)) + \frac{1}{2} \right)$$

- $x_i(t), x_j(t) \in [0,1]$ → Opinions of individuum x_i and its neighbors
- $x_{ij}(t) = x_i(t) - x_j(t) \in [-1,1]$ → difference of opinion
- $w_{ij} \in [0,1]$ → social influence parameter (edge weights, allows unsymmetrical influence)
- $\alpha_i \in [0, +\cdots]$ → controversy of the topic (nonlinearity parameter,)
- $\rho_i \in [0,1]$ → max opinion difference before repulsion

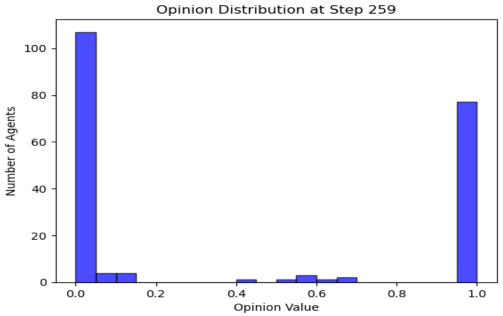


Draft figure 2

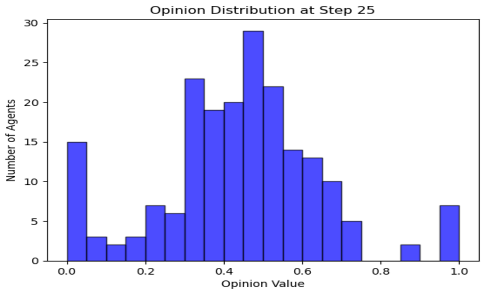
$$x_i(t+1) = x_i + \frac{\beta_i}{\sum_{j\neq i} w_{ij}} \sum_j w_{ij}\;\; x_{ij}\cdot\tanh(-\alpha_i(|x_{ij}| - \;\rho_i))$$



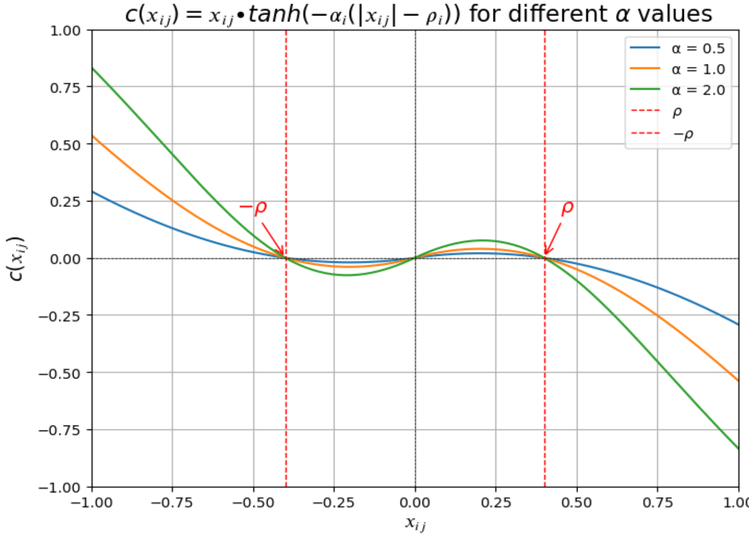
Variance: 0.000618
Bipolarity: 0.0



Variance: 0.22913
Bipolarity: 0.94



Variance: 0.04452
Bipolarity: 0.125



$$x_i(t+1) = \begin{cases} x_j & for \qquad majority(\;\{w_{ij}x_j\,(t): j\in N(i)\}) > \beta_i \\ x_i & else. \end{cases}$$

+compl cont